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GENERATING RELATIONS INVOLVING HYPERGEOMETRIC FUNCTIONS OF THREE VARIABLES

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1. In this paper, an attempt is made to extend certain generating relations involving the Kampé de Fériet function of two variables (see [1]) to hold for the generalized hypergeometric function of three variables, viz. $F^{(3)}[x, y, z]$ defined by Srivastava [2, p. 428] in the form

$$F^{(3)}\left[\begin{matrix}(a)::(b);\ (b');\ (b''):(c);\ (c');\ (c'');\\ (e)::(f);\ (f');\ (f''):(g);\ (g');\ (g'');\end{matrix}\ x, y, z\right]=$$

$$\sum_{m,n,p=0}^{\infty}\frac{((a))_{m+n+p}\,((b))_{m+n}\,((b'))_{n+p}\,((b''))_{m+p}\,((c))_m\,((c'))_n\,((c''))_p}{((e))_{m+n+p}\,((f))_{m+n}\,((f'))_{n+p}\,((f''))_{m+p}\,((g))_m\,((g'))_n\,((g''))_p}\,\frac{x^m\,y^n\,z^p}{m!\,n!\,p!} \qquad \dots(1.1)$$

where (a) denotes the sequence of A parameters $a_1, \dots, a_A$, with similar interpretations for (b), (b'), etc. It will be assumed throughout the paper that there are A of the a parameters, B of the b parameters, and so on. Thus $((a))_m$ is to be interpreted as $\prod_{j=1}^{A}(a_j)_m$, with similar interpretations for $((b))_m$, etc. The Srivastava function $F^{(3)}[x, y, z]$, defined by (1.1) above, is a generalization of Lauricella's fourteen hypergeometric functions of three variables, viz. $F_1, \dots, F_{14}$ (see [3], p. 114) and of the three additional functions H_A, H_B, H_C defined by Srivastava himself ([4, p. 97] ; see also [5]). Thus the results obtained in this paper are general in character, and a large number of special cases, known or new, can be obtained by specialising the parameters or the variables, or both.

We require the following formula in our proof. From [6] we have

$$F^{(3)}\left[\begin{matrix} \alpha :: - : - ; - : \beta ; \beta' ; \beta'' ; \\ - :: - ; \delta ; - : \nu ; - ; - ; \end{matrix} x, y, z\right]$$

$$= \frac{\Gamma(\nu)}{\Gamma(\beta)} x^{1-\nu} D_x^{\beta-\nu} \left[x^{\beta-1} (1-x)^{-\alpha} F_1\left(\alpha, \beta', \beta'' ; \delta ; \frac{y}{1-x}, \frac{z}{1-x}\right)\right] \quad \ldots(1.2)$$

where the function on the left-hand side is the same as the Lauricella function F_8 [3, p. 114] of three variables or the function F_G in the subsequent notation of Saran [7], and F_1 is the Appell function defined in [8, p. 14].

Also D_x^{λ} is an operator of fractional derivative defined by

$$D_x^{\lambda} (x^{\mu-1}) = \frac{\Gamma(\mu)}{\Gamma(\mu-\lambda)} x^{\mu-\lambda-1}, \quad \mu \neq \lambda.$$

2. In this section, we obtain the following generating relations involving Lauricella's function F_8 [3, p. 114] of three variables in the notation of Srivastava [2, p. 428].

$$\sum_{n=0}^{\infty} \binom{\nu+n}{n} F^{(3)}\left[\begin{matrix} a :: - ; - ; - : c'' ; \rho-n ; \sigma-n ; \\ - :: - ; g ; - : h'' ; - ; - ; \end{matrix} z, x, y\right] t^n$$

$$= (1-t)^{-\nu-1} \sum_{m, n, p=0}^{\infty} \binom{\nu+n}{n} \frac{(a)_{2n+m+p} (\rho)_m (\sigma)_p}{(g)_{2n+m+p}} \frac{x^m y^p}{m! \, p!} \left[\frac{xyt}{(1-t)^2}\right]^n$$

$$\times F^{(3)}\left[\begin{matrix} a+2n+m+p :: - ; \text{———} ; - : c'' ; \nu+n+1 ; \nu+n+1 ; \\ \text{———} :: - ; g+2n+m+p ; - : h'' ; \text{———} ; \text{———} ; \end{matrix} z, \frac{xt}{t-1}, \frac{yt}{t-1}\right] \quad \ldots(2.1)$$

