Jñānābha, Vol. 50(1) (2020), 102-114
(Dedicated to Honor Professor H.M. Srivastava on His 80th Birth Anniversary Celebrations)

ON SOFT AND FUZZY SOFT RELATIONS WITH THEIR APPLICATIONS

By
Seema Singh
Department of Statistics, M. D. University, Rohtak-124001, Haryana, India
Email:seemamotsara2@gmail.com
D. S. Hooda
Honorary Professor of Mathematics, G. J. University of Science and Technology, Hisar- 125001, Haryana, India
Corresponding Author D.S. Hooda
Email:ds_hooda@rediffmail.com
S. C. Malik
Department of Statistics, M. D. University, Rohtak-124001, Haryana, India
Email:sc_malik@rediffmail.com
(*Received : March* 08, 2020 *; Revised: May* 05, 2020)
DOI: https://doi.org/10.58250/jnanabha.2020.50112

Abstract

In our day to day life we face many problems which are abstract or vague in nature. These problems cannot be solved only by using simple mathematical tools. To deal with such kind of problems a new technique popularly known as Fuzzy Set Theory was discovered. Fuzzy set is the generalization of crisp set and is used in almost every field of life including Medical Sciences, Business, Administration, Social Science and Operation Research. Later on, a new concept of parameterization of power set of the universal set was introduced. Consequently, Fuzzy Soft Set Theory was defined by embedding Fuzzy Set and Soft Set. In the present communication fuzzy binary relation is described and its applications are studied. The concepts of soft and fuzzy soft relations are also defined with their applications in decision making problems.
2010 Mathematics Subject Classifications: 03E72, 03E75, 62C86, 11M50.
Keywords and phrases: Fuzzy Set, Soft Set, Fuzzy Soft Set, Soft Relation and Fuzzy Soft Relation, Decision making.

1 Introduction

In real life there exist many problems which contain vague or linguistic data and these data cannot be analysed merely by mathematical tools. However, other concepts, like Game Theory, Fuzzy Set, Soft Set, Fuzzy Soft Set and Rough Set, etc. have been developed to deal with such types of imprecise data. Fuzzy Set theory is very popular now these days and so several studies have been carried out in this area during past few years.

L.A. Zadeh [10] defined the concept of Fuzzy Set to handle the imprecise data. Fuzzy Set theory is the extension of crisp set. In addition to this, Rough Set theory was developed by Computer Scientist Pawlak [8] and that was known as Pawlak Rough Set Theory. These theories have been successfully applied to the fields of Decision Making, Dimension Reduction, Data Mining, etc.

Besides the Fuzzy Set Theory, the Soft Set Theory is another mathematical tool to deal with vague data. Soft Set is the parameterization of power set of fuzzy subsets of universal set.

Molodstov [4] was the first to introduce soft set theory for modelling uncertainties. He not only defined fundamental concepts of soft set theory, but also showed how this theory had removed parameterization insufficiency which was existing in the case of game theory, probability theory, rough set and fuzzy set theories. Many models are unique in case of soft set theory. Soft set has its wide applications in the fields of medical and environmental sciences, economic and business management.

Extending soft set theory a new approach known as Fuzzy Soft Set was introduced by Maji et al. [6]. He proposed Fuzzy Soft Set theory by embedding the concept of fuzzy set and soft set. Later on this concept was generalized by Majumdar and Samanta [7] in many ways and frequently used in decision making problems. Recently, Hooda and Kumari [4] have applied this theory in dimension reduction and medical diagnosis.

Aktas and Cagman [1] compared soft sets with the related concepts of fuzzy sets and rough sets. Yang et al. [10] worked on different operations for fuzzy soft sets. Zou and Xiao [11] introduced the soft set and fuzzy soft set into incomplete environment.

Decision making problems are the centre of attraction in every field. Optimum decision must be taken on the basis of uncertain or vague data. Fuzzy set theory and soft set theory are the best technique to handle such situations. Also, there may be the situation where two or more soft sets or fuzzy soft sets are given and we have to form a relation between them. On the basis of that relation decision is taken.

The concepts of soft relation and fuzzy soft relation are applied in forming relations between two attributes, like the relation of weight with height. A person is considered to be fit if his/her weight is within the range prescribed corresponding to his/her height. Otherwise he/she is considered as overweight or underweight according to the situation. The binary relation of height and weight is one of the important studies in health. The study of fuzzy relation was extended to soft and fuzzy soft relations which have found interesting and useful applications in decision making problems and medical diagnosis.

In the present paper basic concepts and definitions are described in Section 2, Section 3 fuzzy binary relation and its applications are studied. Soft and fuzzy soft relations with their applications are discussed in Sections 4 and 5 respectively. The conclusion and future scope are given in Section 6 with the references in the end.

