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(Dedicated to Honor Dr. R. C. Singh Chandel on His 75th Birth Anniversary Celebrations)

A COMPARATIVE STUDY OF PURCHASING *EOQ* (ECONOMIC ORDER QUANTITY) MODELS FOR NON-DETERIORATION AND DETERIORATING ITEMS UNDER STOCK-LINKED AND EXPONENTIAL DEMAND

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Abstract

Demand plays an important role for smooth running of any type of business. Most of the inventory modelers considered steady demand rate in their models. While in actual practice demand rate is in fluctuating state. In this study, a purchasing *EOQ* models for non-deteriorating and deteriorating items with stock-linked and exponential demand is considered. Three models are measured. In ***Model* I**, a purchasing *EOQ* model for non-deteriorating item with stock-dependent demand is assumed. In the second model an *EOQ* model for deteriorating item with stock-sensitive demand is considered. In the third model an *EOQ* model for decay item with exponential demand is considered. The mathematical models are established of all these three cases. Optimality conditions are also taken in account. Numerical examples and sensitivity analysis is also conversed. Taylor's series approximation is used for finding numerical results.

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1 Introduction

Inventory is an essential part for running profitable business. Demand plays a crucial and important role in making *EOQ* strategy. Several research papers published by researchers considering variable demand. Large pile of goods in the warehouse is also attracting customers to purchase more items. Dave and Patel [7] developed an *EOQ* for liner demand changeable demand with deterioration. Xu and Wang [28] designed an inventory model for decaying commodities with time changeable demand and limited shortage cost. Guchhait et al. [11] presented an *EPQ* (Economic Production Quantity) models for breakable items with variable demand, being dependent on time or on-hand stock. Tripathi and Pandey [21] analyzed an *EOQ* model for decaying products with Weibull distribution time-sensitive demand under trade credits. Some notable research papers with stock-sensitive and variable demand are published by Tripathi [22], Baker and Urban [2], Pal et al. [16], Teng and Chang [23], Soni and Shah [18], Musa and Sani [14], Tripathi et al. [24].

The problem of framing *EOQ* models for deteriorating items has received considerable attention in recent years. Most of the researchers assumed constant deterioration rate. Mandal and Phujdar [15] established an economic production quantity (*EPQ*) model for decaying commodities with steady production rate and stock-associated demand. Ghiami and Williams [17] presented EPQ model when a manufacturer delivered a deteriorating products to retailers. Sicilia et al. [19] designed a deterministic *EOQ* system for commodities with constant decay rate .Lee and Kim [12] developed the optimal ordering strategy considering both deteriorating and defecting items in an integrated production distribution model for a single –vendor, single – buyer supply chain. Tripathi [25] considered an *EOQ* model of deteriorating products with stock – sensitive demand under inflation. Dye [8] presented an *EOQ* model over a finite time for non-instantaneous deteriorating items using preservation technology. Researchers including Benkherouf and Balkhi [3], Chakrabarty et al. [5], Dye [9], Sarkar [20], Taleizadeh [26]. Wu and Sarkar [28], Yang et al. [29], Wang et al. [27], Liao [13], Chang et al. [6], Dye and Hsieh [10], Atici et al. [1], Bakker et al. [4] developed *EOQ* models that focused on deterioration rate.

The remainder of the study is prepared as follows. ***Section* 2** covers notations and assumptions of the model. In ***Section* 3**, the models are formulated with optimal solution, numerical examples and sensitivity studies are conversed. In this section the condition of total cost is minimized is obtained. ***Section* 4** , provides the comparative study of models I, II and III. Finally, some conclusions and future research lines are given in ***Section* 5**.

2 Notations and Assumptions

2.1 Notations:

The following notations are used in this study:

D	Demand rate
Q	Order quantity
C_h	Carrying cost / unit time
C_0	Ordering cost/ order
T	Cycle time
θ	Deterioration rate, $0 \le \theta < 1$
OC and HC	Ordering cost and holding cost
Z	Total cost
OC^*, HC^* and Z^*	Optimal OC, HC, and $Z(T)$ respectively.

