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(Dedicated to Honor Professor H.M. Srivastava on His 80th Birth Anniversary Celebrations)

GENERALIZED FRACTIONAL CALCULUS OF *I*-FUNCTION OF TWO VARIABLES

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Abstract

This paper is devoted to study and develop the generalized fractional calculus of arbitrary order for the *I*-function of two variables which is based on generalized fractional integration and differentiation operators of arbitrary complex order involving Appell hypergeometric function F_3 as a kernel due to Saigo and Maeda. On account of general nature of the Saigo-Maeda operators, a large number of results involving Saigo and Riemann-Liouville operetors are found as corollaries. Some special cases also have been considered.
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1 Introduction

In last some decades, considerable amount of research work in fractional calculus is published due to its applicability in the various fields of science and engineering such as dynamical system in control theory, electrical circuits, viscoelasticity, electrochemistry, fluid mechanics, mathematical biology, image processing, astrophysics and quantum mechanics.There is no doubt that fractional calculus has become an important mathematical tool to solve diverse problems of mathematics, science and engineering. The fractional calculus operators involving various special functions have been successfully applied to frame relevant system in various fields of science and engineering. see [2], [3], [19], [20]. Therefore number of authors have investigated different unifications and extentions of various fractional calculus operators. For more detail about fractional calculus operators, reader may refer to the research monograph by Miller and Ross [16], Samko *et al.*[22], and Kiryakova [9].

The image formulas for special functions of one and more variables under various fractional calculus operators have been obtained by number of authors such as Gupta *et al.*[7] obtained the image formulas of the product of two H functions using Saigo operators, Agarwal [1] studied and developed the generalized fractional integration of the product of $\overline{H}$-function and a general class of polynomials in Saigo operators, Kumar [10] established some new unified integral and differential formulas associated with $\overline{H}$-function applying Saigo and Maeda operator. For more information, we may also refer to Chandel [4]; Chandel and Vishwakarma [5]; Chandel and Gupta [6]; Kumar, Purohit and Choi [12]; Kumar [13]; Kumar, Chandel and Srivastava [14]; Kumar, Pathan and Kumari [15]; Mathai, Saxena and Houbold [17]; Pathan, Kumar, Srivastava and Chandel [28]; Srivastava, Saxena and Ram [28]; Srivastava, Chandel and vishwakarma [29]. In order to stimulate more interest in the subject, we have established some image formulas concerning to *I*-function of two variables.

In 1995, Goyal and Agrawal [8] introduced I-function of two variables by means of Mellin-Barnes type integrals in the following manner

$$(1.1)\quad I^{m_1,n_1:m_2,n_2;m_3,n_3}_{p,q:p_i^{(1)},q_i^{(1)};p_i^{(2)},q_i^{(2)}:r}\left[\begin{array}{c|c} z_1 & [(e_p:E_p,E_p^{'})]:[(a_j,\alpha_j)_{1,n_2}],[(a_{ji},\alpha_{ji})_{n_2+1,p_i^{(1)}}];[(c_j,\gamma_j)_{1,n_3}],[(c_{ji},\gamma_{ji})_{n_3+1,p_i^{(2)}}] \\ z_2 & [(f_q:F_q,F_q^{'})]:[(b_j,\beta_j)_{1,m_2}],[(b_{ji},\beta_{ji})_{m_2+1,q_i^{(1)}}];[(d_j,\delta_j)_{1,m_3}],[(d_{ji},\delta_{ji})_{m_3+1,q_i^{(2)}}] \end{array}\right]$$

$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi_1(\xi)\,\phi_2(\eta)\,\psi(\xi,\eta)\,z_1^{\xi}\,z_2^{\eta}\,d\xi\,d\eta,$$

where $\omega = \sqrt{-1}$ and $\phi_1(\xi)$, $\phi_2(\eta)$, $\psi(\xi,\eta)$ are given by

$$(1.2)\qquad \phi_1(\xi) = \frac{\prod_{j=1}^{m_2}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n_2}\Gamma(1-a_j+\alpha_j\xi)}{\sum_{i=1}^{r}\left[\prod_{j=m_2+1}^{q_i^{(1)}}\Gamma(1-b_{ji}+\beta_{ji}\xi)\prod_{j=n_2+1}^{p_i^{(1)}}\Gamma(a_{ji}-\alpha_{ji}\xi)\right]},$$

$$(1.3)\qquad \phi_2(\eta) = \frac{\prod_{j=1}^{m_3}\Gamma(d_j-\delta_j\eta)\prod_{j=1}^{n_3}\Gamma(1-c_j+\gamma_j\eta)}{\sum_{i=1}^{r}\left[\prod_{j=m_3+1}^{q_i^{(2)}}\Gamma(1-d_{ji}+\delta_{ji}\eta)\prod_{j=n_3+1}^{p_i^{(2)}}\Gamma(c_{ji}-\gamma_{ji}\eta)\right]},$$

$$(1.4)\qquad \psi(\xi,\eta) = \frac{\prod_{j=1}^{m_1}\Gamma(f_j-F_j\xi-F_j^{'}\eta)\prod_{j=1}^{n_1}\Gamma(1-e_j+E_j\xi+E_j^{'}\eta)}{\prod_{j=m_1+1}^{q}\Gamma(1-f_j+F_j\xi+F_j^{'}\eta)\prod_{j=n_1+1}^{p}\Gamma(e_j-E_j\xi-E_j^{'}\eta)},$$

and an empty product is interpreted as unity. z_1, z_2 are two non zero complex variables, L_1, L_2 are two Mellin-Barnes type contour integrals and $m_1,n_1;\ m_2,n_2;\ m_3,n_3,\ p,q;\ p_i^{(1)}$, $q_i^{(1)};\ p_i^{(2)},q_i^{(2)}$ are non-negative integers satisfying the conditions $0 \le n_1 \le p$, $0 \le n_2 \le p_i^{(1)}$, $0 \le n_3 \le p_i^{(2)}$, $0 \le m_1 \le q$, $0 \le m_2 \le q_i^{(1)}$, $0 \le m_3 \le q_i^{(2)}$ for all $i = 1,2,3,\cdots,r$ where r is also a positive integer. $\alpha_j(j = 1,\cdots,n_2),\beta_j(j = 1,\cdots,m_2),\gamma_j(j = 1,\cdots,n_3),\delta_j(j = 1,\cdots,m_3),\alpha_{ji}(j = n_2+1,\cdots,p_i^{(1)}),\beta_{ji}(j = m_2+1,\cdots,q_i^{(1)}),\gamma_{ji}(j = n_3+1,\cdots,p_i^{(2)}),\delta_{ji}(j = m_3+1,\cdots,q_i^{(2)})$ are assumed to be positive quantities for standardization purposes. $E_j,E_j^{'},F_j,F_j^{'}$ are also positive. $a_j(j = 1,\cdots,n_2),b_j(j = 1,\cdots,m_2),c_j(j = 1,\cdots,n_3),d_j(j = 1,\cdots,m_3),a_{ji}(j = n_2+1,\cdots,p_i^{(1)}),b_{ji}(j = m_2+1,\cdots,q_i^{(1)}),c_{ji}(j = n_3+1,\cdots,p_i^{(2)}),d_{ji}(j = m_3+1,\cdots,q_i^{(2)})$ are complex for all $i = 1,2,3,\cdots,r$.

The contour L_1 lies in the complex ξ-plane and runs from $-\omega\infty$ to $+\omega\infty$ with loops, if necessary, to ensure that the poles of $\Gamma(b_j-\beta_j\xi)(j = 1,\cdots,m_2),\Gamma(f_j-F_j\xi-F_j^{'}\eta)(j = 1,\cdots,m_1)$

lies to the right and the poles of $\Gamma(1-a_j+\alpha_j\xi)(j=1,\cdots,n_2), \Gamma(1-e_j+E_j\xi+E_j^{'}\eta)(j=1,\cdots,n_1)$ to the left of the contour L_1.The contour L_2 lies in the complex η plane and runs from $-\omega\infty$ to $+\omega\infty$ with loops, if necessary, to ensure that the poles of $\Gamma(d_j-\delta_j\eta)(j=1,\cdots,m_3), \Gamma(f_j-F_j\xi-F_j^{'}\eta)(j=1,\cdots,m_1)$ lies to the right and the poles of $\Gamma(1-c_j+\gamma_j\xi)(j=1,\cdots,n_3), \Gamma(1-e_j+E_j\xi+E_j^{'}\eta)(j=1,\cdots,n_1)$ to the left of the contour L_2. All the poles are simple poles.