2 Basic Concepts and Definitions

In this section we define some basic concepts and definitions which are used later on development of the paper.

2.1 Fuzzy Set

***Definition* 2.1.** *Let us consider X as a set of universe, then a fuzzy subset "A" of X is defined as a set of ordered pair given by*

$$A = \{< x, \mu_A(x) > / x \epsilon X\} \tag{2.1}$$

where, $\mu_A(x)$ *is called membership function from X to* $[0, 1]$ *with the following properties:*

$$\mu_A(x) = \begin{cases} 0, & \text{if } x \notin A \text{ there is no ambiguity} \\ 1, & x \epsilon A \text{ there is no ambiguity} \\ 0.5 & \text{whether } x \epsilon A \text{ or } x \notin A, \text{ there is maximum ambiguity .} \end{cases} \tag{2.2}$$

***Example* 2.1.** *A possible membership function defined for the set of real number close to* 9 *is*

$$\mu_A(x) = \frac{1}{1 + (x - 9)^2} \ ; (x, \mu_A(x)) \epsilon R^2. \tag{2.3}$$

Here, number 7 *is assigned a membership value* 0.2*,* 11 *is assigned a membership value* 0.2*,* 21 *is assigned a membership value* 0.0068 *and for* 9 *membership value is* 1*. Thus the set A given by* $A = \{(7, 0.2), (11, 0.2), (21, 0.0068), (9, 1)\}$ *is a fuzzy set.*

2.2 Soft Set

***Definition* 2.2.** *Soft set is the extension of fuzzy set theory proposed by Molodstov* [5] *to handle uncertainty of the non-probabilistic approach.*

Let's consider 'U' as the universal set and T be the parametric set, then the pair (π, T) is defined as soft set over U iff π is a function of T into power set of U i.e. $\pi : T \to P(U)$.

***Example* 2.2.** *Assuming $S = \{s_1, s_2, s_3, s_4, s_5\}$ be the set of scooters under study and let 'T' be the parametric set given as $T = \{t_1 = expensive, t_2 = beautiful, t_3 = cheap, t_4 = in\ good\ repair, t_5 = latest\}$. Then the soft set is given by (π, T) describe the attractiveness of the scooters and given by*

$$(\pi, T) = \begin{cases} expensive = s_1, s_2, s_3, \\ beautiful = s_3, s_5, \\ cheap = s_3, s_4, s_5 \\ in\ good\ repair = s_1, s_2, s_3, s_4, s_5 \\ latest = s_1, s_3, s_4, s_5. \end{cases} \tag{2.4}$$

***Table* 2.1:** Tabular representation of (π, T)

X/E	t_1	t_2	t_3	t_4	t_5
s_1	1	0	0	1	1
s_2	1	0	0	1	0
s_3	0	1	1	1	1
s_4	0	0	1	1	1
s_5	1	1	1	1	1

2.3 Fuzzy Soft Set

***Definition* 2.3.** [6] *Fuzzy soft set was defined was by Maji et al. as a hybridization of soft set and fuzzy set. Let X be a universal set and E be the set of parameters and $A \subset E$. Let $F(x)$ be the set of all fuzzy subsets of X, then the pair (F, A) is called fuzzy soft subset of X, where F is a mapping from A to fuzzy set $F(x)$.*

***Example* 2.3.** *Let $X = \{c_1, c_2, c_3\}$ be set of* 3 *cars and $E = \{costly(e_1), getup(e_2), colour(e_3)\}$ be the set of parameters and let $A = \{e_1, e_2\} \epsilon E$. Then,*

$$(F, A) = \begin{cases} F(e_1) = \{c_1/0.6, c_2/0.4, c_3/0.4\} \\ F(e_2) = \{c_1/0.6, c_2/0.3, c_3/0.8\}, \end{cases} \tag{2.5}$$

(F, A) is a Fuzzy Soft Set over U to give the "attractiveness of cars".

3 Fuzzy Relations

A relation is a subset of $X \times Y$, where X and Y are crisp sets, where fuzzy relation is a fuzzy subset of $X \times Y$ i.e. a mapping from X to Y. There are many applications of fuzzy relations.

***Definition* 3.1.** [2] *A fuzzy relation is as a fuzzy set defined on Cartesian product of crisp set* $(X_1, X_2, , X_n)$ *with membership grade* $(x_1, x_2, , x_n)$. *The membership grade indicates the strength of the relation present between the elements of the tuples.*

A fuzzy relation can also be conveniently represented by n-dimensional membership array whose entries correspond to n-tuples in the universal set. These entries take values representing the membership grades of the corresponding n-tuples. In other words, an n- dimensional fuzzy relation R is a fuzzy set of Cartesian product $(X_1 \times X_2 \times \cdots \times X_n)$, *where* $(X_1, X_2, , X_n)$ *are domain.*

Definition* 3.2.** [2] ***Binary Fuzzy Relation
When fuzzy relation is taken over only two crisp sets i.e. between X and Y is known as binary fuzzy relation.

