

**A GENERALIZED SUBCLASS OF ALPHA CONVEX BI-UNIVALENT FUNCTIONS OF COMPLEX ORDER**

By

**Gurmeet Singh<sup>1</sup>, Gagandeep Singh<sup>2</sup> and Gurcharanjit Singh<sup>3</sup>**

<sup>1</sup> Department of Mathematics, GSSDOS Khalsa College, Patiala-147001, Punjab, India

Email: meetgur111@gmail.com

<sup>2</sup> Department of Mathematics, Khalsa College, Amritsar-143001, Punjab, India

Email: kamboj.gagandeep@yahoo.in

<sup>3</sup> Department of Mathematics, Punjabi University, Patiala-147002, Punjab, India

Department of Mathematics, GNDU College, Chungh-143001, Punjab, India

Email: dhillongs82@yahoo.com

(Received : January 22, 2020 ; Revised: May 02, 2020)

**Abstract**

In this present investigation a subclass of alpha convex bi-univalent functions of complex order in the open unit disc  $U = \{z : |z| < 1\}$ , defined by Salagean operator and quasi-subordination is discussed. The estimates on the initial coefficients  $|a_2|$  and  $|a_3|$  for the functions in this subclass are studied. The results obtained in this paper would generalise those already proved by various authors.

**2010 Mathematics Subject Classifications:** 30C45.

**Keywords and phrases:** Bi-univalent functions, Salagean operator, Quasi-subordination, Coefficient estimate, Univalent functions.

**1 Introduction and Preliminaries**

Let  $A$  be the class of functions of the form

$$(1.1) \quad f(z) = z + \sum_{k=2}^{\infty} a_k z^k,$$

which are analytic in the open unit disc  $U = \{z : |z| < 1\}$ . By  $S$ , we denote the class of functions  $f(z) \in A$  and univalent in  $U$ .

Let us denote by  $B$ , the class of bounded or Schwarz functions  $w(z)$  satisfying  $w(0) = 0$  and  $|w(z)| \leq 1$  which are analytic in the open unit disc  $U$  and of the form

$$w(z) = \sum_{n=1}^{\infty} c_n z^n, z \in U.$$

A function  $f \in S$  is said to be starlike if it satisfies the inequality

$$\operatorname{Re} \left( \frac{zf'(z)}{f(z)} \right) > 0 (z \in U).$$

The class of starlike functions is denoted by  $S^*$ .

A function  $f \in S$  is said to be convex if it satisfies the inequality

$$\operatorname{Re} \left( \frac{(zf'(z))'}{f'(z)} \right) > 0 (z \in U).$$

The class of convex functions is denoted by  $K$ .

A function  $f \in S$  is said to be  $\alpha$ -convex if it satisfies the inequality

$$\operatorname{Re} \left( (1 - \alpha) \frac{zf'(z)}{f(z)} + \alpha \frac{(zf'(z))'}{f'(z)} \right) > 0 \quad (0 \leq \alpha \leq 1, z \in U).$$

The class of  $\alpha$ -convex functions is denoted by  $M(\alpha)$  and was introduced by Mocanu [8]. In particular  $M(0) \equiv S^*$  and  $M(1) \equiv K$ .

For  $f \in A$ , Salagean [14] introduced the following operator:

$$D^0 f(z) = f(z), D^1 f(z) = zf'(z),$$

and in general,

$$D^n f(z) = D(D^{n-1} f(z)), n \in N$$

or equivalent to

$$D^n f(z) = z + \sum_{k=2}^{\infty} k^n a_k z^k, n \in N_0 = N \cup (0).$$

The inverse functions of the functions in the class  $S$  may not be defined on the entire unit disc  $U$  although the functions in the class  $S$  are invertible. However using Koebe-one quarter theorem [4] it is obvious that the image of  $U$  under every function  $f \in S$  contains a disc of radius  $\frac{1}{4}$ . Hence every univalent function  $f$  has an inverse  $f^{-1}$ , defined by

$$f^{-1}(f(z)) = z (z \in U)$$

and

$$f(f^{-1}(w)) = w \left( |w| < r_0(f) : r_0(f) \geq \frac{1}{4} \right),$$

where

$$(1.2) \quad g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3) w^3 - (5a_2^3 - 5a_2 a_3 + a_4) w^4 + \dots$$

A function  $f \in A$  is said to be bi-univalent in  $U$  if both  $f$  and  $f^{-1}$  are univalent in  $U$ .

