

CUBICAL POLYNOMIAL TIME FUNCTION DEMAND RATE AND PARETO TYPE PERISHABLE RATE BASED INVENTORY MODEL WITH PERMISSIBLE DELAY IN PAYMENTS UNDER PARTIAL BACKLOGGING

By

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Abstract

In present paper an inventory model is produced for immediate transient things with cubical polynomial time function demand rate and pareto type perishable rate with permissible delay in payments. Deficiencies are permitted and partially backlogged for the next replenishment cycle. Holding cost is linear function of time. The fundamental motivation of this paper is to examine the retailer's ideal strategy that minimizes the retailer's yearly total cost per unit time under reasonable deferral in instalments inside the EOQ structure. Numerical results are shown for proposed model. Sensitivity investigation of the ideal arrangements regarding various parameters is examined.

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1 Introduction

It is day to day challenge that concerns the need and availability of products relevant to distributors, wholesalers and consumers. On large scale inventory problems can be divided into raw materials, process work and finished products. The basic EOQ model (1915) recognized three categories of costs, such as inventory prices, purchasing/set up cost and keeping/holding/carrying costs. Deterioration of commodities is unavoidable and a growing occurrence of every day life. Deterioration plays significant role in the control of inventories. Instant and non-instant are two categories of deterioration. Stock actions under the permissible delay in payments is another crucial function in actual world scenarios. Partial backlogging is another factor which effect a slight decline in profit if there is waiting for shipment. Significant studies performed by a number of researchers in the field. Goyal [9] established an EOQ model that provided a reasonable pause in payments. Aggarwal and Jaggi [1] developed an EOQ model to achieve optimum order quantity of deteriorating products. Chen and Kang [5] have established an optimized inventory model with a pricing approach focussed on the permitted pause in payments. Prasad and Kansal [17] also established a dealer EOQ model where the manufacturer has an incremental interest rate within the allowable delay in payments. A lot size model for decaying items was provided by Shah [24, 25] in order to assess the optimum specific turnaround duration where the manufacturer only gave a specific extended payments for one duration. Optimum cost and lot size was evaluated by Teng et al.[30] under delay in payments. Shalini Singh [27] developed a procuring strategy of an inventory model for single item and multi supplier with allowing shortages.

An inventory model for deteriorating products with power dependent demand and linear deterioration was discussed by Sharma and Vijay [26]. Mohanty et al.[19] considered random review period and discounts for deteriorating items. An EOQ model for instantaneous items with cubic demand and constant deterioration rate was analyzed by Rangarajan and Karthikeyan [21]. Jaggi et al.[8] had taken ramp type demand to develop inventory model. Incremental holding cost were considered under partial backlogging by Singh et al. [28]. Jain et al.[14] produced an inventory model for a supplier. They assumed stock dependent demand rate for perishable items in inflationary environment. Ideal purchasing strategies carried out in [3, 18] with delay in payments. Chang and Dye [4] produced an EOQ model for deteriorating products with time dependent demand and marginally backlogged. When the ordered quantity is less than the prescribed quantity hypothetical theory developed in [11, 20]. Time dependent demand and perishable rate based EOQ model was designed by Sarkar [23] under permissible delay in payments. Strategies of inventory replenishment were discussed and Goyal's model modified by [6, 7]. Jamal et al. [15, 16] constructed an EOQ model for perishable goods and permissible delay in payments. Salameh et al.[22] discussed the continuous inventory analysis paradigm. Teng [29] revised Goyal's model by recognizing the gap between product price and product expenses. An EOQ model with time dependent perishable cost and constant demand rate was presented by Amutha and Chandrasekaran [2]. Teng et al.[31] extended the existing models by introducing some additional parameters. Geetha and Uthayakumar [32] provided an EOQ model for non-immediate depreciation and allowable delay in payments. Inventory models for decaying goods with delay in payments were established in [12, 13, 25].

In this paper, we develop an inventory model with cubical polynomial time function demand rate, perishable rate is taken pareto type, shortages are allowed and are partially backlogged with permissible delay in payments. Holding cost has been taken linear function of time. Replenishment rate is taken infinite and instantaneous. This paper is structured as follows: some notations and assumptions are mentioned in Section 2. Mathematical formulation and solution of model carried out in Section 3. Numerical investigation by assigning values of parameters is performed in Section 4. In Section 5, we present sensitivity analysis of the developed model by varying parameters. Results and observations are reported in Section 6. In Section 7, consequences of the paper are concluded.

2 Notations and Assumptions

We have used the following postulates to develop this inventory model:

1. Lead time is assigned to zero.
2. The replenishment rate is taken as infinite and instantaneous.
3. The finite planning horizon is reckoned.
4. The demand for the aspect is snatched as a cubical polynomial function of time.
5. Perishable rate is put up within the Pareto type.
6. The supplier didn't have provided the replacement or return strategy. Entities that have terminated will be demolished.
7. Shortages are tolerated and are partially backlogged. The backlogging rate is dangling on the customer's waiting duration for the subsequent replenishment, throughout the stock-out duration i.e. for the negative inventory, the backlogging rate is distinguished as $B(t) = \frac{1}{1+\delta(T-t)}$; $\delta > 0$ denotes the backlogging parameter and $t_1 \leq t \leq T$.

8. During the permissible delay period of payments, the retailer does not have to settle down the account with the supplier. The retailer deposits the generated sales revenue in an interest-bearing account. The supplier starts charging interest as soon as the deadline ends.

The following notations have been used in compiling this inventory model:

$$D(t): \text{The demand rate } D(t) = \begin{cases} a + bt + ct^2 + dt^3, & 0 \leq t \leq t_1 \\ D_0(\text{constant}), & t_1 \leq t \leq T \end{cases}$$

where a =initial demand, b =initial rate of change of demand,

c =acceleration of demand,

d =rate of change of acceleration of demand,

$$\theta(t): \theta(t) = \frac{\theta_1 \theta_2}{1 + \theta_2 t} \text{ (Pareto type),}$$

where $\theta_1 > 0$ and $0 < \theta_2 < 1$,

HC : Holding cost has taken as a linear function of time. $HC = \alpha + \beta t$, $\alpha > 0$, $\beta > 0$,

t_1 : Time to exhaust stock within a replenishment cycle, $0 < t_1 < T$,

T : Length of a replenishment cycle,

M : Permissible delay period,

A : Fixed ordering cost per unit,

$p(t)$: The selling price at time t , $p(0) = p$,

C_b : Unit shortage cost of an item,

C_p : Unit purchasing cost of an item,

C_l : Unit lost sale cost of an item,

I_p : The interest charged per unit of money per year by the supplier, $0 < I_p < 1$,

I_e : The interest earned per unit of money per year, $0 < I_e < 1$.

3 Mathematical Formulation and Solution of the Model

The inventory system for instantaneous deteriorating items with shortages is portrayed in the following **Figure 3.1**

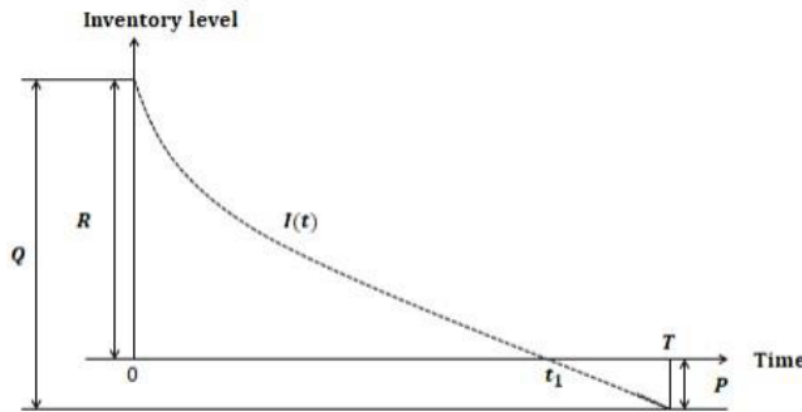


Figure 3.1: Inventory System for Instantaneous Deteriorating Items with Shortages

The instantaneous inventory level $I(t)$ at any time ' t ' during the cycle time $[0, t_1]$ is represented by the following governing differential equation:

$$(3.1) \quad \frac{dI(t)}{dt} + \theta(t)I(t) = -D(t), \quad 0 \leq t \leq t_1$$

$$\Rightarrow \frac{dI(t)}{dt} + \left(\frac{\theta_1 \theta_2}{1 + \theta_2 t} \right) I(t) = -(a + bt + ct^2 + dt^3), 0 \leq t \leq t_1.$$