Convergence conditions are as follows:

$$|arg\, z_1| < \frac{A_i\pi}{2},\ |arg\, z_2| < \frac{B_i\pi}{2}, \tag{1.5}$$

where

$$A_i = \sum_{j=1}^{n_1} E_j - \sum_{j=n_1+1}^{p} E_j + \sum_{j=1}^{m_1} F_j - \sum_{j=m_1+1}^{q} F_j + \sum_{j=1}^{m_2}\beta_j - \sum_{j=m_2+1}^{q_i^{(1)}}\beta_{ji} + \sum_{j=1}^{n_2}\alpha_j - \sum_{j=n_2+1}^{p_i^{(1)}}\alpha_{ji} > 0, \tag{1.6}$$

and

$$B_i = \sum_{j=1}^{n_1} E_j^{'} - \sum_{j=n_1+1}^{p} E_j^{'} + \sum_{j=1}^{m_1} F_j^{'} - \sum_{j=m_1+1}^{q} F_j^{'} + \sum_{j=1}^{m_3}\delta_j - \sum_{j=m_3+1}^{q_i^{(2)}}\delta_{ji} + \sum_{j=1}^{n_3}\gamma_j - \sum_{j=n_3+1}^{p_i^{(2)}}\gamma_{ji} > 0, \tag{1.7}$$

for $i = 1, ..., r$.

By considering the behaviour of the Gamma functions involved in $I[z_1, z_2]$ defined by (1.1), it can be shown that $I[z_1, z_2]$ is of certain algebraic order of z_1, z_2 for large values of z_1, z_2, if the validity conditions (1.5)-(1.7) are satisfied.

Also, for small values of z_1 and z_2

$$I\left[\begin{array}{c} z_1 \\ z_2 \end{array}\right] = o\left(|z_1|^{\lambda_j}|z_2|^{\mu_k}\right) \text{ for all } j = 1, 2, \cdots, m_2;\ k = 1, 2, \cdots, m_3,$$

where $\lambda_j = min\, Re\left(\frac{b_j}{\beta_j}\right),\ \mu_k = min\, Re\left(\frac{d_k}{\delta_k}\right)$ provided $A_i > 0,\ B_i > 0,$

Further, we observe, for large values of z_1 and z_2, that

$$I\left[\begin{array}{c} z_1 \\ z_2 \end{array}\right] = o\left(|z_1|^{\lambda_j^{'}}|z_2|^{\mu_k^{'}}\right) \text{ for all } j = 1, 2, \cdots, n_2;\ k = 1, 2, \cdots, n_3,$$

where $\lambda_j^{'} = max\, Re\left(\frac{a_j-1}{\alpha_j}\right),\ \mu_k^{'} = max\, Re\left(\frac{c_k-1}{\gamma_k}\right)$ provided $A_i > 0,\ B_i > 0,$

For the sake of brevity throughout the paper we shall use following notations:
$P = m_2, n_2; m_3, n_3,$
$Q = p_i^{(1)}, q_i^{(1)}; p_i^{(2)}, q_i^{(2)} : r,$
$U = [(a_j, \alpha_j)_{1,n_2}], [(a_{ji}, \alpha_{ji})_{n_2+1, p_i^{(1)}}]; [(c_j, \gamma_j)_{1,n_3}], [(c_{ji}, \gamma_{ji})_{n_3+1, p_i^{(2)}}],$
$V = [(b_j, \beta_j)_{1,m_2}], [(b_{ji}, \beta_{ji})_{m_2+1, q_i^{(1)}}]; [(d_j, \delta_j)_{1,m_3}], [(d_{ji}, \delta_{ji})_{m_3+1, q_i^{(2)}}],$

If $\alpha, \alpha', \beta, \beta', \gamma \in C$ and $x > 0$, then the generalized fractional calculus operators containing Appell hypergeometric function F_3 given by Saigo and Maeda [23] are defined in the following manner:

$$\left(I_{0_+}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x) = \frac{x^{-\alpha}}{\Gamma(\gamma)}\int_0^x t^{-\alpha'}(x-t)^{\gamma-1} F_3\left(\alpha, \alpha', \beta, \beta'; \gamma; 1-\frac{t}{x}, 1-\frac{x}{t}\right) f(t)\, dt,\ \ Re(\gamma) > 0, \tag{1.8}$$

$$(1.9)\qquad = \left(\frac{d}{dx}\right)^k \left(I_{0_+}^{\alpha,\alpha',\beta+k,\beta',\gamma+k} f\right)(x),\ Re(\gamma) \leq 0; k = [-Re(\gamma)+1], \left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x)$$

$$(1.10)\qquad = \frac{x^{-\alpha'}}{\Gamma(\gamma)} \int_x^\infty t^{-\alpha}(t-x)^{\gamma-1} F_3\left(\alpha,\alpha',\beta,\beta';\gamma;1-\frac{x}{t},1-\frac{t}{x}\right) f(t)\,dt,\ \ Re(\gamma) > 0,$$

$$(1.11)\qquad = (-1)^k \left(\frac{d}{dx}\right)^k \left(I_{-}^{\alpha,\alpha',\beta,\beta'+k,\gamma+k} f\right)(x),\ Re(\gamma) \leq 0; k = [-Re(\gamma)+1],$$

(1.12)

$$\left(D_{0_+}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x) = \left(I_{0_+}^{-\alpha',-\alpha,-\beta',-\beta,-\gamma} f\right)(x),\ \ Re(\gamma) > 0,$$

$$(1.13)\qquad = \left(\frac{d}{dx}\right)^k \left(I_{0_+}^{-\alpha',-\alpha,-\beta'+k,-\beta,-\gamma+k} f\right)(x),\ Re(\gamma) > 0; k = [Re(\gamma)+1],$$

(1.14)

$$\left(D_{-}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x) = \left(I_{-}^{-\alpha',-\alpha,-\beta',-\beta,-\gamma} f\right)(x),\ \ Re(\gamma) > 0,$$

$$(1.15)\qquad = (-1)^k \left(\frac{d}{dx}\right)^k \left(I_{-}^{-\alpha',-\alpha,-\beta',-\beta+k,-\gamma+k} f\right)(x),\ Re(\gamma) > 0; k = [Re(\gamma)+1],$$

These generalized fractional calculus operators reduces to Saigo's [24] fractional calculus operators by means of the following relationship:

$$(1.16)\qquad \left(I_{0_+}^{\alpha,0,\beta,\beta',\gamma} f\right)(x) = \left(I_{0_+}^{\gamma,\alpha-\gamma,-\beta} f\right)(x),\ \ (\gamma \in C),$$

$$(1.17)\qquad \left(I_{-}^{\alpha,0,\beta,\beta',\gamma} f\right)(x) = \left(I_{-}^{\gamma,\alpha-\gamma,-\beta} f\right)(x),\ \ (\gamma \in C),$$

$$(1.18)\qquad \left(D_{0_+}^{0,\alpha',\beta,\beta',\gamma} f\right)(x) = \left(D_{0_+}^{\gamma,\alpha'-\gamma,\beta'-\gamma} f\right)(x),\ \ Re(\gamma) > 0,$$

$$(1.19)\qquad \left(D_{-}^{0,\alpha',\beta,\beta',\gamma} f\right)(x) = \left(D_{-}^{\gamma,\alpha'-\gamma,\beta'-\gamma} f\right)(x),\ \ Re(\gamma) > 0.$$

Our main findings in the next section are based on the following composition formula due to Saigo-Maeda [23].

***Lemma* 1.1.** *If* $\alpha,\alpha',\beta,\beta',\gamma \in C, Re(\gamma) > 0$ *and* $Re(\rho) > max[0, Re(\alpha+\alpha'+\beta-\gamma), Re(\alpha'-\beta')]$ *then there hold the formula*

$$(1.20)\qquad \left(I_{0_+}^{\alpha,\alpha',\beta,\beta',\gamma} t^{\rho-1}\right)(x) = x^{\rho-\alpha-\alpha'+\gamma-1} \frac{\Gamma(\rho)\Gamma(\rho+\gamma-\alpha-\alpha'-\beta)\Gamma(\rho+\beta'-\alpha')}{\Gamma(\rho+\gamma-\alpha-\alpha')\Gamma(\rho+\gamma-\alpha'-\beta)\Gamma(\rho+\beta')},$$

***Lemma* 1.2.** *If* $\alpha,\alpha',\beta,\beta',\gamma \in C, Re(\gamma) > 0$ *and* $Re(\rho) < 1 + min[Re(-\beta), Re(\alpha+\alpha'-\gamma), Re(\alpha+\beta'-\gamma)]$ *then there hold the formula*

$$(1.21)\quad \left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma} t^{\rho-1}\right)(x) = x^{\rho-\alpha-\alpha'+\gamma-1} \frac{\Gamma(1+\alpha+\alpha'-\gamma-\rho)\Gamma(1+\alpha+\beta'-\gamma-\rho)\Gamma(1-\beta-\rho)}{\Gamma(1-\rho)\Gamma(1+\alpha+\alpha'+\beta'-\gamma-\rho)\Gamma(1+\alpha-\beta-\rho)},$$

2 Main Results

In this section we have established fractional calculus formulas associated to I-function of two variables with the help of Saigo-Maeda generalized fractional calculus operators. Further by specializing the parameters, we have found some corollaries concerning to Saigo fractional calculus operators and Riemann-Liouville fractional calculus operators.The results are presented in the form of theorems stated below.

