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(Dedicated to Honor Professor H.M. Srivastava on His 80th Birth Anniversary Celebrations)

MODELING AND ANALYSIS THE EFFECT OF GLOBAL WARMING ON THE SPREAD OF CARRIER DEPENDENT INFECTIOUS DISEASES

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Abstract

In this paper, a nonlinear SIS model is considered to study the effect of global warming on the spread of carrier dependent infectious diseases. In the model, we deal with five dependent variables, namely, the density of susceptibles, the density of infectives, the density of carrier population, the amount of carbon dioxide in the environment causing global warming and the global warming temperature. In the model, it is assumed that the density of carriers increases with constant growth rate as well as proportional to the global warming temperature. The amount of CO_2 in the environment increases due to human population activities as well as natural factors. It is also assumed that the global warming temperature is proportional to the amount of CO_2 in the environment. The model is studied and investigated with the help of stability theory of differential equations and numerical simulation. The investigation shows that, if global warming temperature increases, then the density of carrier population and the spread of carrier dependent infectious diseases increase.
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1 Introduction

Global warming is the most crucial subject matter to be taken under scrutiny by researchers in this century. Certain greenhouse gases in the Earth's atmosphere, like carbon dioxide (CO_2) and methane (CH_4) trap the sun's heat and do not let it escape. The increase of greenhouse gases in the atmosphere over long period leads to an increase in Earth's surface temperature causing global warming. Over the past 130 years, the world has warmed up to 0.85^{0}C (approx.). The last three decades have been progressively warmer than any preceding decades since 1850, IPCC [10]. Further, it is important to note that detrimental activities of growing human population such as deforestation, throwing waste in rivers and oceans and releasing smoke from factories and car fumes into air etc., has discharged enough amount of carbon dioxide (CO_2) in the environment. Because of the global warming, carriers such as ticks, flies, mites, mosquitoes, cockroaches survive in the warmer environment and can cause the spread of carrier dependent infectious diseases, Zhou et al. [18]. The worldwide distribution of carrier dependent infectious diseases shows the impact of climate change, IPCC [8, 9]. The prediction of World Health Organization is that, between 2030 and 2050 there will be an additional 0.25 million deaths per year from many infectious diseases as measles, smallpox, mumps, malaria, diarrhea etc., due to global warming, WHO [15-17].

It is noteworthy that the effect of global warming on the carrier population with a constant growth rate and it's role on the spread of infectious diseases has not been done till now. Many researchers have studied the spread of infectious diseases by using mathematical models in the past, as Anderson and May [1], Bailey [2], Ghosh et al. [3, 4], Greenhalgh [5] , Hethcote [6, 7], May and Anderson [11], Singh [12] and Singh et al. [13, 14]. In this paper, therefore the effect of global warming on carrier population with constant growth rate and its role on the spread of infectious diseases is modeled and studied.

The following assumptions are made in the modeling process:

(i) The rate of density of carrier population increases with a constant rate.

(ii) The rate of density of carrier population is proportional to the global warming temperature.

(iii) The rate of amount of CO_2 in the environment increases by human activities as well as natural factors.

(iv) The rate of global warming temperature is proportional to amount of CO_2 in the environment.

2 SIS Model

Let $N(t)$ be the total human population density which is divided into two categories, namely susceptible human population density $X(t)$ and infective human population density $Y(t)$. Let $C_r(t)$ be the carrier population density which grows with a constant growth rate coefficient "s". Let $C(t)$ be the amount of CO_2 in the environment and $T(t)$ be the global warming temperature of the environment due to discharge of carbon dioxide(CO_2). The model is governed by the following non linear differential equations:

$$
\begin{aligned}
\frac{dX}{dt} &= A - \beta XY - \lambda XC_r - dX + \nu Y, \\
\frac{dY}{dt} &= \beta XY + \lambda XC_r - (\nu + \alpha + d)Y, \\
\frac{dC_r}{dt} &= s + s_1(T - T_0) - s_0 C_r, \\
\frac{dC}{dt} &= Q_o + \alpha_1(A - dN) - \alpha_o C, \\
\frac{dT}{dt} &= \theta(C - C_o) - \theta_0(T - T_o),
\end{aligned}
\tag{2.1}
$$

with $X + Y = N$, where $X(0) = X_0 > 0, Y(0) = Y_0 \geq 0, N(0) = N_0 > 0, C_r(0) = C_{r0} \geq 0, C(0) = C_0 > 0$ and $T(0) = T_0 > 0$ and take $C_0 = \frac{Q_o}{\alpha_o}$.

In the above model system (2.1), the used parameters are positive real numbers, described as follows:

A : The constant immigration rate of human population from outside the region
β : Coefficient of transmission by infective human population density
λ : Coefficient of transmission by carrier population density
d : Natural death rate constant of human population
α : Coefficient of death rate of infective human population due to disease related factors
ν : Coefficient of recovery rate of infective human population density
Q_0 : Discharge rate of CO_2 from natural factors
α_1 : Discharge rate coefficient of CO_2 from man- made sources
α_0 : Natural reduction rate coefficient of CO_2
θ : Growth rate coefficient of temperature of the region due to rise in amount of CO_2
θ_0 : Natural reduction rate coefficient of temperature in the region
s_0 : Reduction rate coefficient of carriers
s : Constant growth rate coefficient of carrier population density
s_1 : Growth rate of carriers due to global warming temperature
T_0 : The equilibrium level of global warming temperature of the environment
C_0 : The equilibrium amount of CO_2

For analyzing the model (2.1), we take the following reduced system by using $X = N - Y$,

$$\frac{dY}{dt} = \beta(N-Y)Y + \lambda(N-Y)C_r - (\nu+\alpha+d)Y, \tag{2.2}$$

$$\frac{dN}{dt} = A - dN - \alpha Y, \tag{2.3}$$

$$\frac{dC_r}{dt} = s + s_1(T-T_0) - s_0C_r, \tag{2.4}$$

$$\frac{dC}{dt} = Q_o + \alpha_1(A-dN) - \alpha_oC, \tag{2.5}$$

$$\frac{dT}{dt} = \theta(C-C_o) - \theta_0(T-T_o), \tag{2.6}$$

with initial conditions

$$Y(0) = Y_0 \geq 0, N(0) = N_0 > 0, C_r(0) = C_{r0} \geq 0, C(0) = C_0 > 0 \text{ and } T(0) = T_0 > 0.$$

Region of Attraction

The region of attraction of the model system (2.2) - (2.6) is given by the set

$$\Omega = \{(Y, N, C_r, C, T) \in R_+^5 : 0 \leq Y \leq N \leq \frac{A}{d}, 0 \leq C_r \leq C_{rm}, 0 < C \leq C_m, 0 < T \leq T_m\},$$

where, $C_{rm} = \dfrac{s\theta_0\alpha_0 + s_1\theta\alpha_1A}{s_0\theta_0\alpha_0}, C_m = C_0 + \dfrac{\alpha_1A}{\alpha_o}$ and $T_m = T_o + \dfrac{\theta\alpha_1A}{\theta_0\alpha_o}$.

It attracts all the solution initiating in the interior of the positive octant of the region.

3 Equilibrium Analysis

The model system (2.2) - (2.6) has only one non-negative equilibria:
(i) $E(Y^*, N^*, C_r^*, C^*, T^*)$.

Proof. To find the equilibrium point $E(Y^*, N^*, C_r^*, C^*, T^*)$ we solve the following set of equations

$$\beta(N-Y)Y + \lambda(N-Y)C_r - (\nu+\alpha+d)Y = 0, \tag{3.1}$$

$$A - dN - \alpha Y = 0, \tag{3.2}$$

$$s + s_1(T-T_0) - s_0C_r = 0, \tag{3.3}$$

$$Q_o + \alpha_1(A - dN) - \alpha_o C = 0, \tag{3.4}$$

$$\theta(C - C_o) - \theta_0(T - T_o) = 0. \tag{3.5}$$

Using above equations we get $C_r = \dfrac{s\theta_0\alpha_0 + s_1\theta\alpha_1\alpha Y}{s_0\theta_0\alpha_0}$.

