Jñanabha Sect. A, Vol. 4, July 1974.

SOME THEOREMS ON A DOUBLE INTEGRAL TRANSFORM

By

Y. N. Prasad AND **A. Siddiqui**
Applied Mathematics Section, Institute of Technology, Banaras Hindu University, Varanasi 221005, *India*

(Received : June 2, 1974 ; in revised form : September 5, 1974)

1. Introduction. Fox [3] defined the H-function as a Mellin-Barnes type contour integral, which is symbolically denoted as

$$H_{p,q}^{m,n}\left[x\,\middle|\,\begin{matrix}\{(a_p,\alpha_p)\}\\ \{(b_q,\beta_q)\}\end{matrix}\right]$$

$$=\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\prod_{j=m+1}^{q}\Gamma(1-b_j+\beta_j s)\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j s)}x^s\,ds, \quad (1{\cdot}1)$$

where $\{(f_r,\gamma_r)\}$ stands for a set of parameters $(f_1,\gamma_1),\ldots,(f_r,\gamma_r)$; $x\neq 0$ and all other conditions are the same as given in [7, p. 310].

In this paper, we discuss the chain properties, connecting the double integral transform

$$\phi(t)=\int_0^\infty\int_0^\infty x^{\alpha-1}y^{\beta-1}(x+y)^\sigma H_{u,v}^{f,g}\left[\lambda(x+y)\,\middle|\,\begin{matrix}\{(A_u,\eta_u)\}\\ \{(B_v,\xi_v)\}\end{matrix}\right]$$

$$H_{p,q}^{m,n}\left[t\,x^{\alpha_1}y^{\beta_1}(x+y)^{\sigma_1}\,\middle|\,\begin{matrix}\{(c_p,\gamma_p)\}\\ \{(d_q,\delta_q)\}\end{matrix}\right]f(x,y)\,dx\,dy, \quad (1{\cdot}2)$$

provided that $0\leqslant m\leqslant q$, $0\leqslant n\leqslant p$, $0\leqslant f\leqslant v$, $0\leqslant g\leqslant u$, $R(\alpha)>0$, $R(\beta)>0$, α_1, β_1, $\sigma_1\geqslant 0$ $-\min_{1\leqslant j\leqslant f} R(B_j/\xi_j)<R(\alpha+\beta+\sigma)<-\max_{1\leqslant i\leqslant g} R\{(A_i-1)/\eta_i\}$, m, n, p, q, f, g, u, v are integers, with various known transforms, *e.g.* the Laplace transform, the Hankel transform, the

$J^{\mu}_{\nu,\lambda}$ —transform due to Pathak [5], and Ψ_{ν_1, k_1, m_1} —transform due to Narain [6]. For the sake of brevity, we denote the double integral transform (1·2) as

$$\phi(t)=DT\,[f(x,\ y)].$$

The motivation of this paper has come from the result (2·2) below given recently by Srivastava and Panda ([7], p. 312). We prove three theorems, frequently using the power series expansion of various special functions appearing in the integrand, and we give several examples based on these theorems, thereby evaluating a few known or new integrals involving the products of H-functions and other special functions.

2. **Theorem 1.** If $\phi(t)=DT[\,f(x)]$ and $f(x)$ is the Laplace transform of $g(z)$, then

$$\phi(t)=\lambda^{-\alpha-\beta-\sigma}\sum_{r=0}^{\infty}\frac{(-1)^r}{r!}\lambda^{-r}$$

$$H^{m+g,\,2+f+n}_{p+\nu+2,\,q+u+1}\left[t\,\lambda^{-\theta}\,\middle|\,\begin{matrix}(1-\alpha-r,\ \alpha_1),\ (1-\beta,\ \beta_1),\ \{(\varepsilon_f,\ \theta\xi_f)\},\ \{(c_p,\ \gamma_p)\},\ (\varepsilon_{f+1},\ \theta\xi_{f+1}),\ \ldots,\ (\varepsilon_\nu,\ \theta\xi_\nu)\\ \{(\Psi_g,\ \theta\eta_g)\},\ \{(d_q,\ \delta_q)\},\ (1-\alpha-\beta-r,\ \alpha_1+\beta_1),\ (\Psi_{g+1},\ \theta\eta_{g+1}),\ \ldots,\ (\Psi_u,\ \theta\eta_u)\end{matrix}\right]\cdot\int_0^\infty z^r\,g(z)\,dz, \tag{2·1}$$

