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FOURIER SERIES FOR MODIFIED MULTI-VARIABLE *H*-FUNCTION

By

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ABSTRACT

The object of this paper is to derive integral involving modified *H*-function of several variables. These integrals are used to establish the Fourier series for generlized function. By suitably specializing the coefficients and the parameters we can obtain many (new and known) interesting results involving multi-variable *H*-functions [8]. The results obtained by Srivastava and Panda [8], Kaul [1,2], Mac Robert [3] and Sneddon [7] follow as particular cases of our results.

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1. Introduction. The modified multi-variable *H*-function employed as kernel of multi-dimentional transform defined by Prasad and Singh [6] on the lines of Srivastava and Panda [8], Prasad and Maurya [5] is as follows:

$$H^{m,n:|R':m_1,n_1;\ldots;m_r,n_r}_{p,q:|R:p_1q_1;\ldots;p_r,q_r}\left[\begin{matrix} Z_1 \\ \vdots \\ Z_r \end{matrix}\left|\begin{matrix} \left(a_j;\alpha'_j,\ldots,\alpha^{(r)}_j\right)_{1,p} : \left(e_j : u'_j g'_j,\ldots,u^{(r)}_j g^{(r)}_j\right)_{1,IR'} : \left(c'_j,\gamma'_j\right)_{1,p_1};\ldots;\left(c^{(r)}_j \gamma^{(r)}_j\right)_{1,p_r} \\ \left(b_j : \beta'_j,\ldots,\beta^{(r)}_j\right)_{1,q} : \left(l_j; U'_j f'_j,\ldots,U^{(r)}_j f^{(r)}_j\right)_{1,IR} : \left(d'_j,\delta'_j\right)_{1,q_1};\ldots;\left(d^{(r)}_j \delta^{(r)}_j\right)_{1,q_r} \end{matrix}\right.\right]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\ldots\int_{L_r}\phi_1(\xi_1)\ldots\phi_r(\xi_r)\times\psi(\xi_1,\ldots,\xi_r)\, z_1^{\xi_1}\ldots z_r^{\xi_r}\, d\xi_1\ldots d\xi_r \qquad \ldots(1.1)$$

where

$$\phi_i(\xi_i) = \frac{\prod_{j=1}^{m_i}\Gamma\left(d^{(i)}_j\xi_i\right)\prod_{j=1}^{n_i}\Gamma\left(1-c^{(i)}_j-\gamma^{(i)}_j\xi_i\right)}{\prod_{j=m_i+1}^{q_i}\Gamma\left(1-d^{(i)}_j-\delta^{(i)}_j\xi_i\right)\prod_{j=n_i+1}^{p_i}\Gamma\left(c^{(i)}_j-\gamma^{(i)}_j\xi_i\right)} \qquad (i=1,2,\ldots,r) \qquad \ldots(1.2)$$

$$\psi(\xi_1,\ldots,\xi_r)=\frac{\prod_{j=1}^{m}\Gamma\left(b_j-\sum_{i=1}^{r}\beta_j^{(i)}\xi_i\right)\prod_{j=1}^{n}\Gamma\left(1-a_j-\sum_{i=1}^{r}\alpha_j^{(i)}\xi_i\right)}{\prod_{j=m+1}^{q}\Gamma\left(1-b_j^{(i)}+\sum_{i=1}^{r}\beta_j^{(i)}\xi_i\right)\prod_{j=n+1}^{p}\Gamma\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}\xi_i\right)}\times\frac{\prod_{j=1}^{IR'}\Gamma\left(e_j+\sum_{i=1}^{r}u_j^{(i)}g_j^{(i)}\xi_i\right)}{\prod_{j=1}^{IR'}\Gamma\left(I_j^{(i)}+\sum_{i=1}^{r}U_j^{(i)}f_j^{(i)}\xi_i\right)}.$$

...(1.3)

The multiple integral (1.1) converges absolutely if

$$|arg\, z_i|<\frac{1}{2}U_i\pi, \qquad (i=1,2,\ldots,r).$$

where

$$U_i=\sum_{j=1}^{m}\beta_j^{(i)}-\sum_{j=m+1}^{q}\beta_j^{(i)}+\sum_{j=1}^{n}\alpha_j^{(i)}-\sum_{j=n+1}^{p}\alpha_j^{(i)}+\sum_{j=1}^{m_i}\delta_j^{(i)}-\sum_{j=m_i+1}^{q_i}\delta_j^{(i)}+\sum_{j=1}^{n_i}\gamma_j^{(i)}$$

$$-\sum_{j=n_i+1}^{p_i}\gamma_j^{(i)}+\sum_{j=1}^{IR'}g_j^{(i)}-\sum_{j=1}^{IR}f_j^{(i)}>0 \qquad (i=1,2,\ldots,r).$$

We shall require the following known integrals.

