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(Dedicated to Honor Professor H.M. Srivastava on His 80th Birth Anniversary Celebrations)

NUMERICAL SOLUTION OF MAGNETOHYDRODYNAMIC *(MHD)* RADIATIVE BOUNDARY LAYER FLOW AND HEAT TRANSFER ALONG A WEDGE IN THE PRESENCE OF SUCTION/INJECTION

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Abstract

An approximate numerical solution for the two dimensional laminar *MHD* radiative boundary layer flow along a wall of wedge with appropriate suction and injection in the presence of viscous dissipation and porous medium is considered in the present article. The fluid is considered to be viscous and incompressible. By applying appropriate similarity transformation, the governing flow equations have been transformed into corresponding higher order ordinary differential equations. The flow model is shown to be controlled by a number of flow parameters, viz. the radiation parameter, magnetic parameter, the permeability parameter, Prandtl number, Eckert number, the Hartree pressure gradient parameter and suction-injection parameter. The system of governing differential equations is solved numerically by shooting method and numerical calculations are carried out for different values of above mentioned dimensionless parameters. An analysis of the drawn results predicts that the velocity boundary layer and thermal boundary layer are influenced appreciably by the suction-injection, radiation and viscous dissipation at the wall of wedge.

2010 Mathematics Subject Classifications: 76D10, 76E25, 76S05, 76W05, 80A20.
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1 Introduction

The flow problems combined with heat transfer over a wedge shaped configuration is encountered in numerous thermal engineering applications such as geothermal systems, heat exchangers, crude oil extraction, the flow in the desert cooler, nuclear waste management and thermal insulation. Historically in such types of flow problems, the popular Falkner-Skan transformation is obeyed to convert the boundary layer partial differential equations into the ordinary differential equations. In fact, a model of such fluid flow over a wedge shaped bodies was first formulated by Falkner and Skan[12] to illustrate the applications of Prandtls boundary layer phenomena. In the later years Hartree [13] studied the similar problem and gave the numerical results for shear stress with different wedge angles. The other pioneer was Eckert[10], who investigated Falkner-Skan flow past an isothermal wedge and gave initial heat transfer values. In *MHD*, we consider the electrically conducting fluid flow with magnetic characteristics. *MHD* plays significant role in plasma studies, *MHD* power generators, construction of heat exchangers etc. There is plenty of literature available in which fluid flows have been studied with or without *MHD* under different fluid properties using the prominent Falkner-Skan transformations. The following studies give the clear insight of the fundamental problem of the wedge flow and associated applications with different fluid properties.

Thermal characteristics in boundary layer wedge flow under different fluid parameters were extensively investigated by Chen [9] and Watanabe [27]. Kafoussias and Nanousis [15] studied the laminar boundary layer wedge flow with *MHD* and suction injection effects. Yih [28] reported the uniform blowing effect on forced convection flow along a wedge with heat flux. Further Anjalidevi et al. [3] and Kandasamy et al. [16] investigated the thermal stratification and chemical reaction effects with heat source and concentration in the presence of suction/injection. Martin and Boyd [18] reported the Falkner-Skan flow with slip conditions and derived the significant effects on flow. On the similar lines Sattar [24] used unsteady fluid flow past a wedge and draw important conclusions. Ashwani et al. [4], Abbasbandy et al. [2] and Khan et al. [17] also provided the significant contribution to the *MHD* wedge flow with different governing fluid parameters. Arthur et al. [5] studied the stagnation point flow over a porous surface with heat transfer effects and viscous dissipation. El-Dabe et al. [11] and Srinivasharya et al. [23] considered non-Newtonian fluid including Casson fluid and nanofluid in their respective studies and gave substantial contribution to the wedge flow literature. Stretching wedge and convective heat transfer on the boundaries have been studied by Nagendramma et al. [20]. Increasing technical importance and variety of applications of wedge flow leads many young minds of globe such as Majety et al. [19], Ullah et al. [26], Alam et al. [1] and Ramesh et al. [22]. Recently Ibrahim and Tulu [14] reported the *MHD* boundary layer flow and heat transfer considering nanofluid and viscous dissipation, in porous media.