$$\sum_{n=0}^{\infty}\binom{\nu+n}{n}F^{(3)}\left[\begin{matrix} a::-;-;-:c'';\rho-n;\sigma+n;\\ -::-;g;-:h'';-;-;\end{matrix} z,x,y\right]t^n$$

$$=(1-t)^{-\nu-1}\sum_{m,n,p=0}^{\infty}\binom{\nu+n}{n}\frac{(a)_{2n+m+p}\,(\rho)_m\,(\sigma-\nu-1)_p\,x^m\,y^p}{(g)_{2n+m+p}\qquad m!\,p!}$$

$$\left[\frac{-xyt}{(1-t)^2}\right]^n$$

$$\times F^{(3)}\left[\begin{matrix} a+2n+m+p::-;\text{———};-:c'';\nu+n+1;\nu+n+1;\\ \text{———}::-;g+2n+m+p;-:h'';\text{———};\text{———};\end{matrix}\right.$$

$$\left. z,\frac{xt}{t-1},\frac{y}{t-1}\right] \qquad \ldots(2.2)$$

$$\sum_{n=0}^{\infty}\binom{\nu+n}{n}F^{(3)}\left[\begin{matrix} a::-;-;-:c'';\rho+n;\sigma+n;\\ -::-;g;-:h'';-;-;\end{matrix} z,x,y\right]t^n$$

$$=(1-t)^{-\nu-1}\sum_{m,n,p=0}^{\infty}\binom{\nu+n}{n}\frac{(a)_{2n+m+p}\,(\rho-\nu-1)_m\,(\sigma-\nu-1)_p\,x^m\,y^p}{(g)_{2n+m+p}\qquad m!\,p!}$$

$$\left[\frac{xyt}{(1-t)^2}\right]^n$$

$$\times F^{(3)}\left[\begin{matrix} a+2n+m+p::-;\text{———};-:c'';\nu+n+1;\nu+n+1;\\ \text{———}::-;g+2n+m+p;-:h'';\text{———};\text{———};\end{matrix}\right.$$

$$\left. z,\frac{x}{1-t},\frac{y}{1-t}\right] \qquad \ldots(2.3)$$

and

$$\sum_{n=0}^{\infty}\binom{\nu+n}{n}F^{(3)}\left[\begin{matrix} a::-;-;-:-n;-n;c'';\\ -::g;-;-:-;-\nu-n;h'';\end{matrix} x,y,z\right]t^n$$

$$=(1-t)^{-\nu-1}\sum_{n=0}^{\infty}\frac{(a)_{2n}\,(-xyt)^n}{(g)_{2n}\qquad n!}$$

$$.\,F^{(3)}\left[\begin{matrix} a+2n::\text{——};-;-:\nu+1;-;c'';\\ \text{——}::g+2n;-;-:\text{——};-;h'';\end{matrix}\frac{xt}{t-1},yt,z\right] \ldots(2.4)$$

Proof of (2.1)—In [1, p. 36, (1.3)], substituting $A=C=1$, $B=B'=D=D'=0$, replacing z by t, and c by g, we have

$$\sum_{n=0}^{\infty}\binom{\nu+n}{n}F_1(a,\rho-n,\sigma-n\,;g\,;x,y)t^n$$

$$=(1-t)^{-\nu-1}\sum_{m,n,p=0}^{\infty}\binom{\nu+n}{n}\frac{(a)_{2n+m+p}(\rho)_m(\sigma)_p\,x^m y^p}{(g)_{2n+m+p}\qquad m!\;p!}\left[\frac{xyt}{(1-t)^2}\right]^n$$

$$.\,F_1\left(a+2n+m+p,\nu+n+1,\nu+n+1\,;g+2n+m+p\,;\frac{xt}{t-1},\frac{yt}{t-1}\right).$$