The solution of above equation (3.1) with boundary condition $I(t_1) = 0$ is

$$(3.2) \quad I(t) = \left[a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{c}{3}(t_1^3 - t^3) + \frac{d}{4}(t_1^4 - t^4) \right. \\ \left. + \frac{a\theta_1\theta_2}{2}(t_1^2 - 2tt_1 + t^2) + \frac{b\theta_1\theta_2}{6}(2t_1^3 - 3tt_1^2 + t^3) \right. \\ \left. + \frac{c\theta_1\theta_2}{12}(3t_1^4 - 4tt_1^3 + t^4) + \frac{d\theta_1\theta_2}{20}(4t_1^5 - 5tt_1^4 + t^5) \right], 0 \leq t \leq t_1.$$

and by using boundary condition $I(0) = R$, we get the maximum positive inventory

$$(3.3) \quad R = \left[\frac{at_1}{2}(2 + \theta_1\theta_2t_1) + \frac{bt_1^2}{6}(3 + 2\theta_1\theta_2t_1) \right. \\ \left. + \frac{ct_1^3}{12}(4 + 3\theta_1\theta_2t_1) + \frac{dt_1^4}{20}(5 + 4\theta_1\theta_2t_1) \right].$$

Partial Backlogging Model

The instantaneous inventory level $I_1(t)$ at any time ' t ' during the shortage period $[t_1, T]$ is represented by the governing differential equation

$$(3.4) \quad \frac{dI_1(t)}{dt} = -\frac{D_0}{1 + \delta(T - t)}, t_1 \leq t \leq T.$$

The solution of above equation (3.4) with boundary condition $I_1(t_1) = 0$ is

$$(3.5) \quad I_1(t) = \frac{D_0}{\delta} \log \left[\frac{1 + \delta(T - t)}{1 + \delta(T - t_1)} \right], t_1 \leq t \leq T.$$

Using boundary condition $-I_1(T) = P$, we get the negative inventory

$$(3.6) \quad -I_1(T) = P = \frac{D_0}{\delta} \log \{1 + \delta(T - t_1)\}.$$

Total inventory, $Q = R + P$

$$(3.7) \quad \Rightarrow Q = \left[\frac{at_1}{2}(2 + \theta_1\theta_2t_1) + \frac{bt_1^2}{6}(3 + 2\theta_1\theta_2t_1) \right. \\ \left. + \frac{ct_1^3}{12}(4 + 3\theta_1\theta_2t_1) + \frac{dt_1^4}{20}(5 + 4\theta_1\theta_2t_1) + \frac{D_0}{\delta} \log \{1 + \delta(T - t_1)\} \right].$$

The total average inventory cost (TC) per cycle consists of the following costs:

(i) Ordering cost per cycle:

$$(3.8) \quad OC = \frac{A}{T}.$$

(ii) Holding cost per cycle:

$$(3.9) \quad HC = \frac{1}{T} \int_0^{t_1} (\alpha + \beta t) I(t) dt \\ = \frac{\alpha}{T} \left\{ \frac{at_1^2}{6}(3 + \theta_1\theta_2t_1) + \frac{bt_1^3}{24}(8 + 3\theta_1\theta_2t_1) + \frac{ct_1^4}{20}(5 + 2\theta_1\theta_2t_1) + \frac{dt_1^5}{60}(12 + 5\theta_1\theta_2t_1) \right\}$$

$$+ \frac{\beta}{T} \left\{ \frac{at_1^3}{24}(4 + \theta_1\theta_2t_1) + \frac{bt_1^4}{120}(15 + 4\theta_1\theta_2t_1) + \frac{ct_1^5}{180}(18 + 5\theta_1\theta_2t_1) + \frac{dt_1^6}{84}(7 + 2\theta_1\theta_2t_1) \right\}.$$

(iii) Shortage cost per cycle:

$$(3.10) \quad \begin{aligned} SC &= \frac{C_b}{T} \int_{t_1}^T [-I_1(t)] dt \\ &= \frac{C_b D_0}{\delta^2 T} [\delta(T - t_1) - \log\{1 + \delta(T - t_1)\}]. \end{aligned}$$

(iv) Perishable cost per cycle:

$$(3.11) \quad \begin{aligned} PC &= \frac{C_p}{T} \left\{ R - \int_0^{t_1} D(t) dt \right\} \\ &= \frac{C_p}{T} \left\{ \frac{a}{2}(\theta_1\theta_2t_1^2) + \frac{b}{3}(\theta_1\theta_2t_1^3) + \frac{c}{4}(\theta_1\theta_2t_1^4) + \frac{d}{5}(\theta_1\theta_2t_1^5) \right\}. \end{aligned}$$

(v) Cost due to lost sales per cycle:

$$(3.12) \quad \begin{aligned} CLS &= \frac{C_l D_0}{T} \int_{t_1}^T \left[1 - \frac{1}{1 + \delta(T - t)} \right] dt \\ &= \frac{C_l D_0}{T} \left[T - t_1 - \frac{1}{\delta} \log\{1 + \delta(T - t_1)\} \right]. \end{aligned}$$

(vi) Interest earned per cycle: ($t_1 < M$)

$$(3.13) \quad \begin{aligned} I_n E &= \frac{pI_e}{T} \int_0^M tD(t) dt \\ &= \frac{pI_e}{T} \left[\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{ct_1^4}{4} + \frac{dt_1^5}{5} + \frac{D_0}{2}(M^2 - t_1^2) \right]. \end{aligned}$$

(vii) Interest payable per cycle: ($t_1 \geq M$)

$$(3.14) \quad \begin{aligned} I_n P_1 &= \frac{C_p I_p}{T} \int_M^{t_1} I(t) dt \\ &= \frac{C_p I_p}{T} \left[\frac{a}{2}(t_1^2 - 2Mt_1 + M^2) + \frac{b}{6}(2t_1^3 - 3Mt_1^2 + M^3) \right. \\ &\quad + \frac{c}{12}(3t_1^4 - 4Mt_1^3 + M^4) + \frac{d}{20}(4t_1^5 - 5Mt_1^4 + M^5) \\ &\quad + \frac{a\theta_1\theta_2}{6}(t_1^3 - 3Mt_1^2 + 3M^2t_1 - M^3) \\ &\quad + \frac{b\theta_1\theta_2}{24}(3t_1^4 - 8Mt_1^3 + 6M^2t_1^2 - M^4) \\ &\quad + \frac{c\theta_1\theta_2}{60}(6t_1^5 - 15Mt_1^4 + 10M^2t_1^3 - M^5) \\ &\quad \left. + \frac{d\theta_1\theta_2}{120}(10t_1^6 - 24Mt_1^5 + 15M^2t_1^4 - M^6) \right]. \end{aligned}$$

Total average inventory cost per cycle,

$$(3.15) \quad TC = OC + HC + PC + SC + CLS + I_n P_1 - I_n E.$$

$$(3.16) \quad TC = \frac{A}{T} + \frac{\alpha}{T} \left\{ \frac{at_1^2}{6}(3 + \theta_1\theta_2t_1) + \frac{bt_1^3}{24}(8 + 3\theta_1\theta_2t_1) + \frac{ct_1^4}{20}(5 + 2\theta_1\theta_2t_1) + \frac{dt_1^5}{60}(12 + 5\theta_1\theta_2t_1) \right\}$$

$$\begin{aligned}
& + \frac{\beta}{T} \left\{ \frac{at_1^3}{24}(4 + \theta_1\theta_2t_1) + \frac{bt_1^4}{120}(15 + 4\theta_1\theta_2t_1) + \frac{ct_1^5}{180}(18 + 5\theta_1\theta_2t_1) + \frac{dt_1^6}{84}(7 + 2\theta_1\theta_2t_1) \right\} \\
& + \frac{C_p}{T} \left\{ \frac{a}{2}(\theta_1\theta_2t_1^2) + \frac{b}{3}(\theta_1\theta_2t_1^3) + \frac{c}{4}(\theta_1\theta_2t_1^4) + \frac{d}{5}(\theta_1\theta_2t_1^5) \right\} \\
& + \frac{C_bD_0}{\delta^2T} [\delta(T - t_1) - \log\{1 + \delta(T - t_1)\}] \\
& + \frac{C_lD_0}{T} \left[T - t_1 - \frac{1}{\delta} \log\{1 + \delta(T - t_1)\} \right] \\
& + \frac{C_pI_p}{T} \left[\frac{a}{2}(t_1^2 - 2Mt_1 + M^2) + \frac{b}{6}(2t_1^3 - 3Mt_1^2 + M^3) \right. \\
& + \frac{c}{12}(3t_1^4 - 4Mt_1^3 + M^4) + \frac{d}{20}(4t_1^5 - 5Mt_1^4 + M^5) \\
& + \frac{a\theta_1\theta_2}{6}(t_1^3 - 3Mt_1^2 + 3M^2t_1 - M^3) \\
& + \frac{b\theta_1\theta_2}{24}(3t_1^4 - 8Mt_1^3 + 6M^2t_1^2 - M^4) \\
& + \frac{c\theta_1\theta_2}{60}(6t_1^5 - 15Mt_1^4 + 10M^2t_1^3 - M^5) \\
& \left. + \frac{d\theta_1\theta_2}{120}(10t_1^6 - 24Mt_1^5 + 15M^2t_1^4 - M^6) \right] \\
& - \frac{pI_e}{T} \left[\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{ct_1^4}{4} + \frac{dt_1^5}{5} + \frac{D_0}{2}(M^2 - t_1^2) \right].
\end{aligned}$$

Our objective is to minimize the total average inventory cost per cycle. Firstly we consider the derivative of TC with respect to the decision variable t_1 i.e. $\frac{d(TC)}{dt_1}$. Setting the derivative equal to zero, we have, $\frac{d(TC)}{dt_1} = 0$.