By  $\Sigma$ , we denote the class of bi-univalent functions in  $U$  defined by (1.1).

Consider two functions  $f$  and  $g$  analytic in  $U$ . We say that  $f$  is subordinate to  $g$  (symbolically  $f < g$ ) if there exists a bounded function  $u(z) \in B$  for which  $f(z) = g(u(z))$ . This result is known as principle of subordination.

Robertson [13] introduced the concept of quasi-subordination in 1970. If  $f$  and  $\phi$  are analytic functions, then we say that  $f$  is quasi-subordinate to  $\phi$  (symbolically  $f <_q \phi$ ) if there exists analytic functions  $k$  and  $\omega$  with  $|k(z)| \leq 1$ ,  $\omega(0) = 0$  and  $|\omega(z)| < 1$  such that

$$\frac{f(z)}{k(z)} < \phi(z),$$

or it is equivalent to

$$(1.3) \quad f(z) = k(z)\phi(\omega(z)).$$

In particular for  $k(z) = 1$ ,  $f(z) = \phi(\omega(z))$ , so that  $f(z) < \phi(z)$  in  $U$ . It is obvious to see that the quasi-subordination is a generalization of the usual subordination. The work on quasi-subordination is quite extensive which finds interesting dimensions in some recent investigations [1,5,7,12].

Lewin [6] discussed the class  $\Sigma$  of bi-univalent functions and obtained the bound for the second coefficient. Brannan and Taha [2] investigated certain subclasses of bi-univalent functions, similar to the familiar subclasses of univalent functions consisting of strongly starlike, starlike and convex functions. They introduced bi-starlike functions and bi-convex functions and obtained estimates on the initial coefficients. Also the subclasses of bi-univalent functions defined by Salagean operator were studied by various authors [3,9,11,15].

The earlier work on bi-univalent functions defined by quasi-subordination and Salagean operator motivate us to define the following subclass:

Also we assume that  $\phi(z)$  is analytic in  $U$  with  $\phi(0) = 1$  and let

$$(1.4) \quad \phi(z) = 1 + B_1z + B_2z^2 + \dots (B_1 \in R^+)$$

and

$$(1.5) \quad k(z) = A_0 + A_1z + A_2z^2 + \dots (|k(z)| \leq 1, z \in U).$$

To avoid repetition, throughout the paper we assume that  $0 \leq \alpha \leq 1$  and  $z \in U$ .

**Definition 1.1.** A function  $f \in \Sigma$  given by (1.1) is said to be in the class  $M_\Sigma(n, \alpha, \gamma, \phi)$  if it satisfy the following conditions:

$$(1.6) \quad \frac{1}{\gamma} \left[ (1 - \alpha) \frac{z(D^{n-1}f(z))'}{D^{n-1}f(z)} + \alpha \frac{z(D^n f(z))'}{D^n f(z)} - 1 \right] <_q (\phi(z) - 1)$$

and

$$(1.7) \quad \frac{1}{\gamma} \left[ (1 - \alpha) \frac{w(D^{n-1}g(w))'}{D^{n-1}g(w)} + \alpha \frac{w(D^n g(w))'}{D^n g(w)} - 1 \right] <_q (\phi(w) - 1),$$

where  $g = f^{-1}$  and  $z, w \in U$ .

The following observations are obvious:

- (i)  $M_\Sigma(n, \alpha, 1, \phi) \equiv M_\Sigma(n, \alpha, \phi)$ .
- (ii)  $M_\Sigma(1, \alpha, 1, \phi) \equiv M_\Sigma(\alpha, \phi)$ .
- (iii)  $M_\Sigma(1, 0, 1, \phi) \equiv S_\Sigma^*(\phi)$ , the class of bi-starlike functions defined with quasi subordination.
- (iv)  $M_\Sigma(1, 1, 1, \phi) \equiv K_\Sigma(\phi)$ , the class of bi-convex functions defined with quasi subordination.

For deriving our main results, we need the following lemma:

**Lemma 1.1.** [10] If  $p \in P$  be family of all functions  $p$  analytic in  $U$  for which  $Re[p(z)] > 0$  and have the form  $p(z) = 1 + p_1z + p_2z^2 + \dots$  for  $z \in U$ , then  $|p_n| \leq 2$  for each  $n$ .