***Theorem* 2.1.** *Let* $\alpha,\alpha',\beta,\beta',\gamma,\rho \in C$, $z_1,z_2 \in C$, $Re(\gamma) > 0$, $\mu,\nu \in R_+$. *Further let the constants* $m_1,n_1,p,q \in N_0$, $a_j,b_j,a_{ji},b_{ji} \in C$, $\alpha_j,\beta_j,\alpha_{ji},\beta_{ji} \in R_+(i = 1,\cdots,p_i^{(1)};\ j = 1,\cdots,q_i^{(1)})$, $c_j,d_j,c_{ji},d_{ji} \in C$, $\gamma_j,\delta_j,\gamma_{ji},\delta_{ji} \in R_+(i = 1,\cdots,p_i^{(2)};\ j = 1,\cdots,q_i^{(2)})$, $|arg\, z_1| < \frac{A_i\pi}{2}$, $|arg\, z_2| < \frac{B_i\pi}{2}$, $A_i > 0$, $B_i > 0$ *and satisfy the condition*

$$Re(\rho) + \mu \min_{1\le j\le m_2} Re\left(\frac{b_j}{\beta_j}\right) + \nu \min_{1\le j\le m_3} Re\left(\frac{d_j}{\delta_j}\right) > max\left[0, Re(\alpha+\alpha'+\beta-\gamma), Re(\alpha'-\beta')\right].$$

Then the fractional integration $I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}$ *of the* I*-function of two variables exists and the following relation holds:*

$$\left\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} t^{\rho-1} I_{p,q:Q}^{m_1,n_1:P}\left[\begin{array}{c|c} z_1 t^{\mu} & [(e_p : E_p, E_p^{'})] : U \\ z_2 t^{\nu} & [(f_q : F_q, F_q^{'})] : V \end{array}\right]\right\}(x) \tag{2.1}$$

$$= x^{\rho+\gamma-\alpha-\alpha'-1} I_{p+3,q+3:Q}^{m_1,n_1+3:P}\left[\begin{array}{c|c} z_1 x^{\mu} & X_1, [(e_p : E_p, E_p^{'})] : U \\ z_2 x^{\nu} & [(f_q : F_q, F_q^{'})], X_2 : V \end{array}\right],$$

where

$$X_1 = [(1-\rho : \mu,\nu)], [(1-\rho+\alpha+\alpha^{'}+\beta-\gamma : \mu,\nu)], [(1-\rho+\alpha^{'}-\beta^{'} : \mu,\nu)],$$
$$X_2 = [(1-\rho+\alpha+\alpha^{'}-\gamma : \mu,\nu)], [(1-\rho+\alpha^{'}+\beta-\gamma : \mu,\nu)], [(1-\rho-\beta^{'} : \mu,\nu)].$$

Proof. In order to prove (2.1), we first express I-function of two variables occurring on the left hand side of (2.1) in terms of Mellin-Barnes contour integral with the help of equation (1.1) and interchanging the order of integration, which is justified under the conditions stated with the ***Theorem***, we obtain (say I_1):

$$I_1 = \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2} \phi_1(\xi)\,\phi_2(\eta)\,\psi(\xi,\eta)\, z_1^{\xi} z_2^{\eta}\, (I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\, t^{\rho+\mu\xi+\nu\eta-1})(x)\, d\xi\, d\eta, \tag{2.2}$$

Now by applying ***Lemma* 1.1**, we arrive at

$$I_1 = x^{\rho+\gamma-\alpha-\alpha'-1}\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2} \phi_1(\xi)\,\phi_2(\eta)\,\psi(\xi,\eta)\,(z_1x^{\mu})^{\xi}\,(z_2x^{\nu})^{\eta}$$
$$\times \frac{\Gamma(\rho+\mu\xi+\nu\eta)\Gamma(\rho+\mu\xi+\nu\eta+\gamma-\alpha-\alpha'-\beta)\Gamma(\rho+\mu\xi+\nu\eta+\beta'-\alpha')}{\Gamma(\rho+\mu\xi+\nu\eta+\gamma-\alpha-\alpha')\Gamma(\rho+\mu\xi+\nu\eta+\gamma-\alpha'-\beta)\Gamma(\rho+\mu\xi+\nu\eta+\beta')}\, d\xi\, d\eta.$$

By re-interpreting the Mellin-Barnes contour integral in terms of I-function of two variables defined by (1.1) , we obtain the right hand side of (2.1) after little simplifications. This completes proof of ***Theorem* 2.1**.
In view of the relation (1.16), we get the following corollary concerning left-sided Saigo fractional integral operator [24].

***Corollary* 2.1.** *Let* $\alpha,\beta,\gamma,\rho \in C$, $z_1,z_2 \in C$, $Re(\alpha) > 0$, $\mu,\nu \in R_+$. *Further let the constants* $m_1,n_1,p,q \in N_0$, $a_j,b_j,a_{ji},b_{ji} \in C$, $\alpha_j,\beta_j,\alpha_{ji},\beta_{ji} \in R_+(i = 1,\cdots,p_i^{(1)};\ j = 1,\cdots,q_i^{(1)})$, $c_j,d_j,c_{ji},d_{ji} \in C$, $\gamma_j,\delta_j,\gamma_{ji},\delta_{ji} \in R_+(i = 1,\cdots,p_i^{(2)};\ j = 1,\cdots,q_i^{(2)})$, $|arg\, z_1| < \frac{A_i\pi}{2}$, $|arg\, z_2| < \frac{B_i\pi}{2}$, $A_i > 0$, $B_i > 0$ *and satisfy the condition*

$$Re(\rho) + \mu \min_{1\le j\le m_2} Re\left(\frac{b_j}{\beta_j}\right) + \nu \min_{1\le j\le m_3} Re\left(\frac{d_j}{\delta_j}\right) > max\left[0, Re(\beta-\gamma)\right].$$

Then the fractional integration $I_{0+}^{\alpha,\beta,\gamma}$ of the I-function of two variables exists and the following relation holds:

(2.3)
$$\left\{I_{0+}^{\alpha,\beta,\gamma} t^{\rho-1} I_{p,q:Q}^{m_1,n_1:P}\left[\begin{array}{c|c} z_1 t^{\mu} & [(e_p : E_p, E_p^{'})] : U \\ z_2 t^{\nu} & [(f_q : F_q, F_q^{'})] : V \end{array}\right]\right\}(x)$$

$$= x^{\rho-\beta-1} I_{p+2,q+2:Q}^{m_1,n_1+2:P}\left[\begin{array}{c|c} z_1 x^{\mu} & [(1-\rho : \mu,\nu)], [(1-\rho-\gamma+\beta : \mu,\nu)], [(e_p : E_p, E_p^{'})] : U \\ z_2 x^{\nu} & [(f_q : F_q, F_q^{'})], [(1-\rho+\beta : \mu,\nu)], [(1-\rho-\alpha-\gamma : \mu,\nu)] : V \end{array}\right].$$

Now if we set $\beta = -\alpha$ in (2.3), we obtain the following result concerning left-sided Riemann-Liouville fractional integral operator [24].

Corollary 2.2. *Let $\alpha,\rho \in C$, $z_1, z_2 \in C$, $Re(\alpha) > 0$, $\mu,\nu \in R_+$. Further let the constants $m_1, n_1, p, q \in N_0$, $a_j, b_j, a_{ji}, b_{ji} \in C$, $\alpha_j, \beta_j, \alpha_{ji}, \beta_{ji} \in R_+ (i = 1, \cdots, p_i^{(1)};\ j = 1, \cdots, q_i^{(1)})$, $c_j, d_j, c_{ji}, d_{ji} \in C$, $\gamma_j, \delta_j, \gamma_{ji}, \delta_{ji} \in R_+ (i = 1, \cdots, p_i^{(2)};\ j = 1, \cdots, q_i^{(2)})$, $|arg\, z_1| < \frac{A_i \pi}{2}$, $|arg\, z_2| < \frac{B_i \pi}{2}$, $A_i > 0$, $B_i > 0$ and satisfy the condition*

$$Re(\rho) + \mu \min_{1\le j\le m_2} Re\left(\frac{b_j}{\beta_j}\right) + \nu \min_{1\le j\le m_3} Re\left(\frac{d_j}{\delta_j}\right) > 0.$$

Then the fractional integration I_{0+}^{α} of the I-function of two variables exists and the following relation holds:

(2.4)
$$\left\{I_{0+}^{\alpha} t^{\rho-1} I_{p,q:Q}^{m_1,n_1:P}\left[\begin{array}{c|c} z_1 t^{\mu} & [(e_p : E_p, E_p^{'})] : U \\ z_2 t^{\nu} & [(f_q : F_q, F_q^{'})] : V \end{array}\right]\right\}(x)$$

$$= x^{\rho-\alpha-1} I_{p+1,q+1:Q}^{m_1,n_1+1:P}\left[\begin{array}{c|c} z_1 x^{\mu} & [(1-\rho : \mu,\nu)], [(e_p : E_p, E_p^{'})] : U \\ z_2 x^{\nu} & [(f_q : F_q, F_q^{'})], [(1-\rho-\alpha : \mu,\nu)] : V \end{array}\right].$$