2.2 Assumptions

The following assumptions are used to build up the model:

(i).The demand rate is stock- dependent for models I and II (i.e. $D = \alpha + \beta I(t)$, $\alpha > 0$, $0 < \beta < 1$) , and exponential demand for model III (i.e. $D = \alpha e^{\beta t}$).

(ii).Models are considered for non- deteriorating in model I and deteriorating items in models II and III.

(iii). Shortages are not allowed.

(iv).There is no renovate or replenishment of the deteriorating commodities.

(v).The replenishment takes place immediately at an endless rate.

3 Mathematical Formulation

In this study three models are considered. In the first model demand rate is considered stock dependent. In the second model deterioration and stock- dependent demand both are considered. In the third model deterioration and exponential demand is considered.

3.1 *Model* I: Stock- dependent demand

In this model, it is assumed that demand rate for the item is stock- dependent. The inventory of commodities, decreases due to purchases and stock- linked demand during $[0, T]$. Therefore, the differential equation of the state is given by:

(3.1) $\dfrac{dI(t)}{dt} = -\{\alpha + \beta I(t)\},\ 0 \le t \le T,$

with

(3.2) $I(0) = Q$ and $I(T) = 0.$

Solution of (3.1) using (3.2) is:

(3.3) $I(t) = \dfrac{\alpha}{\beta}\{e^{(T-t)\beta} - 1\}$

and

(3.4) $Q = I(0) = \dfrac{\alpha}{\beta}(e^{\beta T} - 1) = \alpha T\left(1 + \dfrac{\beta T}{2}\right)$ (approx.).

The Total cost consists of OC and HC:

(3.5) $OC = \dfrac{C_0}{T}$

(3.6) $HC = \dfrac{C_h}{T}\int_0^T I(t)\ dt = \frac{C_h \alpha T}{2}\left(1 + \frac{\beta T}{3}\right)$, (approx.).

Therefore

(3.7) $Z = \dfrac{C_0}{T} + \dfrac{C_h \alpha T}{2}\left(1 + \dfrac{\beta T}{3}\right).$

Optimality Condition

Differentiating (3.7) w.r.t. T, we get

$$\frac{dZ}{dT} = -\frac{C_0}{T^2} + \frac{C_h\alpha}{2}\left(1 + \frac{2\beta T}{3}\right)$$

and

$$\frac{d^2Z}{dT^2} = \frac{2C_0}{T^3} + \frac{\alpha\beta C_h}{3} > 0.$$

It is seen that Z is a convex function in T. We can also the condition of minimization by graph shown below:

