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(Dedicated to Honor Professor H.M. Srivastava on His 80th Birth Anniversary Celebrations)

FIXED POINT OF SUZUKI-TYPE GENERALIZED MULTIVALUED CONTRACTION MAPPINGS ON WEAK PARTIAL METRIC SPACES

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Abstract

Motivated by the recent result of Aydi et al.[4], we establish a fixed point theorem of Suzuki-type generalized multivalued contraction mappings in the framework of weak partial metric space. An example is also given to show the significance of our result.
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1 Introduction

The notion of partial metric space was introduced in 1994 by Matthews[7] as a generalization of the metric space. Such spaces are useful for modeling the problems occurring in computer science. He also proved the famous Banach Contraction Principle in partial metric space. Later on many fixed point results in partial metric space have been proved (see, for instance [1], [2], [6] and references therein). Further Aydi et al.[3] obtained an analogue of Nadler's fixed point theorem in partial metric space using the concept of a partial Hausdorff metric. Recently, Beg and Pathak[5] introduced a weaker form of partial metric called weak partial metric and gave a fixed point theorem. This result extended and generalized by Negi and Gairola[8].
The aim of this paper is to generalize the result of Aydi et al. [4] by introducing generalized Suzuki type multivalued contraction mapping in weak partial metric space. Our result extend various known comparable results in the literature.

2 Preliminaries

The following definitions and results are followed by Beg and Pathak[5].

***Definition* 2.1.** *Let X be a nonempty set. A function $q : X\times X \to \mathbb{R}^+$ is called a weak partial metric on X if for all $x, y, z \in X$, the following conditions hold:*
(Q1) $q(x, x) = q(x, y) \Leftrightarrow x = y$;
(Q2) $q(x, x) \le q(x, y)$;
(Q3) $q(x, y) = q(y, x)$;
(Q4) $q(x, y) \le q(x, z) + q(z, y)$.
The pair (X, q) is called a weak partial metric space.

Let $CB^q(X)$ be the family of all non-empty, closed and bounded subsets of weak partial metric space (X, q). For $E, F \in CB^q(X)$ and $x \in X$, define $\delta_q : CB^q(X) \times CB^q(X) \to [0,\infty)$ and

$q(x, E) = \inf\{q(x, a) : a \in E\}$,
$\delta_q(E, F) = \sup\{q(a, F) : a \in E\}$,
$\delta_q(F, E) = \sup\{q(b, E) : b \in F\}$.

Each weak partial metric q on X generates a T_0 topology τ_q on X which has as a base the family of open q-balls $\{B_q(x, \epsilon) : x \in X,\ \epsilon > 0\}$, where $B_q(x, \epsilon) = \{y \in X : q(x, y) < q(x, x) + \epsilon\}$ for all $x \in X$ and $\epsilon > 0$.

***Remark* 2.1.** *Let $\phi \neq E$ be a set in (X, q). Then*
$a \in \overline{E}$ if and only if $q(a, E) = q(a, a)$,
where $\overline{E}$ denotes the closure of E with respect to the weak partial metric space.
Note that E is closed in (X, q) if and only if $E = \overline{E}$.

***Definition* 2.2.** *A sequence $\{x_n\}$ in (X, q) converges to a point $x \in X$ with respect to τ_q if and only if $q(x, x) = \lim_{n\to\infty} q(x, x_n)$.*

***Remark* 2.2.** *If q is a weak partial metric on X, the function $q^s : X \times X \to \mathbb{R}^+$ given by $q^s(x, y) = q(x, y) - \frac{1}{2}[q(x, x) + q(y, y)]$, defines a metric on X. Further, a sequence $\{x_n\}$ converges in (X, q^s) to a point $x \in X$ if and only if*

$$q(x, x) = \lim_{n\to\infty} q(x_n, x) = \lim_{n,m\to\infty} q(x_n, x_m). \tag{2.1}$$

***Proposition* 2.1.** *Let (X, q) be a weak partial metric space. For any $E, F, H \in CB^q(X)$, the following holds:*
(i) $\delta_q(E, E) = \sup\{q(a, a) : a \in E\}$;
(ii) $\delta_q(E, E) \leq \delta_q(E, F)$;
(iii) $\delta_q(E, F) = 0 \Rightarrow E \subseteq F$;
(iv) $\delta_q(E, F) \leq \delta_q(E, H) + \delta_q(H, F)$.