Secondly we consider the second order derivative of TC with respect to the decision variable t_1 . Provided that TC satisfies the following condition: $\frac{d^2(TC)}{dt_1^2} > 0$.

By solving above non-linear equation by MATLAB software, the value of t_1^* can be obtained and then from Eqns. (3.7) and (3.16), the optimal values of Q^* and TC^* can be found out respectively. Assume suitable values for A , T , a , b , c , d , α , β , p , θ_1 , θ_2 , D_0 , δ , M , C_b , C_l , C_p , I_e and I_p with appropriate units.

4 Numerical Example

Suppose that there is a product of pareto type decreasing function $\theta(t) = \left(\frac{\theta_1\theta_2}{1+\theta_2t}\right)$, where $\theta_1 > 0$ and $0 < \theta_2 < 1$. The parameters of the inventory system are $A = 1000$, $T = 2$, $a = 10$, $b = 12$, $c = 15$, $d = 20$, $\alpha = 5$, $\beta = 0.5$, $p = 50$, $\theta_1 = 0.2$, $\theta_2 = 0.4$, $D_0 = 0.5$, $\delta = 0.5$, $M = 0.5$, $C_b = 2$, $C_l = 4$, $C_p = 5$, $I_e = 0.12$, $I_p = 0.15$.

Under the above given parameters we obtain the optimum solutions $t_1^* = 0.7087$, $TC^* = 499.7216$ and $Q^* = 14.0885$.

5 Sensitivity Analysis

On the basis of the data given in above example the sensitivity analysis is studied by changing the values of parameters a , b , c , d , θ_1 , θ_2 , δ , α , β and M by +50%, +25%, -25% and -50% and supervising the halting parameters at their original values

Table 5.1: Sensitivity Analysis of Partial Backlogging Inventory Model

Changing parameter	Changing %	Optimal values		
		t_1^*	TC^*	Q^*
a	-50 %	0.7003	499.9223	10.1999
	-25 %	0.7049	499.8220	12.1445
	+25 %	0.7120	499.6210	16.0322
	+50 %	0.7148	499.5202	17.9756
b	-50 %	0.7019	499.8175	12.3190
	-25 %	0.7055	499.7696	13.2034
	+25 %	0.7115	499.6734	14.9743
	+50 %	0.7140	499.6251	15.8605
c	-50 %	0.7028	499.7771	12.9799
	-25 %	0.7059	499.7494	13.5335
	+25 %	0.7112	499.6937	14.6447
	+50 %	0.7135	499.6657	15.2018
d	-50 %	0.7032	499.7589	13.2595
	-25 %	0.7061	499.7403	13.6730
	+25 %	0.7111	499.7028	14.5057
	+50 %	0.7133	499.6839	14.9241
δ	-50 %	0.6960	499.5806	13.7148
	-25 %	0.7027	499.6524	13.9103
	+25 %	0.7147	499.7945	14.2725
	+50 %	0.7212	499.8772	14.4804
θ_1	-50 %	1.0409	498.3046	29.5772
	-25 %	0.8457	499.2445	19.3630
	+25 %	0.6091	500.0015	11.0248
	+50 %	0.5341	500.1836	9.0719
θ_2	-50 %	1.0409	498.3046	29.5772
	-25 %	0.8457	499.2445	19.3630
	+25 %	0.6091	500.0015	11.0248
	+50 %	0.5341	500.1836	9.0719
α	-50 %	2.0000	262.7969	183.1858
	-25 %	2.0000	422.3803	183.1858
	+25 %	0.2546	500.7201	3.6986
	+50 %	0.1526	500.8670	2.3513
β	-50 %	0.8028	499.4838	17.6810
	-25 %	0.7508	499.6180	15.6120
	+25 %	0.6737	499.8046	12.9142
	+50 %	0.6440	499.8731	11.9795
M	-50 %	0.4589	500.3020	7.2639
	-25 %	0.5895	500.0489	10.4045
	+25 %	0.8173	499.3249	18.2999
	+50 %	0.9167	498.8731	23.0253

6 Results and Observations

Graphical representations and effect of different parameters on t_1^* , TC^* , Q^* is as follows:

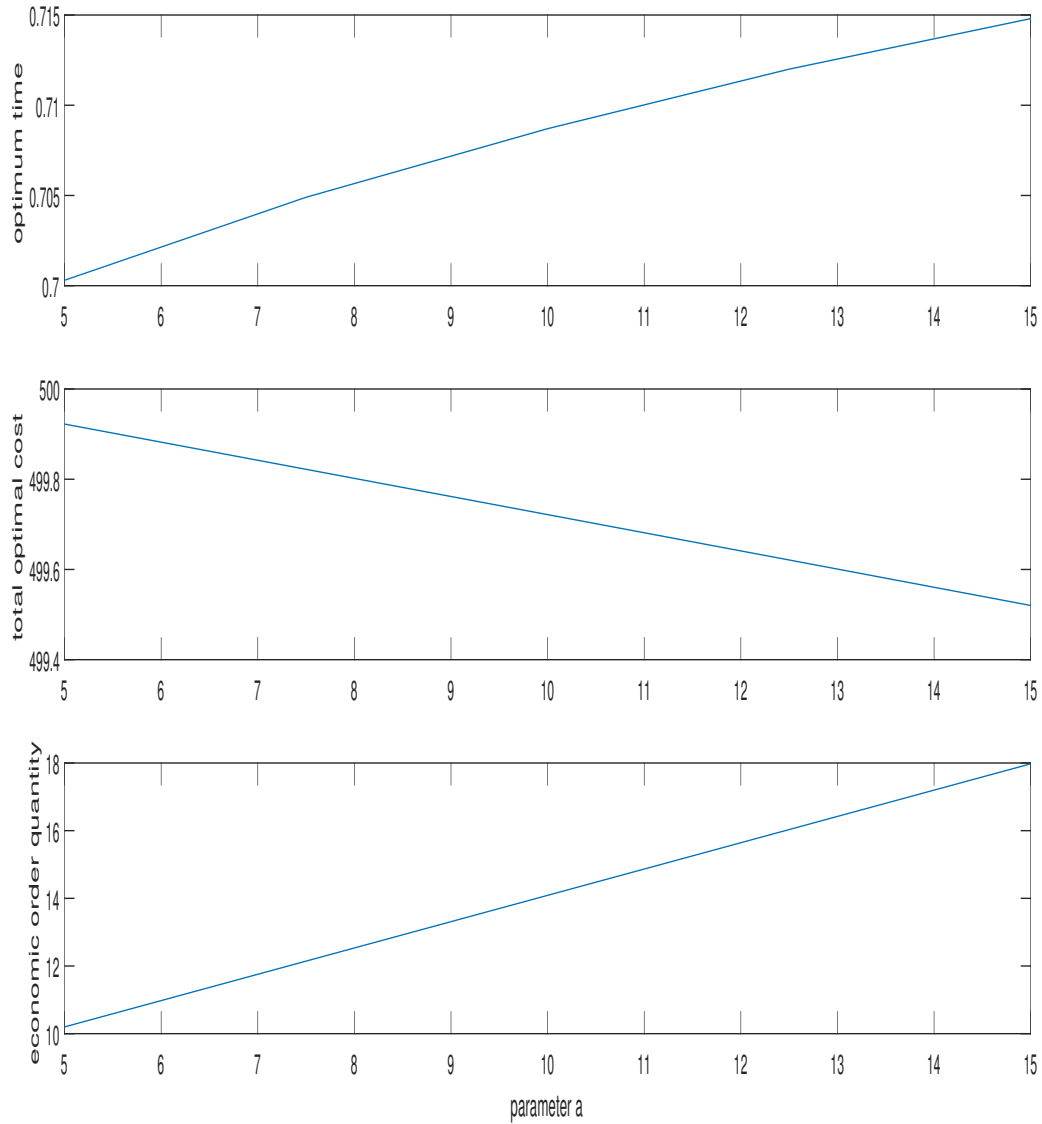


Figure 6.1: graph of percentage change in optimum time, total optimal cost and economic order quantity with respect to parameter a

From above graph we observe that the values of t_1^* and Q^* increase linearly and TC^* decreases linearly with respect to the parameter a .

