MODELING AND ANALYSIS THE EFFECT OF GLOBAL WARMING ON THE SPREAD OF CARRIER DEPENDENT INFECTIOUS DISEASES

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(Received: May 07, 2019; Revised: June 09, 2020)

DOI: https://doi.org/10.58250/jnanabha.2020.50119

Abstract

In this paper, a nonlinear SIS model is considered to study the effect of global warming on the spread of carrier dependent infectious diseases. In the model, we deal with five dependent variables, namely, the density of susceptibles, the density of infectives, the density of carrier population, the amount of carbon dioxide in the environment causing global warming and the global warming temperature. In the model, it is assumed that the density of carriers increases with constant growth rate as well as proportional to the global warming temperature. The amount of $CO_2$ in the environment increases due to human population activities as well as natural factors. It is also assumed that the global warming temperature is proportional to the amount of $CO_2$ in the environment. The model is studied and investigated with the help of stability theory of differential equations and numerical simulation. The investigation shows that, if global warming temperature increases, then the density of carrier population and the spread of carrier dependent infectious diseases increase.

2010 Mathematics Subject Classifications: 37C75, 92B05
Keywords and phrases: Global Warming, SIS Model, Stability, Carriers.

1 Introduction

Global warming is the most crucial subject matter to be taken under scrutiny by researchers in this century. Certain greenhouse gases in the Earth’s atmosphere, like carbon dioxide ($CO_2$) and methane ($CH_4$) trap the sun’s heat and do not let it escape. The increase of greenhouse gases in the atmosphere over long period leads to an increase in Earth’s surface temperature causing global warming. Over the past 130 years, the world has warmed up to 0.85°C (approx.). The last three decades have been progressively warmer than any preceding decades since 1850, IPCC [10]. Further, it is important to note that detrimental activities of growing human population such as deforestation, throwing waste in rivers and oceans and releasing smoke from factories and car fumes into air etc., has discharged enough amount of carbon dioxide ($CO_2$) in the environment. Because of the global warming, carriers such as ticks, flies, mites, mosquitoes, cockroaches survive in the warmer environment and can cause the spread of carrier dependent infectious diseases, Zhou et al. [18]. The worldwide distribution of carrier dependent infectious diseases shows the impact of climate change, IPCC [8, 9]. The prediction of World Health Organization is that, between 2030 and 2050 there will be an additional 0.25 million deaths per year from many infectious diseases as measles, smallpox, mumps, malaria, diarrhea etc., due to global warming, WHO [15-17].

It is noteworthy that the effect of global warming on the carrier population with a constant growth rate and it's role on the spread of infectious diseases has not been done till now. Many
researchers have studied the spread of infectious diseases by using mathematical models in the past, as Anderson and May [1], Bailey [2], Ghosh et al. [3, 4], Greenhalgh [5], Hethcote [6, 7], May and Anderson [11], Singh [12] and Singh et al. [13, 14]. In this paper, therefore the effect of global warming on carrier population with constant growth rate and its role on the spread of infectious diseases is modeled and studied.

The following assumptions are made in the modeling process:

(i) The rate of density of carrier population increases with a constant rate.

(ii) The rate of density of carrier population is proportional to the global warming temperature.

(iii) The rate of amount of $CO_2$ in the environment increases by human activities as well as natural factors.

(iv) The rate of global warming temperature is proportional to amount of $CO_2$ in the environment.

2 SIS Model

Let $N(t)$ be the total human population density which is divided into two categories, namely susceptible human population density $X(t)$ and infective human population density $Y(t)$. Let $C_r(t)$ be the carrier population density which grows with a constant growth rate coefficient "s". Let $C(t)$ be the amount of $CO_2$ in the environment and $T(t)$ be the global warming temperature of the environment due to discharge of carbon dioxide($CO_2$). The model is governed by the following non linear differential equations:

\[
\begin{align*}
\frac{dX}{dt} &= A - \beta XY - \lambda XC_r - dX + \nu Y, \\
\frac{dY}{dt} &= \beta XY + \lambda XC_r - (\nu + \alpha + d)Y, \\
\frac{dC_r}{dt} &= s + s_1(T - T_0) - s_0C_r, \\
\frac{dC}{dt} &= Q_o + \alpha_1(A - dN) - \alpha_oC, \\
\frac{dT}{dt} &= \theta(C - C_o) - \theta_0(T - T_o),
\end{align*}
\]

with $X + Y = N$, where $X(0) = X_0 > 0, Y(0) = Y_0 \geq 0, N(0) = N_0 > 0, C_r(0) = C_{r0} \geq 0, C(0) = C_0 > 0$ and $T(0) = T_0 > 0$ and take $C_0 = \frac{Q_o}{\alpha_o}$.