***Proposition* 2.2.** *Let (X, q) be a weak partial metric space. For all $E, F, H \in CB^q(X)$, we have*
(wp1) $H_q^+(E, E) \leq H_q^+(E, F)$;
(wp2) $H_q^+(E, F) = H_q^+(F, E)$;
(wp3) $H_q^+(E, F) \leq H_q^+(E, H) + H_q^+(H, F)$.

***Definition* 2.3.** *Let (X, q) be a weak partial metric space. For $E, F \in CB^q(X)$, define*

$$H_q^+(E, F) = \frac{1}{2}\{\delta_q(E, F) + \delta_q(F, E)\}.$$

The mapping $H_q^+ : CB^q(X) \times CB^q(X) \to [0, +\infty)$, is called H_q^+-type Hausdorff metric induced by q.

***Definition* 2.4.** *Let (X, q) be a complete weak partial metric space. A multi-valued map $T : X \to CB^q(X)$ is called H_q^+ -contraction if for every $x, y \in X$,*
(i) there exists r in $(0, 1)$ such that
$H_q^+(T(x) \setminus \{x\}, T(y) \setminus \{y\}) \leq r\, q(x, y)$,
(ii) for every x in X, y in $T(x)$ and $\epsilon > 0$, there exists z in $T(y)$ such that
$q(y, z) \leq H_q^+(T(y), T(x)) + \epsilon$.

Beg and Pathak[5] gave the following variant of Nadler's fixed point theorem.

***Theorem* 2.1.** [5] *Every H_q^+- type multivalued contraction on a complete weak partial metric space (X, q) has a fixed point.*

Recently Aydi et al.[4] introduced H_q^+- type Suzuki multivalued contraction mappings and prove the following theorem.

***Theorem* 2.2.** [4] *Let (X, q) be a complete weak partial metric space, and let $T : X \to CB^q(X)$ be a multivalued mapping. Let $\psi : [0, 1) \to (0, 1]$ be the non increasing function defined by*

$$\psi(r) = \begin{cases} 1 & \text{if } 0 \le r < \frac{1}{2} \\ 1 - r & \text{if } \frac{1}{2} \le r < 1. \end{cases} \tag{2.2}$$

Suppose that there exists $0 \le r < 1$ such that T satisfies the condition $\psi(r)q(x, Tx) \le q(x, y)$ implies

$$H_q^+(Tx \setminus \{x\}, Ty \setminus \{y\}) \le rq(x, y),$$

for all $x, y \in X$.

Suppose also that, for all x in X, y in Tx , and $t > 1$, there exists z in Ty such that

$$q(y, z) \le tH_q^+(Ty, Tx).$$

Then T has a fixed point.

3 Main result

Now we state our main result.

***Theorem* 3.1.** *Let (X, q) be a complete weak partial metric space and let $T : X \to CB^q(X)$ be a multivalued mapping and $\psi : [0, 1) \to (0, 1]$ be the non-increasing function defined by (2.2). If there exists $0 \le r < 1$ such that T satisfies the condition*

$$\psi(r)q(x, Tx) \le q(x, y) \ \text{ implies } \ H_q^+(Tx \setminus \{x\}, Ty \setminus \{y\}) \le rM(x, y), \tag{3.1}$$

where $M(x, y) = \max\{q(x, y), q(x, Tx), q(y, Ty)\}$ for all $x, y \in X$.

Suppose also that for every $x \in X, y \in Tx$ and $t > 1, \exists\ z \in Ty$ such that

$$q(y, z) \le tH_q^+(Ty, Tx). \tag{3.2}$$

Then T has a fixed point.