where

$$\begin{cases}\theta=\alpha_1+\beta_1+\sigma_1,\\ \varepsilon_j=1-B_j-(\alpha+\beta+\sigma)\,\xi_j,\quad j=1,\ \ldots,\ \nu,\\ \Psi_j=1-A_j-(\alpha+\beta+\sigma)\,\eta_j,\ j=1,\ \ldots,\ u,\end{cases}$$

provided that the integrals $\int_0^\infty g(z)\,dz$ and $\int_0^\infty z^r\,g(z)\,dz$, $\quad r\geqslant 1$, exist and all other conditions given with (1·2) are satisfied.

Proof. We have

$$f(x)=\int_0^\infty e^{-xz}\,g(z)\,dz.$$

On substituting for $f(x)$ in (1·2) and changing the order of integration, which is justifiable under the given conditions, we have

$$\phi(t)=\int_0^\infty g(z)\,dz \int_0^\infty \int_0^\infty e^{-xz}\, x^{\alpha-1}\, y^{\beta-1}\,(x+y)^{\sigma}$$

$$H_{u,v}^{f,g}\left[\lambda(x+y)\,\middle|\, \begin{matrix}\{(A_u,\eta_u)\}\\ \{(B_v,\xi_v)\}\end{matrix}\right]$$

$$H_{p,q}^{m,n}\left[tx^{\alpha_1}\, y^{\beta_1}\,(x+y)^{\sigma_1}\,\middle|\, \begin{matrix}\{(c_p,\gamma_p)\}\\ \{(d_q,\delta_q)\}\end{matrix}\right]dxdy.$$

Now expanding e^{-xz} in powers of xz and integrating term by term by using the following known integral due to Srivastava and Panda [7, p. 312]

$$\int_0^\infty \int_0^\infty x^{\alpha-1}\, y^{\beta-1}\,(x+y)^{\sigma}\, H_{u,v}^{f,g}\left[\lambda(x+y)\,\middle|\, \begin{matrix}\{(A_u,\eta_u)\}\\ \{(B_v,\xi_v)\}\end{matrix}\right]$$

$$.\, H_{p,q}^{m,n}\left[tx^{\alpha_1}\, y^{\beta_1}\,(x+y)^{\sigma_1}\,\middle|\, \begin{matrix}\{(c_p,\gamma_p)\}\\ \{(d_q,\delta_q)\}\end{matrix}\right]dxdy$$

$$=\lambda^{-\alpha-\beta-\sigma}\, H_{p+v+2,\,q+u+1}^{m+g,\,2+f+n}\left[t\lambda^{-\theta}\,\middle|\, \begin{matrix}(1-\alpha,\alpha_1),(1-\beta,\beta_1),\\ \{(\Psi_g,\theta\eta_g)\},\{(d_q,\delta_q)\},\end{matrix}\right.$$

$$\left.\begin{matrix}\{(\varepsilon_f,\theta\xi_f)\},\{(c_p,\gamma_p)\},(\varepsilon_{f+1},\theta\xi_{f+1}),\ldots,(\varepsilon_v,\theta\xi_v)\\ (1-\alpha-\beta,\alpha_1+\beta_1),(\Psi_{g+1},\theta\eta_{g+1}),\ldots,(\Psi_u,\theta\eta_u)\end{matrix}\right] \tag{2·2}$$

where all the conditions given in (2·1) are satisfied, we obtain the theorem.

3. **Example.** Let $g(z)=z^{-\mu-\frac{1}{2}}\, K_{\nu+\frac{1}{2}}(bz)$.