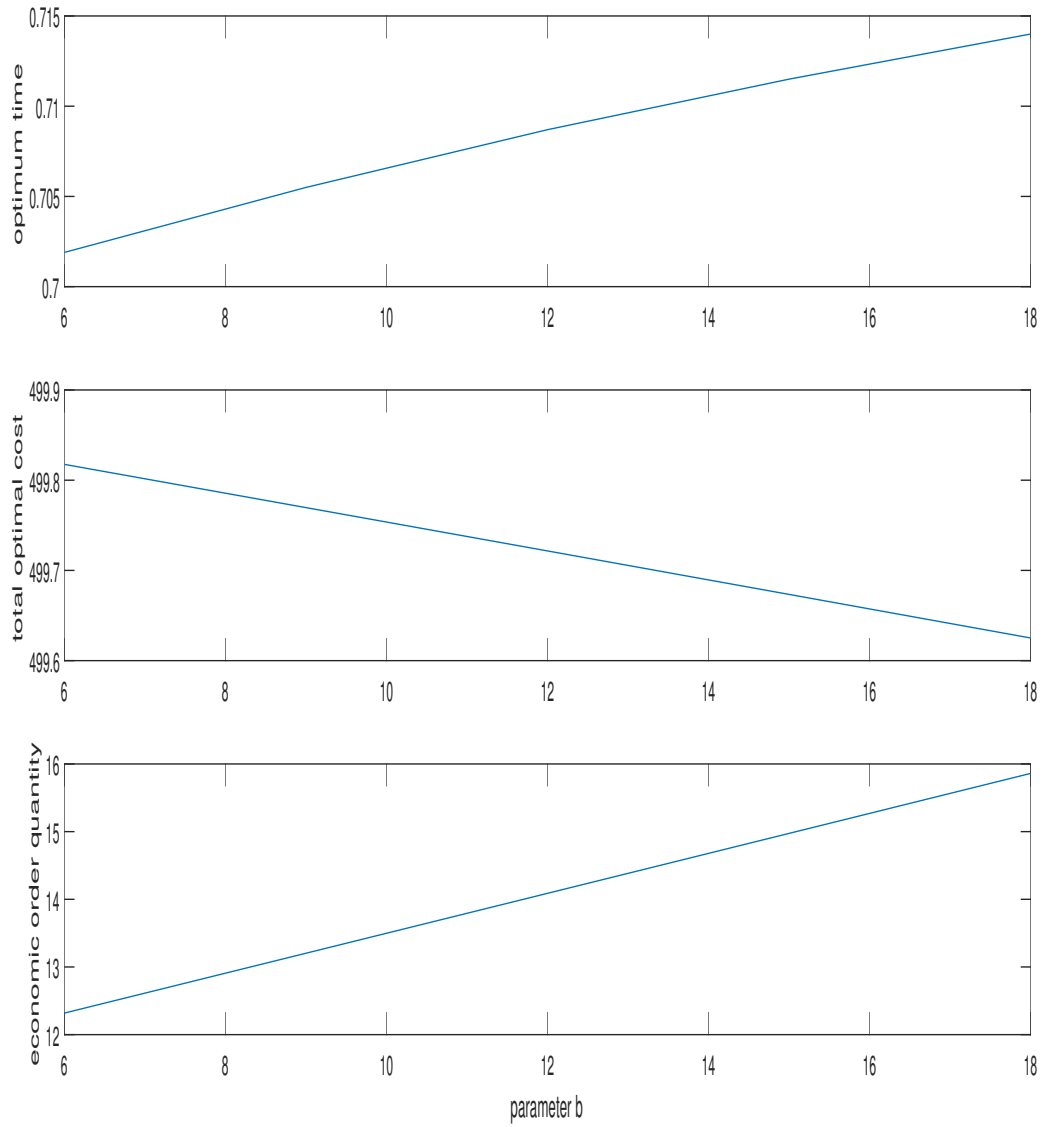


Figure 6.2: graph of percentage change in optimum time, total optimal cost and economic order quantity with respect to parameter b

From above graph we observe that the values of t_1^* and Q^* increase linearly and TC^* decreases linearly with respect to the parameter b .

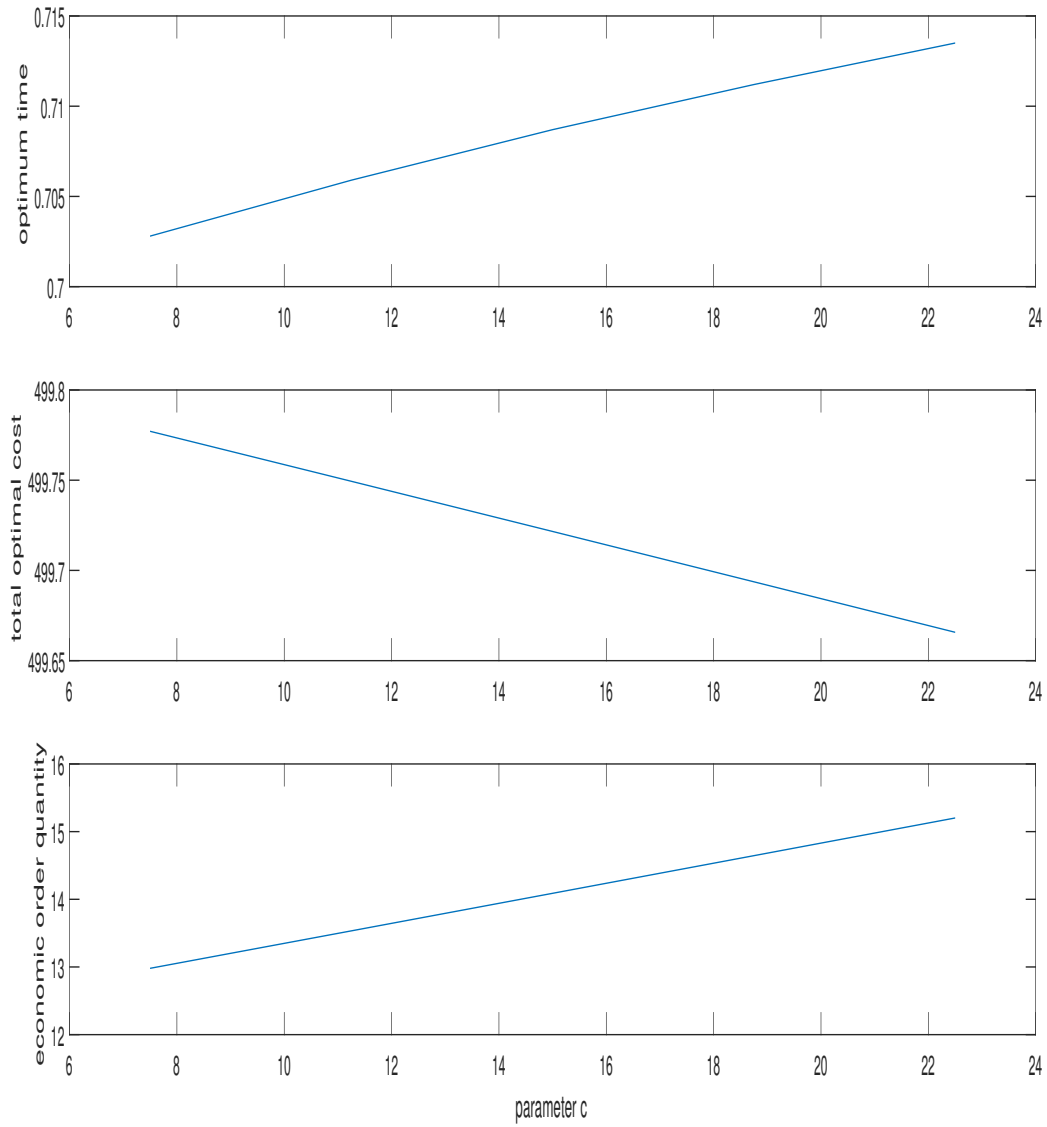


Figure 6.3: graph of percentage change in optimum time, total optimal cost and economic order quantity with respect to parameter c

From above graph we observe that the values of t_1^* and Q^* increase linearly and TC^* decreases linearly with respect to the parameter c .