Replacing x by $\frac{x}{1-z}$, y by $\frac{y}{1-z}$, multiplying both sides by $z^{c''-1}$. $(1-z)^{-a}$ and operating on both sides by $D_z^{c''-h''}$, we obtain

$$\sum_{n=0}^{\infty}\binom{\nu+n}{n}D_z^{c''-h''}\left[z^{c''-1}(1-z)^{-a}F_1\left(a,\rho-n,\sigma-n\,;g\,;\frac{x}{1-z},\frac{y}{1-z}\right)\right]t^n$$

$$=(1-t)^{-\nu-1}\sum_{m,n,p=0}^{\infty}\binom{\nu+n}{n}\frac{(a)_{2n+m+p}(\rho)_m(\sigma)_p\,x^m y^p}{(g)_{2n+m+p}\qquad m!\;p!}\left[\frac{xyt}{(1-t)^2}\right]^n$$

$$.\,D_z^{c''-h''}\left[z^{c''-1}(1-z)^{-a-2n-m-p}F_1\left(a+2n+m+p,\nu+n+1,\nu+n+1;\right.\right.$$

$$\left.\left.g+2n+m+p;\frac{xt}{(t-1)(1-z)},\frac{yt}{(t-1)(1-z)}\right)\right].$$

Now employing the formula (1.2), we arrive at (2.1).

In the general results (2.3), (2.4) and (2.2) of Srivastava [1, p. 39 and p. 40], if we substitute $A=C=1$, $B=B'=D=D'=0$, replace z by t, c by g, and proceed on the same lines as above, our formulas (2.2), (2.3) and (2.4) will follow.

3. Extensions of the Generating Relations (2.1) to (2.4). In this section, by making use of the principle of multidimensional mathematical induction and the Laplace and the inverse Laplace transform techniques, we extend the results of the last section to generalised hypergeometric functions of three variables such as $F^{(3)}[x, y, z]$ defined by (1.1).

If we put

$$F^{(5)}\begin{bmatrix} a :::b\,;b'\,;b''::\alpha, c\,;\beta, c'\,;c'':\rho,\sigma\,; \\ d:::e\,;e'\,;e''::\alpha', f\,;\beta', f'\,;f'':\rho',\sigma'\,; \end{bmatrix} x, y, z, t, u \Bigg] =$$

$$\sum_{m,n,p=0}^{\infty} \frac{(a)_{m+p}\,(b)_{m+p}\,(b')_m\,(b'')_p\,(c)_m\,(c')_p\,(\rho)_m\,(\sigma)_p}{(d)_{m+p}\,(e)_{m+p}\,(e')_m\,(e'')_p\,(f)_m\,(f')_p\,(\rho')_m\,(\sigma)_p}\,\frac{x^m\,y^p}{m!\,p!}$$

$$.\,F^{(3)}\begin{bmatrix} a+m+p::b+m+p;\,b''+p;\,b'+m:\alpha,\,c+m;\,\beta,\,c'+p;\,c''; \\ d+m+p::e+m+p;\,e''+p;\,e'+m:\alpha',f+m;\beta',f'+p;f''; \end{bmatrix} z, t, u \Bigg] \quad ...(3.1)$$

and denote the quotient

$$\frac{((a))_{2n}\,((b))_{2n}\,((b'))_n\,((b''))_n\,((c))_n\,((c'))_n}{((e))_{2n}\,((g))_{2n}\,((g'))_n\,((g''))_n\,((h))_n\,((h'))_n}$$

by

$$\delta_n\begin{bmatrix} a, b, b', b'', c, c' \\ e, g, g', g'', h, h' \end{bmatrix}$$

then the results to be established here are

$$\sum_{n=0}^{\infty}\binom{\nu+n}{n}F^{(3)}\begin{bmatrix} (a)::(b);\,(b');\,(b''):\rho-n,\,(c);\,\sigma-n,\,(c');\,(c''); \\ (e)::(g);\,(g');\,(g''):\quad (h);\quad (h');\,(h''); \end{bmatrix} x, y, z \Bigg] t^n$$

$$=(1-t)^{-\nu-1}\sum_{n=0}^{\infty}\binom{\nu+n}{n}\delta_n\begin{bmatrix} a, b, b', b'', c, c' \\ e, g, g', g'', h, h' \end{bmatrix}\left[\frac{xyt}{(1-t)^2}\right]^n$$

$$.\,F^{(5)}\begin{bmatrix} (a)+2n:::(b)+2n;\,(b'')+n;\,(b')+n::\nu+n+1, \\ (e)+2n:::(g)+2n;\,(g'')+n;\,(g')+n:: \end{bmatrix}$$

$$\begin{matrix} (c)+n;\,\nu+n+1,\;(c')+n;\,(c'');\;\rho;\;\sigma; \\ (h)+n;\qquad\qquad (h')+n;\,(h'');\;-;\;-; \end{matrix}\; x, y, \frac{xt}{t-1}, \frac{yt}{t-1}, z \Bigg] \quad ...(3.2)$$