On using [2, p. 270 (2)],

$$f(x)=\frac{\sqrt{\pi}\,\Gamma(-\mu+\nu+1)\,\Gamma(-\mu-\nu)}{(2b)^{\frac{1}{2}}}\,(x^2-b^2)^{\mu/2}\, P_\nu^\mu\,(x/b), \tag{3·1}$$

where $P_\nu^\mu(z)$ is the associated Legendre function, with $R(\mu)-1 < R(\nu) < -R(\mu)$.

Also using [1, p. 331 (26)], we obtain

$$\int_0^\infty z^r\, g(z)\,dz=b^{-r+\mu-\frac{1}{2}}.\, 2^{r-\lambda-\frac{3}{2}}\,\Gamma(\tfrac{1}{2}r-\tfrac{1}{2}u-\tfrac{1}{2}\nu)$$

$$\Gamma(\tfrac{1}{2}r-\tfrac{1}{2}\mu+\tfrac{1}{2}\nu+\tfrac{1}{2}). \tag{3·2}$$

Finally, on putting the values from (3·1) and (3·2) in the theorem, we obtain

$$\int_0^\infty \int_0^\infty x^{\alpha-1}\, y^{\beta-1}\, (x^2-b^2)^{\frac{1}{2}\mu}\, P_\nu^\mu\,(x/b)\,(x+y)^\sigma$$

$$H_{u,v}^{f,g}\left[\lambda(x+y)\,\middle|\,\begin{matrix}\{(A_u,\eta_u)\}\\ \{(B_v,\xi_v)\}\end{matrix}\right] H_{p,q}^{m,n}\left[tx^{\alpha_1}\,y^{\beta_1}\,(x+y)^{\sigma_1}\,\middle|\,\begin{matrix}\{(c_p,\gamma_p)\}\\ \{(d_q,\delta_q)\}\end{matrix}\right] dxdy$$

$$=\frac{\lambda^{-\alpha-\beta-\sigma}\,2^{-\mu-1}\,b^\mu}{\sqrt{\pi}\;\Gamma(-\mu+\nu+1)\,\Gamma(-\mu-\nu)}$$

$$\sum_{r=0}^\infty \frac{(-2/b\lambda)^r}{r!}\cdot\;\Gamma(\tfrac{1}{2}r-\tfrac{1}{2}\mu-\tfrac{1}{2}\nu)\,\Gamma(\tfrac{1}{2}r-\tfrac{1}{2}\mu+\tfrac{1}{2}\nu+\tfrac{1}{2})$$

$$\cdot\, H_{p+v+2,\,q+u+1}^{m+g,\,2+f+n}\left[t\lambda^{-\theta}\,\middle|\,\begin{matrix}(1-\alpha-r,\alpha_1),(1-\beta,\beta_1),\{(\varepsilon_f,\theta\xi_f)\},\\ \{(\Psi_g,\theta\eta_g)\},\{(d_q,\delta_q)\},(1-\alpha-\beta-r,\alpha_1+\beta_1),\end{matrix}\right.$$

$$\left.\begin{matrix}\{(c_p,\gamma_p)\},(\varepsilon_{f+1},\theta\xi_{f+1}),\ldots,(\varepsilon_v,\theta\xi_v)\\ (\Psi_{g+1},\theta\eta_{g+1}),\ldots,(\Psi_u,\theta\eta_u)\end{matrix}\right] \qquad (3\cdot3)$$

provided that $R(\mu)-1<R(\nu)<-R(\mu)$ and the conditions given with (2·1) are satisfied.

4. **Theorem 2.** If

$$\phi(t)=DT\,[f(x)], \qquad (4\cdot1)$$

and

$$f(x)=J_{\nu,\lambda_1}^\mu-\text{transform of } g(z). \qquad (4\cdot2)$$

where

$$J_{\nu,\lambda_1}^\mu(x)=\sum_{r=0}^\infty \frac{(-1)^r\,(\frac{1}{2}r)^{\nu+2r+2\lambda_1}}{\Gamma(1+\lambda_1+r)\,\Gamma(1+\lambda_1+\nu+\mu_r)},\ \mu>0 \qquad (4\cdot3)$$