***Example* 3.1.** *Let* $X = (a, b, c)$ *and* $Y = (x, y)$, *then binary fuzzy relation* 'R' *on* $X \times Y$ *is given in* ***Table* 3.1.**

***Table* 3.1**

R	x	y
a	0.6	1.0
b	0.3	0.5
c	0.4	0.2

Definition* 3.3. *Ternary Binary Relation

When fuzzy relation is taken over three crisp sets i.e. between X, Y and Z is known as Ternary Binary Relation.

***Example* 3.2.** *Let* $X = (x_1, x_2, x_3)$, $Y = (y_1, y_2)$ *and* $Z = (z_1, z_2)$, *then fuzzy relation* $X \times Y \times Z$ *is given as*

$$R = \frac{0.21}{< x_1, y_1, z_1 >} + \frac{0.38}{< x_2, y_2, z_1 >} + \frac{0.9}{< x_1, y_2, z_2 >}. \tag{3.1}$$

The tabular representation of the fuzzy relation R is given in ***Tables* 3.2** *and* **3.3**.

***Table* 3.2**

R	y_1	y_2
x_1	0	0.9
x_2	0	0
x_3	0	0

***Table* 3.3**

R	y_1	y_2
x_1	0.21	0
x_2	0	0.38
x_3	0	0

Definition* 3.4. *Height of a Fuzzy Relation
It is a number denoted by $h(R)$, *where R is a fuzzy relation given as:*

$$h(R) = \max_{x \epsilon X, y \epsilon Y} R(x, y) \tag{3.2}$$

i.e. it is the largest membership grade attained be any pair (x, y) *in fuzzy relation R. If* $h(R) = 1$, *then it is a normal fuzzy relation*

Definition* 3.5. *Inverse of a Fuzzy Relation
Inverse of a fuzzy relation $R(x, y)$ is denoted by $R^{-1}(x, y)$, a fuzzy relation over $Y \times X$ is given as

$$R^{-1}(x, y) = R(x, y); x\epsilon X, y\epsilon Y. \tag{3.3}$$

Definition* 3.6. *Max-Min Composition of Two Fuzzy Relations
Let us consider R be a binary fuzzy relation on $X \times Y$ and S be a binary fuzzy relation over $Y \times Z$. Then, max-min composition of R followed by S is a binary fuzzy relation on $X \times Z$. It is denoted by $S \circ R$, given by

$$(S \circ R)(x, z) = \max[\min(R(x, y), R(y, z))], \tag{3.4}$$

where max is taken over all y in Y.

***Example* 3.3.** *Consider the fuzzy relation 'R' on $X \times Y$ and 'S' on $Y \times Z$. X, Y and Z are given as: $X = (a, b, c), Y = (d, e, f)$ and $Z = (\star, \#)$. Fuzzy relations 'R' and 'S' in matrix form are given in the following* ***Tables* 3.4** *and* **3.5** *respectively:*

***Table* 3.4**

R	d	e	F
a	1.0	0.4	0.5
b	0.3	0.0	0.4
c	0.6	0.3	0.2

***Table* 3.5**

S	$*$	#
d	0.7	0.1
e	0.2	0.9
f	0.3	0.4

Then the composition $S \circ R$ is defined and that is also a fuzzy relation over $X \times Z$ described as follows:

$$\begin{aligned}
S \circ R(a, \star) &= \max[\min(1, 0.7), \min(0.4, 0.2), \min(0.5, 0.3)] \\
&= max[0.7, 0.2, 0.3] \\
&= 0.7 \\
S \circ R(a, \#) &= \max[\min(1, 0.1), \min(0.4, 0.9), \min(0.5, 0.4)] \\
&= \max[0.1, 0.4, 0.4] \\
&= 0.4 \\
S \circ R(b, \star) &= \max[\min(0.3, 0.7), \min(0.0, 0.2), \min(0.4, 0.3)] \\
&= \max[0.3, 0.0, 0.3] \\
&= 0.3 \\
S \circ R(b, \#) &= \max[\min(0.3, 0.1), \min(0.0, 0.9), \min(0.4, 0.4)] \\
&= \max[0.1, 0.0, 0.4] \\
&= 0.4 \\
S \circ R(c, \star) &= \max[\min(0.6, 0.7), \min(0.3, 0.2), \min(0.2, 0.3)] \\
&= max[0.6, 0.2, 0.2] \\
&= 0.6 \\
S \circ R(c, \#) &= \max[\min(0.6, 0.1), \min(0.3, 0.9), min(0.2, 0.4)] \\
&= \max[0.1, 0.3, 0.2] = 0.3.
\end{aligned}$$