Theorem 2.2. *Let $\alpha, \alpha', \beta, \beta', \gamma, \rho \in C$, $z_1, z_2 \in C$, $Re(\gamma) > 0$, $\mu,\nu \in R_+$. Further let the constants $m_1, n_1, p, q \in N_0$, $a_j, b_j, a_{ji}, b_{ji} \in C$, $\alpha_j, \beta_j, \alpha_{ji}, \beta_{ji} \in R_+ (i = 1, \cdots, p_i^{(1)};\ j = 1, \cdots, q_i^{(1)})$, $c_j, d_j, c_{ji}, d_{ji} \in C$, $\gamma_j, \delta_j, \gamma_{ji}, \delta_{ji} \in R_+ (i = 1, \cdots, p_i^{(2)};\ j = 1, \cdots, q_i^{(2)})$, $|arg\, z_1| < \frac{A_i \pi}{2}$, $|arg\, z_2| < \frac{B_i \pi}{2}$, $A_i > 0$, $B_i > 0$ and satisfy the condition*

$$Re(\rho) + \mu \max_{1\le j\le n_2}\left[\frac{Re(a_j)-1}{\alpha_j}\right] + \nu \max_{1\le j\le n_3}\left[\frac{Re(c_j)-1}{\gamma_j}\right] < 1 + min\left[Re(-\beta), Re(\alpha+\alpha'-\gamma), Re(\alpha+\beta'-\gamma)\right]$$

Then the fractional integration $I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}$ of the I-function of two variables exists and the following relation holds:

(2.5)
$$\left\{I_{-}^{\alpha,\alpha',\beta,\beta',\gamma} t^{\rho-1} I_{p,q:Q}^{m_1,n_1:P}\left[\begin{array}{c|c} z_1 t^{\mu} & [(e_p : E_p, E_p^{'})] : U \\ z_2 t^{\nu} & [(f_q : F_q, F_q^{'})] : V \end{array}\right]\right\}(x)$$

$$= x^{\rho-\alpha-\alpha^{'}+\gamma-1} I_{p+3,q+3:Q}^{m_1+3,n_1:P}\left[\begin{array}{c|c} z_1 x^{\mu} & [(e_p : E_p, E_p^{'})], X_3 : U \\ z_2 x^{\nu} & X_4, [(f_q : F_q, F_q^{'})] : V \end{array}\right],$$

where

$$X_3 = [(1-\rho : \mu,\nu)], [(1-\rho+\alpha+\alpha^{'}+\beta'-\gamma : \mu,\nu)], [(1-\rho+\alpha-\beta : \mu,\nu)],$$
$$X_4 = [(1-\rho+\alpha+\alpha^{'}-\gamma : \mu,\nu)], [(1-\rho+\alpha+\beta'-\gamma : \mu,\nu)], [(1-\rho-\beta : \mu,\nu)].$$

Proof. In order to prove (2.5), we first express I-function of two variables occurring on the left hand side of (2.5) in terms of Mellin-Barnes contour integral with the help of equation (1.1) and interchanging the order of integration, which is justified under the conditions stated with the ***Theorem***, we obtain (say I_2):

$$I_2 = \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2} \phi_1(\xi)\,\phi_2(\eta)\,\psi(\xi,\eta)\,z_1^{\xi}\,z_2^{\eta}\,(I_-^{\alpha,\alpha',\beta,\beta',\gamma}\, t^{\rho+\mu\xi+\nu\eta-1})(x)\,d\xi\,d\eta, \tag{2.6}$$

Now by applying ***Lemma*** 1.2, we arrive at

$$I_2 = x^{\rho-\alpha-\alpha'+\gamma-1}\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2} \phi_1(\xi)\,\phi_2(\eta)\,\psi(\xi,\eta)\,(z_1x^{\mu})^{\xi}\,(z_2x^{\nu})^{\eta}$$
$$\times\frac{\Gamma(1-\rho+\alpha+\alpha'-\gamma-\mu\xi-\nu\eta)\Gamma(1-\rho+\alpha+\beta'-\gamma-\mu\xi-\nu\eta)\Gamma(1-\rho-\beta-\mu\xi-\nu\eta)}{\Gamma(1-\rho-\mu\xi-\nu\eta)\Gamma(1-\rho+\alpha+\alpha'+\beta'-\gamma-\mu\xi-\nu\eta)\Gamma(1-\rho+\alpha-\beta-\mu\xi-\nu\eta)}\,d\xi\,d\eta.$$

By re-interpreting the Mellin-Barnes contour integral in terms of I-function of two variables defined by (1.1) , we obtain the right hand side of (2.5) after little simplifications. This completes proof of ***Theorem 2.2.***
In view of the relation (1.17), we get following corollary concerning right-sided Saigo fractional integral operator [24].

Corollary 2.3. *Let $\alpha,\beta,\gamma,\rho \in C$, $z_1,z_2 \in C$, $Re(\alpha) > 0$, $\mu,\nu \in R_+$. Further let the constants $m_1,n_1,p,q \in N_0$, $a_j,b_j,a_{ji},b_{ji} \in C$, $\alpha_j,\beta_j,\alpha_{ji},\beta_{ji} \in R_+(i = 1,\cdots,p_i^{(1)};\ j = 1,\cdots,q_i^{(1)})$, $c_j,d_j,c_{ji},d_{ji} \in C$, $\gamma_j,\delta_j,\gamma_{ji},\delta_{ji} \in R_+(i = 1,\cdots,p_i^{(2)};\ j = 1,\cdots,q_i^{(2)})$, $|arg\, z_1| < \frac{A_i\pi}{2}$, $|arg\, z_2| < \frac{B_i\pi}{2}$, $A_i > 0$, $B_i > 0$ and satisfy the condition*

$$Re(\rho) + \mu\max_{1\le j\le n_2}\left[\frac{Re(a_j)-1}{\alpha_j}\right] + \nu\max_{1\le j\le n_3}\left[\frac{Re(c_j)-1}{\gamma_j}\right] < 1 + min\left[Re(\beta), Re(\gamma)\right].$$

Then the fractional integration $I_-^{\alpha,\beta,\gamma}$ of the I-function of two variables exists and the following relation holds:

$$\left\{I_-^{\alpha,\beta,\gamma} t^{\rho-1} I_{p,q:Q}^{m_1,n_1:P}\left[\begin{array}{c} z_1t^{\mu} \\ z_2t^{\nu}\end{array}\middle|\begin{array}{c}[(e_p : E_p, E_p^{'})] : U \\ [(f_q : F_q, F_q^{'})] : V\end{array}\right]\right\}(x) \tag{2.7}$$

$$= x^{\rho-\beta-1} I_{p+2,q+2:Q}^{m_1+2,n_1:P}\left[\begin{array}{c} z_1x^{\mu} \\ z_2x^{\nu}\end{array}\middle|\begin{array}{c}[(e_p : E_p, E_p^{'})], [(1-\rho:\mu,\nu)], [(1-\rho+\gamma:\mu,\nu)], : U \\ [(1-\rho+\beta:\mu,\nu)], [(1-\rho+\gamma:\mu,\nu)][(f_q : F_q, F_q^{'})] : V\end{array}\right].$$

Further, if we set $\beta = -\alpha$ in (2.7), we get following corollary concerning right-sided Riemann Liouville fractional integral operator [24].

Corollary 2.4. *Let $\alpha,\rho \in C$, $z_1,z_2 \in C$, $Re(\alpha) > 0$, $\mu,\nu \in R_+$. Further let the constants $m_1,n_1,p,q \in N_0$, $a_j,b_j,a_{ji},b_{ji} \in C$, $\alpha_j,\beta_j,\alpha_{ji},\beta_{ji} \in R_+(i = 1,\cdots,p_i^{(1)};\ j = 1,\cdots,q_i^{(1)})$, $c_j,d_j,c_{ji},d_{ji} \in C$, $\gamma_j,\delta_j,\gamma_{ji},\delta_{ji} \in R_+(i = 1,\cdots,p_i^{(2)};\ j = 1,\cdots,q_i^{(2)})$, $|arg\, z_1| < \frac{A_i\pi}{2}$, $|arg\, z_2| < \frac{B_i\pi}{2}$, $A_i > 0$, $B_i > 0$ and satisfy the condition*

$$Re(\alpha) + Re(\rho) + \mu\max_{1\le j\le n_2}\left[\frac{Re(a_j)-1}{\alpha_j}\right] + \nu\max_{1\le j\le n_3}\left[\frac{Re(c_j)-1}{\gamma_j}\right] < 1.$$