Now, using the above value of C_r and equations (3.1), (3.2), we get Y^* as the root of the following equation

$$F(Y) = \frac{\beta Y[A - (\alpha + d)Y]}{d} + \frac{\lambda[A - (\alpha + d)Y]}{d}\left(\frac{s\theta_0\alpha_0 + s_1\theta\alpha_1\alpha Y}{s_0\theta_0\alpha_0}\right) - (\nu + \alpha + d)Y = 0. \tag{3.6}$$

So we have, $F(0) = \dfrac{\lambda A s}{d s_0} > 0$ and $F\left(\dfrac{A}{\alpha + d}\right) = -(\nu + \alpha + d)\left(\dfrac{A}{\alpha + d}\right) < 0$

i.e. at least one root of the equation $F(Y) = 0$ lies in the range, $0 < Y < \dfrac{A}{\alpha + d}$.

Rewriting (3.6) as follows

$$F(Y) = \frac{\beta Y[A - (\alpha + d)Y]}{d} + \frac{\lambda[A - (\alpha + d)Y]}{d}(a_0 + b_0 Y) - (\nu + \alpha + d)Y = 0, \tag{3.7}$$

where $a_0 = \frac{s}{s_0}$ and $b_0 = \frac{s_1\theta\alpha_1\alpha}{s_0\theta_0\alpha_0}$.

On differentiating (3.7) with respect to Y, we get

$$F'(Y) = \beta\frac{[A - (\alpha + d)Y]}{d} - \beta Y\left(\frac{\alpha + d}{d}\right) + \frac{\lambda[A - (\alpha + d)Y]}{d}b_0 - \lambda(a_0 + b_0 Y)\left(\frac{\alpha + d}{d}\right) - (\nu + \alpha + d).$$

Then, for $Y > 0$

$$YF'(Y) = \beta\frac{[A - (\alpha + d)Y]}{d}Y - \beta Y^2\left(\frac{\alpha + d}{d}\right) + \frac{\lambda[A - (\alpha + d)Y]}{d}b_0 Y - \lambda Y(a_0 + b_0 Y)\left(\frac{\alpha + d}{d}\right) - (\nu + \alpha + d)Y.$$

$$YF'(Y) = -\beta Y^2\left(\frac{\alpha + d}{d}\right) - \frac{\lambda(\alpha + d)}{d}b_0 Y^2 - a_0\frac{\lambda A}{d} < 0.$$

Hence the equation $F(Y) = 0$ has unique root in the interval $0 < Y < \frac{A}{(\alpha + d)}$

***Remark* 3.1.** *Here we noted that* $\frac{dY}{dQ_0}|_E > 0$

From the model system (2.2) - (2.6)*, we have*

$$\left[\frac{\beta Y(\alpha + d)}{d} - \frac{\beta}{d}\left\{A - (\alpha + d)Y\right\} + \frac{\lambda(\alpha + d)C_r}{d} + (\nu + \alpha + d)\right] \times \frac{dY}{dQ_0} = \frac{\lambda[A - (\alpha + d)Y]}{d}\frac{dC_r}{dQ_0}, \tag{3.8}$$

$$s_1\frac{dT}{dQ_0} = s_0\frac{dC_r}{dQ_0}, \tag{3.9}$$

$$\frac{dC}{dQ_0} = \frac{1}{\alpha_o} + \frac{\alpha_1\alpha}{\alpha_o}\frac{dY}{dQ_0}, \tag{3.10}$$

$$\theta\frac{dC}{dQ_0} = \theta_0\frac{dT}{dQ_0}. \tag{3.11}$$

On writing (3.8) with the help of (3.9), (3.10) *and* (3.11)*, we get*

$$\frac{dY}{dQ_0} = \frac{\lambda s_1\theta[A - (\alpha + d)Y]}{d s_0\theta_0\alpha_o\left[\frac{\beta Y(\alpha+d)}{d} + \frac{\lambda(\alpha+d)C_r}{d} - \left\{\frac{\beta\{A-(\alpha+d)Y\}}{d} + \frac{\lambda}{d}b_0\{A - (\alpha + d)Y\} - (\nu + \alpha + d)\right\}\right]}. \tag{3.12}$$

By equation (3.7)*, we have*

$$\left[\frac{\beta\{A-(\alpha+d)Y\}}{d}+\frac{\lambda\{A-(\alpha+d)Y\}}{d}b_0-(\nu+\alpha+d)\right]=-\frac{\lambda[A-(\alpha+d)Y]}{dY}a_0,$$

which on substituting in equation (3.12)*, gives*

$$\frac{dY}{dQ_0}=\frac{\lambda s_1\theta[A-(\alpha+d)Y]}{ds_0\theta_0\alpha_o\left[\frac{\beta Y(\alpha+d)}{d}+\frac{\lambda(\alpha+d)C_r}{d}+\frac{\lambda\{A-(\alpha+d)Y\}}{dY}a_0\right]}. \tag{3.13}$$

So we have, $\frac{dY}{dQ_0}|_E > 0$*, which shows that the density of infective human population density increases as the discharge rate of* CO_2 *from natural factors increases at the equilibrium point E.*

Since $C_r=\dfrac{s\theta_0\alpha_0+s_1\theta\alpha_1\alpha Y}{s_0\theta_0\alpha_0}=\dfrac{s}{s_0}+\dfrac{s_1\theta\alpha_1\alpha}{s_0\theta_0\alpha_0}Y=a_0+b_0Y$ *, using in* (3.8)*, we get*

(3.14)

$$\left[\frac{\beta Y(\alpha+d)}{d}-\frac{\beta}{d}\{A-(\alpha+d)Y\}+\frac{\lambda(\alpha+d)(a_0+b_0Y)}{d}+(\nu+\alpha+d)\right]\frac{dY}{dQ_0}=\frac{\lambda[A-(\alpha+d)Y]}{d}\frac{dC_r}{dQ_0}.$$

By equation (3.7)*, we have*

$$-\frac{\beta\{A-(\alpha+d)Y\}}{d}+\frac{\lambda(\alpha+d)}{d}(a_0+b_0Y)+(\nu+\alpha+d)=\frac{\lambda A}{dY}(a_0+b_0Y),$$

which on substituting in (3.14)*, we get*

$$\left[\frac{\beta Y(\alpha+d)}{d}+\frac{\lambda A}{dY}(a_0+b_0Y)\right]\frac{dY}{dQ_0}=\frac{\lambda[A-(\alpha+d)Y]}{d}\frac{dC_r}{dQ_0}$$

Since $\frac{dY}{dQ_0}|_E > 0$ *therefore* $\frac{dC_r}{dQ_0}|_E > 0$*. Thus, we have* $\frac{dC_r}{dQ_0}|_E > 0$ *, which shows that the density of carrier population increases as the discharge rate of* CO_2*from natural factors increases at the equilibrium point E.*

Remark 3.2. *It is also noted that* $\frac{dY}{d\theta}|_E > 0$

By equation (3.7)*, we have*

$$\frac{\beta Y[A-(\alpha+d)Y]}{d}+\frac{\lambda[A-(\alpha+d)Y]}{d}(a_0+d_0\theta Y)-(\nu+\alpha+d)Y=0, \tag{3.15}$$

where $d_0=\frac{s_1\alpha_1\alpha}{s_0\theta_0\alpha_0}$.