$$\sum_{n=0}^{\infty}\binom{\nu+n}{n}F^{(3)}\begin{bmatrix}(a)::(b);(b');(b''):\rho-n,(c);\sigma+n,(c');(c'');\\(e)::(g);(g');(g''):\quad(h);\quad(h');(h'');\end{bmatrix}$$

$$\left.\begin{matrix}\\ x,y,z\end{matrix}\right]t^n$$

$$=(1-t)^{-\nu-1}\sum_{n=0}^{\infty}\binom{\nu+n}{n}\delta_n\begin{bmatrix}a,b,b',b'',c,c'\\e,g,g',g'',h,h'\end{bmatrix}\left[\frac{-xyt}{(1-t)^2}\right]^n$$

$$.\,F^{(5)}\begin{bmatrix}(a)+2n:::(b)+2n;(b'')+n;(b')+n::\nu+n+1,\\(e)+2n:::(g)+2n;(g'')+n;(g')+n::\end{bmatrix}$$

$$\left.\begin{matrix}(c)+n;\nu+n+1,(c')+n;(c'');\quad\rho;\sigma-\nu-1;\\(h)+n;\quad(h')+n;(h'');\ -;\ \text{———};\end{matrix}\ x,y,\frac{xt}{t-1},\frac{y}{1-t},z\right]$$

...(3.3)

$$\sum_{n=0}^{\infty}\binom{\nu+n}{n}F^{(3)}\begin{bmatrix}(a)::(b);(b');(b''):\rho+n,(c);\sigma+n,(c');(c'');\\(e)::(g);(g');(g''):\quad(h);\quad(h');(h'');\end{bmatrix}$$

$$\left.\begin{matrix}\\ x,y,z\end{matrix}\right]t^n$$

$$=(1-t)^{-\nu-1}\sum_{n=0}^{\infty}\binom{\nu+n}{n}\delta_n\begin{bmatrix}a,b,b',b'',c,c'\\e,g,g',g'',h,h'\end{bmatrix}\left[\frac{xyt}{(1-t)^2}\right]^n$$

$$.\,F^{(5)}\begin{bmatrix}(a)+2n:::(b)+2n;(b'')+n;(b')+n::\nu+n+1,\\(e)+2n:::(g)+2n;(g'')+n;(g')+n::\end{bmatrix}$$

$$\left.\begin{matrix}(c)+n;\nu+n+1,(c')+n;(c''):\rho-\nu-1;\sigma-\nu-1;\\(h)+n;\quad(h')+n;(h''):\ \text{———};\ \text{———};\end{matrix}\ x,y,\frac{x}{1-t},\frac{y}{1-t},z\right]$$

...(3.4)

and

$$\sum_{n=0}^{\infty}\binom{\nu+n}{n}F^{(3)}\begin{bmatrix}(a)::(b);(b');(b''):-n,(c);-n,\quad(c');(c'');\\(e)::(g);(g');(g''):-,\ (h);-\nu-n,(h');(h'');\end{bmatrix}$$

$$\left.\begin{matrix}\\ x,y,z\end{matrix}\right]t^n$$

$$=(1-t)^{-\nu-1}\sum_{n=0}^{\infty}\delta_n\begin{bmatrix}a, b, b', b'', c, c'\\ e, g, g', g'', h, h'\end{bmatrix}\frac{(-xyt)^n}{n!}$$

$$\cdot F^{(3)}\begin{bmatrix}(a)+2n :: (b)+2n; (b')+n; (b'')+n : \nu+1, (c)+n; (c')+n; (c''); \\ (e)+2n :: (g)+2n; (g')+n; (g'')+n : \quad (h)+n; (h')+n; (h''); \end{bmatrix}\frac{xt}{t-1}, yt, z\Bigg] \quad ...(3.5)$$

Proof of (3.2)—In view of (2.1), formula (3.2) holds good when $A=C''=G=H''=1$ and $B=B'=B''=C=C'=E=G'=G''=H=H'=0$. Assuming that it remains true for some values of the non negative integers $A, B, B', ..., H''$, if we replace x by xt_1, y by yt_1 z by zt_1, multiply both sides by $(t_1)^{a_{A+1}-1}$ and take their Laplace transforms with respect to t_1, we observe that A is replaced by $A+1$. Again in (3.2), replacing x by x/t_1, y by y/t_1, z by z/t_1, multiplying both sides by $(t_1)^{-e_{E+1}}$, and taking their inverse Laplace transforms, we find that E is replaced by $E+1$. Thus the inductions on A and E are completed. Similarly, inductions on other parameters can be performed, and the formal proof of (3.2) by induction is completed.