Then the fractional integration I_-^{α} of the I-function of two variables exists and the following relation holds:

$$\left\{I_-^{\alpha} t^{\rho-1} I_{p,q:Q}^{m_1,n_1:P}\left[\begin{array}{c} z_1t^{\mu} \\ z_2t^{\nu}\end{array}\middle|\begin{array}{c}[(e_p : E_p, E_p^{'})] : U \\ [(f_q : F_q, F_q^{'})] : V\end{array}\right]\right\}(x) \tag{2.8}$$

$$= x^{\rho-\alpha-1} I^{m_1+1,n_1:P}_{p+1,q+1:Q}\left[\begin{array}{c|c} z_1x^{\mu} & [(e_p:E_p,E'_p)],[(1-\rho:\mu,\nu)]:U \\ z_2x^{\nu} & [(1-\rho-\alpha:\mu,\nu)],[(f_q:F_q,F'_q)]:V \end{array}\right].$$

Theorem 2.3. *Let* $\alpha,\alpha',\beta,\beta',\gamma,\rho \in C$, $z_1,z_2 \in C$, $Re(\gamma) > 0$, $\mu,\nu \in R_+$. *Further let the constants* $m_1,n_1,p,q \in N_0$, $a_j,b_j,a_{ji},b_{ji} \in C$, $\alpha_j,\beta_j,\alpha_{ji},\beta_{ji} \in R_+(i=1,\cdots,p_i^{(1)};\ j=1,\cdots,q_i^{(1)})$, $c_j,d_j,c_{ji},d_{ji} \in C$, $\gamma_j,\delta_j,\gamma_{ji},\delta_{ji} \in R_+(i=1,\cdots,p_i^{(2)};\ j=1,\cdots,q_i^{(2)})$, $|arg\, z_1| < \frac{A_i\pi}{2}$, $|arg\, z_2| < \frac{B_i\pi}{2}$, $A_i > 0$, $B_i > 0$ *and satisfy the condition*

$$Re(\rho) + \mu \min_{1\le j\le m_2} Re\left(\frac{b_j}{\beta_j}\right) + \nu \min_{1\le j\le m_3} Re\left(\frac{d_j}{\delta_j}\right) > max\left[0, Re(-\alpha-\alpha'-\beta'+\gamma), Re(\beta-\alpha)\right]$$

Then the fractional derivative $D^{\alpha,\alpha',\beta,\beta',\gamma}_{0+}$ *of the I-function of two variables exists and the following relation holds:*

$$\left\{D^{\alpha,\alpha',\beta,\beta',\gamma}_{0+} t^{\rho-1} I^{m_1,n_1:P}_{p,q:Q}\left[\begin{array}{c|c} z_1t^{\mu} & [(e_p:E_p,E'_p)]:U \\ z_2t^{\nu} & [(f_q:F_q,F'_q)]:V \end{array}\right]\right\}(x) \tag{2.9}$$

$$= x^{\rho+\alpha+\alpha'-\gamma-1} I^{m_1,n_1+3:P}_{p+3,q+3:Q}\left[\begin{array}{c|c} z_1x^{\mu} & X_5,[(e_p:E_p,E'_p)]:U \\ z_2x^{\nu} & [(f_q:F_q,F'_q)],X_6:V \end{array}\right],$$

where

$$X_5 = [(1-\rho:\mu,\nu)],[(1-\rho-\alpha-\alpha'-\beta'+\gamma:\mu,\nu)],[(1-\rho-\alpha+\beta:\mu,\nu)],$$
$$X_6 = [(1-\rho-\alpha-\beta'+\gamma:\mu,\nu)],[(1-\rho+\beta:\mu,\nu)],[(1-\rho-\alpha-\alpha'+\gamma:\mu,\nu)].$$

Proof. To prove the fractional differential formula (2.9) we express I-function of two variables occurring on the left hand side of (2.9) in terms of double Mellin-Barnes contour integral with the help of equations (1.1),we obtain the following form after little simplification:

$$\left\{D^{\alpha,\alpha',\beta,\beta',\gamma}_{0+} t^{\rho-1} I^{m_1,n_1:P}_{p,q:Q}\left[\begin{array}{c|c} z_1t^{\mu} & [(e_p:E_p,E'_p)]:U \\ z_2t^{\nu} & [(f_q:F_q,F'_q)]:V \end{array}\right]\right\}(x) \tag{2.10}$$
$$= \frac{d^k}{dx^k}\left\{I^{-\alpha',-\alpha,-\beta'+k,-\beta,-\gamma+k}_{0+} t^{\rho-1} I^{m_1,n_1:P}_{p,q:Q}\left[\begin{array}{c|c} z_1t^{\mu} & [(e_p:E_p,E'_p)]:U \\ z_2t^{\nu} & [(f_q:F_q,F'_q)]:V \end{array}\right]\right\}(x),$$
$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2} \phi_1(\xi)\,\phi_2(\eta)\,\psi(\xi,\eta)\, z_1^{\xi}\, z_2^{\eta}\, \frac{d^k}{dx^k}(I^{-\alpha',-\alpha,-\beta'+k,-\beta,-\gamma+k}_{0+}\, t^{\rho+\mu\xi+\nu\eta-1})(x)\, d\xi\, d\eta,$$

where $k = [Re(\gamma)+1]$

Applying ***Lemma* 1.1** to (2.10),we obtain

$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2} \phi_1(\xi)\,\phi_2(\eta)\,\psi(\xi,\eta)\, z_1^{\xi}\, z_2^{\eta}$$
$$\times \frac{\Gamma(\rho+\mu\xi+\nu\eta)\Gamma(\rho+\mu\xi+\nu\eta-\gamma+\alpha'+\alpha+\beta')\Gamma(\rho+\mu\xi+\nu\eta-\beta+\alpha)}{\Gamma(\rho+\mu\xi+\nu\eta+\alpha'+\alpha-\gamma+k)\Gamma(\rho+\mu\xi+\nu\eta-\gamma+\alpha+\beta')\Gamma(\rho+\mu\xi+\nu\eta-\beta)}$$
$$\times \frac{d^k}{dx^k} x^{\rho+\mu\xi+\nu\eta+\alpha'+\alpha-\gamma+k-1}\, d\xi\, d\eta,$$

Using $\frac{d^n}{dx^n}x^m = \frac{\Gamma(m+1)}{\Gamma(m-n+1)}x^{m-n}$ where $m \ge n$ in the above expression, we obtain

$$= x^{\rho+\alpha+\alpha'-\gamma-1}\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2} \phi_1(\xi)\,\phi_2(\eta)\,\psi(\xi,\eta)\,(z_1x)^{\xi}\,(z_2x)^{\eta}$$

$$\times\frac{\Gamma(\rho+\mu\xi+\nu\eta)\Gamma(\rho+\mu\xi+\nu\eta-\gamma+\alpha'+\alpha+\beta')\Gamma(\rho+\mu\xi+\nu\eta-\beta+\alpha)}{\Gamma(\rho+\mu\xi+\nu\eta-\gamma+\alpha+\beta')\Gamma(\rho+\mu\xi+\nu\eta-\beta)\Gamma(\rho+\mu\xi+\nu\eta+\alpha'+\alpha-\gamma)}\,d\xi\,d\eta.$$

By re-interpreting the Mellin-Barnes contour integral in terms of I-function of two variables defined by (1.1), we obtain the right hand side of (2.9) after little simplifications. This completes proof of ***Theorem* 2.3**.

In view of the relation(1.18), we get following corollary concerning left-sided Saigo fractional derivative operator [24].

***Corollary* 2.5.** *Let* $\alpha,\beta,\gamma,\rho \in C$, $z_1,z_2 \in C$, $Re(\alpha) > 0$, $\mu,\nu \in R_+$. *Further let the constants* $m_1,n_1,p,q \in N_0$, $a_j,b_j,a_{ji},b_{ji} \in C$, $\alpha_j,\beta_j,\alpha_{ji},\beta_{ji} \in R_+(i = 1,\cdots,p_i^{(1)};\ j = 1,\cdots,q_i^{(1)})$, $c_j,d_j,c_{ji},d_{ji} \in C$, $\gamma_j,\delta_j,\gamma_{ji},\delta_{ji} \in R_+(i = 1,\cdots,p_i^{(2)};\ j = 1,\cdots,q_i^{(2)})$, $|arg\, z_1| < \frac{A_i\pi}{2}$, $|arg\, z_2| < \frac{B_i\pi}{2}$, $A_i > 0$, $B_i > 0$ *and satisfy the condition*

$$Re(\rho)+\mu\min_{1\le j\le m_2} Re\left(\frac{b_j}{\beta_j}\right)+\nu\min_{1\le j\le m_3} Re\left(\frac{d_j}{\delta_j}\right) > max\,[0, Re(-\alpha-\beta-\gamma)].$$

Then the fractional derivative $D_{0+}^{\alpha,\beta,\gamma}$ *of the I-function of two variables exists and the following relation holds:*

$$(2.11)\quad \left\{D_{0+}^{\alpha,\beta,\gamma} t^{\rho-1} I_{p,q:Q}^{m_1,n_1:P}\left[\begin{array}{c} z_1 t^{\mu} \\ z_2 t^{\nu}\end{array}\middle|\begin{array}{c}[(e_p:E_p,E_p')]:U\\ [(f_q:F_q,F_q')]:V\end{array}\right]\right\}(x)$$
$$= x^{\rho+\beta-1} I_{p+2,q+2:Q}^{m_1,n_1+2:P}\left[\begin{array}{c} z_1 x^{\mu} \\ z_2 x^{\nu}\end{array}\middle|\begin{array}{c}[(1-\rho:\mu,\nu)],[(1-\rho-\alpha-\beta-\gamma:\mu,\nu)],[(e_p:E_p,E_p')]:U\\ [(f_q:F_q,F_q')],[(1-\rho-\gamma:\mu,\nu)],[(1-\rho-\beta:\mu,\nu)]:V\end{array}\right].$$

Next, if we set $\beta = -\alpha$ in the above result, we obtain following result concerning left-sided Riemann-Liouville fractional derivative operator [24].

