Jñānabha Sect. A, Vol. 4, July 1974

ON THE REDUCIBILITY OF LAURICELLA'S FUNCTION F_D

by

B. M. Singhal
Department of Mathematics,
Government Science College, Gwalior, M. P., India

(Received : May 28, 1974; in final form : September 27, 1974)

1. Recently, Srivastava [2] gave several identities involving double series with arbitrary coefficients; in particular, he gave the series transformation [2, p. 297].

$$\sum_{m,\, n=0}^{\infty} C_{m+n}\, (\nu)_m\, (\sigma)_n\, \frac{x^{m+n}}{m!\, n!} = \sum_{n=0}^{\infty} C_n\, (\nu+\sigma)_n\, \frac{x^n}{n!} \tag{1}$$

where $\{C_n\}$ is a sequence of arbitrary complex numbers and it is assumed that the series involved converge absolutely.

In this note we shall show that the identity (1) is also true for the corresponding multiple summation.

2. Now we shall prove the following identity :

$$\sum_{m_1, \ldots, m_n=0}^{\infty} f\,(m_1+\ldots+m_n)\, (\alpha_1)_{m_1} \ldots (\alpha_n)_{m_n} \frac{x^{m_1+\ldots+m_n}}{m_1! \ldots m_n!}$$

$$= \sum_{N=0}^{\infty} f(N)\, (\alpha_1+\ldots+\alpha_n)_N\, \frac{x^N}{N!}, \tag{2}$$

provided that the series involved are absolutely convergent.

In order to prove this identity, we consider

$$I \equiv \sum_{m_1, \ldots, m_n=0}^{\infty} f(\alpha+m_1+\ldots+m_n)\, (\alpha_1)_{m_1} \ldots (\alpha_n)_{m_n} \frac{x^{m_1+\ldots+m_n}}{m_1! \ldots m_n!}$$

$$= \sum_{m_1, \ldots, m_n=0}^{\infty} \frac{(\alpha_1)_{m_1}}{m_1!} \ldots \frac{(\alpha_n)_{m_n}}{m_n!}\, x^{m_1+\ldots+m_n}\, E_{\alpha}^{m_1+\ldots+m_n} \{f(\alpha)\},$$

where $\overset{n}{\underset{\alpha}{E}}\{f(\alpha)\}=f(\alpha+n)$, n being a non-negative integer. Therefore,

$$I=(1-E_\alpha\ x)^{-\alpha_1}\dots(1-E_\alpha\ x)^{-\alpha_n}\{f(\alpha)\}$$

$$=(1-E_\alpha\ x)^{(-\alpha_1-\dots-\alpha_n)}\{f(\alpha)\}$$

$$=\sum_{N=0}^{\infty}\frac{(\alpha_1+\dots+\alpha_n)_N}{N!}x^N\overset{N}{\underset{\alpha}{E}}\{f(\alpha)\}$$

$$=\sum_{N=0}^{\infty}f(\alpha+N)\ (\alpha_1+\dots+\alpha_n)_N\frac{x^N}{N!}.$$

On putting $\alpha=0$, we at once arrive at (2).

3. Evidently, when $n=2$, the identity (2) reduces to (1). Moreover, by appropriately specializing the function $f(N)$, (2) can readily be rewritten as reduction formulas for hypergeometric functions of several variables. In particular, for the Lauricella function F_D of n variables, defined by [1, p. 113],

$$F_D[a, b_1, \dots, b_n; c; x_1, \dots, x_n]$$

$$=\sum_{m_1, \dots, m_n=0}^{\infty}\frac{(a)_{m_1+\dots+m_n}\ (b_1)_{m_1}\dots(b_n)_{m_n}}{(c)_{m_1+\dots+m_n}}\frac{x_1^{m_1}}{m_1!}\dots\frac{x_n^{m_n}}{m_n!},\tag{3}$$

where $\max\{|x_1|, \dots, |x_n|\}<1$, we are led to the following known special case of (2).

$$F_D[a, \alpha_1, \dots, \alpha_n; b; x, \dots, x]$$

$$={}_2F_1[a, \alpha_1+\dots+\alpha_n; b; x],\ |x|<1,\tag{4}$$

which is due to Lauricella himself [1, p. 150].

Acknowledgments. The author is thankful to Dr. B. M. Agrawal for his guidance and also to Professor H. M. Srivastava for his valuable comments, which have been of great help in the improvement of this paper.

REFERENCES

[1] G. Lauricella, *Sulle funzioni ipergeometriche a più variabili*, Rend. Circ. Mat. Palermo. 7 (1893), 111-158.

[2] H. M. Srivastava, *On the reducibility of Appell's function F_4*, Canad. Math. Bull. 16 (1973), 295-298.