1. $\int_0^\pi cos\,P\theta(cos\,\theta/2)^{2\rho}(sin\,\theta/2)^{2\rho_1}\,d\theta$

$$=\frac{\Gamma(P+\rho+1/2)\Gamma(\rho_1+1/2)}{2\Gamma(P+\rho+\rho_1+1)}\,{}_3F_2\left[\begin{matrix}\rho_1+1/2,-P,-P+1/2;\\-P-\rho+1/2,1/2;\end{matrix}\,1\right] \quad \ldots(1.4)$$

provided that $Re(2\rho+1)>0$, $Re(\rho_1+1)>0$ and $P=0,1,2,\ldots$.

2. $\int_0^\pi sin(2s+1)\theta(cos\,\theta)^{2\rho}(sin\,\theta)^{2\rho_1}\,d\theta$

$$=\frac{\Gamma(2s+2)\Gamma(s+\rho+1/2)\Gamma(\rho_1+1)}{\Gamma(s+\rho+\rho_1+3/2)\Gamma(2s+1)}\,{}_3F_2\left[\begin{matrix}\rho_1+1,-s,(-s+1/2);\\(-s-\rho+1/2),3/2;\end{matrix}\,1\right],$$

provided that $Re(2\rho+1)>0$, $Re(\rho_1+1)>0$ and $s=0,1,2,\ldots$.

2. Main Integrals.

(I) $$\int_0^\pi\left(cos\frac{\theta}{2}\right)^{2\rho}\left(sin\frac{\theta}{2}\right)^{2\rho_1}H\left[\left\{\left(cos\frac{\theta}{2}\right)^{2h_1}\left(sin\frac{\theta}{2}\right)^{2k_1}z_1,\ldots,\left(cos\frac{\theta}{2}\right)^{2h_r}\left(sin\frac{\theta}{2}\right)^{2k_r}z_r\right\}\right]d\theta$$

$$= H^{m,n+2:IR,m_1,n_1;\ldots;m_r n_r}_{p+2,q+1:IR',p_1,q_1;\ldots;p_r q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\left|\begin{matrix}(1/2-\rho;h_1,\ldots,h_r),(1/2-\rho_1;k_1,\ldots,k_r), \\ (-\rho-\rho_1;h_1+k_1,\ldots,h_r+k_r),\end{matrix}\right.\right.$$

$$\left.\begin{matrix}\left(a_j;\alpha'_j,\ldots,\alpha^{(r)}_j\right)_{1,p}:\left(e_j:u'_jg'_j,\ldots,u^{(r)}_j g^{(r)}_j\right)_{1,IR'}:\left(c'_j,\gamma'_j\right)_{1,p_1};\ldots;\left(c^{(r)}_j,\gamma^{(r)}_j\right)_{1,p_r} \\ \left(b_j:\beta'_j,\ldots,\beta^{(r)}_j\right)_{1,q}:\left(l_j;U'_jf'_j,\ldots,U^{(r)}_j f^{(r)}_j\right)_{1,IR}:\left(d'_j,\delta'_j\right)_{1,q_1};\ldots;\left(d^{(r)}_j\delta^{(r)}_j\right)_{1,q_r}\end{matrix}\right] \quad \ldots(2.1)$$

provided that $Re\left(2\rho+2\sum_{i=1}^{r}h_i\lambda_i\right)>0$, $Re\left(2\rho_1+2\sum_{i=1}^{r}k_i\lambda_i+1\right)>0$,

where $\lambda_i = min\, Re\left(d^{(i)}_j/\delta^{(i)}_j\right)\ (j=1,\ldots m_i, i=1,\ldots,r)$

(iii) $0\le\theta\le\pi$

along with the conditions for the convergence of modified *H*-function.

(II) $$\int_0^\pi cos P\theta\left(cos\frac{\theta}{2}\right)^{2\rho}\left(sin\frac{\theta}{2}\right)^{2\rho_1} H\left[\left\{\left(cos\frac{\theta}{2}\right)^{2h_1}\left(sin\frac{\theta}{2}\right)^{2k_1}z_1,\ldots,\left(cos\frac{\theta}{2}\right)^{2h_r}\left(sin\frac{\theta}{2}\right)^{2k_r}z_r\right\}\right]d\theta$$

$$=\sum_{N=0}^{P}\frac{(-1)^N(-P)_N(-P+1/2)_N}{(1/2)_N\,N!}H^{m,n+2:IR':m_1,n_1,\ldots,m_r,n_r}_{p+2,q+1:IR:p_1,q_1,\ldots,p_r,q_r}$$