then

$$\phi(t)=\frac{\lambda^{-\alpha-\beta-\sigma-\nu-2\lambda_1+\frac{1}{2}}}{2^{\nu+2\lambda_1}}$$

$$\sum_{r=0}^\infty \frac{(-1)^r\,(2\lambda)^{-2r}}{\Gamma(1+\lambda_1+r)\,\Gamma(1+\lambda_1+\nu+\mu_r)}$$

$$H^{g+m,\,2+f+n}_{p+\nu+2,\,q+u+1}\left[t\ \lambda^{-\theta}\left|\begin{matrix}(\tfrac{1}{2}-\alpha-\nu-2\lambda-2r,\ \alpha_1),\ (1-\beta,\ \beta_1),\ \{(\varepsilon_f,\ \theta\xi_f),\\ \{(\Psi_g,\ \theta\eta_g)\},\ \{(d_q,\ \delta_q)\},\end{matrix}\right.\right.$$

$$\left.\begin{matrix}\{(c_p,\ \gamma_p)\},\ (\varepsilon_{f+1},\ \theta\xi_{f+1}),\ \ldots,\ (\varepsilon_\nu,\ \theta\xi_\nu)\\ (\tfrac{1}{2}-\alpha-\beta-\nu-2\lambda_1-2r,\ \alpha_1+\beta_1),\ (\Psi_{g+1},\ \theta\eta_{g+1}),\ \ldots,\ (\Psi_u,\ \theta\eta_u)\end{matrix}\right].$$

$$\cdot\int_0^\infty z^{\nu+2r+2\lambda_1+\frac{1}{2}}\ g(z)\,dz, \qquad (4{\cdot}4)$$

provided that the integrals $\int_0^\infty z^{\frac{1}{2}}\, g(z)\ dz$ & $\int_0^\infty z^{\nu+2\lambda_1+2r+\frac{1}{2}} g(z)\,dz$ exist, $\alpha_1,\ \beta_1,\ \sigma_1,\quad \mu\geqslant 0,\quad R(\alpha+\nu+2\lambda_1+\frac{1}{2})>0,\quad R(\beta)>0,\quad -\min\limits_{1\leqslant j\leqslant f} R(B_j/\xi_j)<R(\alpha+\beta+\sigma+\nu+2\lambda_1+\frac{1}{2})\ <\ -\max\limits_{1\leqslant i\leqslant g} R\{(A_i-1)/\eta_i\}$, and $\varepsilon_j(j=1,\ 2,\ \ldots,\ \nu)$ and $\psi_j\ (j=1,\ 2,\ \ldots,\ u)$ are same as in (2.1).

Proof. On substituting for

$$f(x)=\int_0^\infty (xz)^{\frac{1}{2}}\quad J^{\mu}_{\nu,\,\lambda_1}(xz)\ g(z)\ dz, \qquad (4{\cdot}5)$$

in the expression for $\phi(t)$ given by (1·2) and changing the order of integration, which is justifiable under the given conditions, we have

$$\phi(t)=\int_0^\infty z^{\frac{1}{2}}\ g(z)dz\int_0^\infty\int_0^\infty x^{\alpha-\frac{1}{2}}y^{\beta-1}\ J^{\mu}_{\nu,\,\lambda_1}(xz)$$

$$(x+y)^{\sigma}\ H^{f,\,g}_{u,\,\nu}\left[\lambda(x+y)\left|\begin{matrix}\{(A_u,\ \eta_u)\}\\ \{(B_\nu,\ \xi_\nu)\}\end{matrix}\right.\right]$$

$$H^{m,\,n}_{p,\,q}\left[t\,x^{\alpha_1}\ y^{\beta_1}\,(x+y)^{\sigma_1}\left|\begin{matrix}\{(c_p,\ \gamma_p)\}\\ \{(d_q,\ \delta_q)\}\end{matrix}\right.\right]dxdy \qquad (4.6)$$

Now on substituting the series expansion for $J^{\mu}_{\nu,\,\lambda_1}(xz)$ from (4·3) and evaluating the inner integral with the help of (2.2), we obtain the theorem stated above.