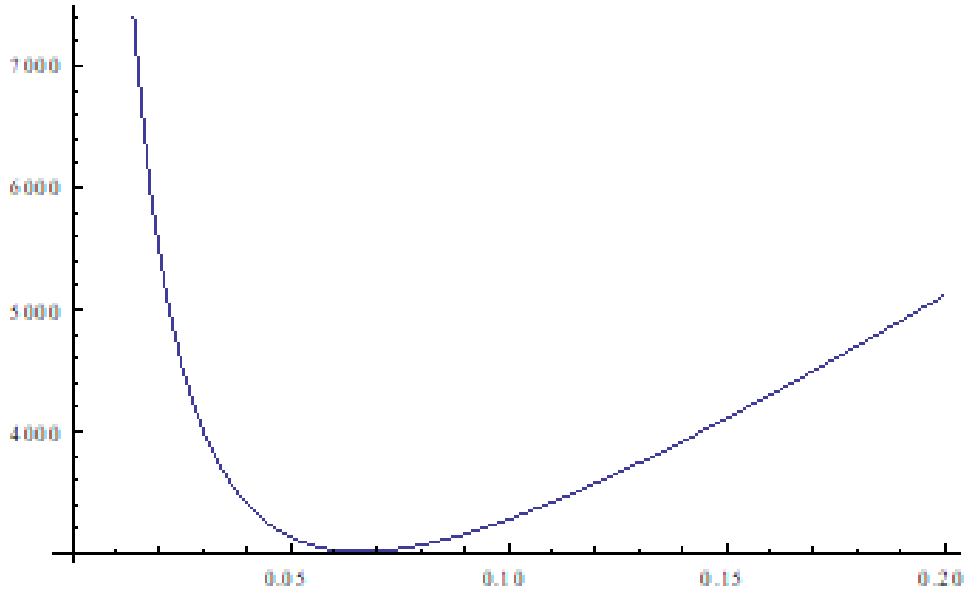

***Figure* 3.1:** Between T (0.00 – 2.00) and Z

T^* is calculated by solving

(3.8) $$\frac{dZ}{dT} = 0 \Rightarrow \alpha C_h (2\beta T + 3) T^2 - 6C_0 = 0.$$

***Example* 3.1** *Let us consider the cost parameters:* $\alpha = 4500$, $\beta = 0.4$, $C_h = 10$, $C_0 = 100$ *in appropriate units. Substituting theses values in* (3.8)*, and solving for T, we get,* $T^* = 0.0660869$ *yrs, corresponding* $Q^* = 301.322$ *units,* $OC^* = 1513.159$*,* $HC^* = 1500.061$ *and* $Z^* = \$3013.22$*.*

Sensitivity Analysis

It is reasonable to study the sensitivity study with respect to constraints over a known optimum solution. It is imperative to get the belongings on dissimilar scheme parameters, such as holding cost, ordering cost, etc. In the following ***Table* 1**, keeping all parameters same, discussed in numerical ***Example* 3.1**, varying one parameter at a time.

***Table* 3.1:** The effect of parameters on T^*, Q^* and OC^*, HC^* and Z^*

Parameters		Optimal values				
		T^*	Q^*	OC^*	HC^*	Z^*
C_0	80	0.0591636	269.386	1352.183	1341.677	2693.86
	90	0.0627232	285.795	1434.876	1423.074	2857.95
	110	0.0692835	316.090	1587.680	1573.280	3160.96
	120	0.0723354	330.218	1658.939	1643.241	3302.18
	130	0.0752602	343.768	1727.341	1710.349	3437.69
C_h	12	0.0603740	274.964	1659.342	1643.218	3299.56
	14	0.0559281	254.492	1788.010	1774.870	3562.88
	16	0.0523406	237.998	1910.563	1897.407	3807.97
	18	0.0493665	224.343	2025.665	2012.505	4038.17
	20	0.0468487	212.794	2134.531	2121.359	4255.89
α	4600	0.0653707	304.637	1529.737	1516.633	3046.37
	4700	0.0646774	307.916	1546.135	1533.025	3079.16
	4800	0.0640058	311.161	1562.358	1555.632	3117.99
	4900	0.0633547	314.372	1578.415	1565.305	3143.72
	5000	0.0627232	317.550	1594.306	1581.194	3175.50
β	0.45	0.0660162	300.995	1514.780	1500.070	3014.85
	0.50	0.0659458	300.670	1516.397	1500.093	3016.49
	0.55	0.0658758	300.347	1518.008	1500.102	3018.11
	0.60	0.0658062	300.025	1519.614	1500.126	3019.74
	0.65	0.0657369	299.705	1521.216	1500.144	3021.36

3.2 *Model* II: Stock linked Demand under deterioration

In this model stock- dependent demand and deterioration both are measured. The majority of items in the universe deteriorate over time. Daily usable products like, bread, milk, green vegetable etc. deteriorate over time. The differential equation of this situation is

$$(3.9)\quad \frac{d}{dt}I(t) + \theta I(t) = -\{\alpha + \beta I(t)\};\ 0 < t < T,$$

with $I(0) = Q$ and $I(T) = 0$.