Proof. Let r_1 be a real number such that $0 \le r \le r_1 < 1$ and $w_0 \in X$. Since Tw_0 is nonempty, it follows that if $w_0 \in Tw_0$, then proof is finished. Let $w_1 \in Tw_0$ be such that $w_0 \neq w_1$. Similarly $\exists\ w_2 \in Tw_1$ such that $w_1 \neq w_2$. From (3.2) we have

$$q(w_1, w_2) \le \frac{1}{\sqrt{r_1}} H_q^+(Tw_0, Tw_1). \tag{3.3}$$

Since $\psi(r) \le 1$, we have

$$\psi(r)q(w_1, Tw_1) \le q(w_1, Tw_1) \le q(w_1, w_2).$$

Using (3.1) in (3.3), we have

$$\begin{aligned} q(w_1, w_2) &\le \frac{1}{\sqrt{r_1}} H_q^+(Tw_0, Tw_1) \\ &\le \frac{1}{\sqrt{r_1}} H_q^+(Tw_0 \setminus \{w_0\}, Tw_1 \setminus \{w_1\}) \end{aligned}$$

$$\begin{aligned}
&\le \frac{r}{\sqrt{r_1}} M(w_0, w_1) < \sqrt{r_1} M(w_0, w_1)\\
&= \sqrt{r_1} \max\{q(w_0, w_1), q(w_0, Tw_0), q(w_1, Tw_1)\}\\
&\le \sqrt{r_1} \max\{q(w_0, w_1), q(w_0, w_1), q(w_1, w_2)\}.
\end{aligned}$$

If $q(w_0, w_1) \le q(w_1, w_2)$ then

$$q(w_1, w_2) \le \sqrt{r_1} q(w_1, w_2)$$

but $\sqrt{r_1} < 1$ then we get a contradiction.

Thus, we have

$$q(w_1, w_2) \le \sqrt{r_1} q(w_0, w_1).$$

Continuing this process, we obtain a sequence $\{w_n\}$ in X such that

$$q(w_n, w_{n+1}) \le (\sqrt{r_1})^n q(w_0, w_1).$$

Now, we prove that $\{w_n\}$ is a Cauchy sequence in (X, q^s).

For all $k \in \mathbb{N}$, we have

$$\begin{aligned}
q^s(w_n, w_{n+k}) &\le q(w_n, w_{n+k})\\
&\le q(w_n, w_{n+1}) + q(w_{n+1}, w_{n+2}) + ... + q(w_{n+k-1}, w_{n+k})\\
&\le [(\sqrt{r_1})^n + (\sqrt{r_1})^{n+1} + ... + (\sqrt{r_1})^{n+k-1}] q(w_0, w_1)\\
&\le \frac{(\sqrt{r_1})^n}{1 - \sqrt{r_1}} q(w_0, w_1)
\end{aligned}$$

$$\longrightarrow 0 \text{ as } n \to \infty.$$

Hence, $\lim_{n\to\infty} q^s(w_n, w_{n+k}) = 0$.

This implies that $\{w_n\}$ is a Cauchy sequence in (X, q^s). Since (X, q) is complete, therefore (X, q^s) is also complete metric space. It follows that there exists $u \in X$ such that $\lim_{n\to\infty} w_n = u$ in (X, q^s).

Therefore, $\lim_{n\to\infty} q^s(w_n, u) = 0$.

From (2.1), we have

$$q(u, u) = \lim_{n\to\infty} q(w_n, u) = \lim_{n,k\to\infty} q(w_n, w_k) = 0.$$

Now, from triangle inequality

$$q(u, Tx) \le q(u, w_{n+1}) + q(w_{n+1}, Tx)$$

and

$$q(w_{n+1}, Tx) \le q(w_{n+1}, w_n) + q(w_n, u) + q(u, Tx).$$

Taking limit in the above inequalities, we get

$$q(u, Tx) = \lim_{n\to\infty} q(w_{n+1}, Tx). \tag{3.4}$$

We claim that

$$q(u, Tx) \le 2r \max\{q(u, x), q(x, Tx)\}, \ \forall\, x \in X \setminus \{u\}.$$

Since, $\lim_{n\to\infty} q(w_n, u) = 0$, $\exists\, n_0 \in \mathbb{N}$ such that $q(w_n, u) \le \frac{1}{3} q(u, x)$, $\forall\, n \ge n_0$.