## 2 Coefficient bounds for the function class $M_\Sigma(n, \alpha, \gamma, \phi)$

**Theorem 2.1.** If  $f \in M_\Sigma(n, \alpha, \gamma, \phi)$ , then

$$(2.1) \quad |a_2| \leq \min. \left[ \frac{|A_0\gamma|B_1}{(n + \alpha + 1)}, \sqrt{\frac{|A_0\gamma|(B_1 + |B_2 - B_1|)}{(n + \alpha + 1)}} \right]$$

and

$$(2.2) \quad |a_3| \leq \min. \left[ \frac{|A_0\gamma|(B_1 + |B_2 - B_1|)}{(n + \alpha + 1)} + \frac{|A_1\gamma|B_1 + |A_0\gamma|B_1}{(n + 2)(n + 2\alpha + 1)} \right. \\ \left. \frac{|\gamma|}{(n + 2)(n + 2\alpha + 1)} \left[ |\gamma| \left[ \frac{(n + 1)^2 + \alpha(2n + 3)}{(n + \alpha + 1)^2} \right] B_1^2 |A_0|^2 + (B_1 + |B_2 - B_1|)|A_0| + |A_1|B_1 \right] \right].$$

**Proof.** As  $f \in M_{\Sigma}(n, \alpha, \gamma, \phi)$ , so by **Definition 1.1**, using the concept of quasi-subordination, there exists Schwarz functions  $r(z)$  and  $s(z)$  and analytic function  $k(z)$  such that

$$(2.3) \quad \frac{1}{\gamma} \left[ (1 - \alpha) \frac{z(D^{n-1}f(z))'}{D^{n-1}f(z)} + \alpha \frac{z(D^n f(z))'}{D^n f(z)} - 1 \right] = k(z)(\phi(r(z)) - 1)$$

and

$$(2.4) \quad \frac{1}{\gamma} \left[ (1 - \alpha) \frac{w(D^{n-1}g(w))'}{D^{n-1}g(w)} + \alpha \frac{w(D^n g(w))'}{D^n g(w)} - 1 \right] = k(w)(\phi(s(w)) - 1),$$

where  $r(z) = 1 + r_1 z + r_2 z^2 + \dots$  and  $s(w) = 1 + s_1 w + s_2 w^2 + \dots$

Define the functions  $p(z)$  and  $q(z)$  by

$$(2.5) \quad r(z) = \frac{p(z) - 1}{p(z) + 1} = \frac{1}{2} \left[ c_1 z + \left( c_2 - \frac{c_1^2}{2} \right) z^2 + \dots \right]$$

and

$$(2.6) \quad s(z) = \frac{q(z) - 1}{q(z) + 1} = \frac{1}{2} \left[ d_1 z + \left( d_2 - \frac{d_1^2}{2} \right) z^2 + \dots \right].$$

Using (2.5) and (2.6) in (2.3) and (2.4) respectively, it yields

$$(2.7) \quad \frac{1}{\gamma} \left[ (1 - \alpha) \frac{z(D^{n-1}f(z))'}{D^{n-1}f(z)} + \alpha \frac{z(D^n f(z))'}{D^n f(z)} - 1 \right] = k(z) \left[ \phi \left( \frac{p(z) - 1}{p(z) + 1} \right) - 1 \right]$$

and

$$(2.8) \quad \frac{1}{\gamma} \left[ (1 - \alpha) \frac{w(D^{n-1}g(w))'}{D^{n-1}g(w)} + \alpha \frac{w(D^n g(w))'}{D^n g(w)} - 1 \right] = k(w) \left[ \phi \left( \frac{q(w) - 1}{q(w) + 1} \right) - 1 \right].$$

But

$$(2.9) \quad \frac{1}{\gamma} \left[ (1 - \alpha) \frac{z(D^{n-1}f(z))'}{D^{n-1}f(z)} + \alpha \frac{z(D^n f(z))'}{D^n f(z)} - 1 \right] = \frac{1}{\gamma} \left[ (n + \alpha + 1)a_2 z + [(n + 2)(n + 2\alpha + 1)a_3 - ((n + 1)^2 + \alpha(2n + 3))a_2^2]z^2 + \dots \right]$$

and

$$(2.10) \quad \frac{1}{\gamma} \left[ (1 - \alpha) \frac{w(D^{n-1}g(w))'}{D^{n-1}g(w)} + \alpha \frac{w(D^n g(w))'}{D^n g(w)} - 1 \right] = \frac{1}{\gamma} \left[ -(n + \alpha + 1)a_2 w + [(n + 2)(n + 2\alpha + 1)(2a_2^2 - a_3) - ((n + 1)^2 + \alpha(2n + 3))a_2^2]w^2 + \dots \right].$$