On differentiating (3.15) *with respect to* θ*, we get*

$$\frac{dY}{d\theta}\Big[\frac{\beta Y(\alpha+d)}{d}-\frac{\beta\{A-(\alpha+d)Y\}}{d}-\frac{\lambda d_0\theta\{A-(\alpha+d)Y\}}{d}+\frac{\lambda(\alpha+d)(a_0+d_0\theta Y)}{d}$$
$$+(\nu+\alpha+d)\Big]=\frac{\lambda d_0Y[A-(\alpha+d)Y]}{d},$$

$$\frac{dY}{d\theta}=\frac{\lambda d_0Y[A-(\alpha+d)Y]}{d[\frac{\beta Y(\alpha+d)}{d}-\frac{\beta\{A-(\alpha+d)Y\}}{d}-\frac{\lambda d_0\theta\{A-(\alpha+d)Y\}}{d}+\frac{\lambda(\alpha+d)(a_0+d_0\theta Y)}{d}+(\nu+\alpha+d)]}. \tag{3.16}$$

By equation (3.15)*, we have*

$$\frac{\beta Y(\alpha+d)}{d}+\frac{\lambda(\alpha+d)}{d}(a_0+d_0\theta Y)+(\nu+\alpha+d)=\frac{\beta A}{d}+\frac{\lambda A}{dY}(a_0+d_0\theta Y).$$

On putting in (3.16)*, we get*

$$\frac{dY}{d\theta}=\frac{\lambda d_0Y[A-(\alpha+d)Y]}{[\beta(\alpha+d)Y+\lambda d_0\theta(\alpha+d)Y+\frac{\lambda Aa_0}{Y}]}.$$

So we have, $\frac{dY}{d\theta}|_E > 0$ *which shows that the density of infective human population increases as the growth rate coefficient of temperature of the region due to rise in amount of* CO_2 *in the environment increases at the equilibrium point E.*

4 Stability Analysis

In this section, we study the stability behavior of the equilibrium point E. The results are stated in the following theorems.

***Theorem* 4.1.** *The equilibrium point $E(Y^*, N^*, C_r^*, C^*, T^*)$ is locally asymptotically stable provided the following conditions are satisfied*

$$a_1a_2a_3 - a_3^2 - a_1^2a_4 > 0, \tag{4.1}$$

$$(a_1a_4 - a_5)(a_1a_2a_3 - a_3^2 - a_1^2a_4) - a_5(a_1a_2 - a_3)^2 - a_1a_5^2 > 0, \tag{4.2}$$

where

$$\begin{aligned}
a_1 &= (\beta Y^* + \frac{\lambda C_r^* N^*}{Y^*}) + (s_0 + \alpha_0 + \theta_0) + d,\\
a_2 &= (\beta Y^* + \frac{\lambda C_r^* N^*}{Y^*})(d + s_0 + \alpha_0 + \theta_0) + \alpha(\beta Y^* + \lambda C_r^*) + d(s_0 + \alpha_0 + \theta_0),\\
&\quad + (s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0),\\
a_3 &= (\beta Y^* + \frac{\lambda C_r^* N^*}{Y^*})\{d(s_0 + \alpha_0 + \theta_0) + (s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0)\},\\
&\quad + \alpha(\beta Y^* + \lambda C_r^*)(s_0 + \alpha_0 + \theta_0) + d(s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0) + s_0\theta_0\alpha_0,\\
a_4 &= (\beta Y^* + \frac{\lambda C_r^* N^*}{Y^*})\{d(s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0) + s_0\theta_0\alpha_0\},\\
&\quad + \alpha(\beta Y^* + \lambda C_r^*)(s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0) + ds_0\theta_0\alpha_0,\\
a_5 &= \left[(\beta Y^* + \lambda C_r^*)(\alpha + d) + \frac{\lambda d(N^* - Y^*)}{Y^*}\right] s_0\theta_0\alpha_0.
\end{aligned}$$

Here it is noted that $a_i > 0,\ \forall i = 1, 2, 3, 4, 5$.

Proof. See the Appendix A.

***Theorem* 4.2.** *The equilibrium point $E(Y^*, N^*, C_r^*, C^*, T^*)$ is globally asymptotically stable in Ω provided the following conditions are satisfied*

$$\alpha\lambda^2 C_{rm}^2 < \beta^2 d Y^{*2}, \tag{4.3}$$

$$4\alpha d\lambda^2\alpha_1^2\theta^2 s_1^2(N^* - Y^*)^2 < \beta^2 Y^{*2} s_0^2\theta_0^2\alpha_0^2, \tag{4.4}$$

where C_{rm} is the maximum value of C_r, which is given by $C_{rm} = \dfrac{s\theta_0\alpha_0 + s_1\theta\alpha_1 A}{s_0\theta_0\alpha_0}$.

Proof. See the Appendix B.

5 Numerical Simulation

Here we discuss the existence and stability of non-trivial equilibrium point $E(Y^*, N^*, C_r^*, C^*, T^*)$ by taking the following set of parameters and using *MAPLE*.

$$\begin{aligned}
&A = 20, d = 0.0004, \alpha = 0.0005, \alpha_0 = 0.016, \alpha_1 = 0.6 \times 10^{-3},\\
&\beta = 6 \times 10^{-7}, \nu = 0.012, \lambda = 2 \times 10^{-8}, s_0 = 0.3, T_0 = 14,\\
&Q_0 = 6, \theta = 0.1, \theta_0 = 0.19, C_0 = 375, s = 2.4 \times 10^4,\\
&s_1 = 2 \times 10^4
\end{aligned}$$

The Jacobian matrix $J(E)$for the above values of parameters at $E(Y^*, N^*, C_r^*, C^*, T^*)$ is

$$\begin{bmatrix} -0.01259970927 & 0.01037645075 & 0.0003558913828 & 0 & 0 \\ -0.0005 & -0.0004 & 0 & 0 & 0 \\ 0 & 0 & -0.3 & 0 & 20000 \\ 0 & -2.40 \quad \times 10^{-7} & 0 & -0.016 & 0 \\ 0 & 0 & 0 & 0.1 & -0.19 \end{bmatrix}$$

The characteristic roots of the Jacobian matrix corresponding to the equilibrium point $E(Y^*, N^*, C_r^*, C^*, T^*)$ are:

$-0.00083244611, \quad -0.01219714633, \quad -0.01596815206,$
$-0.1899940766, \quad -0.3000078875$

Since all characteristic roots are negative real numbers, therefore $E(Y^*, N^*, C_r^*, C^*, T^*)$ is locally stable.

For above values of parameters, the non-trivial equilibrium point $E(Y^*, N^*, C_r^*, C^*, T^*)$ corresponding to (2.2) - (2.6) is obtained as follows:

$Y^* = 14313.52483, \quad N^* = 32108.09397, \quad C_r^* = 89416.79265,$
$C^* = 375.2683786, \quad T^* = 14.14125189$

Eliminating C_r, C, T from (3.1) - (3.5), we get

$$\beta(N - Y)Y + \lambda(N - Y)\left(\frac{s\theta_0\alpha_0 + s_1\theta\alpha_1\alpha Y}{s_0\theta_0\alpha_0}\right) - (\nu + \alpha + d)Y = 0, \tag{5.1}$$

$$A - dN - \alpha Y = 0. \tag{5.2}$$

For above values of parameters the equations (5.1) and (5.2) are plotted in Y-N plane (***Fig.*5.1**), and the intersection point is (Y^*, N^*).

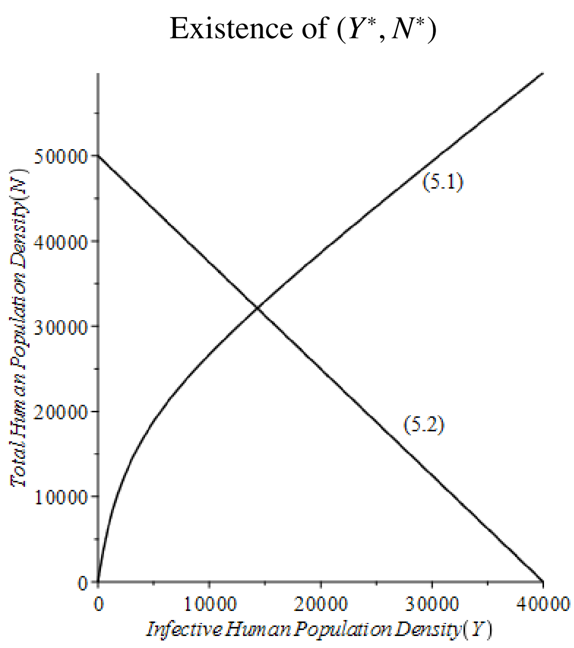

***Figure* 5.1**

With the above mentioned set of values of parameters, we plot the graphs from ***Fig.*5.1** - ***Fig.*5.13**. ***Fig.*5.2**, shows the nonlinear stability behavior between N and Y with different initial conditions tending to equilibrium point (Y^*, N^*) as time increases.