In the above model system (2.1), the used parameters are positive real numbers, described as follows:
Proof. To find the equilibrium point $(Y^*, N^*, C^*, C^*, T^*)$, we solve the following set of equations

\begin{align}
\beta(N - Y)Y + \lambda(N - Y)C_r - (\nu + \alpha + d)Y &= 0, \\
A - dN - \alpha Y &= 0, \\
s + s_1(T - T_0) - s_0C_r &= 0,
\end{align}

where $A = \frac{s\theta_0\alpha_0 + s_1\theta_1A}{s_0\theta_0\alpha_0}$, $C_m = C_0 + \frac{\alpha_1A}{\alpha_0}$ and $T_m = T_o + \frac{\theta_1A}{\theta_0\alpha_0}$.

It attracts all the solution initiating in the interior of the positive octant of the region.

3 Equilibrium Analysis

The model system (2.2) - (2.6) has only one non-negative equilibria:

(i) \( E(Y^*, N^*, C^*, C^*, T^*) \).

**Proof.** To find the equilibrium point \( E(Y^*, N^*, C^*, C^*, T^*) \) we solve the following set of equations

\begin{align}
\beta(N - Y)Y + \lambda(N - Y)C_r - (\nu + \alpha + d)Y &= 0, \\
A - dN - \alpha Y &= 0, \\
s + s_1(T - T_0) - s_0C_r &= 0,
\end{align}

\[ A \text{ : The constant immigration rate of human population from outside the region} \]
\[ \beta \text{ : Coefficient of transmission by infective human population density} \]
\[ \lambda \text{ : Coefficient of transmission by carrier population density} \]
\[ d \text{ : Natural death rate constant of human population} \]
\[ \alpha \text{ : Coefficient of death rate of infective human population due to disease related factors} \]
\[ \nu \text{ : Coefficient of recovery rate of infective human population density} \]
\[ Q_0 \text{ : Discharge rate of CO}_2 \text{ from natural factors} \]
\[ \alpha_1 \text{ : Discharge rate coefficient of CO}_2 \text{ from man- made sources} \]
\[ \alpha_0 \text{ : Natural reduction rate coefficient of CO}_2 \]
\[ \theta \text{ : Growth rate coefficient of temperature of the region due to rise in amount of CO}_2 \]
\[ \theta_0 \text{ : Natural reduction rate coefficient of temperature in the region} \]
\[ s_0 \text{ : Reduction rate coefficient of carriers} \]
\[ s \text{ : Constant growth rate coefficient of carrier population density} \]
\[ s_1 \text{ : Growth rate of carriers due to global warming temperature} \]
\[ T_0 \text{ : The equilibrium level of global warming temperature of the environment} \]
\[ C_0 \text{ : The equilibrium amount of CO}_2 \]

For analyzing the model (2.1), we take the following reduced system by using \( X = N - Y, \)

\begin{align}
\frac{dY}{dt} &= \beta(N - Y)Y + \lambda(N - Y)C_r - (\nu + \alpha + d)Y, \\
\frac{dN}{dt} &= A - dN - \alpha Y, \\
\frac{dC_r}{dt} &= s + s_1(T - T_0) - s_0C_r, \\
\frac{dC}{dt} &= Q_0 + \alpha_1(A - dN) - \alpha_0C, \\
\frac{dT}{dt} &= \theta(C - C_o) - \theta_0(T - T_o),
\end{align}

with initial conditions

\[ Y(0) = Y_0 \geq 0, N(0) = N_0 > 0, C_r(0) = C_{r0} \geq 0, C(0) = C_0 > 0 \text{ and } T(0) = T_0 > 0. \]

**Region of Attraction**

The region of attraction of the model system (2.2) - (2.6) is given by the set

\[ \Omega = \{(Y, N, C_r, C, T) \in R^5_+ : 0 \leq Y \leq N \leq \frac{A}{d}, 0 \leq C_r \leq C_{r_m}, 0 \leq C \leq C_m, 0 < T \leq T_m \}, \]

where, \( C_{r_m} = \frac{s\theta_0\alpha_0 + s_1\theta_1A}{s_0\theta_0\alpha_0} \), \( C_m = C_0 + \frac{\alpha_1A}{\alpha_0} \) and \( T_m = T_o + \frac{\theta_1A}{\theta_0\alpha_0} \).