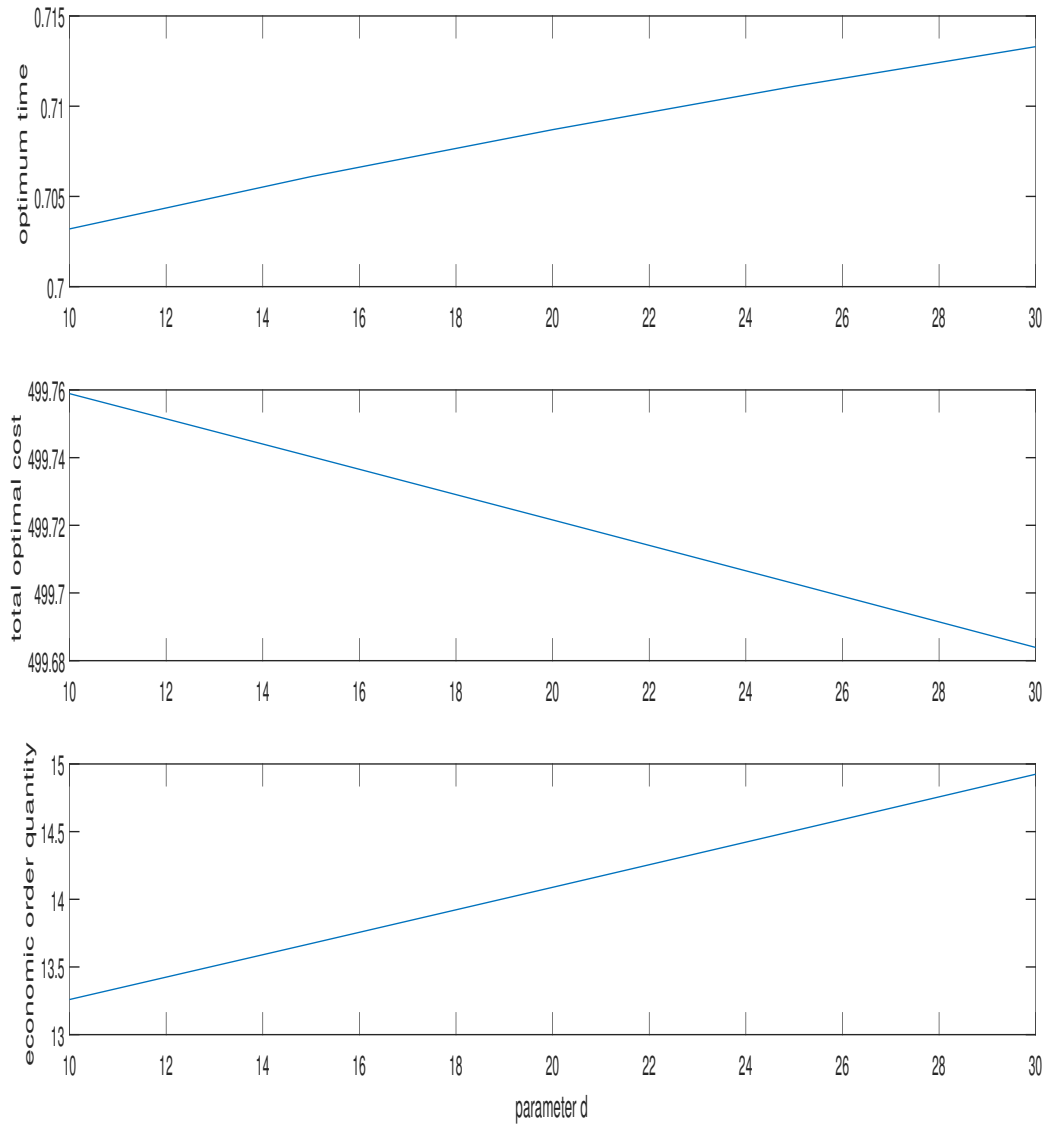


Figure 6.4: graph of percentage change in optimum time, total optimal cost and economic order quantity with respect to parameter d

From above graph we observe that the values of t_1^* and Q^* increase linearly and TC^* decreases linearly with respect to the parameter d .

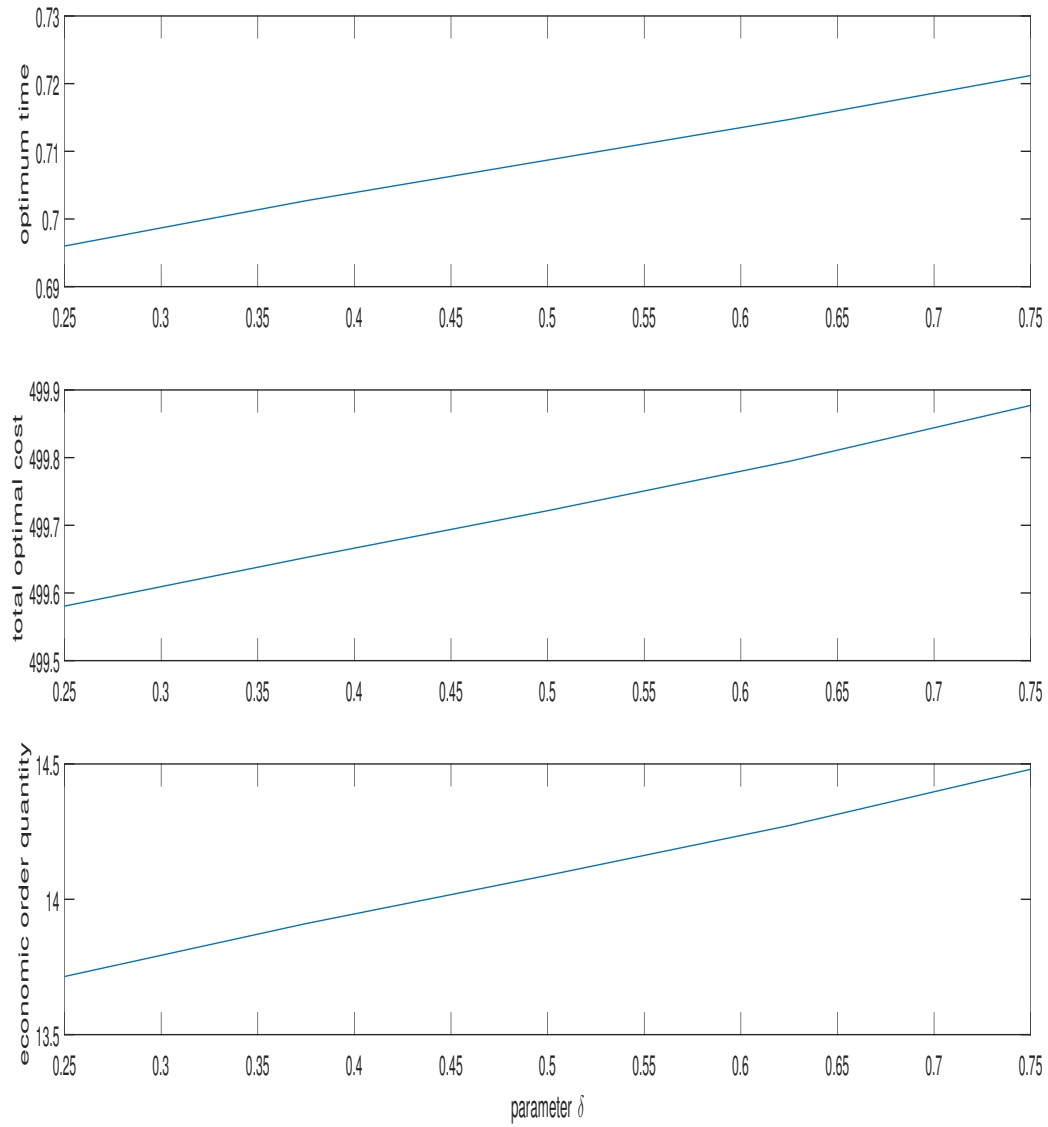


Figure 6.5: graph of percentage change in optimum time, total optimal cost and economic order quantity with respect to parameter δ

From above graph we observe that the values of t_1^* , Q^* and TC^* increase linearly with respect to the parameter δ .