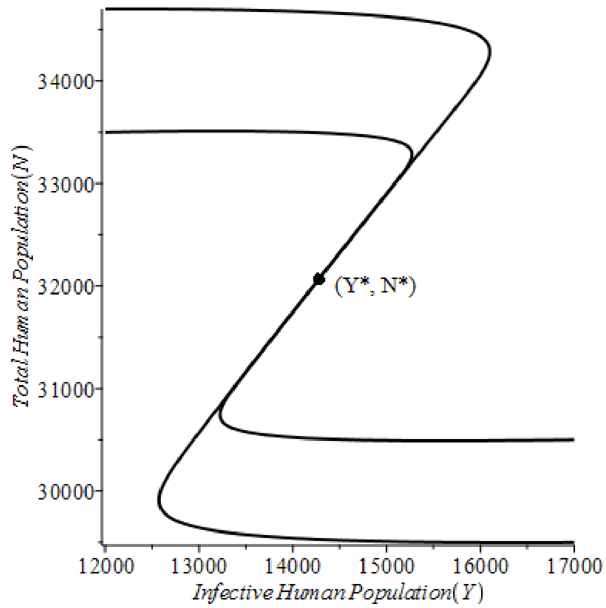

***Figure* 5.2:** Nonlinear stability of (Y^*, N^*) in the Y -N plane.

For solving the system of non-linear ODE, we use Runge-Kutta method in *MAPLE*.
The initial conditions for various quantities are given below:
$C(0) = 375, \quad C_r(0) = 80000, \quad T(0) = 14, \quad N(0) = 31000, \quad Y(0) = 13000$

***Fig.* 5.3** - ***Fig.* 5.13** show the effect of various parameters on infective human population density, carrier population density and amount of carbon dioxide. Every figure contains three curves with the same initial conditions as given above but with different values of corresponding parameter.

From ***Fig.* 5.3**, it is seen that the density of carriers increases if growth rate of carriers due to global warming temperature (s_1) increases. ***Fig.* 5.4** shows that the amount of CO_2 in the environment increases if the discharge rate coefficient of CO_2 from man-made sources(α_1) increases. From ***Fig.* 5.5**, it is seen that the density of infective human population increases as the growth rate coefficient of temperature of the region due to rise in amount of $CO_2(\theta)$ increases. ***Fig.* 5.6** shows that the density of infective human population increases if the discharge rate coefficient of CO_2 from man-made sources (α_1) increases. ***Fig.* 5.7** shows that the density of infective human population decreases if reduction rate coefficient of carriers(s_0) increases. ***Fig.* 5.8** shows that the density of infective human population increases if growth rate of carriers due to global warming temperature (s_1) increases. ***Fig.* 5.9** shows that the density of infective human population increases if constant growth rate coefficient of carrier population density (s) increases. ***Fig.* 5.10** shows that the density of infective human population increases if coefficient of transmission by carrier population density(λ) increases. ***Fig.* 5.11** shows that the density of infective human population increases if constant immigration rate of human population from outside the region under study (A) increases. ***Fig.* 5.12** shows that the density of infective human population increases if the discharge rate of CO_2 from natural factors (Q_0) increases. ***Fig.* 5.13**, shows that the density of carrier population increases if the discharge rate of CO_2 from natural factors (Q_0) increases.