It attracts all the solution initiating in the interior of the positive octant of the region.
(3.4) \[ Q_o + \alpha_1 (A - dN) - \alpha_o C = 0, \]
(3.5) \[ \theta (C - C_o) - \theta_0 (T - T_o) = 0. \]

Using above equations we get \( C_r = \frac{s \theta_0 \alpha_0 + s_1 \theta_1 \alpha_1 Y}{s_0 \theta_0 \alpha_0}. \)

Now, using the above value of \( C_r \) and equations (3.1), (3.2), we get \( Y^* \) as the root of the following equation

\[
(3.6) \quad F(Y) = \frac{\beta Y}{d} \left[ A - (\alpha + d) Y \right] + \frac{\lambda}{d} \left[ A - (\alpha + d) Y \right] \left( \frac{s \theta_0 \alpha_0 + s_1 \theta_1 \alpha_1 Y}{s_0 \theta_0 \alpha_0} \right) - (\nu + \alpha + d) Y = 0.
\]

So we have, \( F(0) = \frac{\lambda A s}{d s_0} > 0 \) and \( F \left( \frac{A}{\alpha + d} \right) = -(\nu + \alpha + d) \left( \frac{A}{\alpha + d} \right) < 0 \)

i.e. at least one root of the equation \( F(Y) = 0 \) lies in the range, \( 0 < Y < \frac{A}{\alpha + d} \).

Rewriting (3.6) as follows

\[
(3.7) \quad F(Y) = \frac{\beta Y}{d} \left[ A - (\alpha + d) Y \right] + \frac{\lambda}{d} \left[ A - (\alpha + d) Y \right] (a_0 + b_0 Y) - (\nu + \alpha + d) Y = 0,
\]

where \( a_0 = \frac{s_1 \theta_1 \alpha_1}{s_0 \theta_0 \alpha_0} \) and \( b_0 = \frac{s \theta_0 \alpha_0}{s_0 \theta_0 \alpha_0} \).

On differentiating (3.7) with respect to \( Y \), we get

\[
F' (Y) = \frac{\beta}{d} \left[ A - (\alpha + d) Y \right] - \beta Y \left( \frac{\alpha + d}{d} \right) + \frac{\lambda}{d} \left[ A - (\alpha + d) Y \right] b_0 - \lambda (a_0 + b_0 Y) \left( \frac{\alpha + d}{d} \right) - (\nu + \alpha + d).
\]

Then, for \( Y > 0 \)

\[
Y F' (Y) = \beta \left( A - (\alpha + d) Y \right) Y - \beta Y^2 \left( \frac{\alpha + d}{d} \right) + \frac{\lambda}{d} \left[ A - (\alpha + d) Y \right] b_0 Y - \lambda Y (a_0 + b_0 Y) \left( \frac{\alpha + d}{d} \right) - (\nu + \alpha + d) Y.
\]

\[
Y F' (Y) = - \beta Y^2 \left( \frac{\alpha + d}{d} \right) - \frac{\lambda (\alpha + d)}{d} b_0 Y^2 - a_0 \frac{\lambda A}{d} < 0.
\]

Hence the equation \( F(Y) = 0 \) has unique root in the interval \( 0 < Y < \frac{A}{\alpha + d} \).

**Remark 3.1.** Here we noted that \( \frac{dY}{dQ_0} |_{Y^*} > 0 \)

From the model system (2.2) - (2.6), we have

\[
(3.8) \quad \left[ \frac{\beta Y (\alpha + d)}{d} - \frac{\beta}{d} \left( A - (\alpha + d) Y \right) + \frac{\lambda (\alpha + d) C_r}{d} + (\nu + \alpha + d) \right] \times \frac{dY}{dQ_0} = \frac{\lambda}{d} \left[ A - (\alpha + d) Y \right] \frac{dC_r}{dQ_0},
\]

\[
(3.9) \quad \frac{s_1}{d} \frac{dT}{dQ_0} = \frac{s_0}{d} \frac{dC_r}{dQ_0},
\]

\[
(3.10) \quad \frac{dC}{dQ_0} = \frac{1}{\alpha_o} + \frac{\alpha_1 \alpha}{\alpha_o} \frac{dY}{dQ_0},
\]

\[
(3.11) \quad \frac{\theta}{dQ_0} = \frac{\theta_0}{dQ_0} \frac{dT}{dQ_0}.
\]