As $w_{n+1} \in Tw_n$ then we have

$$\begin{aligned}
\psi(r) q(w_n, Tw_n) \le q(w_n, Tw_n) &\le q(w_n, w_{n+1})\\
&\le q(w_n, u) + q(u, w_{n+1})\\
&\le \frac{1}{3} q(u, x) + \frac{1}{3} q(u, x) = \frac{2}{3} q(u, x)
\end{aligned}$$

$$\begin{aligned} &\leq q(u,x) - \frac{1}{3}q(u,x) \\ &\leq q(u,x) - q(w_n,u) \\ &\leq q(x,w_n). \end{aligned}$$

Hence, for any $n \geq n_0$ we get

$$\psi(r)q(w_n, Tw_n) \leq q(x, w_n).$$

From (3.1) we have

$$H_q^+(Tw_n, Tx) \leq rM(w_n, x).$$

Since

$$\begin{aligned} q(w_{n+1}, Tx) &\leq \delta_q(Tw_n, Tx) \\ &\leq 2H_q^+(Tw_n, Tx) \\ &\leq 2rM(w_n, x) = 2r\max\{q(w_n, x), q(w_n, Tw_n), q(x, Tx)\} \\ &\leq 2r\max\{q(w_n, u) + q(u, x), q(w_n, w_{n+1}), q(x, Tx)\}. \end{aligned}$$

Taking limit as $n \to \infty$, we get

$$\lim_{n\to\infty} q(w_{n+1}, Tx) \leq 2r\max\{q(u, x), q(x, Tx)\}.$$

From (3.4) we get

$$q(u, Tx) \leq 2r\max\{q(u, x), q(x, Tx)\}, \ \forall \ x \in X \setminus \{u\}. \tag{3.5}$$

Now, we claim that

$$H_q^+(Tx, Tu) \leq r\max\{q(x, u), q(x, Tx), q(u, Tu)\} \tag{3.6}$$

for all $x \in X$ such that $x \neq u$.

For each $n \in \mathbb{N}$, $\exists \ y_n \in Tx$ such that

$$q(u, y_n) \leq q(u, Tx) + \frac{1}{n}q(u, x).$$

Therefore

$$\begin{aligned} q(x, Tx) &\leq q(x, y_n) \\ &\leq q(x, u) + q(u, y_n) \\ &\leq q(x, u) + q(u, Tx) + \frac{1}{n}q(u, x) \\ &\leq q(x, u) + 2r\max\{q(u, x), q(x, Tx)\} + \frac{1}{n}q(u, x). \end{aligned} \tag{3.7}$$

Suppose $\max\{q(u, x), q(x, Tx)\} = q(u, x)$ then

$$q(x, Tx) \leq (1 + 2r + \frac{1}{n})q(u, x),$$

which implies

$$\frac{1}{1 + 2r + \frac{1}{n}}q(x, Tx) \leq q(u, x).$$

This further implies that

$$H_q^+(Tu, Tx) \leq rM(x, u),$$

and equation (3.6) holds.
Now if $\max\{q(x,u), q(x,Tx)\} = q(x,Tx)$ then from (3.7), we get

$$q(x,Tx) \le q(x,u) + 2rq(x,Tx) + \frac{1}{n}q(x,u)$$
$$\frac{(1-2r)}{(1+\frac{1}{n})}q(x,Tx) \le q(x,u).$$

This also implies that
$H_q^+(Tx,Tu) \le rM(x,u)$ and equation (3.6) holds.