Thus, $S \circ R$ in matrix form is given as

0.7	0.4
0.3	0.4
0.6	0.3

Definition* 3.7. *Max Product Composition *If R and S are the fuzzy relation over $X \times Y$ and $Y \times Z$ respectively, then max-product composition of R followed by S is given as*

$$(S \circ R)(x, z) = \max[R(x, y) * S(z, y)] \tag{3.5}$$

where '' is the ordinary product of real numbers and max is taken over all elements y in Y.*

***Definition* 3.8.** *Let we consider R and S two fuzzy relations over $X \times Y$ and $Y \times Z$ respectively given in following Tables*

***Table* 3.6:** Fuzzy Relation R

0.3	0.5	0.8
0.0	0.7	1.0
0.4	0.6	0.5

***Table* 3.7:** Fuzzy Relation S

0.9	0.5	0.7	0.7
0.3	0.2	0.0	0.9
1.0	0.0	0.5	0.5

Then the max-product composition of $S \circ R$ is given in the ***Table* 3.8** *given below:*

***Table* 3.8:** $S \circ R$

0.8	0.15	0.4	0.45
1.0	0.14	0.5	0.63
0.5	0.20	0.28	0.54

4 Soft Relation and its Application

In this section the concept of soft relation is introduced and its application in decision making is studied with an example.

***Definition* 4.1.** *Let us consider U and V as two initial universal sets and E be the parametric set and let (F, E) and (G, E) be two soft set over U and V respectively, then (H, E) is a soft relation between (F, E) and (G, E) over $U \times V$ if*

$$H : E \to 2^{U \times V},$$

where H is a mapping such that

$$H(e) = \begin{cases} (u_i, v_j); & \text{if } u_i \epsilon F(e) \text{ and } v_j \epsilon G(e) \ \forall\ e \epsilon E \\ \phi; & \text{otherwise} \end{cases} \tag{4.1}$$

***Example* 4.1.** *Let $U = (h_1, h_2, h_3, h_4)$ be the set of four houses and let $V = (f_1, f_2, f_3)$, be the set of three farm houses. Also, let 'E' be the parametric set namely*

$E = (e_1(green\ surrounding), e_2(cheap), e_3(wooden))$. Then the soft sets (F, E) and (G, E) over U and V are given by:

$(F, E) = F(e_1) = (h_1, h_3), F(e_2) = (h_2, h_4), F(e_3) = (h_1, h_2)$ and

$(G,E) = G(e_1) = (f_1, f_3), G(e_2) = (f_1, f_2), G(e_3) = (f_2, f_3)$.
Then, the soft relation (H,E) between (F,E) and (G,E) is given as
$H(e_1) = (h_1, f_1), (h_1, f_3), (h_3, f_1), (h_3, f_3)$,
$H(e_2) = (h_2, f_1), (h_2, f_2), (h_4, f_1), (h_4, f_2)$ *and*
$H(e_3) = (h_1, f_2), (h_1, f_3), (h_2, f_2), (h_2, f_3)$.

The tabular representation of soft sets (F, E) *and* (G, E) *are given in* ***Tables* 4.1** *and* **4.2**.

***Table* 4.1:** Soft Set (F, E)

U	e_1	e_2	e_3
h_1	1	0	1
h_2	0	1	1
h_3	1	0	0
h_4	0	1	0

***Table* 4.2:** Soft Set (G, E)

V	e_1	e_2	e_3
f_1	1	1	0
f_2	0	1	1
f_3	1	0	1

The soft relation (H,E) is given in the following Table:

***Table* 4.3:** Soft Relation (H, E)

$U \times V$	e_1	e_2	e_3
(h_1, f_1)	1	0	0
(h_1, f_2)	0	0	1
(h_1, f_3)	1	0	1
(h_2, f_1)	0	1	0
(h_2, f_2)	0	1	1
(h_2, f_3)	0	0	1
(h_3, f_1)	1	0	0
(h_3, f_2)	0	0	0
(h_3, f_3)	1	0	0
(h_4, f_1)	0	1	0
(h_4, f_2)	0	1	0
(h_4, f_3)	0	0	0

4.1 Properties of Soft Relation

Let (H_1, E) and (H_2, E) be the two soft relations between (F, E) and (G, E) over $U \times V$, then the following results hold:

(a) Union of two soft relations is also a soft relation, i.e., $(H, E) = (H_1, E) \cup (H_2, E)$ such that $H(e) = H_1(e) \cup H_2(e) \;\; ; \;\; \forall\, e\, \epsilon E$.
(b) Intersection of soft relation is also a soft relation, i.e., $(K, E) = (H_1, E) \cap (H_2, E)$; such that $K(e) = H_1(e) \cap H_2(e) \;\; ; \;\; \forall e \,\epsilon\, E$.
(c) Complement of soft relation is also a soft relation. Let us consider $(F, E)^C$ and $(G, E)^C$ as the complement of soft set (F, E) and (G, E). Then soft relation between $(F, E)^C$ and $(G, E)^C$ is given by
$H^C = \rceil E \rightarrow 2^{U \times V}$, where H^C is a mapping such that

$$H^C(\rceil, e) = (u_i, v_j), \text{ where } u_i \,\epsilon\, F(\rceil e) \text{ and } v_j\; G(\rceil e), \;\forall\, e\, \epsilon E.$$

The symbol "⌉" stands for "not in".

(d) Composition of soft relation is also a soft relation. Let $(F,E),(G,E)$ and (H,E) are three soft sets over U,V and W respectively. Also let (K_1,E) and (K_2,E) are soft relation between $(F,E)\&(G,E)$ and $(G,E)\&(H,E)$ over $U\times V$ and $V\times W$ respectively. Then composition of (K_1,E) and (K_2,E) is also a soft relation over $U\times W$ given as $K:E\to 2^{U\times W}$: such that

$$K(e)=\{(u,v);\exists v\epsilon V, u\epsilon U \text{ and } w\epsilon W, \text{ also } (u,v)\epsilon(K_1,E) \text{ and } (v,w)\epsilon(K_2,E)\}$$

(e) A soft relation is reflexive iff $(u_i,u_i)\epsilon H(e)\ \forall e\epsilon E$ and$u_i\epsilon U$.

(f) A soft relation is symmetric iff $(u_i,u_j)\epsilon H(e)$ and $(u_j,u_j)\in H(e)\ \forall\ e\epsilon E$ and $u_i,u_j,u_k\epsilon U$

(g) A soft relation is transitive iff $(u_i,u_j)\epsilon H(e)$ and $(u_j,u_i)\in H(e)\ \forall\ e\epsilon E$ and $u_i,u_j\epsilon U$

(h) A soft relation is soft tolerance relation if it is reflexive and symmetric.

(i) A soft relation is soft equivalence relation if it is reflexive, symmetric and transitive.

4.2 Application of Soft Relation in Decision Making Problem

Here we discuss the application of soft relation in decision making problem. Suppose $B=b_1,b_2,b_3,b_4$ is the set of four boys and $G=g_1,g_2,g_3$ is the set of three girls who play badminton. From these players a pair of boy and girl is to be chosen for sponsorship. Also, let 'E' be the set of parameters to judge the capability of a badminton players. E is given as $=e_1$ (physical fitness), e_2 (average matches win), e_3 (judgement capability), e_4 (average matches played), e_5 (height).

Suppose Mr X is interested to sponsor a mixed pair of badminton player on the basis of his choice of parameters.

$A=e_1$ (physical fitness), e_2 (average matches win), e_3 (judgement capability).

4.2.1 Soft Set Formation

Let us consider soft sets (F,A) and (H,A) over B and G respectively, given as

$$(F,A)=F(e_1)=(b_1,b_2),F(e_2)=(b_1,b_2,b_4),F(e_3)=(b_1,b_3)$$
$$(H,A)=H(e_1)=(g_1,g_3),H(e_2)=(g_1,g_2),H(e_3)=(g_1,g_2,g_3)$$

Tabular representation of soft sets (F,A) and (H,A) are given in the following tables respectively.

***Table* 4.4:** Soft Set (F,A)

B	e_1	e_2	e_3
b_1	1	1	1
b_2	1	1	0
b_3	0	0	1
b_4	0	1	0

***Table* 4.5:** Soft Set (H,A)

B	e_1	e_2	e_3
g_1	1	1	1
g_2	0	1	1
g_3	1	0	1

4.2.2 Algorithm for Selection of Mixed Double Players using Soft Relation

(i) Firstly, input the soft set (F,A) and (H,A).

(ii) Secondly, construct the soft relation (I,A) table using soft sets (F,A) and (H,A) w.r.t the choice of the paramters of Mr X.

(iii) Then compute the choice value $r_{i,j}$ i.e $r_{i,j}=\sum_{e\epsilon E}(b_i,g_j)$ for the soft relation (I,A).

(iv) Find $m=\max\limits_{1\le i\le 4,1\le j\le 3} r_{i,j}$,

If two or more values of 'm' are same then Mr X can choose any one of them by his opinion.