On writing (3.8) with the help of (3.9), (3.10) and (3.11), we get

\[
(3.12) \quad \frac{dY}{dQ_0} = \frac{\lambda s_1 \theta [A - (\alpha + d) Y]}{d s_0 \theta_0 \alpha_o \left[ \frac{\beta (\alpha + d)}{d} + \frac{\lambda (\alpha + d) C_r}{d} - \left( \frac{\beta [A - (\alpha + d) Y]}{d} + \frac{\lambda b_0 [A - (\alpha + d) Y] - (\nu + \alpha + d)}{d} \right) \right]}. \]
By equation (3.7), we have
\[ \frac{\beta[A-(\alpha+d)Y]}{d} + \frac{\lambda[A-(\alpha+d)Y]}{d}b_0 - (\nu + \alpha + d) = -\frac{\lambda[A-(\alpha+d)Y]}{d}a_0, \]
which on substituting in equation (3.12), gives
\[ \frac{dY}{dQ_0} = \frac{\lambda s_0 \theta_0 A_0}{d s_0 \theta_0 \alpha_0} \left[ \frac{\beta Y}{d} + \frac{\lambda(\alpha+d)C_r}{d} + \frac{\lambda[A-(\alpha+d)Y]}{d}a_0 \right]. \]

So we have, \( \frac{dY}{dQ_0} \big|_{E} > 0 \), which shows that the density of infective human population density increases as the discharge rate of CO_2 from natural factors increases at the equilibrium point E.

Since \( C_r = \frac{s \theta_0 A_0 + s_1 \theta_0 \alpha_0 Y}{s_0 \theta_0 \alpha_0} = \frac{s}{s_0} + \frac{s_1 \theta_0}{s_0 \theta_0 \alpha_0} Y = a_0 + b_0 Y \), using in (3.8), we get
\[ \left( \frac{\beta Y + \lambda(\alpha + d)}{d} (a_0 + b_0 Y) + (\nu + \alpha + d) \right) \frac{dY}{dQ_0} = \lambda[A-(\alpha+d)Y] \frac{dC_r}{dQ_0}. \]

By equation (3.7), we have
\[ \frac{\beta Y + \lambda(\alpha + d)}{d} (a_0 + b_0 Y) + (\nu + \alpha + d) = \lambda \frac{A-(\alpha+d)Y}{d} (a_0 + b_0 Y), \]
which on substituting in (3.14), we get
\[ \left( \frac{\beta Y + \lambda(\alpha + d)}{d} (a_0 + b_0 Y) \right) \frac{dY}{dQ_0} = \lambda[A-(\alpha+d)Y] \frac{dC_r}{dQ_0}. \]

Since \( \frac{dY}{dQ_0} \big|_{E} > 0 \) therefore \( \frac{dC_r}{dQ_0} \big|_{E} > 0 \). Thus, we have \( \frac{dC_r}{dQ_0} \big|_{E} > 0 \), which shows that the density of carrier population increases as the discharge rate of CO_2 from natural factors increases at the equilibrium point E.

**Remark 3.2.** It is also noted that \( \frac{dY}{d\theta} \big|_{E} > 0 \)

By equation (3.7), we have
\[ \frac{\beta Y [A-(\alpha+d)Y]}{d} + \frac{\lambda[A-(\alpha+d)Y]}{d} (a_0 + d_0 \theta Y) - (\nu + \alpha + d)Y = 0, \]
where \( d_0 = \frac{\frac{\lambda d_0 Y}{d}[A-(\alpha+d)Y]}{\frac{\lambda d_0 Y}{d}[A-(\alpha+d)Y]} \).

On differentiating (3.15) with respect to \( \theta \), we get
\[ \frac{dY}{d\theta} \left[ \frac{\beta Y (\alpha + d)}{d} + \frac{\lambda(\alpha + d)}{d} (a_0 + d_0 \theta Y) + (\nu + \alpha + d) \right] = \frac{\lambda d_0 Y [A-(\alpha+d)Y]}{d}. \]

By equation (3.15), we have
\[ \frac{\beta Y (\alpha + d)}{d} + \frac{\lambda(\alpha + d)}{d} (a_0 + d_0 \theta Y) + (\nu + \alpha + d) = \frac{\beta Y}{d} + \frac{\lambda A}{dY} (a_0 + d_0 \theta Y). \]

On putting in (3.16), we get
\[ \frac{dY}{d\theta} = \frac{\lambda d_0 Y [A-(\alpha+d)Y]}{[\beta(\alpha + d)Y + \lambda d_0 \theta (\alpha + d)Y + \frac{\lambda A}{Y}].} \]

So we have, \( \frac{dY}{d\theta} \big|_{E} > 0 \) which shows that the density of infective human population increases as the growth rate coefficient of temperature of the region due to rise in amount of CO_2 in the environment increases at the equilibrium point E.
4 Stability Analysis

In this section, we study the stability behavior of the equilibrium point $E$. The results are stated in the following theorems.