The solution of (3.9) with above condition is:

$$(3.10)\quad I(t) = \frac{\alpha}{\theta+\beta}\left\{e^{(T-t)(\theta+\beta)} - 1\right\}$$

and

$$(3.11)\quad Q = I(0) = \frac{\alpha}{\theta+\beta}\left\{e^{(\theta+\beta)T} - 1\right\} = \alpha T\left\{1 + \frac{(\theta+\beta)T}{2}\right\}\ \text{(approx.)}.$$

Total Cost

Total cost is given as follows:

$$(3.12)\quad OC = \frac{C_0}{T}$$

$$(3.13)\quad HC = \frac{C_h}{T}\int_0^T I(t)\,dt = \frac{\alpha C_h T}{2}\left\{1 + \frac{(\theta+\beta)T}{3}\right\},$$

and

$$(3.14)\quad Z = \frac{C_0}{T} + \frac{\alpha C_h T}{2}\left\{1 + \frac{(\theta+\beta)T}{3}\right\}.$$

Optimality Condition

Differentiating (3.14) w.r.t. 'T', we get

$$\frac{dZ}{dT} = -\frac{C_0}{T^2} + \frac{\alpha C_h}{2}\left\{1 + \frac{2(\theta+\beta)T}{3}\right\},$$

and

(3.15) $$\frac{d^2Z}{dT^2} = \frac{2C_0}{T^3} + \frac{\alpha C_h(\theta+\beta)}{3} > 0.$$

It can be easily seen that Z is a convex function in T. We can also the condition of minimization by graph shown below:

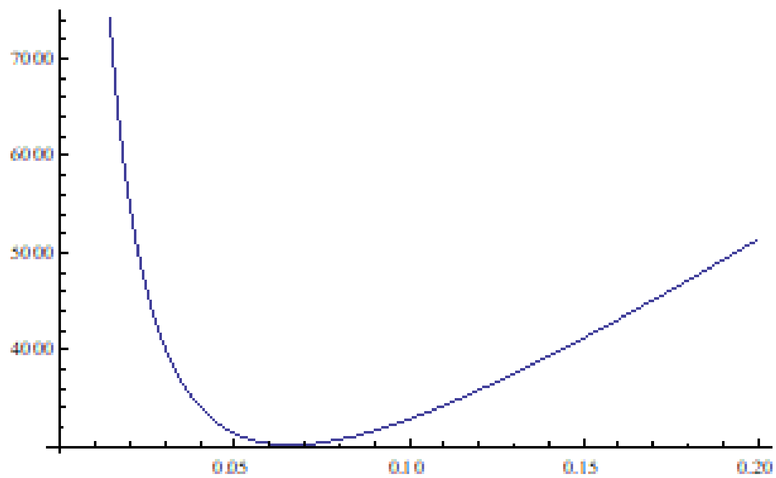

***Figure* 3.2:** Between $T(0.00 - -0.20)$ and Z

Hence, Optimal cycle time T can be calculated by solving

(3.16) $$\frac{dZ}{dT} = 0 \Rightarrow \alpha C_h \{2(\theta+\beta)T + 3\} T^2 - 6C_0 = 0.$$

***Example* 3.2** *Let us consider the cost parameters:* $\alpha = 4500$, $\beta = 0.4$, $C_h = 10$, $C_0 = 100$, $\theta = 0.05$, *in appropriate units. Substituting theses values in* (3.16)*, and solving for* T*, we get,* $T^* = 0.0660162$ *yrs,* $Q^* = 301.486$ *units,* $OC^* = 1514.780$, $HC^* = 1500.07$ *and* $Z^* = \$3014.85$.

Sensitivity Analysis

It is reasonable to study the sensitivity with respect to constraints over an agreed best possible solution. It is important to obtain the belongings on unlike structure parameters, such as holding cost, ordering cost, etc. In the following ***Table* 3.2**, keeping all parameters same, discussed in numerical ***Example* 3.1**, varying one parameter at a time.