Finally, let $b \in Tu$ then

$$q(b,Tw_n) \le \delta_q(Tu,Tw_n). \tag{3.8}$$

Also, we know that

$$\begin{aligned} q(u,Tu) &\le q(u,Tw_n) + q(Tw_n,Tu) \\ &\le q(u,w_{n+1}) + q(b,Tw_n). \end{aligned}$$

From (3.8), we have

$$q(u,Tu) \le q(u,w_{n+1}) + \delta_q(Tu,Tw_n).$$

Taking limit, we get

$$q(u,Tu) \le \lim_{n\to\infty} \delta_q(Tu,Tw_n). \tag{3.9}$$

Also we know that

$$q(w_{n+1},Tu) \le \delta_q(Tw_n,Tu).$$

Taking limit, we have

$$q(u,Tu) \le \lim_{n\to\infty} \delta_q(Tw_n,Tu). \tag{3.10}$$

From the definition(2.3), we know that

$$\frac{1}{2}[\delta_q(Tw_n,Tu) + \delta_q(Tu,Tw_n)] = H_q^+(Tw_n,Tu).$$

Taking limit in the above expression and using (3.9) and (3.10) we get

$$\begin{aligned} \frac{1}{2}[q(u,Tu) + q(u,Tu)] &\le \lim_{n\to\infty} \frac{1}{2}[\delta_q(Tw_n,Tu) + \delta_q(Tu,Tw_n)] \\ &= \lim_{n\to\infty} H_q^+(Tw_n,Tu) \\ &= \lim_{n\to\infty} H_q^+(Tw_n \setminus \{w_n\}, Tu \setminus \{u\}) \\ &\le \lim_{n\to\infty} r\max\{q(w_n,u), q(w_n,Tw_n), q(u,Tu)\} \\ &= rq(u,Tu) \end{aligned}$$

which implies that

$$q(u,Tu) \le rq(u,Tu).$$

Since $r < 1$ then we get a contradiction.
Hence, we have $q(u,Tu) = 0 = q(u,u)$. Since Tu is closed then $u \in \overline{Tu} = Tu$.
Now we give an example to verify our result.

***Example* 3.1.** *Let* $X = \{0, \frac{1}{3}, 1\}$ *and define a weak partial metric* $q : X \times X \to [0, \infty)$ *as follows:* $q(0,0) = 0,\ q(\frac{1}{3}, \frac{1}{3}) = \frac{1}{5},\ q(1,1) = \frac{1}{3},\ q(0,1) = q(1,0) = \frac{2}{5},\ q(1, \frac{1}{3}) = q(\frac{1}{3}, 1) = \frac{3}{5},\ q(0, \frac{1}{3}) = q(\frac{1}{3}, 0) = \frac{1}{4}.$

Define a mapping $T : X \to CB^q(X)$ *by*

$$Tx = \begin{cases} \{0\} & \text{if } x = 0 \\ \{1\} & \text{if } x = \frac{1}{3} \\ \{0, \frac{1}{3}\} & \text{if } x = 1. \end{cases}$$

Clearly (X, q) is a weak partial metric space.

Choose $r = 0.9$. From the definition of ψ we have $\psi(0.9) = 0.1$. To investigate the contraction condition (3.1) holds for all $x, y \in X$, we assume the following cases:

Case (I) When $x = 0$, we have

$$\psi(r)q(0, T(0)) = 0 \le q(0, y),\ \forall\ y \in X.$$

For $y = 0$, we have

$$H_q^+(T(0) \setminus \{0\}, T(0) \setminus \{0\}) = H_q^+(\phi, \phi) = 0 \le rM(0, 0).$$

For $y = \frac{1}{3}$, we have

$$H_q^+(T(0) \setminus \{0\}, T(\tfrac{1}{3}) \setminus \{\tfrac{1}{3}\}) = H_q^+(\phi, \{1\}) = 0 \le rM(0, \tfrac{1}{3}).$$

For $y = 1$, we have

$$H_q^+(T(0) \setminus \{0\}, T(1) \setminus \{1\}) = H_q^+\Big(\phi, \Big\{0, \tfrac{1}{3}\Big\}\Big) = 0 \le rM(0, 1).$$

Case (II) When $x = \frac{1}{3}$, we have

$$\psi(r)q\Big(\frac{1}{3}, T\Big(\frac{1}{3}\Big)\Big) = (0.1)q(\frac{1}{3}, 1) = 0.06 \le q(\frac{1}{3}, y),\ \forall\ y \in X.$$