Example. Let $g(z)=z^{-\frac{1}{2}}\,e^{-az}\,J_\nu(bz)$. On using [2, p. 50(17)], we get

$$f(x)=\frac{1}{\pi}\ b^{-\frac{1}{2}}\,Q_{\nu-\frac{1}{2}}\left(\frac{a^2+b^2+x^2}{2bx}\right), \qquad (4.7)$$

provided that $R(a) > \text{Im}(b) > 0$ and $R(\nu) > -\frac{1}{2}$. Also, using [1, p. 327 (6)], we obtain

$$\int_0^\infty z^{\nu+2r+\frac{1}{2}} g(z)\, dz = \frac{b^\nu\, \Gamma(2r+2\nu+1)}{2^\nu\, a^{2\nu+2r+1}\, \Gamma(\nu+1)}$$

$$ {}_2F_1\left(\nu+r+\tfrac{1}{2}, \nu+r+1\,;\, \nu+1\,;\, -\frac{b^2}{a^2}\right),$$

provided that $R(a) > \text{Im}(b) > 0$, $R(\nu) > -\frac{1}{2}$. Hence using the result (4·4) with $\lambda_1 = 0$ and $\mu = 1$, we have

$$\int_0^\infty \int_0^\infty x^{\alpha-1}\, y^{\beta-1}\, (x+y)^\sigma\, Q_{\nu-\frac{1}{2}}\left(\frac{a^2+b^2+x^2}{2bx}\right) H^{f,\, g}_{u,\, v}\left[\lambda(x+y)\,\middle|\, \begin{matrix}\{(A_u, \eta_u)\}\\ \{(B_v, \xi_v)\}\end{matrix}\right] H^{m,\, n}_{p,\, q}\left[t\, x^{\alpha_1}\, y^{\beta_1}\, (x+y)^{\sigma_1}\,\middle|\, \begin{matrix}\{(c_p, \gamma_p)\}\\ \{(d_q, \delta_q)\}\end{matrix}\right] dxdy$$

$$= \frac{\lambda^{-\alpha-\beta-\sigma-\nu+\frac{1}{2}}\, b^{\nu+\frac{1}{2}}.\pi}{2^{2\nu}\, \Gamma(1+\nu)\, a^{2\nu+1}} \sum_{r=0}^\infty \frac{(-1)^r}{r!}\, \frac{(2\lambda a)^{-2r}\, \Gamma(2\nu+2r+1)}{\Gamma(\nu+r+1)} \,.$$

$$ {}_2F_1\left(\nu+r+\tfrac{1}{2}, \nu+r+1\,;\, \nu+1\,;\, -\frac{b^2}{a^2}\right)$$

$$H^{m+g,\, 2+f+n}_{p+v+2,\, q+u+1}\left[t\lambda^{-\theta}\,\middle|\, \begin{matrix}(\frac{1}{2}-\alpha-\nu-2r, \alpha_1), (1-\beta, \beta_1), \{(\varepsilon_f, \theta\xi_f)\}, \{(c_p, \gamma_p)\}, (\varepsilon_{f+1}, \theta\xi_{f+1}), \ldots, (\varepsilon_v, \theta\xi_v)\\ \{(\psi_g, \theta\eta_g)\}, \{(d_q, \delta_q)\}, (\frac{1}{2}-\alpha-\beta-\nu-2r, \alpha_1+\beta_1), (\psi_{g+1}, \theta\eta_{g+1}), \ldots, (\psi_u, \theta\eta_u)\end{matrix}\right], \quad (4·8)$$

provided that $R(a) > \text{Im}(b) > 0$, $R(\nu) > -\frac{1}{2}$, $\alpha_1, \beta_1, \sigma_1 \geqslant 0$, and the other conditions given with (2·1) are satisfied.