***Corollary* 2.6.** *Let* $\alpha,\rho \in C$, $z_1,z_2 \in C$, $Re(\alpha) > 0$, $\mu,\nu \in R_+$. *Further let the constants* $m_1,n_1,p,q \in N_0$, $a_j,b_j,a_{ji},b_{ji} \in C$, $\alpha_j,\beta_j,\alpha_{ji},\beta_{ji} \in R_+(i = 1,\cdots,p_i^{(1)};\ j = 1,\cdots,q_i^{(1)})$, $c_j,d_j,c_{ji},d_{ji} \in C$, $\gamma_j,\delta_j,\gamma_{ji},\delta_{ji} \in R_+(i = 1,\cdots,p_i^{(2)};\ j = 1,\cdots,q_i^{(2)})$, $|arg\, z_1| < \frac{A_i\pi}{2}$, $|arg\, z_2| < \frac{B_i\pi}{2}$, $A_i > 0$, $B_i > 0$ *and satisfy the condition*

$$Re(\rho)+\mu\min_{1\le j\le m_2} Re\left(\frac{b_j}{\beta_j}\right)+\nu\min_{1\le j\le m_3} Re\left(\frac{d_j}{\delta_j}\right) > 0.$$

Then the fractional derivative D_{0+}^{α} *of the I-function of two variables exists and the following relation holds:*

$$(2.12)\quad \left\{D_{0+}^{\alpha} t^{\rho-1} I_{p,q:Q}^{m_1,n_1:P}\left[\begin{array}{c} z_1 t^{\mu} \\ z_2 t^{\nu}\end{array}\middle|\begin{array}{c}[(e_p:E_p,E_p')]:U\\ [(f_q:F_q,F_q')]:V\end{array}\right]\right\}(x)$$
$$= x^{\rho+\alpha-1} I_{p+1,q+1:Q}^{m_1,n_1+1:P}\left[\begin{array}{c} z_1 x^{\mu} \\ z_2 x^{\nu}\end{array}\middle|\begin{array}{c}[(1-\rho:\mu,\nu)],[(e_p:E_p,E_p')]:U\\ [(f_q:F_q,F_q')],[(1-\rho-\alpha:\mu,\nu)]:V\end{array}\right].$$

***Theorem* 2.4.** *Let* $\alpha,\alpha',\beta,\beta',\gamma,\rho \in C$, $z_1,z_2 \in C$, $Re(\gamma) > 0$, $\mu,\nu \in R_+$. *Further let the constants* $m_1,n_1,p,q \in N_0$, $a_j,b_j,a_{ji},b_{ji} \in C$, $\alpha_j,\beta_j,\alpha_{ji},\beta_{ji} \in R_+(i = 1,\cdots,p_i^{(1)};\ j = 1,\cdots,q_i^{(1)})$, $c_j,d_j,c_{ji},d_{ji} \in C$, $\gamma_j,\delta_j,\gamma_{ji},\delta_{ji} \in R_+(i = 1,\cdots,p_i^{(2)};\ j = 1,\cdots,q_i^{(2)})$, $|arg\, z_1| < \frac{A_i\pi}{2}$, $|arg\, z_2| < \frac{B_i\pi}{2}$, $A_i > 0$, $B_i > 0$ *and satisfy the condition*

$$Re(\rho)+\mu\max_{1\le j\le n_2}\left[\frac{Re(a_j)-1}{\alpha_j}\right]+\nu\max_{1\le j\le n_3}\left[\frac{Re(c_j)-1}{\gamma_j}\right] < 1+min\,[Re(\beta'), Re(-\alpha-\alpha'+\gamma), Re(-\alpha'-\beta+\gamma)].$$

Then the fractional derivative $D_{-}^{\alpha,\alpha',\beta,\beta',\gamma}$ *of the I-function of two variables exists and the following relation holds:*

$$(2.13)\quad \left\{D_{-}^{\alpha,\alpha',\beta,\beta',\gamma} t^{\rho-1} I_{p,q:Q}^{m_1,n_1:P}\left[\begin{array}{c|c} z_1 t^{\mu} & [(e_p : E_p, E_p')] : U \\ z_2 t^{\nu} & [(f_q : F_q, F_q')] : V \end{array}\right]\right\}(x)$$
$$= x^{\rho+\alpha+\alpha'-\gamma-1} I_{p+3,q+3:Q}^{m_1+3,n_1:P}\left[\begin{array}{c|c} z_1 x^{\mu} & [(e_p : E_p, E_p')], X_7 : U \\ z_2 x^{\nu} & X_8, [(f_q : F_q, F_q')] : V \end{array}\right],$$

where

$$X_7 = [(1-\rho : \mu,\nu)], [(1-\rho-\alpha-\alpha'-\beta+\gamma : \mu,\nu)], [(1-\rho-\alpha'+\beta' : \mu,\nu)],$$
$$X_8 = [(1-\rho-\alpha'-\beta+\gamma : \mu,\nu)], [(1-\rho+\beta' : \mu,\nu)], [(1-\rho-\alpha-\alpha'+\gamma : \mu,\nu)].$$

Proof. To prove the fractional differential formula (2.13) we express I-function of two variables occurring on the left hand side of (2.13) in terms of double Mellin-Barnes contour integral with the help of equations (1.1),we obtain the following form after little simplification:

$$(2.14)\quad \left\{D_{-}^{\alpha,\alpha',\beta,\beta',\gamma} t^{\rho-1} I_{p,q:Q}^{m_1,n_1:P}\left[\begin{array}{c|c} z_1 t^{\mu} & [(e_p : E_p, E_p')] : U \\ z_2 t^{\nu} & [(f_q : F_q, F_q')] : V \end{array}\right]\right\}(x)$$
$$= (-1)^k \frac{d^k}{dx^k}\left\{I_{-}^{-\alpha',-\alpha,-\beta',-\beta+k,-\gamma+k} t^{\rho-1} I_{p,q:Q}^{m_1,n_1:P}\left[\begin{array}{c|c} z_1 t^{\mu} & [(e_p : E_p, E_p')] : U \\ z_2 t^{\nu} & [(f_q : F_q, F_q')] : V \end{array}\right]\right\}(x)$$
$$= (-1)^k \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2} \phi_1(\xi)\,\phi_2(\eta)\,\psi(\xi,\eta)\, z_1^{\xi} z_2^{\eta} \frac{d^k}{dx^k}(I_{-}^{-\alpha',-\alpha,-\beta',-\beta+k,-\gamma+k}\, t^{\rho+\mu\xi+\nu\eta-1})(x)\, d\xi\, d\eta,$$

where $k = [Re(\gamma)+1]$

Applying ***Lemma* 1.2** to (2.14),we obtain

$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2} \phi_1(\xi)\,\phi_2(\eta)\,\psi(\xi,\eta)\, z_1^{\xi} z_2^{\eta}$$
$$\times\frac{\Gamma(1-\rho-\alpha-\alpha'+\gamma-k-\mu\xi-\nu\eta)\Gamma(1-\rho-\alpha'-\beta+\gamma-\mu\xi-\nu\eta)\Gamma(1-\rho-\beta'-\mu\xi-\nu\eta)}{\Gamma(1-\rho-\mu\xi-\nu\eta)\Gamma(1-\rho-\alpha-\alpha'-\beta+\gamma-\mu\xi-\nu\eta)\Gamma(1-\rho-\alpha'+\beta'-\mu\xi-\nu\eta)}$$
$$\times(-1)^k \frac{d^k}{dx^k} x^{\rho+\mu\xi+\nu\eta+\alpha'+\alpha-\gamma+k-1}\, d\xi\, d\eta,$$
$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2} \phi_1(\xi)\,\phi_2(\eta)\,\psi(\xi,\eta)\, z_1^{\xi} z_2^{\eta}$$
$$\times\frac{\Gamma(1-\rho-\alpha-\alpha'+\gamma-k-\mu\xi-\nu\eta)\Gamma(1-\rho-\alpha'-\beta+\gamma-\mu\xi-\nu\eta)\Gamma(1-\rho+\beta'-\mu\xi-\nu\eta)}{\Gamma(1-\rho-\mu\xi-\nu\eta)\Gamma(1-\rho-\alpha-\alpha'-\beta+\gamma-\mu\xi-\nu\eta)\Gamma(1-\rho-\alpha'+\beta'-\mu\xi-\nu\eta)}$$
$$\times(1-\rho-\alpha-\alpha'+\gamma-k-\mu\xi-\nu\eta)_k\, x^{\rho+\mu\xi+\nu\eta+\alpha'+\alpha-\gamma-1}\, d\xi\, d\eta,$$
$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2} \phi_1(\xi)\,\phi_2(\eta)\,\psi(\xi,\eta)\, z_1^{\xi} z_2^{\eta}$$
$$\times\frac{\Gamma(1-\rho-\alpha'-\beta+\gamma-\mu\xi-\nu\eta)\Gamma(1-\rho+\beta'-\mu\xi-\nu\eta)\Gamma(1-\rho-\alpha-\alpha'+\gamma-\mu\xi-\nu\eta)}{\Gamma(1-\rho-\mu\xi-\nu\eta)\Gamma(1-\rho-\alpha-\alpha'-\beta+\gamma-\mu\xi-\nu\eta)\Gamma(1-\rho-\alpha'+\beta'-\mu\xi-\nu\eta)}$$
$$\times x^{\rho+\mu\xi+\nu\eta+\alpha'+\alpha-\gamma-1}\, d\xi\, d\eta.$$

By re-interpreting the Mellin-Barnes contour integral in terms of I-function of two variables defined by (1.1) , we obtain the right hand side of (2.13) after little simplifications. This completes proof of ***Theorem* 2.4**.