**Theorem 4.1.** The equilibrium point $E(Y^*, N^*, C^r, C^*, T^*)$ is locally asymptotically stable provided the following conditions are satisfied

\begin{align}
(4.1) & \quad a_1a_2a_3 - a_3^2 - a_1^2a_4 > 0, \\
(4.2) & \quad (a_1a_4 - a_2)(a_1a_2a_3 - a_3^2 - a_1^2a_4) - a_5(a_1a_2 - a_3)^2 - a_1a_5^2 > 0,
\end{align}

where

\[
\begin{align*}
    a_1 &= (\beta Y^* + \frac{\lambda C_r^* N^*}{Y^*}) + (s_0 + \alpha_0 + \theta_0) + d, \\
    a_2 &= (\beta Y^* + \frac{\lambda C_r^* N^*}{Y^*})(d + s_0 + \alpha_0 + \theta_0) + \alpha(\beta Y^* + \lambda C^*_r) + d(s_0 + \alpha_0 + \theta_0), \\
    a_3 &= (\beta Y^* + \frac{\lambda C_r^* N^*}{Y^*}) (d(s_0 + \alpha_0 + \theta_0) + (s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0)), \\
    a_4 &= (\beta Y^* + \frac{\lambda C_r^* N^*}{Y^*})(d(s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0) + s_0\theta_0\alpha_0), \\
    a_5 &= \left[(\beta Y^* + \lambda C^*_r)(\alpha + d) + \frac{\lambda d(N^* - Y^*)}{Y^*}\right] s_0\theta_0\alpha_0.
\end{align*}
\]

Here it is noted that $a_i > 0, \forall i = 1, 2, 3, 4, 5$.

**Proof.** See the Appendix A.

**Theorem 4.2.** The equilibrium point $E(Y^*, N^*, C^r, C^*, T^*)$ is globally asymptotically stable in $\Omega$ provided the following conditions are satisfied

\begin{align}
(4.3) & \quad \alpha\lambda^2 C_{rm}^2 < \beta^2 d Y^2, \\
(4.4) & \quad 4\alpha d\lambda^2 a_1^2 \theta^2 s_1^2 (N^* - Y^*)^2 < \beta^2 Y^2 s_0^2 \theta_0^2 \alpha_0^2,
\end{align}

where $C_{rm}$ is the maximum value of $C_r$, which is given by $C_{rm} = \frac{s\theta_0\alpha_0 + s_1\theta_1\alpha_1 A}{s_0\theta_0\alpha_0}$.

**Proof.** See the Appendix B.

5 Numerical Simulation

Here we discuss the existence and stability of non-trivial equilibrium point $E(Y^*, N^*, C^r, C^*, T^*)$ by taking the following set of parameters and using MAPLE.

\begin{align*}
    A &= 20, d = 0.0004, \alpha = 0.0005, \alpha_0 = 0.016, \alpha_1 = 0.6 \times 10^{-3}, \\
    \beta &= 6 \times 10^{-7}, \nu = 0.012, \lambda = 2 \times 10^{-8}, s_0 = 0.3, T_0 = 14, \\
    Q_0 &= 6, \theta = 0.1, \theta_0 = 0.19, C_0 = 375, s = 2.4 \times 10^4, \\
    s_1 &= 2 \times 10^4
\end{align*}
The Jacobian matrix $J(E)$ for the above values of parameters at $E(Y^*, N^*, C^*, C^r, T^*)$ is

$$
\begin{bmatrix}
-0.01259970927 & 0.01037645075 & 0.0003558913828 & 0 & 0 \\
-0.0005 & -0.0004 & 0 & 0 & 0 \\
0 & 0 & -0.3 & 0 & 20000 \\
0 & -2.40 \times 10^{-7} & 0 & -0.016 & 0 \\
0 & 0 & 0 & 0.1 & -0.19 \\
\end{bmatrix}
$$

The characteristic roots of the Jacobian matrix corresponding to the equilibrium point $E(Y^*, N^*, C^*, C^r, T^*)$ are:

$-0.00083244611$, $-0.01219714633$, $-0.01596815206$, $-0.1899940766$, $-0.3000078875$

Since all characteristic roots are negative real numbers, therefore $E(Y^*, N^*, C^*, C^r, T^*)$ is locally stable.

For above values of parameters, the non-trivial equilibrium point $E(Y^*, N^*, C^*, C^r, T^*)$ corresponding to (2.2) - (2.6) is obtained as follows:

$Y^* = 14313.52483$, $N^* = 32108.09397$, $C^r = 89416.79265$, $C^* = 375.2683786$, $T^* = 14.14125189$

Eliminating $C_r, C, T$ from (3.1) - (3.5), we get

$$
\beta(N - Y)Y + \lambda(N - Y)(\frac{s\theta_0\alpha_0 + s_1\theta\alpha_1\alpha Y}{s_0\theta_0\alpha_0}) - (v + \alpha + d)Y = 0,
$$

(5.1)

$$
A - dN - \alpha Y = 0.
$$

(5.2)

For above values of parameters the equations (5.1) and (5.2) are plotted in $Y$-$N$ plane (Fig.5.1), and the intersection point is $(Y^*, N^*)$.