The results (3.3), (3.4) and (3.5) can be established by the above techniques, using (2.2), (2.3) and (2.4), respectively, instead of the generating relation (2.1).

4. **Particular Cases of** (3.2) **to** (3.5). We observe that the results of the last section are of the form

$$\sum_{n=0}^{\infty}\binom{\nu+n}{n}\phi_n(x, y, z)\, t^n.$$

By specialising the parameters, the function $\phi_n(x, y, z)$ can be reduced to Lauricella's fourteen hypergeometric functions $F_1, ..., F_{14}$, with the only exception of the function F_5 or $F_C^{(3)}$ defined in [3, p. 118]. However, this function $\phi_n(x, y, z)$ cannot be specialized to Srivastava's functions H_A, H_B and H_C.

Some other special cases are mentioned below.

(i) In (3.2), letting $\rho=\sigma=0$, we obtain

$$\sum_{n=0}^{\infty}\binom{\nu+n}{n}F^{(3)}\left[\begin{matrix}(a)::(b);(b');(b''):-n,(c);\ -n,(c');(c'');\\(e)::(g);(g');(g''):\quad (h);\qquad (h');(h'');\end{matrix}\ x,y,z\right]t^n$$

$$=(1-t)^{-\nu-1}\sum_{n=0}^{\infty}\binom{\nu+n}{n}\delta_n\begin{bmatrix}a,b,b',b'',c,c'\\e,g,g',g'',h,h'\end{bmatrix}\left[\frac{xyt}{(1-t)^2}\right]^n$$

$$\cdot F^{(3)}\left[\begin{matrix}(a)+2n::(b)+2n;(b')+n;(b'')+n:\nu+n+1,(c)+n;\\(e)+2n::(g)+2n;(g')+n;(g'')+n:\qquad (h)+n;\end{matrix}\ \begin{matrix}\nu+n+1,(c')+n;(c'');\\(h')+n;(h'');\end{matrix}\ \frac{xt}{t-1},\frac{yt}{t-1},z\right] \quad \text{...(4.1)}$$

(ii) In (3.3) putting $\rho=\sigma-\nu-1=0$, we have

$$\sum_{n=0}^{\infty}\binom{\nu+n}{n}F^{(3)}\left[\begin{matrix}(a)::(b);(b');(b''):-n,(c);\nu+n+1,(c');(c'');\\(e)::(g);(g');(g''):\quad (h);\qquad (h');(h'');\end{matrix}\ x,y,z\right]t^n$$

$$=(1-t)^{-\nu-1}\sum_{n=0}^{\infty}\binom{\nu+n}{n}\delta_n\begin{bmatrix}a,b,b',b'',c,c'\\e,g,g',g'',h,h'\end{bmatrix}\left[\frac{-xyt}{(1-t)^2}\right]^n$$

$$\cdot F^{(3)}\left[\begin{matrix}(a)+2n::(b)+2n;(b')+n;(b'')+n:\nu+n+1,(c)+n;\\(e)+2n::(g)+2n;(g')+n;(g'')+n:\qquad (h)+n;\end{matrix}\ \begin{matrix}\nu+n+1,(c')+n;(c'');\\(h')+n;(h'');\end{matrix}\ \frac{xt}{t-1},\frac{y}{1-t},z\right] \quad \text{...(4.2)}$$

Making use of Vandermonde's theorem [9, p. 69, Ex. 4] the right-hand side of (4.2) above can be simplified, and we get the result in a compact form as

$$\sum_{n=0}^{\infty}\binom{\nu+n}{n}F^{(3)}\left[\begin{matrix}(a)::(b);(b');(b''):-n,(c);\nu+n+1,(c');(c'');\\(e)::(g);(g');(g''):\quad (h);\qquad (h');(h'');\end{matrix}\ x,y,z\right]t^n$$

$$=(1-t)^{-\nu-1}\, F^{(3)}\begin{bmatrix}(a)::\nu+1,(b);(b');(b''):(c);(c');(c'');\\(e)::\quad (g);(g');(g''):(h);(h');(h'');\end{bmatrix}\frac{xt}{t-1},\frac{y}{1-t},z\Big] \quad ...(4.3)$$

Substituting $z=0$, $B=G=C=H=C'=H'=0$, we obtain a known result due to Srivastava [1, p. 42, (3.4)].