5. Theorem 3. If

$$\phi(t) = DT[f(x)], \quad (5·1)$$

and $f(x)$ is self-reciprocal in the ψ_{ν_1}, k_1, m_1 —transform, then

$$\phi(t) = \frac{\lambda^{-\alpha-\beta-\sigma-\nu_1-\frac{1}{2}}}{2^{\nu_1}} \sum_{r=0}^\infty \frac{(-1)^r}{r!}\, \frac{\Gamma(2m_1-r)\, \Gamma(\frac{1}{2}-k_1+m_1+\nu_1+r)}{\Gamma(1+2m_1+\nu_1+r)} \,.$$

2^{-2r}

$$\frac{1}{\Gamma(1+\nu_1+r)\,\Gamma(-k_1+m_1+\frac{1}{2}-r)}$$

$$H^{g+m,\,2+f+u}_{p+v+2,\,q+u+1}\left[t\lambda^{-\theta}\,\middle|\,\begin{matrix}(\frac{1}{2}-\alpha-\nu_1-2r,\,\alpha_1),\\ \{(\psi_g,\,\theta\eta_g)\},\end{matrix}\right.$$

$$\left.\begin{matrix}(1-\beta,\,\beta_1),\,\{(\varepsilon_f,\,\theta\xi_f)\},\,\{(c_p,\,\gamma_p)\};\,(\varepsilon_{f+1},\,\theta\xi_{f+1}),\,\ldots,\,(\varepsilon_v,\,\theta\xi_v)\\ \{(d_q,\,\delta_q)\},\,(\frac{1}{2}-\alpha-\beta-\nu_1-2r,\,\alpha_1+\beta_1),\,(\psi_{g+1},\,\theta\eta_{g+1}),\,\ldots,\,(\psi_u,\,\theta\eta_u)\end{matrix}\right].$$

$$\int_0^\infty z^{\nu_1+2r+\frac{1}{2}}\,f(z)\,dz, \tag{5·2}$$

provided that the integrals $\int_0^\infty z^{\nu_1+2r+\frac{1}{2}} f(z)\,dz$ and $\int_0^\infty z^{-\nu_1+\frac{1}{2}}$ $f(z)\,dz$ exist, $\alpha_1,\,\beta_1,\,\sigma_1 \geqslant 0$, $R(\nu_1-k_1+m_1+\frac{1}{2})>0$, $R(m_1)(>0,\,\nu_1<0,$ $\nu_1<-2m_1$, $2m_1$ is not an integer ; $-\delta'<R(\alpha+\beta+\sigma+\nu_1+\frac{1}{2})<-\beta'$, and θ, ε_j $(j=1,\,\ldots,\,v)$, ψ_j $(j=1,\,\ldots,\,u)$ are the same as in (2.1).

Proof. We have

$$f(x)=2^{\nu_1}\int_0^\infty (xz)^{-\nu_1+\frac{1}{2}}\;H^{2,\,1}_{2,\,4}\left[\frac{x^2z^2}{4}\,\middle|\,\begin{matrix}(k_1-m_1-\frac{1}{2},\,1),\,(\nu_1-k_1+2m_1+\frac{1}{2},\,1)\\ (\nu_1,\,1),\,(\nu_1+2m_1,\,1),\,(-2m_1,\,1),\,(0,\,1)\end{matrix}\right]f(z)dz. \tag{5·3}$$

On substituting for $f(x)$ from (5·3) in (5·1) and changing the order of integration, which is justifiable under the above conditions, we obtain

$$\phi(t)=2^{\nu_1}\int_0^\infty z^{-\nu_1+\frac{1}{2}}f(z)\,dz\int_0^\infty\int_0^\infty x^{\alpha-\nu_1+\frac{1}{2}-1}\,y^{\beta-1}\,(x+y)^{\sigma}$$

$$.\;H^{f,\,g}_{u,\,v}\left[\lambda(x+y)\,\middle|\,\begin{matrix}\{(A_u,\,\eta_u)\}\\ \{(B_v,\,\xi_v)\}\end{matrix}\right]H^{m,\,n}_{p,\,q}\left[tx^{\alpha_1}\,y^{\beta_1}\,(x+y)^{\sigma_1}\,\middle|\,\begin{matrix}\{(c_p,\,\gamma_p)\}\\ \{d_q,\,\delta_q)\}\end{matrix}\right]$$

$$.\;H^{2,\,1}_{2,\,4}\left[\frac{x^2z^2}{4}\,\middle|\,\begin{matrix}(k_1-m_1-\frac{1}{2},\,1),\,(\nu_1-k_1+m_1+\frac{1}{2},\,1)\\ (\nu_1,\,1),\,(\nu_1+2m_1,\,1),\,(-2m_1,\,1),\,(0,\,1)\end{matrix}\right]dx\,dy. \tag{5·4}$$