As we already construct the soft set(F,A) and (H,A) in ***Tables* 4.4** and **4.5**. Now we construct the soft relation (I,A) over $B\times G$ and then compute the choice value$r_{i,j}$. By applying ***Algorithm* 4.2.2**, we get the following soft relation as given below:

Table **4.6:** Soft Relation over $B \times G$

$B \times G$	e_1	e_2	e_3	Choice value ($r_{i,j}$)
(b_1, g_1)	1	1	1	3
(b_1, g_2)	0	1	1	2
(b_1, g_3)	1	0	1	2
(b_2, g_1)	1	1	0	2
(b_2, g_2)	0	1	0	1
(b_2, g_3)	1	0	0	1
(b_3, g_1)	0	0	1	1
(b_3, g_2)	0	0	1	1
(b_3, g_3)	1	0	1	1
(b_4, g_1)	0	1	0	1
(b_4, g_2)	0	1	0	1
(b_4, g_3)	0	0	0	0

Here maximum value of $r_{ij} = 3$ given by r_{11} for pair (b_1, g_1). Hence, Mr X will sponsor the pair (b_1, g_1) for mixed double badminton game

5 Fuzzy Soft Relation

In this section we shall define fuzzy soft relation with its application in decision making problem by considering example.

***Definition* 5.1.** *Let U and V be two initial universal sets and E be the set of parameter. Also, let (F, E) and (G, E) be two fuzzy soft set over U and V respectively and $\phi(U \times V)$ be the set of all fuzzy subset of $U \times V$, then (H, E) is a fuzzy soft relation between (F, E) and (G, E) over $U \times V$ if $H : E \to \phi(U \times V)$, where H is a mapping such that*

(5.1) $$H(e) = \{(u_i, v_j)/u_{ij};\ u_{ij} = \min(u_i, u_j)\ \forall e \in E, (u_i, u_i) \in F(e) \text{ and } (v_j, u_j) \in G(e)\}.$$

***Example* 5.1.** *Let U = {Paris, Berlin, Amsterdam} and V = {Rome, Madrid, Lisbon} are sets of cities and let 'E' be the set of given parameters, where E ={e_1 (far), e_2 (very far), e_3 (near), e_4 (crowded), e_5 (well managed)}. Let H be the fuzzy soft relation over U and V given by*

(H, E) = {$H(e_1)$ = *(Paris, Rome)*/0.60, *(Paris, Madrid)*/0.45,*(Paris, Lisbon)*/0.40, *(Berlin, Rome)*/0.55, *(Berlin, Madrid)*/0.65, *(Amsterdam, Lisbon)*/0.70, *(Amsterdam, Rome)*/0.75, *(Amsterdam, Madrid)*/0.50, *(Amsterdam, Lisbon)*/0.80}

Tabular Representation of Fuzzy Soft Relation over $U \times V$ is given in ***Table* 5.1**.

Table **5.1:** Fuzzy Soft Relation over $U \times V$

H (far)	Rome	Madrid	Lisbon
Paris	0.60	0.45	0.40
Berlin	0.55	0.65	0.70
Amsterdam	0.75	0.50	0.80

5.1 Properties of Fuzzy Soft Relation

Let (H_1, E) and (H_2, E) be the two fuzzy soft relations between (F, E) and (G, E) over $U \times V$, then

(a) Union of two fuzzy soft relations is also a fuzzy soft relation i.e.

$$(H, E) = (H_1, E) \cup (H_2, E), \text{ where } H(e) = H_1(e) \cup H_2(e)\ ; \forall\ e \epsilon E.$$

(b) Intersection of fuzzy soft relation is also a fuzzy soft relation i.e

$$(K, E) = (H_1, E) \cap (H_2, E), \text{ where } K(e) = H_1(e) \cap H_2(e);\ \ \forall\ e \epsilon E.$$

(c) Complement of fuzzy soft relation is also a fuzzy soft relation.
Let us consider $(F, E)^(C)$ and (G, E) as the complement of fuzzy soft set (F, E) and (G, E), then fuzzy soft relation between $(F, E)^C$ and $(G, E)^C$ is given by

$H^C = \rceil E \to 2^{U \times V}$, where H^C is a mapping such that

$$H^C(\rceil e) = \frac{(u_i, v_j)}{u_{ij}};\ u_{jj} = \min(\mu_i, \mu_j), \forall\ e \epsilon E,\ (u_i, \mu_i)\ \epsilon F(\rceil e) \text{ and } (v_j, \mu_j)\ \epsilon G(\rceil e).$$

The symbol "⌉" stands for "not in"

(d) A fuzzy soft relation is reflexive iff $\mu_{H(e)}\ (h_i, h_i) = 1\ \forall\ e \epsilon E$ and $h_i \epsilon U$.