Similarly, a special case of (3.4) with $\rho=\sigma=\nu+1$ can be obtained

5. Srivastava [1, p. 39, (2.3)] has obtained the formula

$$\sum_{n=0}^{\infty}\binom{\nu+n}{n}F^{(2)}\begin{bmatrix}(a):\rho-n,(b);\sigma+n,(b');\\(c):\quad (d);\quad (d');\end{bmatrix} x,y\Big] z^n$$

$$=(1-z)^{-\nu-1}\sum_{n=0}^{\infty}\binom{\nu+n}{n}\frac{((a))_{2n}\,((b))_n\,((b'))_n}{((c))_{2n}\,((d))_n\,((d'))_n}\left[\frac{-xyz}{(1-z)^2}\right]^n$$

$$.\,F^{(4)}\begin{bmatrix}(a)+2n::\rho,(b)+n;\sigma-\nu-1,(b')+n:\nu+n+1,(b)+n;\nu+n+1,(b')+n;\\(c)+2n::\quad (d)+n;\quad (d')+n:\quad (d)+n;\quad (d')+n;\end{bmatrix} x,y,\frac{xz}{z-1},\frac{y}{1-z}\Big] \quad ...(5.1)$$

$A+B\leqslant C+D$, $A+B'\leqslant C+D'$ and $|x|$, $|y|$ and $|z|$ are so constrained that the two sides have a meaning. Here the function $F^{(4)}$ is defined by Srivastava [1, p. 35, (1·1)]as

$$F^{(4)}\begin{bmatrix}\alpha::\beta,\rho;\gamma,\sigma:\beta',\rho;\gamma',\sigma:\\\lambda::\mu,\delta;\nu,\in:\mu',\delta;\nu',\in:\end{bmatrix} x,y,z,t\Big]$$

$$=\sum_{m,n=0}^{\infty}\frac{(\alpha)_{m+n}\,(\beta)_m\,(\gamma)_n\,(\rho)_m\,(\sigma)_n\,x^m\,y^n}{(\lambda)_{m+n}\,(\mu)_m\,(\nu)_n\,(\delta)_m\,(\in)_n\,m!\,n!}$$

$$.\,F^{(2)}\begin{bmatrix}\alpha+m+n:\beta',\rho+m;\gamma',\sigma+n;\\\lambda+m+n:\mu',\delta+m;\nu',\in+n;\end{bmatrix} z,t\Big]$$

where $F^{(2)}[x,y]$ is Kampé de Fériet's double hypergeometric function [8, p. 150] in the notation of Burchnall and Chaundy [10, p.112].

We observe that the right-hand side of (5·1) can be simplified by using Vandermonde's theorem [9, p. 69, Ex. 4] as

$$(1-z)^{-\nu-1}\sum_{r=0}^{\infty}\frac{((a))_r\,((b))_r\,(\rho)_r\,x^r}{((c))_r\,((d))_r\,r!}$$

$$\cdot F^{(3)}\begin{bmatrix}(a)+r :: 1+\nu; (b'); - : (b)+r; -; \sigma-\nu-1; \\ (c)+r :: -; (d'); - : (d)+r; -; -; \end{bmatrix.$$

$$\left.\frac{xz}{z-1}, \frac{y}{1-z}, y\right] \quad \dots(5.2)$$

Proceeding on the same lines as above, the right-hand side of result (3.3) of this paper can be simplified as

$$(1-t)^{-\nu-1}\sum_{r,\,k=0}^{\infty}\frac{((a))_{r+k}\,((b))_r\,((b'))_k\,((b''))_{r+k}\,((c))_r\,(\rho)_r}{((e))_{r+k}\,((g))_r\,((g'))_k\,((g''))_{r+k}\,((h))_r}\frac{x^r\,z^k}{r!\;k!}$$

$$\cdot F^{(3)}\begin{bmatrix}(a)+r+k, (b)+r :: 1+\nu; (b')+k, (c'); - : \\ (e)+r+k, (g)+r :: -; (g')+k, (h'); - : \end{bmatrix.$$

$$\left.\begin{matrix}(b'')+r+k, (c)+r; -; \sigma-\nu-1; \\ (g'')+r+k, (h)+r; -; ---; \end{matrix}\frac{xt}{t-1}, \frac{y}{1-t}, z\right] \quad \dots(5.3)$$

The details are ommited.

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