In the present analysis we consider the numerical study of *MHD* boundary layer flow along a wedge with radiation and suction-injection effects. Suction and injection of a fluid through surfaces can give the desired heat transfer rates. Injection or blowing through porous bounding heated or cooled surfaces is widely used in boundary layer control. The effect of various flow parameters on velocity and temperature fields are derived and analyzed with tabular and graphical mode of representation.

2 Mathematical formulation

In the present analysis, we propose two dimensional *MHD* boundary layer flow along a wedge with radiation and suction-injection effects. As proposed in ***Fig.* 2.1**[9], x-coordinate is considered parallel to the wedge and y-coordinate is taken along the free stream. T_w is temperature at wall of the wedge and T_∞ is ambient temperature. The fluid regime is considered to have constant fluid properties. A constant magnetic field of strength B_0 is assumed in the normal direction to the wall of wedge. The induced magnetic field is not taken into consideration as it is too small to compare with the applied magnetic field as suggested by Ullah et al. [26].

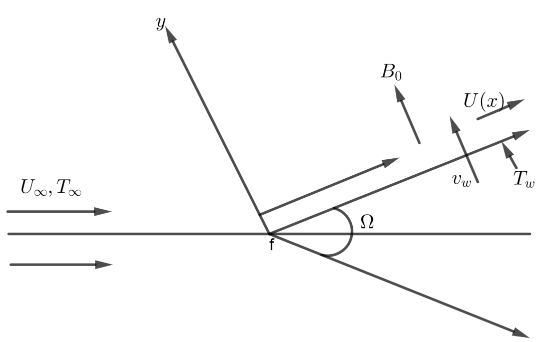

***Figure* 2.1:** Flow analysis along the wall of wedge

Following the above assumptions together with boundary layer approximation, the governing flow equations i.e. continuity, momentum and energy equations can be expressed as (Srinivasacharya et al. [22] and Alam et al. [26]) following:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_f}\frac{\partial p}{\partial x} + \upsilon_f\frac{\partial^2 u}{\partial y^2} - \left(\frac{\sigma B_o^2}{\rho_f} + \frac{\upsilon_f}{K_o}\right)u, \tag{2.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_f\frac{\partial^2 T}{\partial y^2} + \frac{\upsilon_f}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{\partial q_r}{\partial y}. \tag{2.3}$$

The appropriate boundary conditions as per formulation are prescribed as:

$$\begin{aligned} &u = 0 \qquad v = -v_w(x) \qquad T = T_w \qquad at \quad y = 0\\ &u = U(x) = U_\infty x^m \qquad T \to T_\infty \qquad as \quad y \to \infty. \end{aligned} \tag{2.4}$$

Here $v_w(x) > 0$ is the velocity of suction and $v_w(x) < 0$ is the velocity of injection. $v_w(x) = v_0 x^{\frac{m-1}{2}}$ is a prescribed velocity considered at the wall of the wedge and v_0 is the initial strength of suction. Again u and v are velocities in x and y directions respectively and ρ_f is fluid density, υ_f is kinematic viscosity, α_f is thermal diffusivity and C_p is specific heat of the fluid.

Equation (2.2) shows that pressure p in the boundary layer must be equal to that of free stream for any prescribed value of x . As velocity does not change in free stream, so there is no vorticity involved. In such a case simple Bernoullis equation can be used as suggested by Falkner and Skan [1]. Fluid velocity outside the boundary layer is taken as $U(x) = U_\infty x^m$. For a uniform stream, the equation (2.2) can be expressed as (Falkner and Skan [12])

$$-\frac{1}{\rho_f}\frac{\partial p}{\partial x} = U\frac{dU}{dx} + \left(\frac{\sigma B_o^2}{\rho_f} + \frac{\upsilon_f}{K_o}\right)U. \tag{2.5}$$

On using equation (2.5) into equation (2.2), the momentum equation becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = U\frac{dU}{dx} + \upsilon_f\frac{\partial^2 u}{\partial y^2} + \left(\frac{\sigma B_o^2}{\rho_f} + \frac{\upsilon_f}{K_o}\right)(U - u). \tag{2.6}$$

In the above equations, x is measured from tip of the wedge, x is the Falkner-Skan power law parameter, $\beta = \frac{2m}{1+m}$ is the Hartree pressure gradient parameter corresponding to $\beta = \frac{\Omega}{\pi}$ for the total angle Ω of the wedge as mentioned in ***Fig.* 2.1**. Positive Falkner-Skan power law parameter m represents favorable pressure gradient while negative value of m represents adverse pressure gradient. (Nagendramma et al. [20]).