(e) A fuzzy soft relation is symmetric iff $\mu_{H(e)}(h_i, h_j) = \mu_{H(e)}(h_j, h_i)\ \forall\ e\ \epsilon E$ and $h_i, h_j\ \epsilon\ U$.

(f) A fuzzy soft relation is transitive iff $\mu_{H(e)}(h_i, h_j) = \lambda_1$ and $\mu_{H(e)}(h_j, h_k) = \lambda_2 \to \mu_{H(e)}(h_i, h_k) = \lambda\ \forall e\ \epsilon\ E, \lambda \geq \min(\lambda_1, \lambda_2)$ and $h_i, h_j, h_k\ \epsilon\ U$.

(g) A fuzzy soft relation is fuzzy tolerance relation if it is reflexive and symmetric.

(h) A fuzzy soft relation is fuzzy equivalence relation if it is reflexive, symmetric and transitive.

5.2 Application of Fuzzy Soft Relation in Decision Making Problem

Let us consider $B = b_1, b_2, b_3, b_4$ the set of four boys and $G = g_1, g_2, g_3$ the set of three girls who play badminton. We are to choose a pair for double badminton game. Further, let 'E'' be a set of parameters to judge the capability of a badminton players and is given as

={e_1 (physical fitness), e_2 (average matches win), e_3 (judgement capability),

e_4 (average matches played), e_5 (height)}

Suppose Mr X is interested to sponsor a mixed pair of badminton player on the basis of his choice of parameters. Let A be a subset of E given as

={e_1 (physical fitness), e_2 (average matches win), e_3 (judgement capability)}

5.2.1 Algorithm for Selection of Mixed Double Using Fuzzy Soft Relation

(i) Firstly, input the soft sets (F, A) and (H, A) w.r.t. choice of parameters of Mr X.

(ii) Secondly, covert the soft sets (F, A) and (H, A) into fuzzy soft sets using suitable technique.

(iii) Form the fuzzy soft relation (I, A) between fuzzy soft sets (F, A) and (H, A).

(iv) Then compute the comparison table for fuzzy soft relation (I, A).

(v) Then compute row-sum and column-sum of comparison table as

$$s_{ij} = \sum_{e \epsilon A} r_{ij} \text{ and } p_{ij} = \sum_{e \epsilon A} r_{ij}$$

(vi) Then find the score value $S_{ij} = s_{ij} - p_{ij}$.

(vii) Finally, maximum of S_{ij} will be the choice. In case two or more values of $S_i j$ are same, then Mr X can choose any one the pair according to his opinion

5.2.2 Illustration with Example

Let us consider soft sets (F, A) and (H, A) over B and G respectively, given as

$$(F, A) = \{F(e_1) = (b_1, b_2), F(e_2) = (b_1, b_2, b_4), F(e_3) = (b_1, b_3)\}$$
$$(H, A) = \{H(e_1) = (g_1, g_3), H(e_2) = (g_1, g_2), H(e_3) = (g_1, g_2, g_3)\}.$$

Tabular representation of soft sets (F, A) and (H, A) are given in ***Tables* 5.2** and **5.3** respectively.

***Table* 5.2:** Soft Set (F, A)

B	e_1	e_2	e_3
b_1	1	1	1
b_2	1	1	0
b_3	0	0	1
b_4	0	1	0

***Table* 5.3:** Soft Set (H, A)

G	e_1	e_2	e_3
g_1	1	1	1
g_2	0	1	1
g_3	1	0	1

These soft sets are converted respectively to fuzzy soft set by applying ***Algorithm* 5.2.1** given in ***Tables* 5.4** and **5.5** below:

***Table* 5.4:** Fuzzy Soft Set (F, A)

B	e_1	e_2	e_3	
b_1	0.50	0.75	0.50	3/3
b_2	0.34	0.50	0	2/3
b_3	0	0	0.17	1/3
b_4	0	0.26	0	1/3
	2/4	3/4	2/4	

***Table* 5.5:** Fuzzy Soft Set (H, A)

G	e_1	e_2	e_3	
g_1	0.67	0.67	1	3/3
g_2	0	0.45	0.67	2/3
g_3	0.45	0	0.67	2/3
	2/3	2/3	3/3	

By applying the ***Algorithm* 5.2.1** the fuzzy soft relation (I, A) is found out as given below in the ***Table* 5.6**.