For $y = 0$, we have

$$H_q^+(T(\tfrac{1}{3}) \setminus \{\tfrac{1}{3}\}, T(0) \setminus \{0\}) = H_q^+(\{1\}, \phi) = 0 \le rM(\tfrac{1}{3}, 0).$$

For $y = \frac{1}{3}$, we have

$$H_q^+(T(\tfrac{1}{3}) \setminus \{\tfrac{1}{3}\}, T(\tfrac{1}{3}) \setminus \{\tfrac{1}{3}\}) = H_q^+(\{1\}, \{1\}) = \tfrac{1}{3} \le rM(\tfrac{1}{3}, \tfrac{1}{3}) = r\tfrac{3}{5}.$$

For $y = 1$, we have

$$H_q^+(T(\tfrac{1}{3}) \setminus \{\tfrac{1}{3}\}, T(1) \setminus \{1\}) = H_q^+\Big(\ \{1\}, \Big\{0, \tfrac{1}{3}\Big\}\Big) = \tfrac{1}{2} \le rM(\tfrac{1}{3}, 1) = r\tfrac{3}{5}.$$

Case (III) When $x = 1$, we have

$$\psi(r)q(1, T(1)) = (0.1)q\Big(1, \Big\{0, \frac{1}{3}\Big\}\Big) = 0.04 \le q(1, y),\ \forall\ y \in X.$$

For $y = 0$, we have

$$H_q^+(T(0) \setminus \{0\}, T(1) \setminus \{1\}) = H_q^+\Big(\phi, \Big\{0, \tfrac{1}{3}\Big\}\Big) = 0 \le rM(1, 0).$$

For $y = \frac{1}{3}$, we have

$$H_q^+(T(0) \setminus \{0\}, T(\tfrac{1}{3}) \setminus \{\tfrac{1}{3}\}) = H_q^+(\phi, \{1\}) = 0 \le rM(0, \tfrac{1}{3}) = r\tfrac{3}{5}.$$

For $y = 1$, we have

$$H_q^+(T(1) \setminus \{1\}, T(1) \setminus \{1\}) = H_q^+\Big(\Big\{0, \tfrac{1}{3}\Big\}, \Big\{0, \tfrac{1}{3}\Big\}\Big) = \tfrac{1}{5} \le rM(1, 1) = r\tfrac{2}{5}.$$

Finally, we will enquire the condition (3.2) with $t = 2$. For this, we discuss the following situations:

(i) If $x = 0$, then $y \in T(0) = \{0\}$, so $\exists\, z \in T(y) = \{0\}$ such that

$$0 = q(y, z) \le 2H_q^+(T(y), T(x)).$$

(ii) If $x = \frac{1}{3}$, then $y \in T(\frac{1}{3}) = \{1\}$, so $\exists\, z$(say $z = 0$) $\in T(1) = \{0, \frac{1}{3}\}$ such that

$$\frac{2}{5} = q(y, z) \le 2H_q^+\Big(T(1), T\Big(\frac{1}{3}\Big)\Big) = 1.$$

(iii) If $x = 1$, then $y \in T(1) = \{0, \frac{1}{3}\}$. If $y = 0$, then $z = 0$, and condition holds.

Also if $y = \frac{1}{3}$, then $\exists\, z \in T(\frac{1}{3}) = \{1\}$ such that

$$\frac{3}{5} = q(y, z) \le 2H_q^+\Big(\{1\}, \Big\{0, \frac{1}{3}\Big\}\Big) = 1.$$

Hence all the conditions of ***Theorem*** **3.1** are satisfied. Here $x = 0$ is fixed point of T.

On the other hand the result of Aydi et al.[4] is not applicable. We see that

$$H_q^+(T(\tfrac{1}{3}) \setminus \{\tfrac{1}{3}\}, T(\tfrac{1}{3}) \setminus \{\tfrac{1}{3}\}) = H_q^+(\{1\}, \{1\}) = \tfrac{1}{3} \le r\tfrac{1}{5} = rq(\tfrac{1}{3}, \tfrac{1}{3})$$

is not satisfied for any $r \in (0, 1)$.

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