The radiative heat flux q_r mentioned in equation (2.3) is modelled as per Rosseland approximation [21] by the following

$$q_r = -\frac{4\alpha}{3\beta}\frac{\partial T^4}{\partial y}, \tag{2.7}$$

In the above equation, α represents Stefan-Boltzmann constant and β represents the mean absorption constant. The above approximation holds good at points optically far from the boundary surface and fair for intensive absorption. Now we assume that the temperature difference with in the fluid flow varies as a linear function of temperature so that expanding the term T^4 by the well known Taylor series about T_∞ and omitting the higher-order terms

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \tag{2.8}$$

Using (2.8) into the equation (2.7), we get

$$q_r = -\frac{16\alpha T_\infty^3}{3\beta}\frac{\delta T}{\delta y}. \tag{2.9}$$

Now substituting the value of q_r from (2.9) in (2.3), we get

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\alpha_f + \frac{16\alpha T_\infty^3}{3\beta}\right)\frac{\partial^2 T}{\partial y^2} + \frac{\upsilon_f}{C_p}\left(\frac{\partial u}{\partial y}\right)^2. \tag{2.10}$$

3 Similarity Analysis

In order to solve above system of equations, we apply suitable similarity transformation to convert above partial differential equations into higher order ordinary differential equations. We choose stream function ψ in such a manner that continuity equation (2.1) is identically satisfied, for this

$$u = \frac{\partial \psi}{\partial y} \quad and \quad v = -\frac{\partial \psi}{\partial x}. \tag{3.1}$$

Following the transformation suggested by Bansal [6], the corresponding momentum equation (2.6) and energy equation (2.10) can be converted into ordinary differential equation as:

$$\psi(x,\eta) = \sqrt{\frac{2}{1+m}\upsilon_f U_\infty} x^{\frac{m-1}{2}} f(\eta), \tag{3.2}$$

$$\eta = y\sqrt{\frac{1+m}{2}\frac{U_\infty}{\upsilon_f}} x^{\frac{m-1}{2}},$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}.$$

In the above transformation equations, η is dimensionless similarity variable, $f(\eta)$nondimensional stream function, $f'(\eta)$is nondimensional velocity and $\theta(\eta)$ is nondimensional temperature. Now using the above transformations and approximation suggested by Kafoussias et al. [15], we get the following transformed equations together with boundary conditions,

$$\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} + \beta\left[1 - \left(\frac{df}{d\eta}\right)^2\right] + \frac{1}{1+m}(M+K)\left(1 - \frac{df}{d\eta}\right) = 0, \tag{3.3}$$

$$\left[1 + \frac{4N}{3}\right]\frac{d^2\theta}{d\eta^2} + Pr\left[f\frac{d\theta}{d\eta} + Ec\left(\frac{d^2\theta}{d\eta^2}\right)^2\right] = 0, \tag{3.4}$$

with associated boundary conditions:

$$f = \frac{2}{1+m}S, \quad f' = 0 \quad and \quad \theta = 1 \quad at \quad \eta = 0, \tag{3.5}$$

$$f' \to 1 \quad and \quad \theta \to 0 \quad as \quad \eta \to \infty.$$

The non-dimensional parameters involved in the present study can be summarized as follows:

Radiation parameter $N = \frac{4\alpha T_\infty^3}{\beta\kappa}$, Magnetic parameter $M = \frac{2\sigma B_o^2 x^{1-m}}{\rho_f U_\infty}$,
Eckert number $Ec = \frac{U^2}{C_p(T_w - T_\infty)}$, Prandtl number $Pr = \frac{\upsilon_f}{\alpha_f}$,
Permeability parameter $K = \frac{2\upsilon_f x^{1-m}}{K_o U_\infty}$, Suction parameter $S = v_o\sqrt{\frac{2}{\upsilon_f U_\infty (m+1)}} > 0$,
and $S > 0$ for injection parameter.