***Table* 3.2:** The effect of parameters on T^*, Q^* and OC^*, HC^* and Z^*

Parameters		Optimal values				
		T^*	Q^*	OC^*	HC^*	Z^*
C_0	80	0.0591067	269.517	1353.484	1341.696	2695.18
	90	0.0626594	285.943	1436.337	1423.083	2859.42
	110	0.0692059	316.276	1589.460	1573.300	3162.76
	120	0.0722508	330.414	1660.881	1643.259	3304.14
	130	0.0751689	343.981	1729.439	1710.371	3439.81
C_h	12	0.0603148	275.099	1657.768	1643.432	3301.20
	14	0.0558772	254.609	1769.639	1774.881	3564.52
	16	0.0522960	238.101	1912.192	1897.428	3809.62
	18	0.0493268	224.434	2027.296	2012.514	4039.81
	20	0.0468129	212.877	2136.163	2121.377	4257.54
α	4600	0.0653015	304.800	1531.358	1516.642	3048.00
	4700	0.0646097	308.080	1547.755	1533.045	3080.80
	4800	0.0639394	311.324	1563.981	1549.259	3113.24
	4900	0.0632894	314.535	1580.038	1565.312	3145.35
	5000	0.0626594	317.140	1595.930	1581.210	3177.14
β	0.45	0.0659458	301.649	1516.397	1500.093	3016.49
	0.50	0.0658758	301.811	1518.008	1500.102	3018.11
	0.55	0.0658062	301.974	1519.614	1500.126	3019.74
	0.60	0.0657369	302.136	1521.216	1500.144	3021.36
	0.65	0.0656681	302.298	1522.809	1500.171	3022.98
θ	0.06	0.0660021	301.518	1515.103	1500.077	3015.18
	0.07	0.0659880	301.551	1515.427	1500.083	3015.51
	0.08	0.0659739	301.583	1515.751	1500.079	3015.83
	0.09	0.0659598	301.616	1516.075	1500.085	3016.16
	0.10	0.0659458	301.686	1516.397	1500.093	3016.49

3.3 *Model* III: Exponential demand under deterioration

In most of EOQ model demand is considered invariable. While in real situation demand is always in dynamic state. The model developed for deteriorating inventory in which demand is a exponential function of time. The differential equation is:

(3.17) $\dfrac{dI(t)}{dt} + \theta I(t) = -\alpha e^{\beta t},$

with, $I(0) = Q$ and $I(T) = 0$.

Solution of (3.17) with the above condition is

(3.18) $I(t) = \dfrac{\alpha}{\theta+\beta}\left\{e^{\beta T} e^{\theta(T-t)} - e^{\beta t}\right\}.$

also,

$$Q = I(0) = \frac{\alpha}{\theta+\beta}\left\{e^{(\theta+\beta)T} - 1\right\} = \alpha T\left\{1 + \frac{(\theta+\beta)T}{2}\right\}, \quad \text{(approx.)}.$$

Total Cost

Total cost consists of ordering cost and holding cost.

(3.19) $OC = \dfrac{C_0}{T},$

(3.20) $HC = \dfrac{C_h}{T}\int_0^T I(t)\,dt = \frac{C_h}{T}\int_0^T \frac{\alpha}{\theta+\beta}\left\{e^{\beta T} e^{\theta(T-t)} - e^{\beta t}\right\}dt$

$$= \frac{\alpha C_h}{(\theta+\beta)T}\left\{\frac{e^{(\theta+\beta)T} - e^{\beta T}}{\theta} - \frac{e^{\beta T} - 1}{\beta}\right\} = \frac{\alpha C_h T}{2}\left\{1 + \frac{(\theta+2\beta)T}{3}\right\}, (approx.),$$

and

(3.21) $Z = \dfrac{C_0}{T} + = \dfrac{\alpha C_h T}{2}\left\{1 + \dfrac{(\theta+2\beta)T}{3}\right\}.$

Optimal Condition

Differentiating (3.21) w.r.t. 'T', we get

$$\frac{dZ}{dT} = -\frac{C_0}{T^2} + \frac{\alpha C_h}{2}\left\{1 + \frac{2(\theta + 2\beta)T}{3}\right\} \tag{3.22}$$

and

$$\frac{d^2Z}{dT^2} = \frac{2C_0}{T^3} + \frac{\alpha C_h(\theta + 2\beta)}{3} > 0.$$