But from the power series expansion due to Mukherjee and Prasad [4]

$$H^{m+1,\,n}_{p,\,q+1}\left[ax^{\sigma}\,\middle|\,\begin{matrix}\{(a_p,\,\alpha_p)\}\\ (b_0,\,\beta_0),\,\{(b_q,\,\beta_q)\}\end{matrix}\right]$$

$$=\frac{1}{\beta_0}\sum_{r=0}^{\infty}\frac{(-1)^r}{r!}$$

$$\frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\,\rho_r)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\,\rho_r)}{\prod_{j=m+1}^{q}\Gamma(1-b_j+\beta_j\,\rho_r)\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j\,\rho_r)}\,a^{\rho_r}\,x^{\sigma\rho_r}, \tag{5·5}$$

where

$\rho_r=\frac{b_0+r}{\beta_0}$, $\beta<R\left(\frac{b_0}{\beta_0}\right)<\delta$, $|\arg a|<\frac{1}{2}\lambda\pi$, $\lambda>0$, $A>0$, we have

$$H_{2,4}^{2,1}\left[\frac{x^2z^2}{4}\middle|\begin{matrix}(k_1-m_1-\frac{1}{2},1),(\nu_1-k_1+m_1+\frac{1}{2},1)\\(\nu_1,1),(\nu_1+2m_1,1),(-2m_1,1),(0,1)\end{matrix}\right]$$

$$=\sum_{r=0}^{\infty}\frac{(-1)^r}{r!}\frac{\Gamma(2m_1-r)\,\Gamma(\frac{1}{2}-k_1+m_1+\nu_1+r)}{\Gamma(1+2m_1+\nu_1+r)\,\Gamma(1+\nu_1+r)\,\Gamma(-k_1+m_1+\frac{1}{2}-r)}$$

$$\cdot\left(\frac{xz}{2}\right)^{2\nu_1+2r}, \tag{5·6}$$

provided that $R(\nu_1-k_1+m_1+\frac{1}{2})>0$, $R(m_1)>0$, $\nu_1<0$, $-\nu_1<-2m_1$, and $2m_1$ is not a positive integer. On substituting the results (5.6) in (5·4), interchanging the order of integration and summation, and finally using the result (2·2), we obtain the theorem.

Acknowledgement. Our thanks are due to Dr. R. N. Pandey for his valuable suggestions in the preparation of this paper.

REFERENCES

[1] A. Erdélyi et al. *Tables of Integral Transforms*, vol. I, McGraw Hill, New York, 1954.

[2] ————, *Tables of Integral Transforms*, vol. II, McGraw Hill, New York, 1954.

[3] C. Fox, *The G and H functions as symmetrical Fourier kernels*, Trans. Amer. Math. Soc. **98** (1961), 395-429.

[4] S. N. Mukherjee and Y. N. Prasad, *On some infinite integrals involving the products of H-functions*, Math. Education vol, **5**, (1971), p. 60.

[5] R. S. Pathak, *Certain convergence theorems and asymptotic properties of a generalization of Lommel and Maitland transform*, Proc. Nat. Acad. Sci. India. Sect. A **36**, (1966), 81-86.

[6] Roop Narain, *On a generalization of Hankel transform and self-reciprocal functions*, Univ. e Politec. Torino, Rend. Sem. Mat. **16** (1956-57), 269-300.

[7] H. M. Srivastava and Rekha Panda, *Some operational techniques in the theory of special functions*, Nederl. Akad. Wetensch. Proc. Ser. A **76** = Indag. Math. **35** (1973), 308-319.