Again using (1.4) and (1.5) in (2.5) and (2.6) respectively, we get

$$(2.11) \quad k(z) \left[ \phi \left( \frac{p(z) - 1}{p(z) + 1} \right) - 1 \right] = \frac{1}{2} A_0 B_1 c_1 z + \left[ \frac{1}{2} A_1 B_1 c_1 + \frac{1}{2} A_0 B_1 \left( c_2 - \frac{c_1^2}{2} \right) + \frac{A_0 B_2 c_1^2}{4} \right] z^2 + \dots$$

and

$$(2.12) \quad k(w) \left[ \phi \left( \frac{q(w) - 1}{q(w) + 1} \right) - 1 \right] = \frac{1}{2} A_0 B_1 d_1 w + \left[ \frac{1}{2} A_1 B_1 d_1 + \frac{1}{2} A_0 B_1 \left( d_2 - \frac{d_1^2}{2} \right) + \frac{A_0 B_2 d_1^2}{4} \right] w^2 + \dots$$

Using (2.9) and (2.11) in (2.7) and equating the coefficients of  $z$  and  $z^2$ , we get

$$(2.13) \quad \frac{(n + \alpha + 1)}{\gamma} a_2 = \frac{1}{2} A_0 B_1 c_1$$

and

$$(2.14) \quad \frac{(n+2)(n+2\alpha+1)a_3 - ((n+1)^2 + \alpha(2n+3))a_2^2}{\gamma} = \frac{1}{2}A_1B_1c_1 + \frac{1}{2}A_0B_1\left(c_2 - \frac{c_1^2}{2}\right) + \frac{A_0B_2}{4}c_1^2.$$

Again using (2.10) and (2.12) in (2.8) and equating the coefficients of  $w$  and  $w^2$ , we get

$$(2.15) \quad -\frac{(n+\alpha+1)}{\gamma}a_2 = \frac{1}{2}A_0B_1d_1$$

and

$$(2.16) \quad \frac{(n+2)(n+2\alpha+1)(2a_2^2 - a_3) - ((n+1)^2 + \alpha(2n+3))a_2^2}{\gamma} = \frac{1}{2}A_1B_1d_1 + \frac{1}{2}A_0B_1\left(d_2 - \frac{d_1^2}{2}\right) + \frac{A_0B_2}{4}d_1^2.$$

From (2.13) and (2.15), it is clear that

$$(2.17) \quad c_1 = -d_1$$

and

$$(2.18) \quad a_2 = \frac{A_0B_1c_1\gamma}{2(n+\alpha+1)} = -\frac{A_0B_1d_1\gamma}{2(n+\alpha+1)}.$$

Therefore on applying triangle inequality and using **Lemma 1.1**, (2.18) yields

$$(2.19) \quad |a_2| \leq \frac{|A_0\gamma|B_1}{(n+\alpha+1)}.$$

Adding (2.14) and (2.16), it yields

$$(2.20) \quad \frac{2[(n+2)(n+2\alpha+1) - (n+1)^2 - \alpha(2n+3)]a_2^2}{\gamma} = \frac{1}{2}A_0B_1(c_2 + d_2) + \frac{A_0(B_2 - B_1)}{4}(c_1^2 + d_1^2).$$

Using **Lemma 1.1** and on applying triangle inequality in (2.20), we obtain

$$(2.21) \quad |a_2|^2 \leq \frac{|A_0\gamma|(B_1 + |B_2 - B_1|)}{(n+\alpha+1)}.$$

So, the result (2.1) can be easily obtained from (2.19) and (2.21).