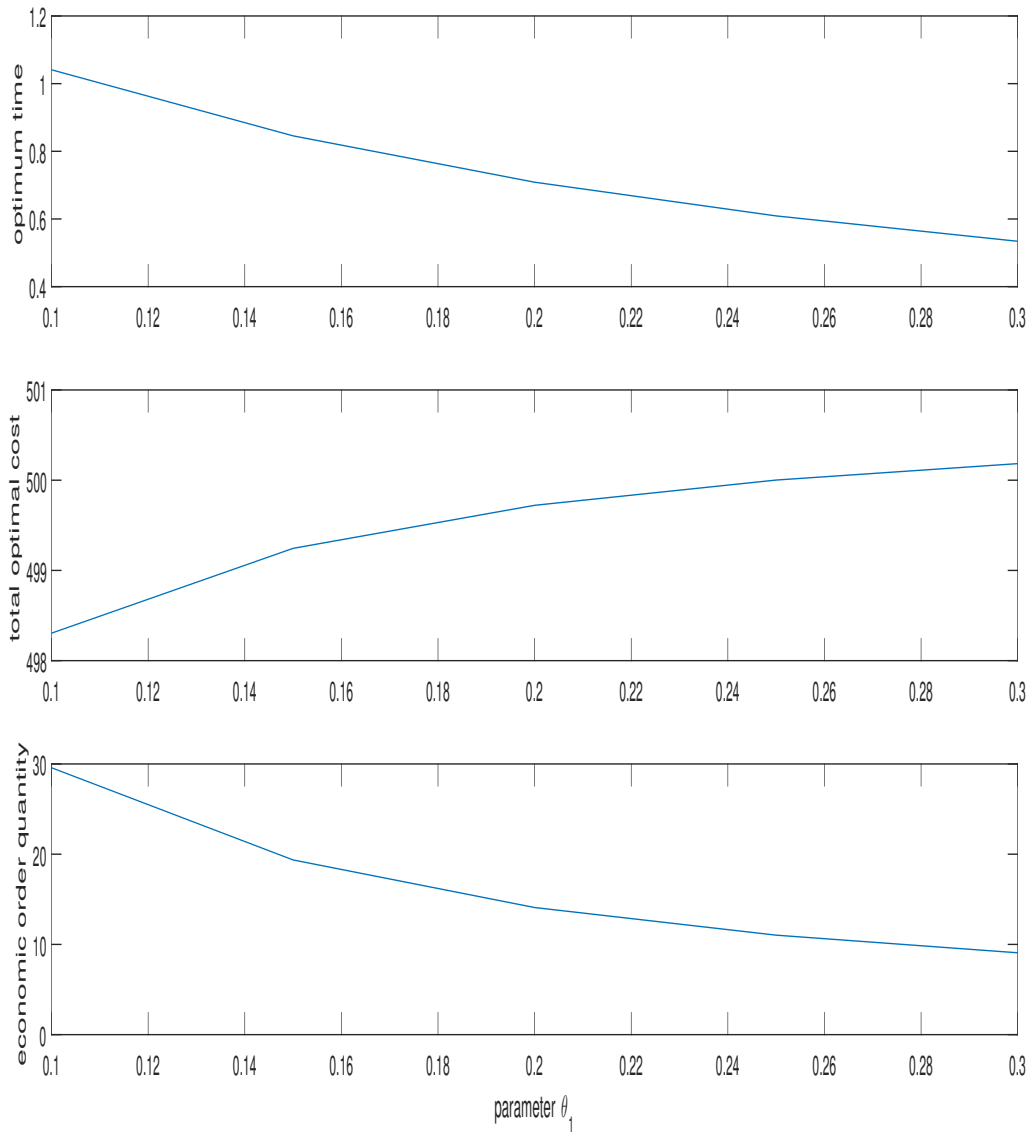


Figure 6.6: graph of percentage change in optimum time, total optimal cost and economic order quantity with respect to parameter θ_1

From above graph we observe that on adjusting the percentage value of the parameter θ_1 , t_1^* decreases approximately linear, TC^* increases initially and after a certain time it goes flat and Q^* decreases initially and after a certain time it goes flat.

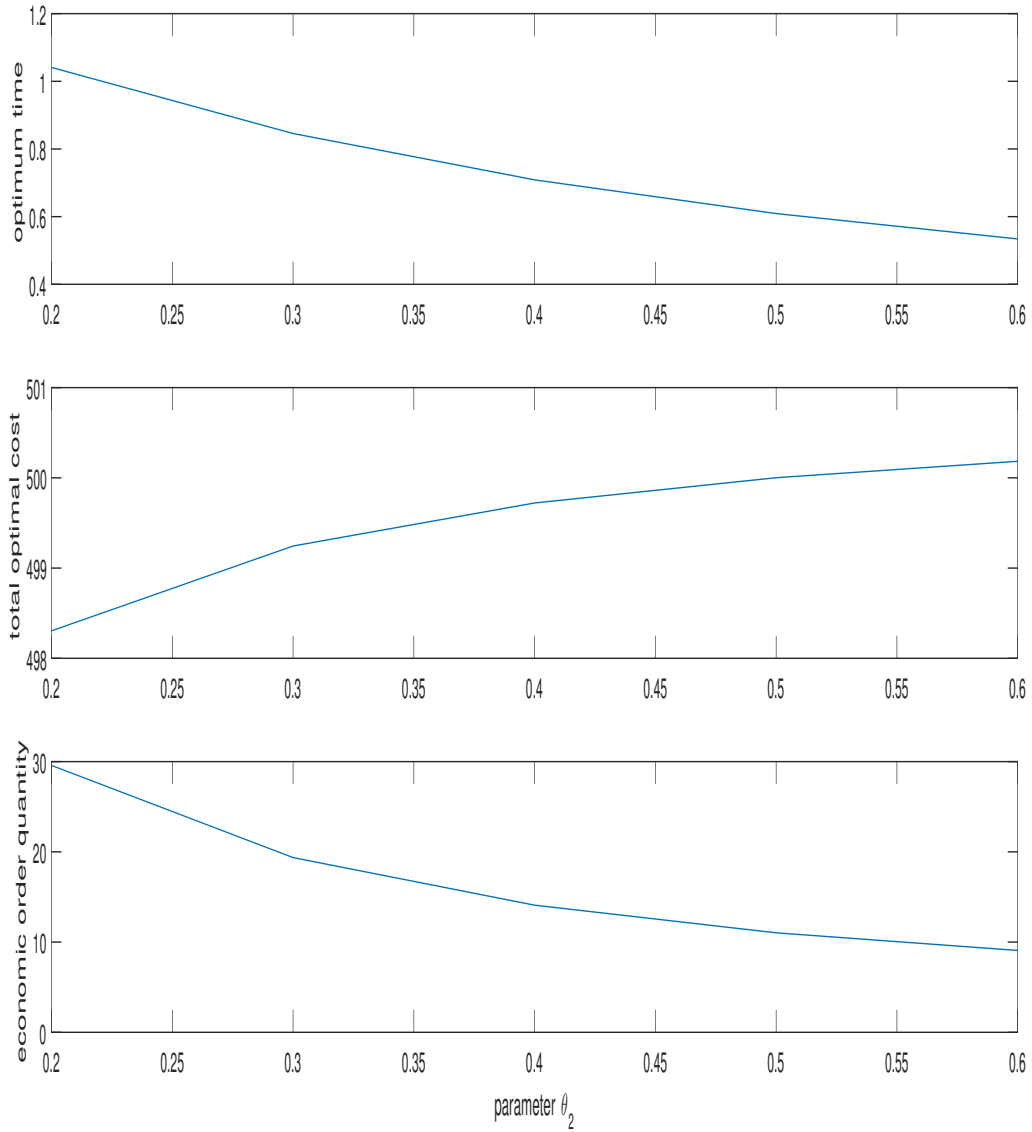


Figure 6.7: graph of percentage change in optimum time, total optimal cost and economic order quantity with respect to parameter θ_2

From above graph we observe that on adjusting the percentage value of the parameter θ_2 , t_1^* decreases approximately linear, TC^* increases initially and after a certain time it goes flat and Q^* decreases initially and after a certain time it goes flat.