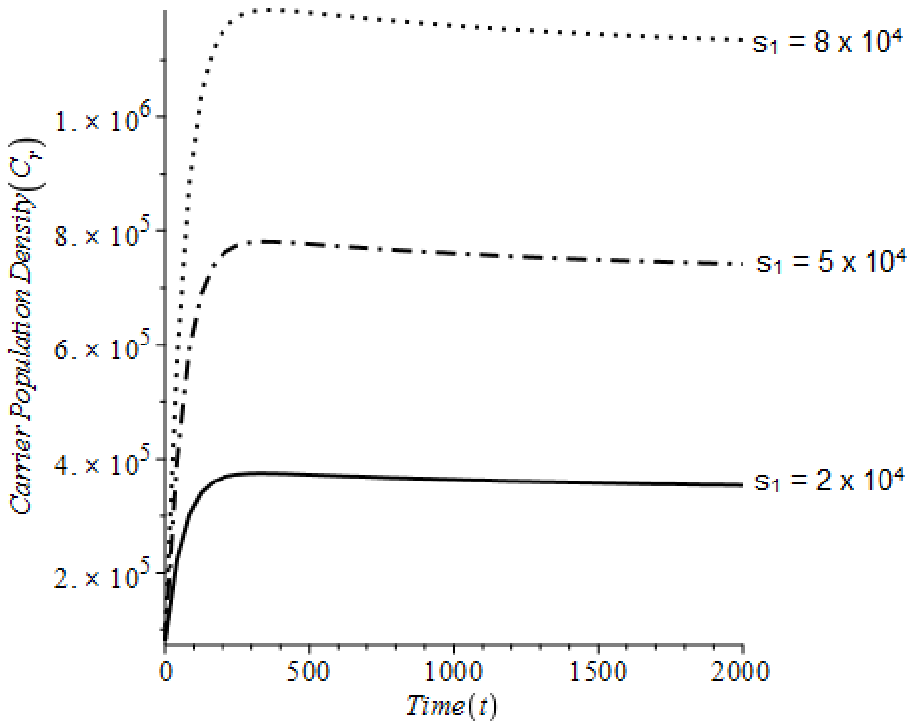

***Figure* 5.3:** Plot between carrier population density C_r and time t for various values of s_1.

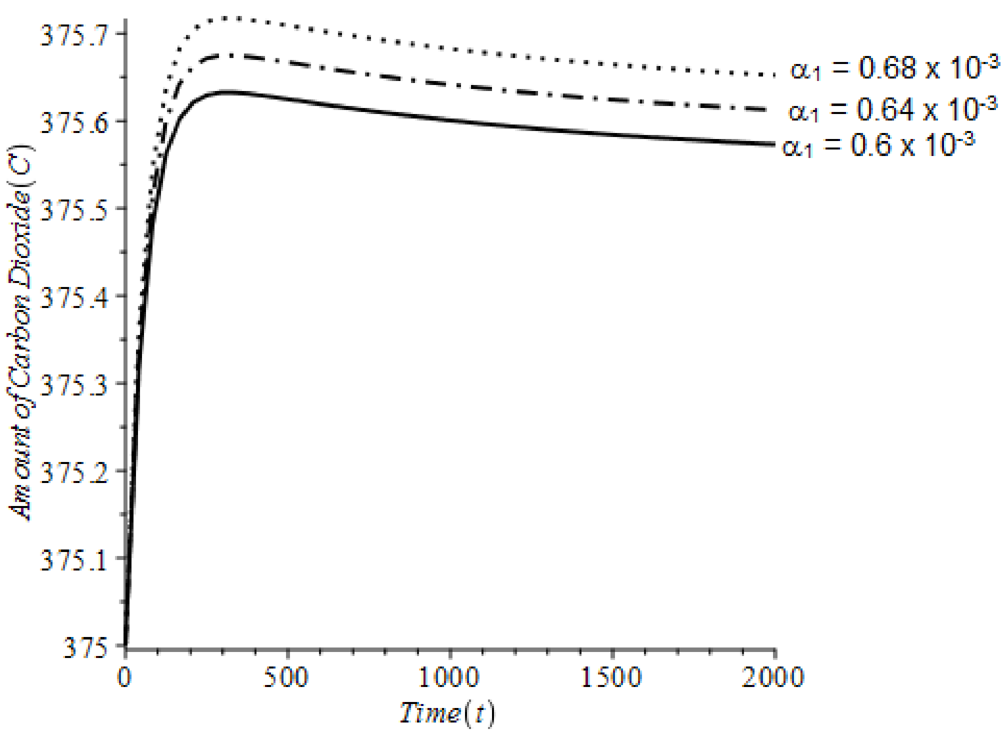

***Figure* 5.4:** Plot between amount of CO_2 and time t for various values of α_1.

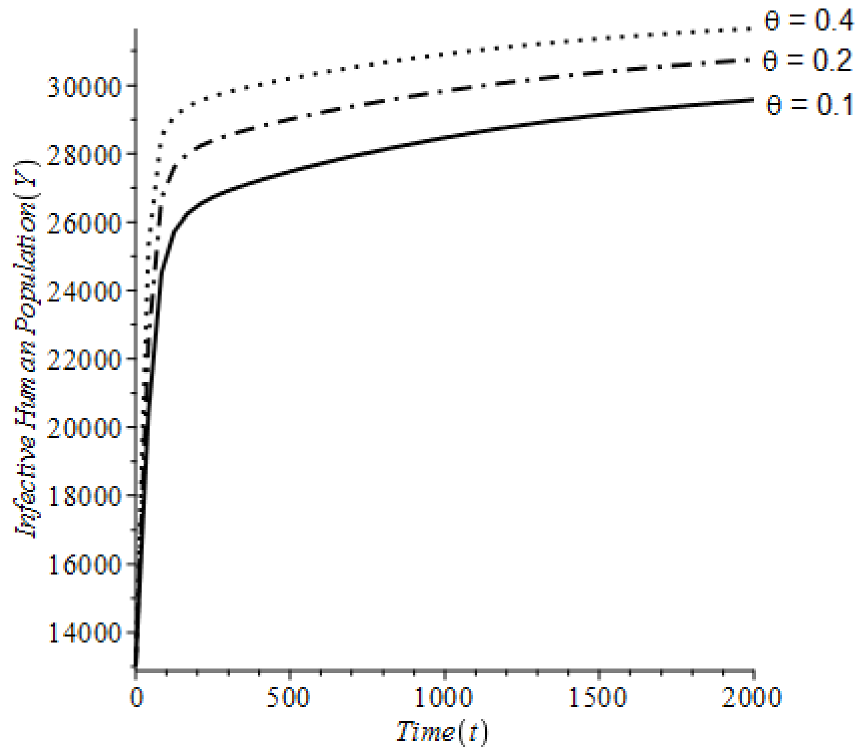

***Figure* 5.5:** Plot between infective human population Y and time t for various values of θ.

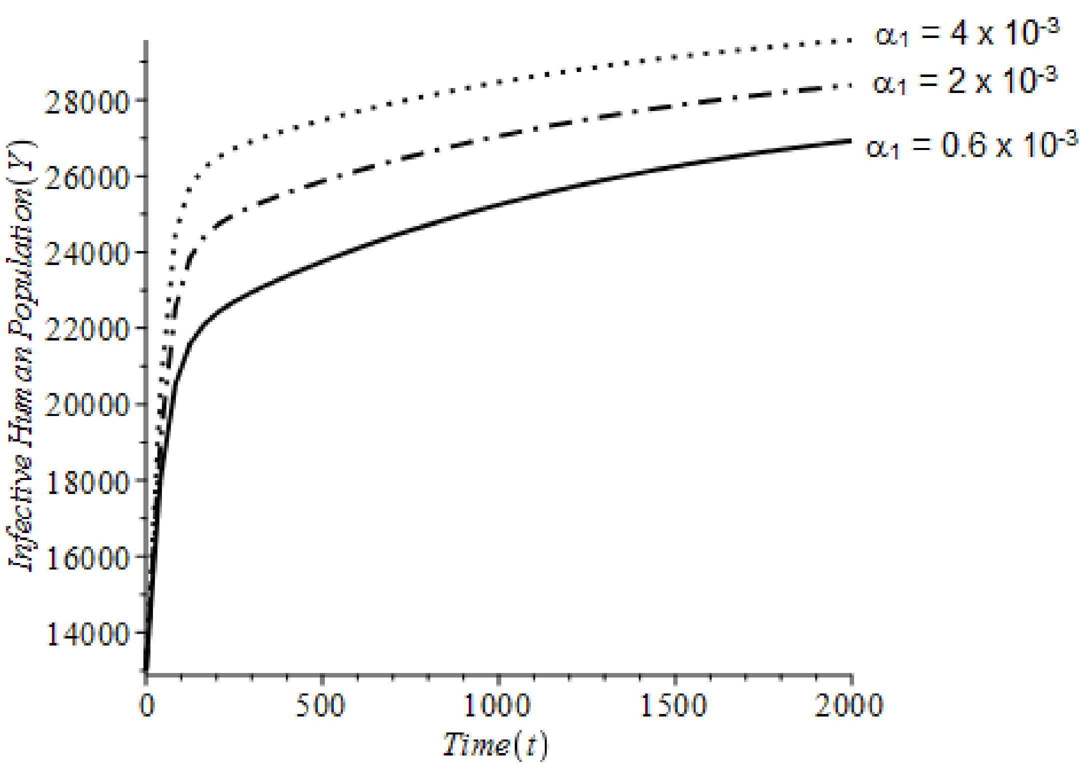

***Figure* 5.6:** Plot between infective human population Y and time t for various values of α_1.

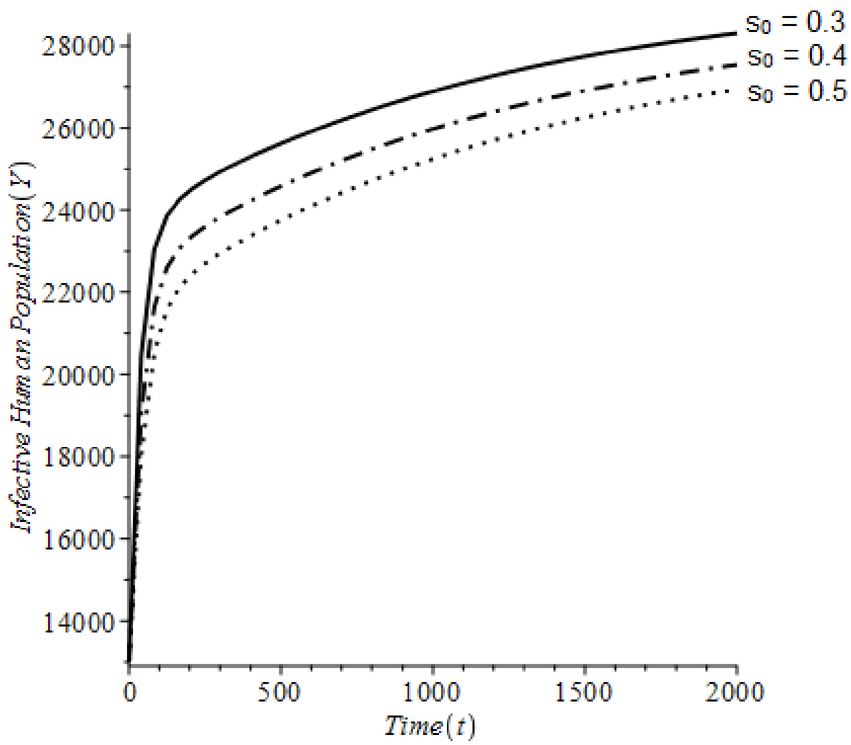