In view of the relation (1.19), we get following corollary concerning right-sided Saigo fractional derivative operator [24].

***Corollary* 2.7.** *Let* $\alpha,\beta,\gamma,\rho \in C$, $z_1, z_2 \in C$, $Re(\alpha) > 0$, $\mu,\nu \in R_+$. *Further let the constants* $m_1, n_1, p, q \in N_0$, $a_j, b_j, a_{ji}, b_{ji} \in C$, $\alpha_j, \beta_j, \alpha_{ji}, \beta_{ji} \in R_+(i = 1,\cdots,p_i^{(1)};\ j = 1,\cdots,q_i^{(1)})$, $c_j, d_j, c_{ji}, d_{ji} \in C$, $\gamma_j, \delta_j, \gamma_{ji}, \delta_{ji} \in R_+(i = 1,\cdots,p_i^{(2)};\ j = 1,\cdots,q_i^{(2)})$, $|arg\, z_1| < \frac{A_i\pi}{2}$, $|arg\, z_2| < \frac{B_i\pi}{2}$, $A_i > 0$, $B_i > 0$ *and satisfy the condition*

$$Re(\rho) + \mu \max_{1\le j\le n_2}\left[\frac{Re(a_j)-1}{\alpha_j}\right] + \nu \max_{1\le j\le n_3}\left[\frac{Re(c_j)-1}{\gamma_j}\right] < 1 + min\left[Re(-\beta), Re(\alpha+\gamma)\right].$$

Then the fractional derivative $D_-^{\alpha,\beta,\gamma}$ *of the I-function of two variables exists and the following relation holds:*

$$\text{(2.15)}\quad \left\{D_-^{\alpha,\beta,\gamma} t^{\rho-1} I_{p,q:Q}^{m_1,n_1:P}\left[\begin{array}{c|c} z_1 t^{\mu} & [(e_p : E_p, E_p')] : U \\ z_2 t^{\nu} & [(f_q : F_q, F_q')] : V \end{array}\right]\right\}(x)$$

$$= x^{\rho+\beta-1} I_{p+2,q+2:Q}^{m_1+2,n_1:P}\left[\begin{array}{c|c} z_1 x^{\mu} & [(e_p : E_p, E_p')], [(1-\rho : \mu,\nu)], [(1-\rho-\beta+\gamma : \mu,\nu)] : U \\ z_2 x^{\nu} & [(1-\rho+\alpha+\gamma : \mu,\nu)], [(1-\rho-\beta' : \mu,\nu)], [(f_q : F_q, F_q')] : V \end{array}\right].$$

Further, if we set $\beta = -\alpha$ in (2.15), we obtain following corollary concerning right-sided Riemann-Liouville derivative operator [24].

***Corollary* 2.8.** *Let* $\alpha,\rho \in C$, $z_1, z_2 \in C$, $Re(\alpha) > 0$, $\mu,\nu \in R_+$. *Further let the constants* $m_1, n_1, p, q \in N_0$, $a_j, b_j, a_{ji}, b_{ji} \in C$, $\alpha_j, \beta_j, \alpha_{ji}, \beta_{ji} \in R_+(i = 1,\cdots,p_i^{(1)};\ j = 1,\cdots,q_i^{(1)})$, $c_j, d_j, c_{ji}, d_{ji} \in C$, $\gamma_j, \delta_j, \gamma_{ji}, \delta_{ji} \in R_+(i = 1,\cdots,p_i^{(2)};\ j = 1,\cdots,q_i^{(2)})$, $|arg\, z_1| < \frac{A_i\pi}{2}$, $|arg\, z_2| < \frac{B_i\pi}{2}$, $A_i > 0$, $B_i > 0$ *and satisfy the condition*

$$Re(\rho) + Re(\alpha) + \mu \max_{1\le j\le n_2}\left[\frac{Re(a_j)-1}{\alpha_j}\right] + \nu \max_{1\le j\le n_3}\left[\frac{Re(c_j)-1}{\gamma_j}\right] < 0.$$

Then the fractional derivative D_-^{α} *of the I-function of two variables exists and the following relation holds:*

$$\text{(2.16)}\quad \left\{D_-^{\alpha} t^{\rho-1} I_{p,q:Q}^{m_1,n_1:P}\left[\begin{array}{c|c} z_1 t^{\mu} & [(e_p : E_p, E_p')] : U \\ z_2 t^{\nu} & [(f_q : F_q, F_q')] : V \end{array}\right]\right\}(x)$$

$$= x^{\rho+\alpha-1} I_{p+1,q+1:Q}^{m_1+1,n_1:P}\left[\begin{array}{c|c} z_1 x^{\mu} & [(e_p : E_p, E_p')], [(1-\rho : \mu,\nu)] : U \\ z_2 x^{\nu} & [(1-\rho-\alpha : \mu,\nu)], [(f_q : F_q, F_q')] : V \end{array}\right].$$

3 Special Cases

The I-function of two variables is a most generalized form of special functions, consequently it can be reduced in a large number of special functions (or product of such functions) by suitably specializing the parameters involved in the function. Here we provide a few special cases of our main results.

(i) If we set $m_1 = n_1 = p = q = 0$ in ***Theorem* 2.1** then we have following known result given by Saxena *et al* [26], p.637, eq.(3.3) in terms of product of I-function of one variable introduced by Saxena [25].

$$\text{(3.1)}\quad \left\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} t^{\rho-1} I_{p_i^{(1)},q_i^{(1)}:r}^{m_2,n_2}\left[z_1 t^{\mu}\left|\begin{array}{c} (a_j,\alpha_j)_{1,n_2}, (a_{ji},\alpha_{ji})_{n_2+1,p_i^{(1)}} \\ (b_j,\beta_j)_{1,m_2}, (b_{ji},\beta_{ji})_{m_2+1,q_i^{(1)}} \end{array}\right.\right]\right.$$

$$\left.\times I_{p_i^{(2)},q_i^{(2)}:r}^{m_3,n_3}\left[z_2 t^{\nu}\left|\begin{array}{c} (c_j,\gamma_j)_{1,n_3}, (c_{ji},\gamma_{ji})_{n_3+1,p_i^{(2)}} \\ (d_j,\delta_j)_{1,m_3}, (d_{ji},\delta_{ji})_{m_3+1,q_i^{(2)}} \end{array}\right.\right]\right\}(x)$$

$$= x^{\rho+\gamma-\alpha-\alpha'-1} I_{3,3:p_i^{(1)},q_i^{(1)};p_i^{(2)},q_i^{(2)}:r}^{0,3:m_2,n_2;m_3,n_3}\left[\begin{array}{c|c} z_1 x^{\mu} & X_1, \ldots : U \\ z_2 x^{\nu} & X_2, \ldots : V \end{array}\right],$$

where X_1 and X_2 are same as given in ***Theorem* 2.1**. The conditions of validity of the above result easily follow from ***Theorem* 2.1**.

(ii) If we set $m_1 = 0$ and $r = 1$ in ***Theorem* 2.1**, the I-function of two variables occurring in L.H.S. reduces into H-function of two variables [27] then we have following known result given by Dinesh Kumar [11], p.1128, eq.(4.2).

$$\left\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} t^{\rho-1} H^{0,n_1:m_2,n_2;m_3,n_3}_{p,q:p_1^{(1)},q_1^{(1)};p_1^{(2)},q_1^{(2)}}\left[\begin{array}{c|c} z_1 t^{\mu} & [(e_p:E_p,E'_p)]:T_1 \\ z_2 t^{\nu} & [(f_q:F_q,F'_q)]:T_2 \end{array}\right]\right\}(x) \tag{3.2}$$

$$= x^{\rho+\gamma-\alpha-\alpha'-1} H^{0,n_1+3:m_2,n_2;m_3,n_3}_{p+3,q+3:p_1^{(1)},q_1^{(1)};p_1^{(2)},q_1^{(2)}}\left[\begin{array}{c|c} z_1 x^{\mu} & X_1,[(e_p:E_p,E'_p)]:T_1 \\ z_2 x^{\nu} & [(f_q:F_q,F'_q)],X_2:T_2 \end{array}\right],$$

where

$$T_1 = [(a_j,\alpha_j)_{1,p_1^{(1)}}];[(c_j,\gamma_j)_{1,p_1^{(2)}}], \quad T_2 = [(b_j,\beta_j)_{1,q_1^{(1)}}];[(d_j,\delta_j)_{1,q_1^{(2)}}].$$

Also X_1 and X_2 are same as given in ***Theorem* 2.1**. The conditions of validity of the above result easily follow from ***Theorem* 2.1**.