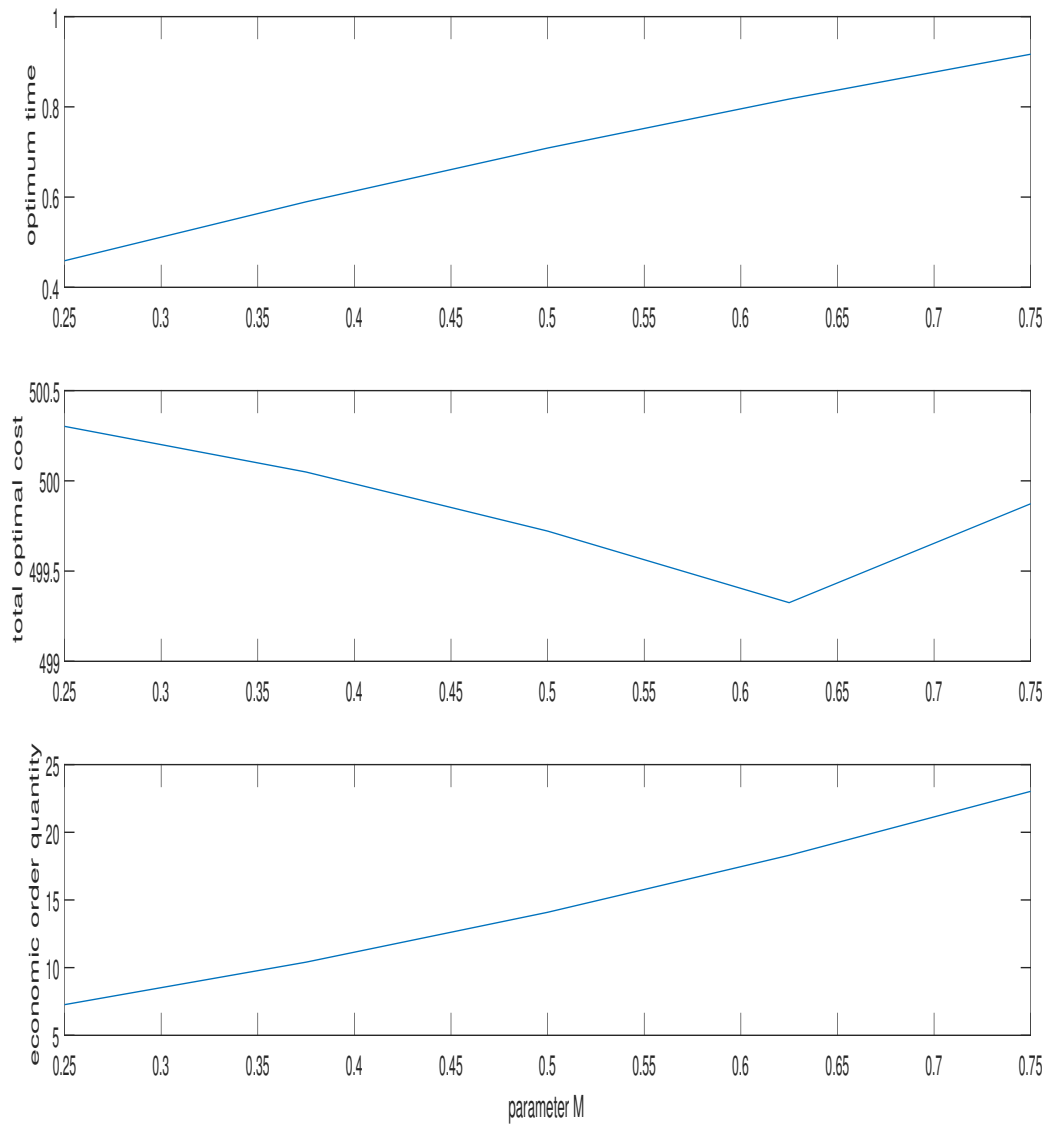


Figure 6.8: graph of percentage change in optimum time, total optimal cost and economic order quantity with respect to parameter M

From above graph we observe that the values of t_1^* and Q^* increase linearly with respect to the parameter M . TC^* decreases linearly and then suddenly it increases with respect to the parameter M .

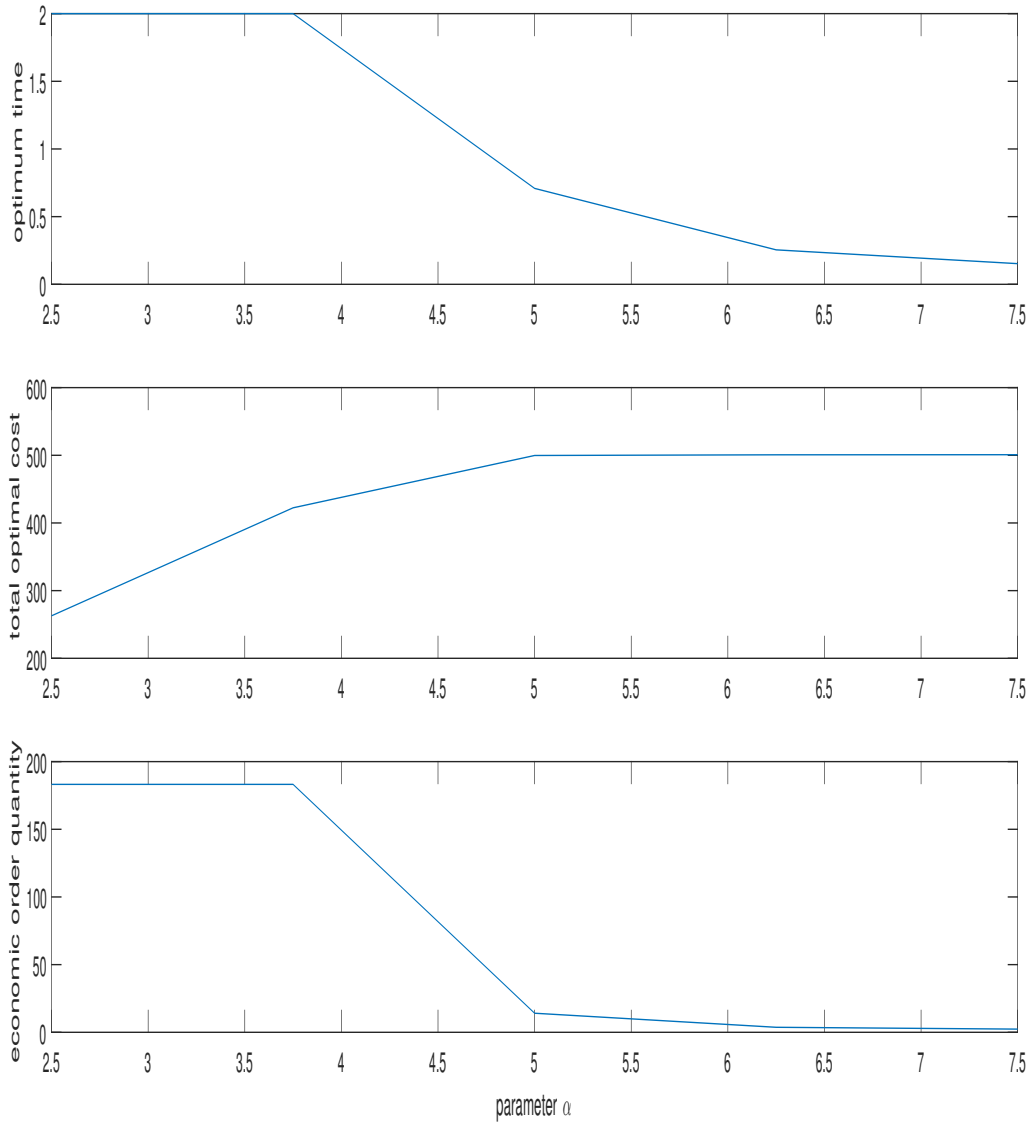


Figure 6.9: graph of percentage change in optimum time, total optimal cost and economic order quantity with respect to parameter α

From above graph we observe that t_1^* decreases with non-constant slop and TC^* increases with non-constant slope with respect to the parameter α . Q^* with respect to α initially remains constant, then decreases linearly and finally goes flat.

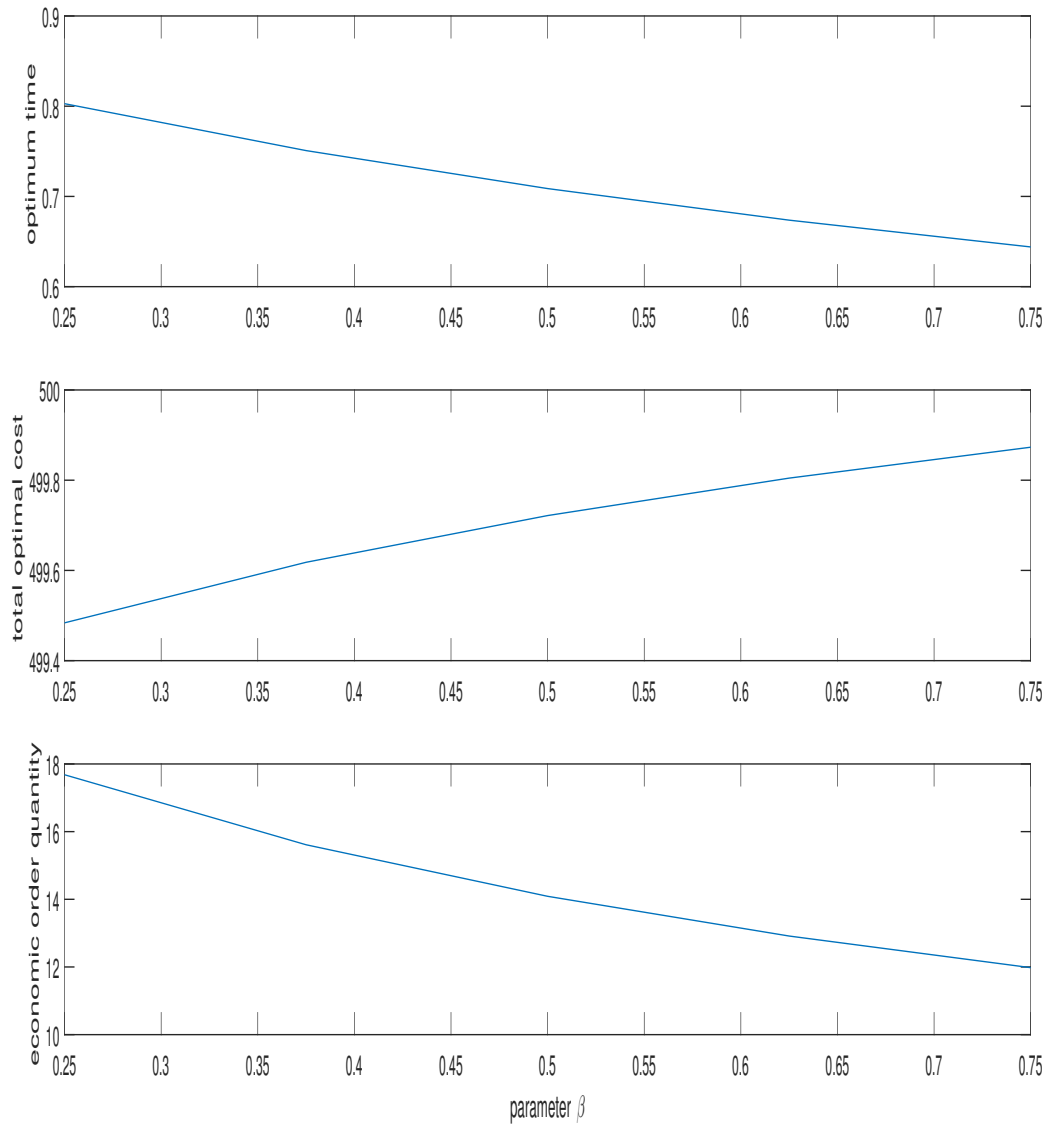


Figure 6.10: graph of percentage change in optimum time, total optimal cost and economic order quantity with respect to parameter β