***Figure* 5.7:** Plot between infective human population Y and time t for various values of s_0.

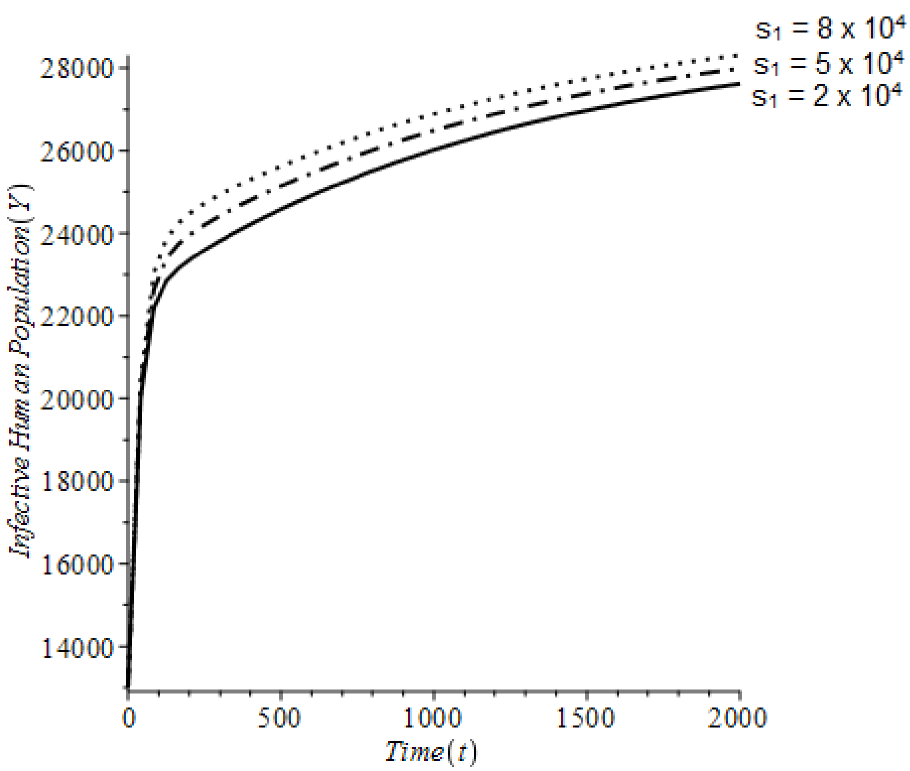

***Figure* 5.8:** Plot between infective human population Y and time t for various values of s_1.

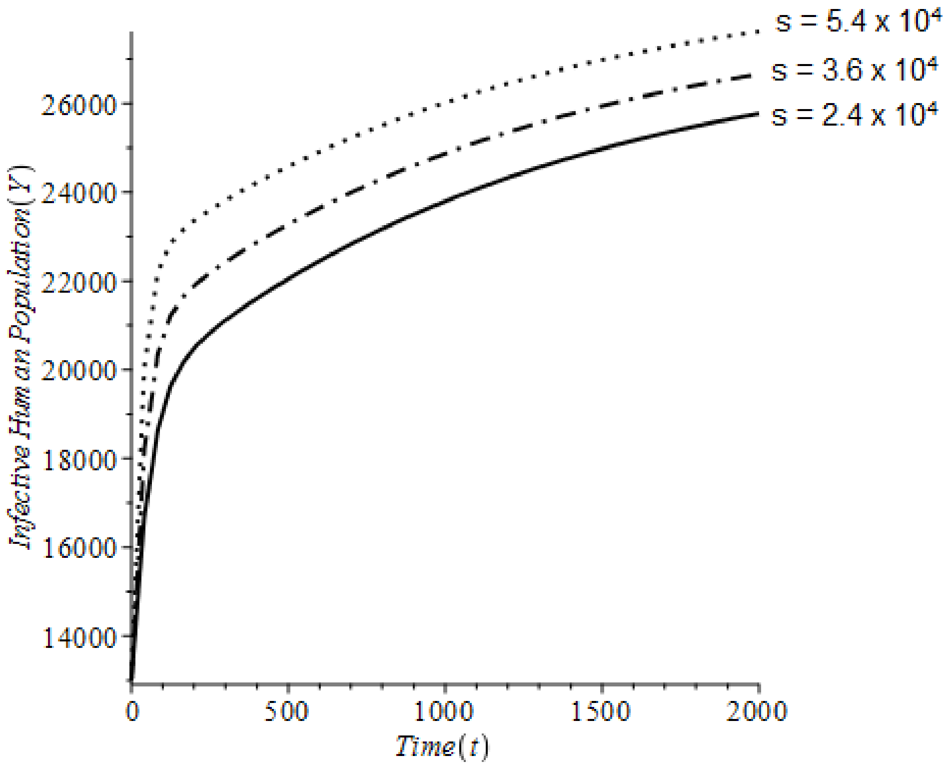

***Figure* 5.9:** Plot between infective human population Y and time t for various values of s.

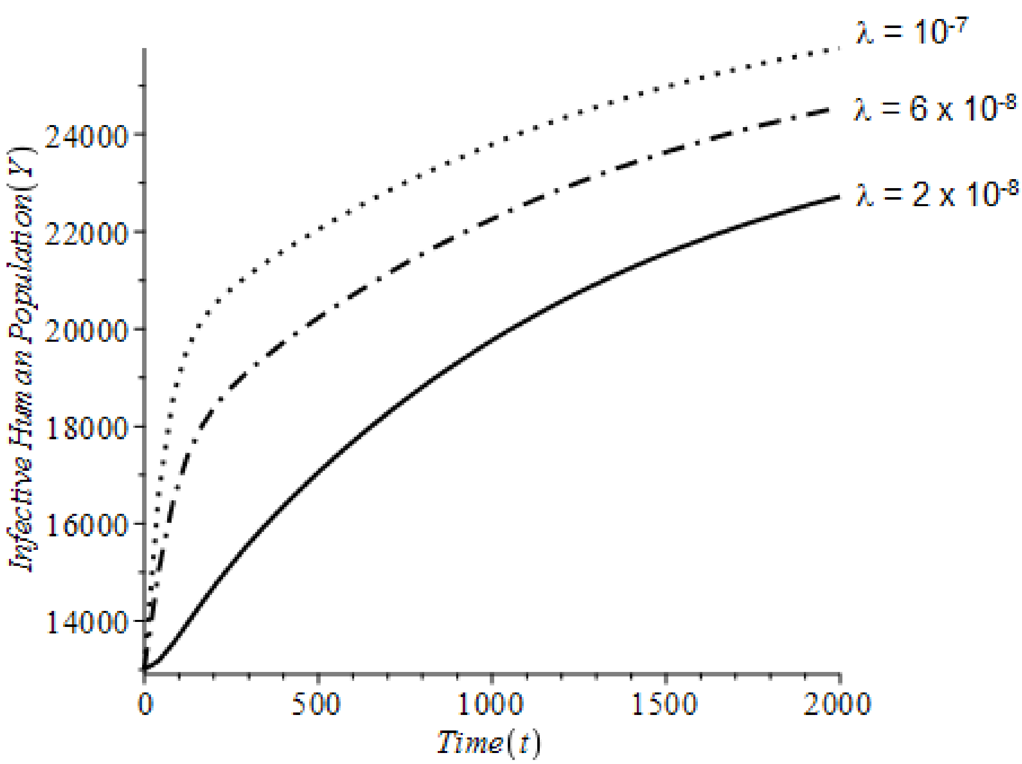

***Figure* 5.10:** Plot between infective human population Y and time t for various values of λ.

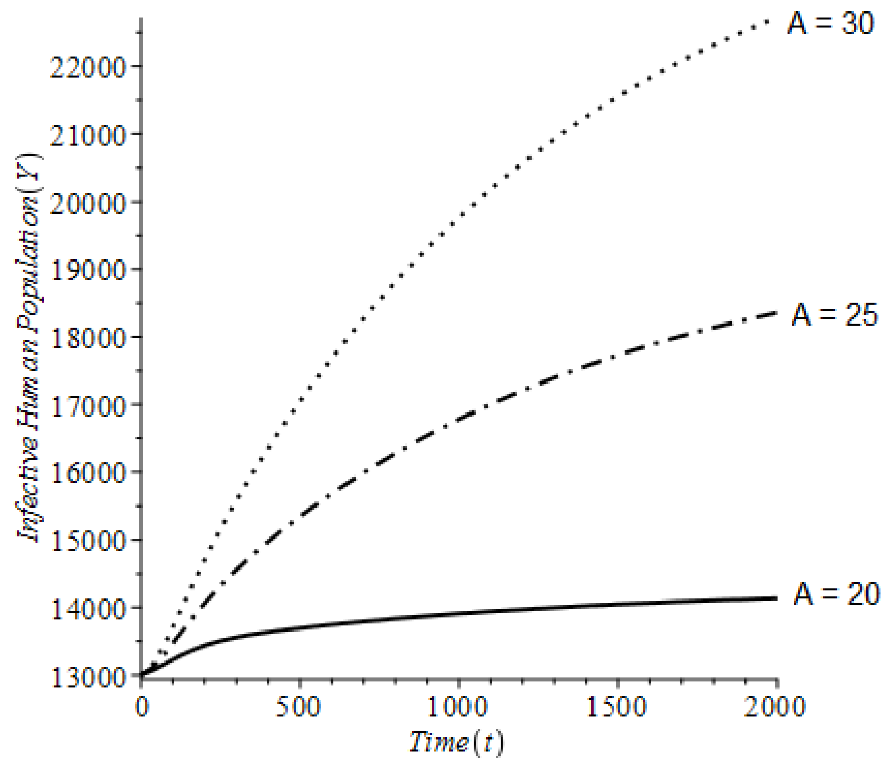

Figure **5.11:** Plot between infective human population Y and time t for various values of A.