Since the second derivative of Z is positive. This shows that Z gives the minimum value at T^*. We can also the condition of minimization by graph shown below:

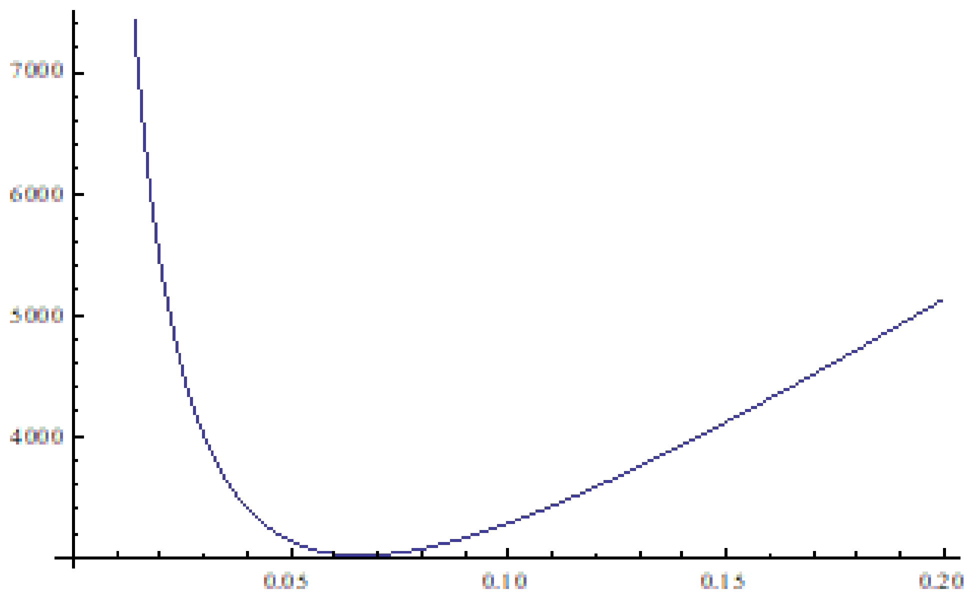

***Figure* 3.3:** Between $T(0.00 - -0.20)$ and Z

Thus, an Optimal cycle time T is obtained by putting

$$\frac{dZ}{dT} = 0 \Rightarrow \alpha C_h\{2(\theta + 2\beta)T + 3\}T^2 - 6C_0 = 0. \tag{3.23}$$

Equations (3.8), (3.16) and (3.23) are cubic in T. The solution of these equations provides three roots. It is seen that these equations having only one change in sign. By Descartes' rule there exists only one positive root.

***Example* 3.3** *Let us consider the cost parameters:* $\alpha = 4500$, $\beta = 0.4$, $C_h = 10$, $C_0 = 100$, $\theta = 0.05$, *in appropriate units. Substituting theses values in* (3.23)*, and solving for* T*, we get,* $T^* = 0.0654635$ *yrs, corresponding* $Q^* = 298.925$ *units,* $OC^* = 1527.569$*,* $HC^* = 1500.251$ *and* $Z^* = 3027.82$*.*

Sensitivity Analysis

It is reasonable to study the sensitivity study with respect to model parameters over a given optimum solution. It is important to get the effects on different system parameters, such as holding cost, ordering cost, etc. In the following ***Table* 3.3**, keeping all parameters same, discussed in numerical ***Example* 1**, varying one parameter at a time.