Existence of $(Y^*, N^*)$

![Figure 5.1](image)

With the above mentioned set of values of parameters, we plot the graphs from Fig.5.1 - Fig.5.13. Fig.5.2, shows the nonlinear stability behavior between $N$ and $Y$ with different initial conditions tending to equilibrium point $(Y^*, N^*)$ as time increases.
For solving the system of non-linear ODE, we use Runge-Kutta method in MAPLE.
The initial conditions for various quantities are given below:
\[ C(0) = 375, \quad C_r(0) = 80000, \quad T(0) = 14, \quad N(0) = 31000, \quad Y(0) = 13000 \]

**Fig. 5.3 - Fig. 5.13** show the effect of various parameters on infective human population density, carrier population density and amount of carbon dioxide. Every figure contains three curves with the same initial conditions as given above but with different values of corresponding parameter.

From **Fig. 5.3**, it is seen that the density of carriers increases if growth rate of carriers due to global warming temperature \((s_1)\) increases. **Fig. 5.4** shows that the amount of \(CO_2\) in the environment increases if the discharge rate coefficient of \(CO_2\) from man-made sources \((\alpha_1)\) increases. From **Fig. 5.5**, it is seen that the density of infective human population increases as the growth rate coefficient of temperature of the region due to rise in amount of \(CO_2(\theta)\) increases. **Fig. 5.6** shows that the density of infective human population increases if the discharge rate coefficient of \(CO_2\) from man-made sources \((\alpha_1)\) increases. **Fig. 5.7** shows that the density of infective human population decreases if reduction rate coefficient of carriers \((s_0)\) increases. **Fig. 5.8** shows that the density of infective human population increases if growth rate of carriers due to global warming temperature \((s_1)\) increases. **Fig. 5.9** shows that the density of infective human population increases if constant growth rate coefficient of carrier population density \((s)\) increases. **Fig. 5.10** shows that the density of infective human population increases if coefficient of transmission by carrier population density \((\lambda)\) increases. **Fig. 5.11** shows that the density of infective human population increases if constant immigration rate of human population from outside the region under study \((A)\) increases. **Fig. 5.12** shows that the density of infective human population increases if the discharge rate of \(CO_2\) from natural factors \((Q_0)\) increases. **Fig. 5.13**, shows that the density of carrier population increases if the discharge rate of \(CO_2\) from natural factors \((Q_0)\) increases.
Figure 5.3: Plot between carrier population density $C_r$ and time $t$ for various values of $s_1$.

Figure 5.4: Plot between amount of $CO_2$ and time $t$ for various values of $\alpha_1$. 
Figure 5.5: Plot between infective human population $Y$ and time $t$ for various values of $\theta$.

Figure 5.6: Plot between infective human population $Y$ and time $t$ for various values of $\alpha_1$. 
Figure 5.7: Plot between infective human population $Y$ and time $t$ for various values of $s_0$.

Figure 5.8: Plot between infective human population $Y$ and time $t$ for various values of $s_1$. 
Figure 5.9: Plot between infective human population $Y$ and time $t$ for various values of $s$.

Figure 5.10: Plot between infective human population $Y$ and time $t$ for various values of $\lambda$. 

\begin{align*}
\text{Infective Human Population (Y)} \\
\text{Time (t)}
\end{align*}

$\lambda = 10^{-7}$

$\lambda = 6 \times 10^{-8}$

$\lambda = 2 \times 10^{-8}$
Figure 5.11: Plot between infective human population $Y$ and time $t$ for various values of $A$.

Figure 5.12: Plot between infective human population $Y$ and time $t$ for various values of $Q_0$. 
6 Conclusions

The larger amount of carbon dioxide (\(CO_2\)) in the environment is accountable for global warming. In this paper, we have studied the effect of global warming on the growth of carrier population with a constant growth rate and its role on the spread of infectious diseases. The model has five dependent variables namely, the density of susceptibles, the density of infectives, the density of carrier population, the amount of \(CO_2\) in the environment causing global warming and the global warming temperature of the environment due to discharge of \(CO_2\). We have assumed that the density of carriers increases due to global warming temperature. The global warming temperature has been assumed to be proportional to the amount of \(CO_2\) in the environment.