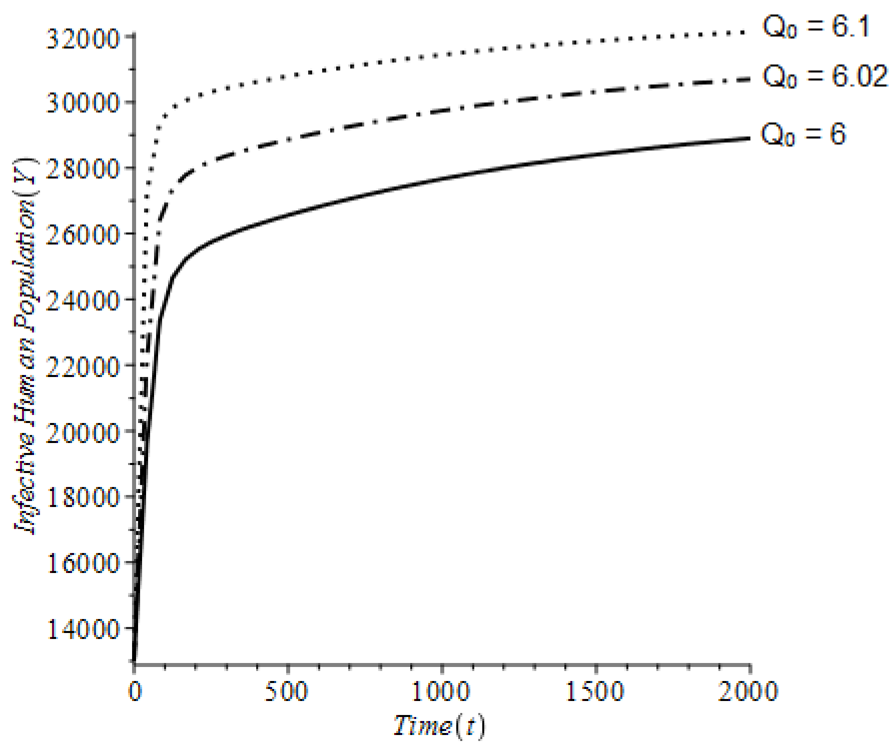

Figure **5.12:** Plot between infective human population Y and time t for various values of Q_0.

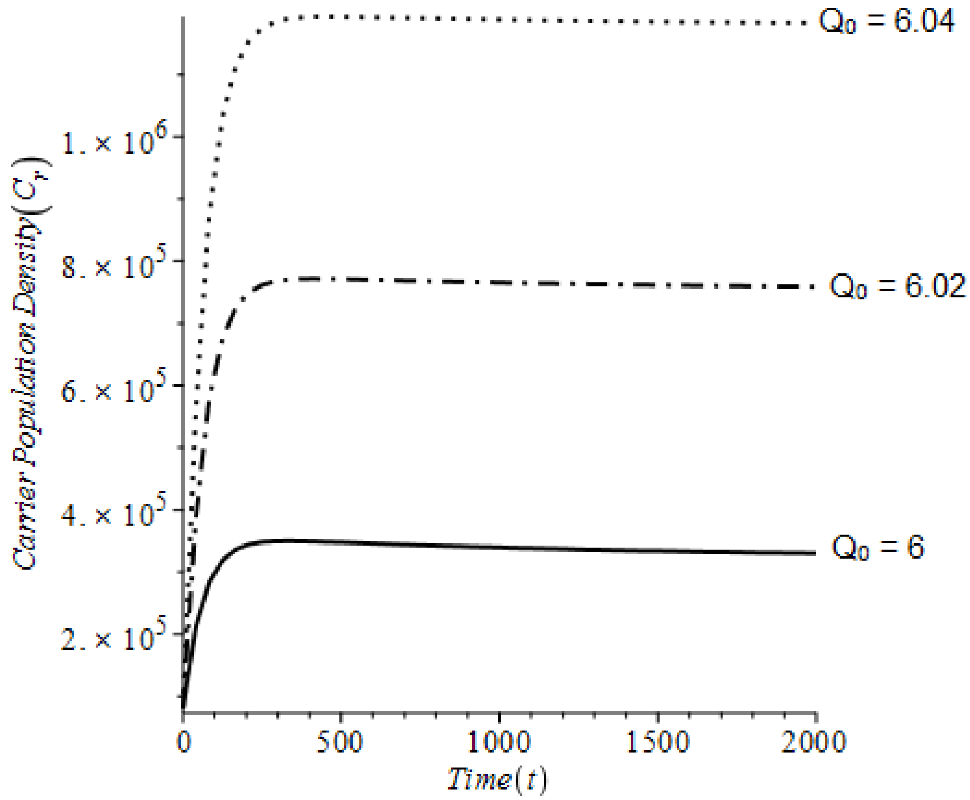

***Figure* 5.13:** Plot between carrier population density C_r and time t for various values of Q_0.

6 Conclusions

The larger amount of carbon dioxide (CO_2) in the environment is accountable for global warming. In this paper, we have studied the effect of global warming on the growth of carrier population with a constant growth rate and its role on the spread of infectious diseases. The model has five dependent variables namely, the density of susceptibles, the density of infectives, the density of carrier population, the amount of CO_2 in the environment causing global warming and the global warming temperature of the environment due to discharge of CO_2 . We have assumed that the density of carriers increases due to global warming temperature. The global warming temperature has been assumed to be proportional to the amount of CO_2 in the environment.

The proposed mathematical model has been studied with the help of stability theory of differential equations and numerical simulation. The local stability and the global stability conditions for non-trivial equilibrium point have been derived. For a set of parameters, numerical simulation proves the analytical results.

It has been found out that as the discharge rate of carbon dioxide from natural factors and man-made sources increases, the number of infectives increases. Also, the infectives and carrier population increase as the global warming temperature increases and thus the prevalence of carrier dependent infectious diseases increases in the environment.

Appendix A. Proof of the *Theorem* 4.1

Proof. The Jacobian matrix for the system (2.2) - (2.6) is

$$J(E) = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 \\ -\alpha & -d & 0 & 0 & 0 \\ 0 & 0 & -s_0 & 0 & s_1 \\ 0 & -\alpha_1 d & 0 & -\alpha_o & 0 \\ 0 & 0 & 0 & \theta & -\theta_0 \end{bmatrix},$$

where

$$a_{11} = \beta(N^* - Y^*) - \beta Y^* - \lambda C_r^* - (\nu + \alpha + d), a_{12} = \beta Y^* + \lambda C_r^*, a_{13} = \lambda(N^* - Y^*).$$

The characteristic equation of above Jacobian matrix is as follows:

(A.1)

$$\begin{aligned} &x^5 + \{-a_{11} + (s_0 + \alpha_0 + \theta_0) + d\}x^4 \\ &\quad + \{-a_{11}(d + s_0 + \alpha_0 + \theta_0) + a_{12}\alpha + d(s_0 + \alpha_0 + \theta_0) + (s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0)\}x^3 \\ &\quad + [-a_{11}\{d(s_0 + \alpha_0 + \theta_0) + (s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0)\} + a_{12}\alpha(s_0 + \alpha_0 + \theta_0) + d(s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0) + s_0\theta_0\alpha_0]x^2 \\ &\quad + [-a_{11}\{d(s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0) + s_0\theta_0\alpha_0\} + a_{12}\alpha(s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0) + ds_0\theta_0\alpha_0]x \\ &\quad + (-a_{11}ds_0\theta_0\alpha_0 + a_{12}\alpha s_0\theta_0\alpha_0 - a_{13}\alpha ds_1\theta\alpha_1) = 0. \end{aligned}$$