Now subtracting (2.16) from (2.14), we obtain

$$(2.22) \quad a_3 = a_2^2 + \frac{A_1B_1(c_1 - d_1) + A_0B_1(c_2 - d_2)}{4(n+2)(n+2\alpha+1)}\gamma.$$

Applying triangle inequality and using **Lemma 1.1** and (2.21) in (2.22), it yields

$$(2.23) \quad |a_3| \leq \frac{|A_0|(B_1 + |B_2 - B_1|)}{(n+\alpha+1)} + \frac{|A_1\gamma|B_1 + |A_0\gamma|B_1}{(n+2)(n+2\alpha+1)}.$$

From (2.13) and (2.14), we have

$$(2.24) \quad |a_3| \leq \frac{|\gamma|}{(n+2)(n+2\alpha+1)} \left[ |\gamma| \left[ \frac{(n+1)^2 + \alpha(2n+3)}{(n+\alpha+1)^2} \right] B_1^2|A_0|^2 + (B_1 + |B_2 - B_1|)|A_0| + |A_1|B_1 \right].$$

Again from (2.15) and (2.17), it gives

$$(2.25) \quad |a_3| \leq \frac{|\gamma|}{(n+2)(n+2\alpha+1)} \left[ |\gamma| \left[ \frac{n^2 + 2n\alpha + 5\alpha + 3}{(n+\alpha+1)^2} \right] B_1^2|A_0|^2 + (B_1 + |B_2 - B_1|)|A_0| + |A_1|B_1 \right].$$

Since R.H.S. of (2.25) is greater than that of (2.24), so result (2.2) is obvious.

For  $\gamma = 1$ , **Theorem 2.1** gives the following result:

**Corollary 2.1.** If  $M_{\Sigma}(n, \alpha, \phi)$ , then

$$|a_2| \leq \min. \left[ \frac{|A_0|B_1}{(n + \alpha + 1)}, \sqrt{\frac{|A_0|(B_1 + |B_2 - B_1|)}{(n + \alpha + 1)}} \right]$$

and

$$|a_3| \leq \min. \left[ \frac{|A_0|(B_1 + |B_2 - B_1|)}{(n + \alpha + 1)} + \frac{|A_1|B_1 + |A_0|B_1}{(n + 2)(n + 2\alpha + 1)}, \right. \\ \left. \frac{1}{(n + 2)(n + 2\alpha + 1)} \left[ \left[ \frac{(n + 1)^2 + \alpha(2n + 3)}{(n + \alpha + 1)^2} \right] B_1^2 |A_0|^2 + (B_1 + |B_2 - B_1|)|A_0| + |A_1|B_1 \right] \right].$$

For  $\gamma = 1$  and  $n = 1$ , the following result is obvious from **Theorem 2.1**:

**Corollary 2.2.** If  $f(z) \in M_{\Sigma}(\alpha, \phi)$ , then

$$|a_2| \leq \min. \left[ \frac{|A_0|B_1}{(2 + \alpha)}, \sqrt{\frac{|A_0|(B_1 + |B_2 - B_1|)}{(2 + \alpha)}} \right]$$

and

$$|a_3| \leq \min. \left[ \frac{|A_0|(B_1 + |B_2 - B_1|)}{(2 + \alpha)} + \frac{|A_1|B_1 + |A_0|B_1}{6(1 + \alpha)}, \right. \\ \left. \frac{1}{6(1 + \alpha)} \left[ \left[ \frac{4 + 5\alpha}{(2 + \alpha)^2} \right] B_1^2 |A_0|^2 + (B_1 + |B_2 - B_1|)|A_0| + |A_1|B_1 \right] \right].$$

For  $\gamma = 1, \alpha = 0$  and  $n = 1$ , **Theorem 2.1** gives the following result:

**Corollary 2.3.** If  $f(z) \in S_{\Sigma}^*(\phi)$ , then

$$|a_2| \leq \min. \left[ \frac{|A_0|B_1}{2}, \sqrt{\frac{|A_0|(B_1 + |B_2 - B_1|)}{2}} \right]$$

and

$$|a_3| \leq \min. \left[ \frac{|A_0|(B_1 + |B_2 - B_1|)}{2} + \frac{|A_1|B_1 + |A_0|B_1}{6}, \right. \\ \left. \frac{1}{6} \left[ B_1^2 |A_0|^2 + (B_1 + |B_2 - B_1|)|A_0| + |A_1|B_1 \right] \right].$$

For  $\gamma = 1, \alpha = 1$  and  $n = 1$ , the following result is obvious from **Theorem 2.1**:

**Corollary 2.4.** If  $f(z) \in K_{\Sigma}(\phi)$ , then

$$|a_2| \leq \min. \left[ \frac{|A_0|B_1}{3}, \sqrt{\frac{|A_0|(B_1 + |B_2 - B_1|)}{3}} \right]$$

and

$$|a_3| \leq \min. \left[ \frac{|A_0|(B_1 + |B_2 - B_1|)}{3} + \frac{|A_1|B_1 + |A_0|B_1}{12}, \right. \\ \left. \frac{1}{12} \left[ B_1^2 |A_0|^2 + (B_1 + |B_2 - B_1|)|A_0| + |A_1|B_1 \right] \right].$$

### 3. Conclusion

This paper is concerned with a very generalized subclass of alpha convex bi-univalent functions of complex order in the open unit disc. The class is associated with Salagean operator and is defined by means of quasi-subordination. We have studied the estimates of the initial coefficients  $|a_2|$  and  $|a_3|$  for the functions in this class. By giving the particular values to the various parameters like  $\alpha$ ,  $\gamma$ ,  $n$  and  $q$ , the results already proved by earlier researchers can be easily obtained. So this paper will work as a milestone to the future researchers in this field.

**Acknowledgement.** The authors are very much grateful to the Editor and referee for their suggestions to bring the paper in its present form.

### References

- [1] O. Altıntaş and S. Owa, Majorizations and quasi-subordinations for certain analytic functions, *Proc. Japan Acad. Ser.A.* **68**(7) (1992), 181-185.
- [2] D. A. Brannan and T. S. Taha, On some classes of bi-univalent functions, in: S. M. Mazhar, A. Hamoni, N. S. Faour (Eds.), *Mathematical Analysis and its Applications*, Kuwait; February 18-21, 1985, in: *KFAS Proceedings Series*, **3**, Pergamon Press, Elsevier Science Limited, Oxford, 1988, pp. 53-60. See also *Studia Univ. Babeş-Bolyai Math.*, **31**(2), (1986), 70-77.
- [3] Murat Çağlar and Erhan Deniz, Initial Coefficients for a subclass of bi-univalent functions defined by Salagean differential operator, *Commun. Fac. Sci. Univ. Auk. Ser. Al Math. Stat.* **66**(1)(2017), 85-91.
- [4] P. L. Duren, *Univalent functions*, Springer-Verlag, New York, 1983.
- [5] S. Y. Lee, Quasi-subordinate functions and coefficient conjectures, *J. Korean Math. Soc.* **12**(1)(1975), 43-50.
- [6] M. Lewin, On a coefficient problem for bi-univalent functions, *Proc. Amer. Math. Soc.*, **18**(1967), 63-68.
- [7] N. Magesh, V. K. Balaji and J. Yamini, Certain subclasses of bi-starlike and bi-convex functions based on quasi-subordination, *Abstract and applied analysis*, Art. ID 3102960, 6 pages, 2016.
- [8] P.T. Mocanu, Une propriété de convexité généralisée dans la théorie de la représentation conforme, *Mathematica (Cluj)*, **11**(34)(1969), 127-133.
- [9] G. Murugusundaramoorthy, C. Selvaraj and O. S. Babu, Coefficient estimates for Pascu-type subclasses of bi-univalent functions based on subordination, *Int. J. of Non-linear Science*, **19**(1)(2015), 47-52.
- [10] Ch. Pommerenke, *Univalent functions*, Vandenhoeck and Ruprecht, Göttingen, 1975.
- [11] C. Ramachandran and D. Kavitha, Coefficient estimates for a subclass of bi-univalent functions defined by Salagean operator using quasi subordination, *Applied Mathematical Sciences*, **11**(35)(2017), 1725-1732.
- [12] F. Y. Ren, S. Owa and S. Fukui, Some inequalities on quasi-subordinate functions, *Bull. Austral. Math. Soc.*, **43**(2)(1991), 317-324.
- [13] M. S. Robertson, Quasi-subordination and coefficient conjecture, *Bull. Amer. Math. Soc.*, **76**(1970), 1-9.
- [14] G. S. Salagean, Subclasses of Univalent Functions, *Chapter in Lecture Notes in Mathematics*, Springer-Verlag, Vol. **1013**, 1983, 362-372.
- [15] Bilal Seker and Veysi Mehmetoglu, Coefficient bounds for new subclasses of bi-univalent functions, *New Trends in Mathematical Sciences*, **4**(3)(2016), 197-203.