Table **3.3:** Effect of several parameters on optimal values

Parameters		Optimal values				
		T^*	Q^*	OC^*	HC^*	Z^*
C_0	80	0.0586615	267.461	1363.756	1341.824	2705.58
	90	0.0621603	283.634	1447.869	1423.241	2871.11
	110	0.0685999	313.464	1603.501	1573.499	3177.00
	120	0.0715919	327.353	1676.167	1643.493	3319.66
	130	0.0744572	340.671	1745.969	1710.631	3456.60
C_h	12	0.0598516	272.959	1670.799	1643.401	3314.20
	14	0.0554783	252.769	1802.506	1775.034	3577.54
	16	0.0519456	236.487	1925.091	1897.569	3822.66
	18	0.0490144	222.997	2040.217	2012.653	4052.87
	20	0.0465310	211.582	2149.105	2121.506	4270.61
A	4600	0.0647605	302.239	1544.151	1516.819	3060.97
	4700	0.0640798	305.517	1560.554	1533.216	3093.77
	4800	0.0634202	308.761	1576.785	1549.435	3126.22
	4900	0.0627807	311.971	1592.846	1565.484	3158.33
	5000	0.0621603	315.148	1608.744	1581.376	3190.12
β	0.45	0.0653289	298.781	1530.716	1500.304	3031.02
	0.50	0.0651956	298.640	1533.846	1500.374	3034.22
	0.55	0.0650637	298.502	1536.955	1500.445	3037.40
	0.60	0.0649330	298.365	1540.049	1500.521	3040.57
	0.65	0.0648037	298.231	1543.122	1500.608	3043.73
θ	0.06	0.0654500	298.959	1527.884	1500.256	3028.14
	0.07	0.0654365	298.992	1528.199	1500.261	3028.46
	0.08	0.0654230	299.026	1528.514	1500.266	3028.78
	0.09	0.0654095	299.060	1528.830	1500.270	3029.10
	0.10	0.0653960	300.056	1529.146	1500.274	3029.42

On comparing all the three models, the sensitivity outcomes show that
(i). On increasing C_0, T^*, Q^*, OC^*, HC^* and Z^* increases. It shows that all optimal values move in the same track with ordering cost/ order.
(ii).On raising C_h; T^* and Q^* diminishes, while OC^*, HC^* and Z^* increases. It means that T^* and Q^* moves in the opposite direction with C_h, while OC^*, HC^* and Z^* moves in the same direction with inventory carrying cost/ unit time.
(iii).On improving 'α'; T^* decreases, while Q^*, OC^*, HC^* and Z^* increases.
(iv).On raising 'θ' and 'β', all the optimal values varies insignificantly (approximately).

4 Comparisons of Optimal Lot- Size and Costs

Table **4.1:** A comparative study is conversed between ***Models*** **I**, **II** and **III**.

	Q^*	OC^*	HC^*	Z^*
Model I	301.322	1513.159	1500.061	3013.22
Model II	301.486	1514.780	1500.070	3014.85
Model III	298.925	1527.569	1500.251	3027.82

From the above ***Table*** **4.1**, it is easily seen that the order quantity obtained from ***Model*** **II** is superior to ***Models*** **I** and **III**. It is also seen that optimal setup cost, holding cost and total cost rises from ***Model*** **I** to **III**. It means that all optimal cost (OCj, HCj, and Zj) in ***Model*** **III** is better to compare to the OCj, HCj and Zj in ***Models*** **I** and **II**.

5 Conclusion

In this study, a purchasing *EOQ* models is established to obtain cycle time using second order and third order approximation in exponential terms. Three dissimilar models are established. In ***Model*** **I**, purchasing *EOQ* ***Model*** with stock- linked demand, in ***Model*** **II** an *EOQ* model with inventory- sensitive demand under deterioration and in ***Model*** **III** an *EOQ* model with exponential demand under decay have been discussed. Mathematical models have been developed for all three models and optimal cycle times are obtained which minimize the total cost. It has been also shown that the conditions of minimization are satisfied in all models. Numerical examples and sensitivity analyses have been conversed to validate the projected model. On comparing all three models, it is easily seen that variations are quite sensitive except 'θ' and 'β'.

Some possible extensions of the model that can be future research topics are: (i) variable deterioration and Weibull deterioration (ii) to suppose a non- linear holding cost (iii) to incorporate discounts in the purchasing cost/ unit; and (iv) to study the case of inflation and time value of money.

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