***Table* 5.6:** Fuzzy Soft Relation (I, A)

$B \times G$	e_1	e_2	e_3
(b_1, g_1)	0.50	0.67	0.50
(b_1, g_2)	0	0.45	0.50
(b_1, g_3)	0.45	0	0.50
(b_2, g_1)	0.34	0.50	0
(b_2, g_2)	0	0.45	0
(b_2, g_3)	0.34	0	0
(b_3, g_1)	0	0	0.17
(b_3, g_2)	0	0	0.17
(b_3, g_3)	0	0	0.17
(b_4, g_1)	0	0.26	0
(b_4, g_2)	0	0.26	0
(b_4, g_3)	0	0	0

Now the comparison table is given below in ***Table* 5.7**

***Table* 5.7:** Comparison Table

$B \times G$	(b_1, g_1)	(b_1, g_2)	(b_1, g_3)	(b_2, g_1)	(b_2, g_2)	(b_2, g_3)	(b_3, g_1)	(b_3, g_2)	(b_3, g_3)	(b_4, g_1)	(b_4, g_2)	(b_4, g_3)
(b_1, g_1)	3	3	3	3	3	3	3	3	3	3	3	3
(b_1, g_2)	1	3	2	1	2	2	3	3	3	3	3	3
(b_1, g_3)	1	2	3	2	2	3	3	3	3	2	2	3
(b_2, g_1)	0	2	1	3	3	3	2	2	2	3	3	3
(b_2, g_2)	0	2	1	1	3	2	2	2	2	3	3	3
(b_2, g_3)	0	1	1	2	2	3	2	2	2	2	2	3
(b_3, g_1)	0	1	1	1	2	2	3	3	3	2	2	3
(b_3, g_2)	0	1	1	1	2	2	3	3	3	2	2	3
(b_3, g_3)	0	1	1	1	2	2	3	3	3	2	2	3
(b_4, g_1)	0	1	1	1	2	2	2	2	2	3	3	3
(b_4, g_2)	0	1	1	1	2	2	2	2	2	3	3	3
(b_4, g_3)	0	1	1	1	2	2	2	2	2	2	2	3

***Table* 5.8:** Comparison between Row-sum and Column-sum

Row-sum	Colum sum	Score Value
36	5	31
29	9	20
29	17	12
27	18	9
24	27	-3
22	28	-6
23	30	-7
23	30	-7
23	30	-7
22	30	-8
22	30	-8
20	36	-16

Here maximum score value $S_{ij} = 31$ corresponding to the pair (b_1, g_1). So Mr X would like to sponsor (b_1, g_1) for mixed double badminton game.

6 Conclusion and Future Scope

The concepts of soft set and fuzzy soft set are recently emerged important topics to deal with uncertainties and ambiguities present in our day to day life. The availability of the parameterization tools in these sets has further enhanced the flexibility of their applications. Thus, soft and fuzzy soft relations which are extensions to crisp and fuzzy relations have been introduced and their applications in decision making problems have been studied with examples.

In our view the theory of soft and fuzzy soft relations based on soft set and fuzzy soft set respectively, can be extended to interval valued soft and fuzzy soft sets. Also, the theory can be extended to intuitionistic soft and fuzzy soft sets, generating a new class of relations. Their application to decision making and medical diagnosis problems can be further considered and studied.

Compliance with Ethical Standards

Conflict of interest: On behalf of all the authors, the corresponding author declares that there is no conflict of interest. This article does not contain any studies with human participants or animals performed by any of the authors.

References

[1] H. Aktas and N. Cagman, Soft Sets and Soft Groups, *Information Sciences*, **177**(2007), 2726-2735.

[2] D. S. Hooda and V. Raich, Fuzzy Set Theory and Fuzzy Controller, *Narora Publishing House, New Delhi*, 2015

[3] D. S Hooda and Rakesh Bajaj, Useful Fuzzy Measures of information, Integrated Ambiguity and Directed Divergence, *International Journal of General System*, **39**(2010), 647-858.

[4] D. S. Hooda and R. Kumari, On Application of Fuzzy Soft Sets in Dimension Reduction and Medical Diagnosis, *Advance in Research*, **12**(2017), 1-9.

[5] D. Molodstov [1999], Soft Set Theory- First Result, *Computer and Mathematics with Application*, **37**(1999), 19-31.

[6] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy Soft Sets, *Journal of Fuzzy Mathematics*, **9**(2001), 589-602.

[7] P. Majumdar and S. K. Samanta, Generalised Fuzzy Soft Sets, *Computer Mathematics Applications*, **59**(2010), 1425-1432.

[8] Z. Pawlak, Rough Sets, *International Journal of Information and Computer Sciences*, **11**(1982), 341-356.

[9] X. Yang, D. Yu, J. Yang and C. Wu, Generalization of Soft Set Theory: From Crisp to Fuzzy Case, *Advance in Soft Computing*, **40**(2007), 345-354.

[10] L. A. Zadeh, Fuzzy Sets, *Information and Control*, **8**(1965), 338-353.

[11] Y. Zou and Z. Xiao, Data Analysis Approaches of Soft Sets under Incomplete Information, *Knowledge- Based System*, **21**(2008), 941-945.