In the present problem, the physical quantities of technical importance are the skin- friction coefficient C_f and local Nusselt number Nu_x, which may be described as follows:
$C_f = \frac{2\tau_w}{\rho U^2(x)}$, where τ_w is the surface shear stress which may be expressed as:
$\tau_w = \mu_f\left(\frac{\partial u}{\partial y}\right)_{y=0}$.

So the nondimensional skin friction coefficient C_f is

$$C_f\sqrt{Re_x} = 2\sqrt{\frac{2}{m+1}} f''(0), \text{ where } Re_x \text{ is reynold number.} \tag{3.6}$$

Similarly local Nusselt number can be expressed as:
$Nu_x = \frac{x q_w}{k(T_w - T_\infty)}$, where q_w is surface heat flux that can be defined as
$q_w = -k_f\left(\frac{\delta T}{\delta y}\right)_{y=0}$.

Hence the nondimensional local Nusselt number is

$$\frac{Nu_x}{\sqrt{Re_x}} = -\sqrt{\frac{m+1}{2}}\theta'(0). \tag{3.7}$$

4 Numerical solution

The system of differential equations (3.3) and (3.4) with boundary conditions (3.5) are solved by shooting method. For this, the above equations are converted into the system of first order differential equations as mentioned below:

$$f_1 = f, \quad f_2 = f', \quad f_3 = f'', \quad f_4 = \theta, \quad f_5 = \theta'. \tag{4.1}$$

Using above notations the first order differential equations are as follows

$$f_3' = f_1 f_3 - \beta\left(1 - f_2^2\right) - \frac{1}{1+m}(M+K)(1-f_2), \tag{4.2}$$

$$f_5' = -\frac{3Pr}{3+4N}\left(f_1 f_5 + Ecf_3^2\right), \tag{4.3}$$

with boundary conditions

$$f_1(0) = \frac{2}{1+m}S, \quad f_2(0) = 1, \quad f_4(0) = 1, \tag{4.4}$$
$$f_2(\infty) = 0 \text{ and } f_4(\infty) = 0.$$

For the solution of above equations, we require $f_3(0)$ and $f_5(0)$, but no such values are available at the boundary. So according to shooting technique, we apply initial guess for $f_3(0)$ and $f_5(0)$. Then we compare the numerical values for f_2 and f_4 at η_∞ with the boundary conditions and adjust the values of f_3 and f_5. Secant method is applied for better approximation. The above procedure is carried out until the proper accuracy is achieved.

5 Result and Discussion

Numerical solutions for the velocity and temperature profiles across the boundary layer for different values of flow parameters have been obtained.

***Table* 5.1** describes the influence of various governing flow parameters on skin friction coefficient and local Nusselt number. It is observed that the skin friction coefficient enhances with increasing Falkner-Skan power law parameter, Magnetic parameter, Permeability parameter and suction parameter while reduces with increasing injection parameter. Also the local Nusselt number enhances with increasing Prandtl number and suction parameter and shows opposite behavior while increasing Falkner-Skan power law parameter, Magnetic parameter, Permeability parameter, Eckert number, Radiation parameter and injection parameter.

Table **5.1:** Numerical computation of skin friction coefficient $-f''(0)$ and local Nusselt number $-\theta'(0)$ for different physical parameters

m	M	K	Pr	Ec	N	S	$-f''(0)$	$-\theta'(0)$
0.0	0.4	.05	0.7	0.5	0.2	0.2	1.066156	0.458095
.14							1.177544	0.432976
.33							1.273992	0.408410
.50							1.332252	0.392289
.14	0.0						0.995269	0.444004
	0.5						1.218687	0.430197
	1.0						1.405526	0.416433
	0.5	0.0					1.198301	0.431582
		0.2					1.277789	0.426039
		0.4					1.352354	0.420533
		0.6					1.422781	0.415079
		0.1	0.2				1.238719	0.358333
			0.7				1.238719	0.433060
			2.0				1.238719	0.586770
			0.7	0.1			1.238719	0.569115
				0.5			1.238719	0.433060
				1.0			1.238719	0.262997
				0.5	0.0		1.238719	0.460142
					0.5		1.238719	0.408067
					2.0		1.238719	0.365363
					0.2	0.5	1.602576	0.602457
						1.0	2.293073	0.919708
						1.5	3.049663	1.261584
						0.0	1.025276	0.333150
						-0.1	0.928828	0.287969
						-0.2	0.839800	0.246321

Table **5.2:** Numerical computation of skin friction coefficient $f''(0)$ for various values of Falkner-Skan power law parameter m for the values $M = K = Ec = N = S = 0$ and Pr=0.73.