The proposed mathematical model has been studied with the help of stability theory of differential equations and numerical simulation. The local stability and the global stability conditions for non-trivial equilibrium point have been derived. For a set of parameters, numerical simulation proves the analytical results.

It has been found out that as the discharge rate of carbon dioxide from natural factors and man-made sources increases, the number of infectives increases. Also, the infectives and carrier population increase as the global warming temperature increases and thus the prevalence of carrier dependent infectious diseases increases in the environment.
Appendix A. Proof of the Theorem 4.1

Proof. The Jacobian matrix for the system (2.2) - (2.6) is

\[
J(E) = \begin{bmatrix}
a_{11} & a_{12} & a_{13} & 0 & 0 \\
-\alpha & -d & 0 & 0 & 0 \\
0 & 0 & -s_0 & 0 & s_1 \\
0 & -\alpha_1 d & 0 & -\alpha_2 & 0 \\
0 & 0 & 0 & \theta & -\theta_0 \\
\end{bmatrix},
\]

where

\[a_{11} = \beta(N^* - Y^*) - \beta Y^* - \lambda C_r^* - (\nu + \alpha + d),\ a_{12} = \beta Y^* + \lambda C_r^*,\ a_{13} = \lambda(N^* - Y^*).\]

The characteristic equation of above Jacobian matrix is as follows:

\[
x^5 + [-a_{11} + (s_0 + \alpha_0 + \theta_0) + d]x^4 \\
+ [-a_{11}(d + s_0 + \alpha_0 + \theta_0) + a_{12}\alpha + d(s_0 + \alpha_0 + \theta_0) + (s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0)]x^3 \\
+ [-a_{11}(d(s_0 + \alpha_0 + \theta_0) + (s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0)) + a_{12}\alpha(s_0 + \alpha_0 + \theta_0) + d(s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0) + s_0\theta_0\alpha_0]x^2 \\
+ [-a_{11}(d(s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0) + s_0\theta_0\alpha_0) + a_{12}\alpha(s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0) + d\theta_0\theta_0\alpha_0 + \theta_0\alpha_0 - \alpha_1 ds_1 \theta_1) = 0.
\]

By (2.2), we have

\[
(A.1) \quad x^5 + [-a_{11} + (s_0 + \alpha_0 + \theta_0) + d]x^4 \\
= \beta(N^* - Y^*)Y^* + \lambda(N^* - Y^*)C_r^* - (\nu + \alpha + d)Y^* = 0 \\
\Rightarrow \quad \{\beta(N^* - Y^*) - (\nu + \alpha + d)\} = \frac{-\lambda(N^* - Y^*)C_r^*}{Y^*}.
\]

Now writing the value of \(a_{11}\), with the help of (A.2), we get

\[
(A.3) \quad a_{11} = -\left(\frac{\lambda N^*C_r^*}{Y^*} + \beta Y^*\right) < 0.
\]

Also we have

\[
(A.4) \quad C_r^* = \frac{s_\theta_0\alpha_0 + s_1\alpha_1\alpha Y^*}{s_0\theta_0\alpha_0} \Rightarrow s_1\alpha_1\alpha = \frac{s_0\theta_0\alpha_0(C_r^* - 1)}{Y^*}.
\]

Use (A.4) and the values of \(a_{11}, a_{12}, a_{13}\) in constant term of (A.1), we get

\[
-a_{11} ds_0 \theta_0 \alpha_0 + a_{12} \alpha s_0 \theta_0 \alpha_0 - a_{13} \alpha d s_1 \theta_1 \alpha_1 = s_0 \theta_0 \alpha_0 [\beta Y^* + \lambda C_r^*](\alpha + d) + \frac{d\lambda(N^* - Y^*)}{Y^*}.
\]

Hence the constant term of (A.1) is positive.

Since \(a_{11} = -\left(\frac{\lambda N^*C_r^*}{Y^*} + \beta Y^*\right) < 0\) and constant term of (A.1) is positive, therefore all coefficients of (A.1) are positive, so the characteristic equation (A.1) can be written as

\[
(A.5) \quad x^5 + a_1 x^4 + a_2 x^3 + a_3 x^2 + a_4 x + a_5 = 0,
\]

where

\[
a_1 = (\beta Y^* + \frac{\lambda C_r^* N^*}{Y^*}) + (s_0 + \alpha_0 + \theta_0) + d, \\
a_2 = (\beta Y^* + \frac{\lambda C_r^* N^*}{Y^*})(d + s_0 + \alpha_0 + \theta_0) + \alpha(\beta Y^* + \lambda C_r^*) + d(s_0 + \alpha_0 + \theta_0), \\
+ (s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0), \\
a_3 = (\beta Y^* + \frac{\lambda C_r^* N^*}{Y^*})[d(s_0 + \alpha_0 + \theta_0) + (s_0\alpha_0 + \alpha_0\theta_0 + s_0\theta_0)],
\]