By (2.2), we have

(A.2)
$$\begin{aligned} &\beta(N^* - Y^*)Y^* + \lambda(N^* - Y^*)C_r^* - (\nu + \alpha + d)Y^* = 0 \\ &\Rightarrow \{\beta(N^* - Y^*) - (\nu + \alpha + d)\} = -\frac{\lambda(N^* - Y^*)C_r^*}{Y^*}. \end{aligned}$$

Now writing the value of a_{11}, with the help of (A.2), we get

(A.3)
$$a_{11} = -\left(\frac{\lambda N^* C_r^*}{Y^*} + \beta Y^*\right) < 0.$$

Also we have

(A.4)
$$C_r^* = \frac{s\theta_0\alpha_0 + s_1\theta\alpha_1\alpha Y^*}{s_0\theta_0\alpha_0} \Rightarrow s_1\theta\alpha_1\alpha = \frac{s_0\theta_0\alpha_0(C_r^* - 1)}{Y^*}.$$

Use (A.4) and the values of a_{11}, a_{12}, a_{13} in constant term of (A.1), we get

$$-a_{11}ds_0\theta_0\alpha_0 + a_{12}\alpha s_0\theta_0\alpha_0 - a_{13}\alpha ds_1\theta\alpha_1 = s_0\theta_0\alpha_0[(\beta Y^* + \lambda C_r^*)(\alpha + d) + \frac{d\lambda(N^* - Y^*)}{Y^*}].$$

Hence the constant term of (A.1) is positive.

Since $a_{11} = -(\frac{\lambda N^* C_r^*}{Y^*} + \beta Y^*) < 0$ and constant term of (A.1) is positive, therefore all coefficients of (A.1) are positive, so the characteristic equation (A.1) can be written as

(A.5)
$$x^5 + a_1x^4 + a_2x^3 + a_3x^2 + a_4x + a_5 = 0,$$

where

$$\begin{aligned} a_1 &= (\beta Y^* + \frac{\lambda C_r^* N^*}{Y^*}) + (s_0 + \alpha_0 + \theta_0) + d, \\ a_2 &= (\beta Y^* + \frac{\lambda C_r^* N^*}{Y^*})(d + s_0 + \alpha_0 + \theta_0) + \alpha(\beta Y^* + \lambda C_r^*) + d(s_0 + \alpha_0 + \theta_0), \\ &\quad + (s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0), \\ a_3 &= (\beta Y^* + \frac{\lambda C_r^* N^*}{Y^*})\{d(s_0 + \alpha_0 + \theta_0) + (s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0)\}, \end{aligned}$$

$$+\,\alpha(\beta Y^* + \lambda C_r^*)(s_0 + \alpha_0 + \theta_0) + d(s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0) + s_0\theta_0\alpha_0,$$

$$a_4 = (\beta Y^* + \frac{\lambda C_r^* N^*}{Y^*})\{d(s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0) + s_0\theta_0\alpha_0\},$$

$$+\,\alpha(\beta Y^* + \lambda C_r^*)(s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0) + ds_0\theta_0\alpha_0,$$

$$a_5 = [(\beta Y^* + \lambda C_r^*)(\alpha + d) + \frac{\lambda d(N^* - Y^*)}{Y^*}]s_0\theta_0\alpha_0.$$

Hence by Routh-Hurwitz criterion the equilibrium point $E(Y^*, N^*, C_r^*, C^*, T^*)$is locally asymptotically stable provided the following conditions are satisfied

$$a_1a_2a_3 - a_3^2 - a_1^2a_4 > 0, \tag{A.6}$$

$$(a_1a_4 - a_5)(a_1a_2a_3 - a_3^2 - a_1^2a_4) - a_5(a_1a_2 - a_3)^2 - a_1a_5^2 > 0. \tag{A.7}$$

Appendix B. Proof of the *Theorem* 4.2

Proof. For the proof of this theorem we use Lyapunov direct method. For this first we let the following positive definite Lyapunov function

$$W = K_0(Y - Y^* - Y^* \ln\frac{Y}{Y^*}) + \frac{K_1}{2}(N - N^*)^2 + \frac{K_2}{2}(C_r - C_r^*)^2 + \frac{K_3}{2}(C - C^*)^2 + \frac{K_4}{2}(T - T^*)^2, \tag{B.1}$$

where K_o, K_1, K_2, K_3 and K_4 are positive constants to be chosen appropriately.

Differentiate (B.1) with respect to t, we get

$$\dot W = K_o\frac{\dot Y}{Y}(Y - Y^*) + K_1(N - N^*)\dot N + K_2(C_r - C_r^*)\dot C_r + K_3(C - C^*)\dot C + K_4(T - T^*)\dot T. \tag{B.2}$$

Using model system (2.2) - (2.6), we get

$$\begin{aligned}\dot W = &- K_o\frac{C_r\lambda N}{YY^*}(Y - Y^*)^2 - K_o\beta(Y - Y^*)^2 - K_1d(N - N^*)^2 - K_2s_0(C_r - C_r^*)^2\\ &- K_3\alpha_0(C - C^*)^2 - K_4\theta_0(T - T^*)^2 + (K_o\beta - K_1\alpha)(N - N^*)(Y - Y^*)\\ &+ K_o\frac{C_r\lambda}{Y^*}(N - N^*)(Y - Y^*) + K_0\lambda(\frac{N^* - Y^*}{Y^*})(C_r - C_r^*)(Y - Y^*)\\ &+ K_2s_1(C_r - C_r^*)(T - T^*) - K_3\alpha_1d(N - N^*)(C - C^*) + K_4\theta(T - T^*)(C - C^*).\end{aligned}$$

Here we take the constants K_0 and K_1 such that $K_0\,\beta - K_1\alpha = 0$.

Further we take

$$K_1 = 1 \text{ then } K_0 = \frac{\alpha}{\beta}. \tag{B.3}$$

We have

$$\begin{aligned}\dot W = &- K_o\frac{C_r\lambda N}{YY^*}(Y - Y^*)^2 - \Big[\frac{K_o\beta}{2}(Y - Y^*)^2 - \frac{K_0\lambda C_r}{Y^*}(N - N^*)(Y - Y^*) + \frac{K_1d}{2}(N - N^*)^2\Big]\\ &- \Big[\frac{K_1d}{2}(N - N^*)^2 + K_3\alpha_1d(N - N^*)(C - C^*) + \frac{K_3\alpha_0}{2}(C - C^*)^2\Big]\\ &- \Big[\frac{K_o\beta}{2}(Y - Y^*)^2 - K_0\lambda(\frac{N^* - Y^*}{Y^*})(C_r - C_r^*)(Y - Y^*) + \frac{K_2s_0}{2}(C_r - C_r^*)^2\Big]\\ &- \Big[\frac{K_2s_0}{2}(C_r - C_r^*)^2 - K_2s_1(C_r - C_r^*)(T - T^*) + \frac{K_4\theta_0}{2}(T - T^*)^2\Big]\\ &- \Big[\frac{K_3\alpha_0}{2}(C - C^*)^2 - K_4\theta(T - T^*)(C - C^*) + \frac{K_4\theta_0}{2}(T - T^*)^2\Big].\end{aligned}$$

The derivative $\dot{W}$ is negative definite if the following conditions are satisfied:

(B.4) $$\alpha\lambda^2 C_{rm}^2 < d\beta^2 Y^{*2},$$

where C_{rm} is the maximum value of C_r

(B.5) $$K_3 < \frac{\alpha_o}{d\alpha_1^2},$$

(B.6) $$K_2 > \frac{\alpha\lambda^2(N^* - Y^*)^2}{s_0\beta^2 Y^{*2}},$$

(B.7) $$K_2 < \frac{s_0\theta_0}{s_1^2}K_4,$$

(B.8) $$K_4\theta^2 < K_3\alpha_o\theta_0.$$

Combining the inequalities (B.5), (B.6), (B.7) and (B.8), we get

(B.9) $$4\alpha d\lambda^2\alpha_1^2\theta^2 s_1^2(N^* - Y^*)^2 < \beta^2 Y^{*2} s_0^2\theta_0^2\alpha_0^2.$$

The inequalities (B.4) and (B.9) are the required conditions.

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