$f''(0)$				
m	Present Value	Ashwani[16]	Ullah[25]	Wubshet[28]
1429	0.7322	0.7320	0.7320	0.73200
.2000	0.8022	0.8021	0.8021	0.80213
.3333	0.9278	0.9277	0.9277	0.92765
1.000	1.2327	1.2326	1.2326	1.23258

***Table* 5.2** reveals that value of for various different values of Falkner-Skan power law parameter finds excellent agreement with the previous published results such as Ashwani et al.[4], Ullah et al.[26] and Wubshet et al.[14]. The above confirmation shows that present results are also accurate.

***Figure* 5.1** shows the effect of Falkner-Skan power law parameter on velocity profile. It is observed that while increasing the Falkner-Skan power law parameter velocity boundary layer accelerates. The similar behavior is seen in the case of Hartree pressure gradient parameter β.

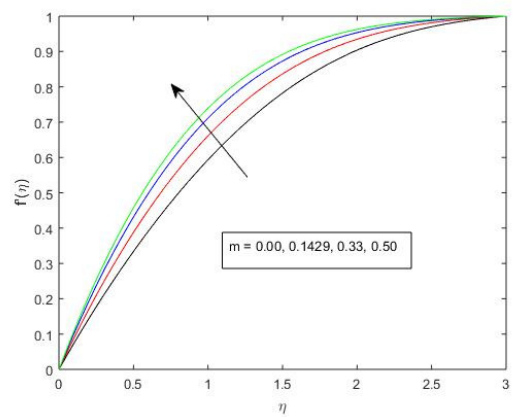

***Figure* 5.1:** Effect of m on velocity profile for $M = 0.4$ and $K = 0.5$

Increment in wedge angle β results into lesser space for fluid flow. Hence velocity boundary layer thickness affects accordingly. ***Figure* 5.2** reports the significant effect of Magnetic parameter$M$ on the velocity field which shows that increasing the value of magnetic parameter results into enhancement on fluid velocity. The effect of permeability parameter on velocity field is described in ***Figure* 5.3**. According to this while increasing the permeability parameter velocity is slightly increased on the porous surface and reduces the boundary layer thickness. The similar effects of m, M, K on velocity have been reported by Wubshet et al. [14].

The effect of Prandtl number on temperature profile is reported in ***Figure* 5.4**. It clearly describes that increasing the value of Prandtl number results into decrease in temperature field. This is because of increasing the Prandtl number tends to reduce the thermal diffusivity of the fluid and causes the weak penetration of heat inside the fluid. The influence of viscous dissipation i.e. Eckert number on the temperature field is shown in ***Figure* 5.5**. The Eckert number describes the conversion the kinetic number into internal energy by work done against the viscous fluid stress. It is observed that increasing the value of Eckert number causes the rise in the temperature. The effect of radiation parameter on temperature field is reported in ***Figure* 5.6**. It is observed from figure that on increasing the value of results into the significant increment of temperature.

The effect of suction and injection parameters on velocity as well as in temperature fields are reported in ***Figures* 5.7-5.10**. Suction and injection are effective tools to control the flow field. Fluid flow can be made laminar using the suction injection. The effect of suction S on velocity field is reported in ***Figure* 5.7** which describes that increasing the amount of suction tends into significant increment in fluid velocity. This is due to creation of more space for fluid particles in which they can move with greater velocity. Similarly the effect of suction parameter S on temperature field is shown in ***Figure* 5.8**. It is observed that increasing the value of S results into decrease in the temperature which can be used to cool the fluid flow. Similarly the effect of injection parameter S on velocity field is reported in ***Figure* 5.9**. It is shown that increasing the value of injection parameter tends to lower the velocity. Also injection causes significant rise in temperature field which is described in ***Figure* 5.10**