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Appendix B. Proof of the Theorem 4.2

Proof. For the proof of this theorem we use Lyapunov direct method. For this first we let the following positive definite Lyapunov function

\( W = K_0(Y - Y^* - Y^* \ln \frac{Y}{Y^*} + \frac{K_1}{2}(N - N^*)^2 + \frac{K_2}{2}(C_r - C^r)^2 + \frac{K_3}{2}(C - C^*)^2 + \frac{K_4}{2}(T - T^*)^2, \)

where \( K_0, K_1, K_2, K_3 \) and \( K_4 \) are positive constants to be chosen appropriately.

Differentiate (B.1) with respect to \( t \), we get

\( \dot{W} = K_0 \dot{Y}(Y - Y^*) + K_1(N - N^*)\dot{N} + K_2(C_r - C^r)\dot{C_r} + K_3(C - C^*)\dot{C} + K_4(T - T^*)\dot{T}. \)

Using model system (2.2) - (2.6), we get

\[
\dot{W} = -K_0 \frac{C_r \alpha N}{Y Y^*} (Y - Y^*)^2 - K_d \beta (Y - Y^*)^2 - K_1 d(N - N^*)^2 - K_2 s_0 (C_r - C^r)^2
\]
\[
- K_3 \alpha_0 (C - C^*)^2 - K_4 \theta_0 (T - T^*)^2 + (K_d \beta - K_1 \alpha) (N - N^*) (Y - Y^*)
\]
\[
+ K_0 \frac{C_r \alpha N}{Y Y^*} (Y - Y^*) - K_0 d \left( \frac{N^* - Y^*}{Y^*} \right) (C_r - C^r)(Y - Y^*)
\]
\[
+ K_3 \alpha_1 (C_r - C^r)(T - T^*) - K_3 \alpha_1 d(N - N^*) (C - C^*) + K_4 \theta (T - T^*) (C - C^*).
\]

Here we take the constants \( K_0 \) and \( K_1 \) such that \( K_0 \beta - K_1 \alpha = 0 \).

Further we take

\[ K_1 = 1 \text{ then } K_0 = \frac{\alpha}{\beta}. \]

We have

\[
\dot{W} = -K_0 \frac{C_r \alpha N}{Y Y^*} (Y - Y^*)^2 - \left[ \frac{K_d \beta}{2} (Y - Y^*)^2 - \frac{K_0 \alpha C_r}{Y^*} (N - N^*) (Y - Y^*) + \frac{K_1 d}{2} (N - N^*)^2 \right]
\]
\[
- \left[ \frac{K_3 d}{2} (N - N^*)^2 + K_3 \alpha_1 d(N - N^*) (C - C^*) + \frac{K_2 s_0}{2} (C_r - C^r)^2 \right]
\]
\[
- \left[ K_3 \alpha_0 (C - C^*)^2 - K_3 \alpha_1 d(N - N^*) (C - C^*) + \frac{K_4 \theta_0}{2} (T - T^*)^2 \right]
\]
\[
- \left[ \frac{K_3 \alpha_0}{2} (C - C^*)^2 - K_4 \theta (T - T^*) (C - C^*) + \frac{K_4 \theta_0}{2} (T - T^*)^2 \right].
\]
The derivative $\dot{W}$ is negative definite if the following conditions are satisfied:

\[(B.4) \quad \alpha \lambda^2 C_{rm}^2 < d \beta^2 Y^r,\]

where $C_{rm}$ is the maximum value of $C_r$

\[(B.5) \quad K_3 < \frac{\alpha_o}{d \alpha_1^2},\]

\[(B.6) \quad K_2 > \frac{\alpha \lambda^2 (N^* - Y^r)^2}{\delta_0 \beta^2 Y^r},\]

\[(B.7) \quad K_2 < \frac{s_0 \theta_0}{s_1} K_4,\]

\[(B.8) \quad K_4 \theta^2 < K_3 \alpha_o \theta_0.\]

Combining the inequalities (B.5), (B.6), (B.7) and (B.8), we get

\[(B.9) \quad 4 \alpha \lambda^2 \alpha_1^2 \theta^2 \delta_1^2 (N^* - Y^r)^2 < \beta^2 Y^r \delta_0 \theta_0 \alpha_0^2.\]

The inequalities (B.4) and (B.9) are the required conditions.

**Acknowledgement.** We are very much thankful to wortly refree and Editor for their valuable suggestions to improve the paper in its present form.

**References**