From above graph we observe that the values of t_1^* and Q^* increase linearly and TC^* decreases linearly with respect to the parameter β .

7 Conclusion

The suggested model offers an effective path for the management of a company enterprise where customer's demand rate is cubical polynomial time function. Pareto type perishable rate is taken. Delay in payment is allowed and shortages are taken partially backlogged. The model is solved analytically by minimizing the total inventory cost. Numerical example of the parameters is also presented to illustrate the model. By sensitivity analysis the decision maker can plan for the optimal value for total cost and for other related parameters. The proposed model can further be extended by taking more realistic assumptions such as probabilistic demand rate, finite replenishment rate

etc.

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References

- [1] S. P. Aggarwal and C. K. Jaggi, Ordering Policies of Deteriorating Items under Conditions of Permissible Delay in Payments, *J. Operat. Res.Soc.*, 46 (1995), 658-662.
- [2] R. Amutha and E. Chandrasekaran, An Inventory Model for Constant Demand with Shortages under Permissible Delay in Payments, *IOSR Journal of Mathematics*, **6(5)** (2013), 28-33.
- [3] B. Bhaula, J. K. Dash and M. R. Kumar, An Optimal Inventory Model for Perishable Items under Succesive Price Discounts with Permissible Delay in Payments, *Operational Research Society of India*, **56** (2019), 261-281.
- [4] H. J. Chang and C. Y. Dye, An Inventorymodel for Deteriorating Items with Partial Backlogging and Permissible Delay in Payments, *International Journal of Systems Science*, **32(3)** (2001), 345-352.
- [5] L. H. Chen and F. S. Kang, Integrated Inventory Models Considering Permissible Delay in Payments and Variant Pricing Strategy, *Applied Mathematical Modelling*, **34(1)** (2010), 36-46.
- [6] K. J. Chung, The Inventory replenishment Policy for Deteriorating Items under Permissible Delay in Payments, *OPSEARCH*, **37** (2000), 267-281.
- [7] K. J. Chung, Y. F. Huang and C. K. Huang, The Replenishment Decision for EOQ Inventory Model under Permissible Delay in Payments, *OPSEARCH*, **39(5-6)** (2002), 327-340.
- [8] C. K. Jaggi, S. Pareek, S. K. Goel and Nidhi, An Inventory Model for Deteriorating Items with Ramp Type Demand under Fuzzy Environment, *International Journal of Logistics Systems and Management*, **22(4)** (2015), 436-463.
- [9] S. K. Goyal, Economic Order Quantity under Conditions of Permissible Delay in Payments, *J. Operat. Res.Soc.*, **36** (1985), 335-338.
- [10] Y. F. Huang, Optimal Retailers Ordering Policies in the EOQ Model under Trade Credit Financing, *Journal of the Operational Research Society*, **54** (2003), 1011-1015.
- [11] Y. F. Huang, Retailers Inventory Policy under Supplier's Partial Trade Credit Policy, *Journal of the Operations Research Society of Japan*, **48(3)** (2005), 173-182.
- [12] Y. F. Huang and K. J. Chung, Optimal Replenishment and Payment Policies in the EOQ Model under Cash Discount and Trade Credit, *Asia-Pacific Journal of Operational Research*, **20** (2003), 177-190.
- [13] H. Hwang and S. W. Shinn, Retailers Pricing and Lot Sizing Policy for Exponentially Deteriorating Product under Condition of Permissible Delay in Payments, *Computer & Operations Research*, **24** (1997), 539-547.
- [14] M. Jain, G. C. Sharma and V. Rani, Cost Analysis for a Supplier in an Inflationary Environment with Stock Dependent Demand Rate for Perishable Items, *Advances in Decision Sciences*, ID457276 (2014), 1-10.
- [15] A. M. Jamal, B. R. Sarker and S. Wang, An Ordering Policy for Deteriorating Items with Allowable Shortage and Permissible Delay in Payments, *Journal of the Operational Research Society*, **48** (1997), 826-833.

- [16] A. M. Jamal, B. R. Sarker and S. Wang, Optimal Payment Time for a Retailer under Permitted Delay of Payment by the Wholesaler, *International Journal of Production Economics*, **66** (2000), 59-66.
- [17] A. Kansal and D. Prasad, An Inventory Model for Deteriorating Items with Linearly Time Varying Demand Rate and Shortages under Inflation, *International Journal of Applied Mathematics and Applications*, **2(2)** (2010), 207-216.
- [18] H.C. Liao, C. H. Tsai and C. T. Su, An Inventory Model for Deteriorating Items under Inflation When a Delay in Payment is Permissible, *International Journal of Production Economics*, **63** (2001), 207-214.
- [19] D. J. Mohanty, R. S. Kumar and A. Goswami, An Improved Inventory Model with Random Review Period and Temporary Price Discount for Deteriorating Items, *International Journal Syst. Assur. Eng. Manag.*, **7(1)** (2016), 62-72.
- [20] M. Pal and S. K. Ghosh, An Inventory Model with Shortage and Quantity Dependent Permissible Delay in Payment, *Aust. Soc. Operat. Res. Bull.*, **25(3)** (2006), 1-12.
- [21] K. Rangarajan and K. Karthikeyan, Analysis of an EOQ Inventory Model for Instantaneous Deteriorating Items with Various Time Dependent Demand Rates, *International Journal of Pure and Applied Mathematics*, **106(7)** (2016), 77-86.
- [22] M. K. Salameh, N .E. Abboud, A. N. El-Kassar and R. E. Ghattas, Continuous Review Inventory Model with Delay in Payments, *International Journal of Production Economics*, **85(1)** (2003), 91-95.
- [23] B. Sarkar, An EOQ Model with Delay in Payments and Time Varying Deterioration Rate, *Mathematical and Computer Modelling*, **55(3-4)** (2012), 367-377.
- [24] N. H. Shah, A Probabilistic Order Level System When Delay in Payments is Permissible, *Journal of the Korean Operations Research and Management Science*, **18(2)** (1993), 175-183.
- [25] N. H. Shah and Y. K. Shah, A Discrete-in-Time Probabilistic Inventory Model for Deteriorating Items under Conditions of Permissible Delay in Payments, *Inter. J. Syst. Sci.*, **29** (1998), 121-126.
- [26] S. C. Sharma and V. Vijay, Replenishment Policy for Deteriorating Inventory System with Power Demand and Backlogging, *Jñānābha*, **47(1)** (2017), 7-16.
- [27] S. Singh, Inventory Model for Single Item-Multi Supplier with Shortages, *Jñānābha*, **48(2)** (2018), 71-80.
- [28] S. Singh, S. Sharma and S. R. Singh, Inventory Model for Deteriorating Items with Incremental Holding Cost under Partial Backlogging, *International Journal of Mathematics in Operations Research*, **15(1)** (2019), 110-126.
- [29] J. T. Teng, On the Economic Order Quantity Under Conditions of Permissible Delay in Payments, *Journal of the Operational Research Society*, **53** (2002), 915-918.
- [30] J.T. Teng, C.T. Chang and S.K. Goyal, Optimal Pricing and Ordering Policy under Permissible Delay in Payments, *International Journal of Production Economics*, **97(2)** (2005), 121-129.
- [31] J. T. Teng, H-L. Yang and L-Y Ouyang, On an EOQ Model for Deteriorating Items with Time-Varying Demand and Partial Backlogging, *Journal of the Operational Research Society*, **54** (2003), 432-436.
- [32] R. Uthayakumar and K.V. Geetha, Replenishment Policy for Single Item Inventory Model with Money Inflation, *OPSEARCH*, **46(3)** (2009), 345-357.