(iii) If we set $m_1 = 0$ and $r = 2$ in ***Theorem* 2.1**, then we obtain a result in terms of a particular case of the I-function of two variables.

$$\left\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} t^{\rho-1} I^{0,n_1:m_2,n_2;m_3,n_3}_{p,q:p_i^{(1)},q_i^{(1)};p_i^{(2)},q_i^{(2)}:2}\left[\begin{array}{c|c} z_1 t^{\mu} & [(e_p:E_p,E'_p)]:U \\ z_2 t^{\nu} & [(f_q:F_q,F'_q)]:V \end{array}\right]\right\}(x) \tag{3.3}$$

$$= x^{\rho+\gamma-\alpha-\alpha'-1} I^{0,n_1+3:m_2,n_2;m_3,n_3}_{p+3,q+3:p_i^{(1)},q_i^{(1)};p_i^{(2)},q_i^{(2)}:2}\left[\begin{array}{c|c} z_1 x^{\mu} & X_1,[(e_p:E_p,E'_p)]:U \\ z_2 x^{\nu} & [(f_q:F_q,F'_q)],X_2:V \end{array}\right],$$

Also X_1 and X_2 are same as given in ***Theorem* 2.1**. The conditions of validity of the above result easily follow from ***Theorem* 2.1**.

(iv) If we set $m_1 = n_1 = p = q = 0$ and $r = 1$ in ***Theorem* 2.1**, then we have following known result given by J. Ram and D. Kumar [21], p.36, eq.(17) in terms of product of H-functions

$$\left\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} t^{\rho-1} H^{m_2,n_2}_{p_1^{(1)},q_1^{(1)}}\left[z_1 t^{\mu}\left|\begin{array}{c} (a_j,\alpha_j)_{1,p_1^{(1)}} \\ (b_j,\beta_j)_{1,q_1^{(1)}} \end{array}\right.\right] \times H^{m_3,n_3}_{p_1^{(2)},q_1^{(2)}}\left[z_2 t^{\nu}\left|\begin{array}{c} (c_j,\gamma_j)_{1,p_1^{(2)}} \\ (d_j,\delta_j)_{1,q_1^{(2)}} \end{array}\right.\right]\right\}(x) \tag{3.4}$$

$$= x^{\rho+\gamma-\alpha-\alpha'-1} H^{0,3:m_2,n_2;m_3,n_3}_{3,3:p_1^{(1)},q_1^{(1)};p_1^{(2)},q_1^{(2)}}\left[\begin{array}{c|c} z_1 x^{\mu} & X_1\ldots:T_1 \\ z_2 x^{\nu} & X_2\ldots:T_2 \end{array}\right],$$

where X_1 and X_2 are same as given in ***Theorem* 2.1**, T_1 and T_2 are also same as given in (3.2). The conditions of validity of the above result easily follow from ***Theorem* 2.1**.

(v) On putting $m_1 = n_1 = p = q = 0, r = 1, \mu = 1, p_1^{(1)} = 0, m_2 = q_1^{(1)} = 1, b_1 = 0$ and $\beta_1 = 1$ in ***Theorem* 2.1** then by virtue of the relation $H^{1,0}_{0,1}[z_1 t|(0,1)] = e^{-z_1 t}$ we have following known result given by Saxena *et al.* [26], p.643, eq.(5.1).

$$\left\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} t^{\rho-1} e^{-z_1 t} H^{m_3,n_3}_{p_1^{(2)},q_1^{(2)}}\left[z_2 t^{\nu}\left|\begin{array}{c} (c_j,\gamma_j)_{1,p_1^{(2)}} \\ (d_j,\delta_j)_{1,q_1^{(2)}} \end{array}\right.\right]\right\}(x) \tag{3.5}$$

$$= x^{\rho+\gamma-\alpha-\alpha'-1} H^{0,3:1,0;m_3,n_3}_{3,3:0,1;p_1^{(2)},q_1^{(2)}} \left[\begin{matrix} z_1 x \\ z_2 x^{\nu} \end{matrix} \middle| \begin{matrix} X_9 ... : -; (c_j, \gamma_j)_{1,p_1^{(2)}} \\ X_{10} ... : (0,1); (d_j, \delta_j)_{1,q_1^{(2)}} \end{matrix} \right],$$

where

$$X_9 = [(1-\rho : 1, \nu)], [(1-\rho+\alpha+\alpha' +\beta-\gamma : 1, \nu)], [(1-\rho+\alpha' -\beta' : 1, \nu)],$$
$$X_{10} = [(1-\rho+\alpha+\alpha' -\gamma : 1, \nu)], [(1-\rho+\alpha' +\beta-\gamma : 1, \nu)], [(1-\rho-\beta' : 1, \nu)].$$

The conditions of validity of the above result easily follow from ***Theorem* 2.1**.
(vi) On setting $z_1 = 0$ in (3.5), we have following result.

(3.6) $$\left\{ I^{\alpha,\alpha',\beta,\beta',\gamma}_{0+} t^{\rho-1} H^{m_3,n_3}_{p_1^{(2)},q_1^{(2)}} \left[z_2 t^{\nu} \middle| \begin{matrix} (c_j, \gamma_j)_{1,p_1^{(2)}} \\ (d_j, \delta_j)_{1,q_1^{(2)}} \end{matrix} \right] \right\}(x)$$

$$= x^{\rho+\gamma-\alpha-\alpha'-1} H^{m_3,n_3+3}_{p_1^{(2)}+3,q_1^{(2)}+3} \left[z_2 x^{\nu} \middle| \begin{matrix} X_{11}, (c_j, \gamma_j)_{1,p_1^{(2)}} \\ (d_j, \delta_j)_{1,q_1^{(2)}}, X_{12} \end{matrix} \right],$$

where

$$X_{11} = [(1-\rho : \nu)], [(1-\rho+\alpha+\alpha' +\beta-\gamma : \nu)], [(1-\rho+\alpha' -\beta' : \nu)],$$
$$X_{12} = [(1-\rho+\alpha+\alpha' -\gamma : \nu)], [(1-\rho+\alpha' +\beta-\gamma : \nu)], [(1-\rho-\beta' : \nu)].$$

The conditions of validity of the above result easily follow from ***Theorem* 2.1.**
(vi) Further on reducing H-function to Wright generalized hypergeometric function in (3.6) due to the relation

$${}_p\psi_q \left[\begin{matrix} (c_1, \gamma_1), \cdots, (c_p, \gamma_p); \\ (d_1, \delta_1), \cdots, (d_q, \delta_q); \end{matrix} \middle| z \right] = \sum_{k=0}^{\infty} \frac{\prod_{j=1}^{p} \Gamma(c_j + \gamma_j k)}{\prod_{j=1}^{q} \Gamma(d_j + \delta_j k)} \frac{z^k}{k!}$$

$$= H^{1,p}_{p,q+1} \left[-z \middle| \begin{matrix} (1-c_1, \gamma_1), \cdots, \left(1-c_p, \gamma_p\right) \\ (0,1), (1-d_1, \delta_1), \cdots, \left(1-d_q, \delta_q\right) \end{matrix} \right].$$

We obtain following result

(3.7) $$\left\{ I^{\alpha,\alpha',\beta,\beta',\gamma}_{0+} t^{\rho-1} {}_p\psi_q \left[z t^{\nu} \middle| \begin{matrix} (c_j, \gamma_j)_{1,p} \\ (d_j, \delta_j)_{1,q} \end{matrix} \right] \right\}(x)$$

$$= x^{\rho+\gamma-\alpha-\alpha'-1} {}_{p+3}\psi_{q+3} \left[z x^{\nu} \middle| \begin{matrix} (c_j, \gamma_j)_{1,p}, X_{13} \\ (d_j, \delta_j)_{1,q}, X_{14} \end{matrix} \right],$$

where

$$X_{13} = (\rho, \nu), (\rho-\alpha-\alpha' -\beta+\gamma, \nu), (\rho-\alpha' +\beta', \nu),$$
$$X_{14} = (\rho-\alpha-\alpha' +\gamma, \nu), (\rho-\alpha' -\beta+\gamma, \nu), (\rho+\beta', \nu).$$

The conditions of validity of the above result easily follow from ***Theorem* 2.1**.
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