6 Conclusion

Numerical solution for the two dimensional laminar *MHD* boundary layer flow along a wall of wedge with uniform suction and injection in the presence of radiation and porous medium has been discussed. Using the suitable similarity transformation, the governing partial differential equations

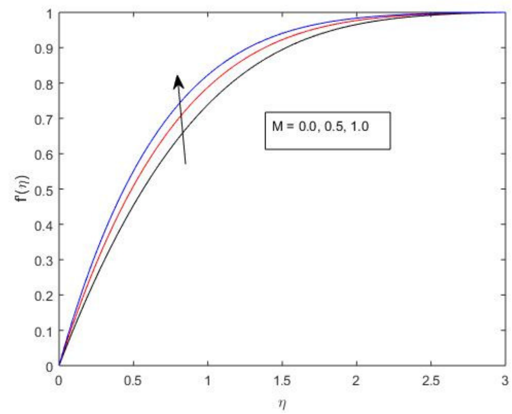

***Figure* 5.2:** Effect of M on velocity profile for m = .14 and K = .05

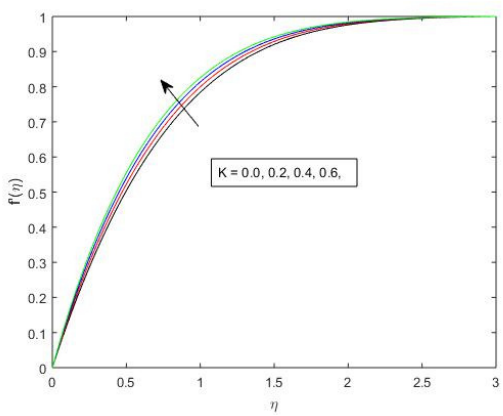

***Figure* 5.3:** Effect of K on velocity profile for M = 0.4 and K = .14

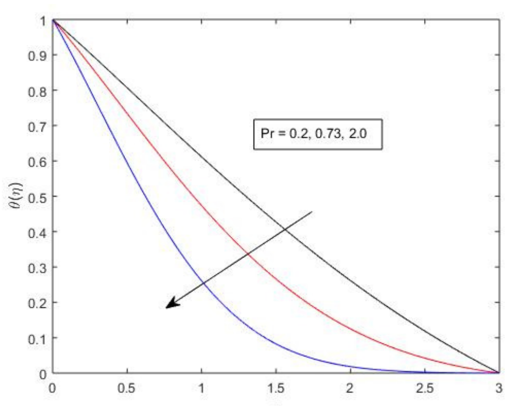

***Figure* 5.4:** Effect of Pr on temperature profile for Ec = 0.5 and N = 0.2

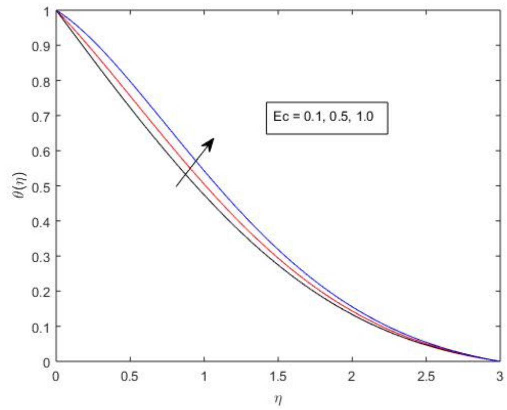

***Figure* 5.5:** Effect of Ec on temperature profile for $Pr = 0.73$ and $N = 0.2$

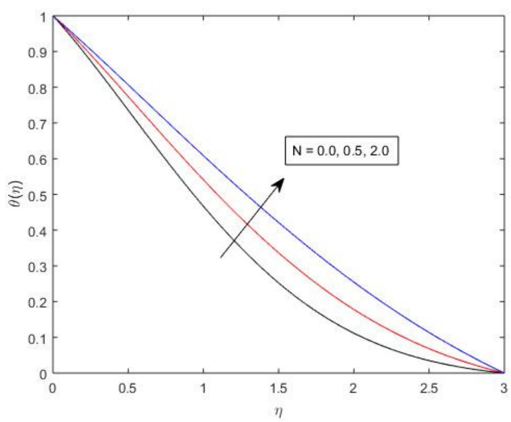

***Figure* 5.6:** Effect of N on temperature profile for $Pr = 0.73$ and $Ec = 0.5$

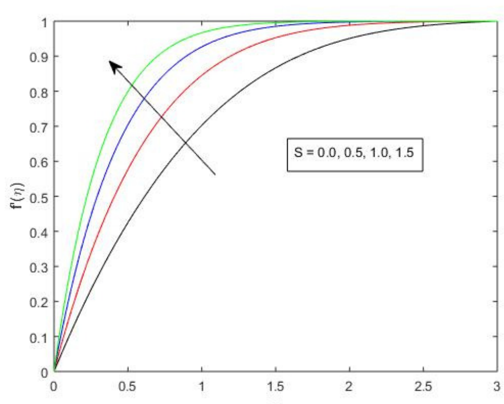

***Figure* 5.7:** Effect of S on velocity profile for $M = 0.4$ and $K = 0.5$

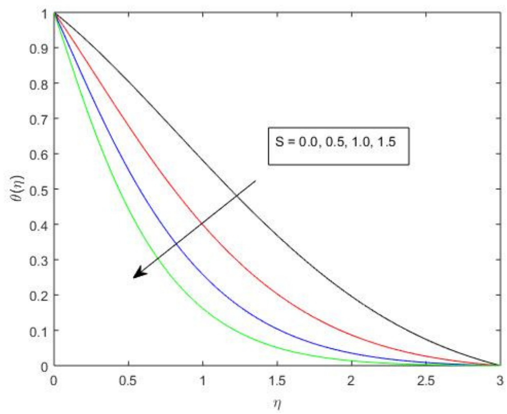

***Figure* 5.8:** Effect of S on temperature profile for $Pr = 0.73$ and $N = 0.2$

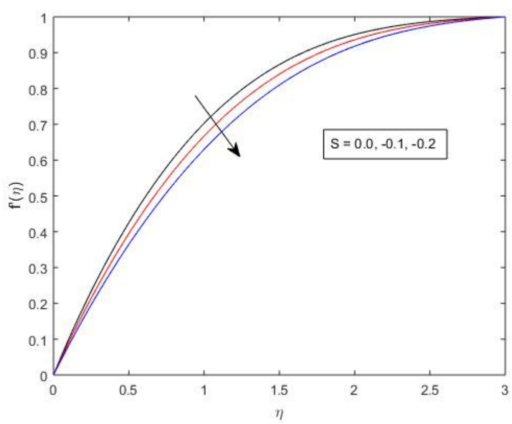

***Figure* 5.9:** Effect of S on velocity profile for $M = 0.4$ and $K = 0.5$

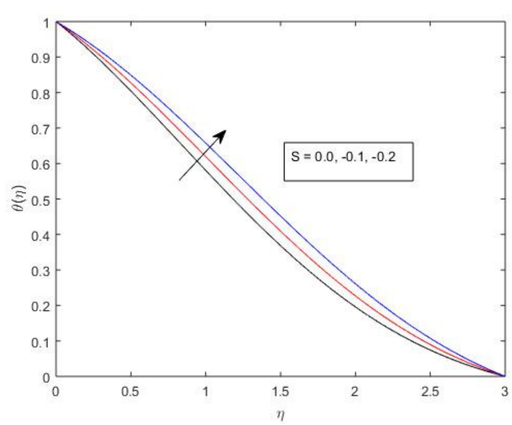

***Figure* 5.10:** Effect of S on temperature profile for $Pr = 0.73$ and $N = 0.2$

are converted into the higher order ordinary differential equations and then solved numerically applying shooting technique. From the aforesaid discussion following conclusions are made:

1. The velocity of fluid enhances with increase in suction parameter, permeability parameter, magnetic parameter and Falkner-Skan power law parameter while reduces with increase in injection parameter.
2. The temperature of fluid enhances with increase in injection parameter, radiation and Eckert number while reduces with increase in Prandtl number and suction parameter.
3. The skin friction coefficient enhances with increasing magnetic parameter, permeability parameter, Falkner-Skan power law parameter and suction parameter.
4. The Nusselt number is a decreasing function for the permeability parameter , Falkner-Skan power law parameter, magnetic parameter, radiation parameter, Eckert number and injection parameter.

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