$$\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\left|\begin{matrix}(1/2+N-P-\rho;h_1,\ldots,h_r),(1/2-N-\rho_1;k_1,\ldots,k_r),\left(a_j;\alpha'_j,\ldots,\alpha^{(r)}_j\right)_{1,p}: \\ (-P-\rho-\rho_1;h_1+k_1,\ldots,h_r+k_r),\qquad\qquad \left(b_j:\beta'_j,\ldots,\beta^{(r)}_j\right)_{1,q}:\end{matrix}\right.\right.$$

$$\left.\begin{matrix}\left(e_j:u'_jg'_j,\ldots,u^{(r)}_j g^{(r)}_j\right)_{1,IR'}:\left(c'_j,\gamma'_j\right)_{1,p_1};\ldots;\left(c^{(r)}_j,\gamma^{(r)}_j\right)_{1,p_r} \\ \left(l_j;U'_jf'_j,\ldots,U^{(r)}_j f^{(r)}_j\right)_{1,IR}:\left(d'_j,\delta'_j\right)_{1,q_1};\ldots;\left(d^{(r)}_j,\delta^{(r)}_j\right)_{1,q_r}\end{matrix}\right] \quad \ldots(2.2)$$

with the conditions of (2.1) and P=0,1,2,... .

(III) $$\int_0^\pi sin(2P+1)\theta(cos\theta)^{2\rho}(sin\theta)^{2\rho_1} H\left[\left\{(cos\theta)^{2h_1}(sin\theta)^{2k_1}z_1,\ldots,(cos\theta)^{2h_r}(sin\theta)^{2k_r}z_r\right\}\right]d\theta$$

$$=(2P+1)\sum_{N=0}^{P}\frac{(-1)^N(-P)_N(-P+1/2)N}{(3/2)_N\,N!}\times H^{m,n+2:IR':m_1,n_1,\ldots,m_r,n_r}_{p+2,q+1:IR:p_1,q_1,\ldots,p_r,q_r}$$

$$\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix} (1/2+N-P-\rho;h_1,...,h_r),(-N-\rho_1;k_1,...,k_r)\left(a_j;\alpha'_j,...,\alpha_j^{(r)}\right)_{1,p}: \\ (-1/2-P-\rho-\rho_1;h_1+k_1,...,h_r+k_r) \qquad \left(b_j:\beta'_j,...,\beta_j^{(r)}\right)_{1,q}: \end{matrix}\right.$$

$$\left.\begin{matrix} \left(e_j:u'_jg'_j,...,u_j^{(r)}g_j^{(r)}\right)_{1,IR'}:\left(c'_j,\gamma'_j\right)_{1,p_1};...;\left(c_j^{(r)}\gamma_j^{(r)}\right)_{1,p_r} \\ \left(l_j;U'_jf'_j,...,U_j^{(r)}f_j^{(r)}\right)_{1,IR}:\left(d'_j,\delta'_j\right)_{1,q_1};...;\left(d_j^{(r)}\delta_j^{(r)}\right)_{1,q_r} \end{matrix}\right] \qquad ...(2.3)$$

provided that $Re\left(\rho_1+\sum_{i=1}^{r}k_i\alpha_i+1\right)>0$,

where $\alpha_i = min\,Re\left(d_j^{(i)}/\delta_j^{(i)}\right),(j=1,2,...,m_i,i=1,2,...,r)$ and $P=0,1,2...$

3. Proof. In order to prove (2.1), we substitute for the modified multi-variable H-function in terms of its contour integral of Mellin-Barenes type and change the order of integration (which is permissible). We then evaluate the inner integral using Gamma function. On interpreting the resulting contour integral by means of (1.1), we obtain (2.1). Proceeding in the similar way, using (1.4) and (1.5) we obtain (2.2) and (2.3).

4. Fourier Series. We shall now establish following Fourier series for mdified multi-variable H-functions :

$$(cos\theta/2)^{2\rho}(sin\theta/2)^{2\rho_1}H\left[\left\{(cos\theta/2)^{2h_1}(sin\theta/2)^{2k_1}z_1,...,(cos\theta/2)^{2h_r}(sin\theta/2)^{2k_r}z_r\right\}\right]$$

$$=\frac{1}{\pi}H^{m,n+1:IR':m_1,n_1;...;m_r,n_r}_{p+1,q+1:IR:p_1,q_1;...;p_r q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix} \left(\frac{1}{2}-\rho;h_1,...,h_r\right), \qquad \left(a_j;\alpha'_j,...,\alpha_j^{(r)}\right)_{1,p}: \\ (-\rho-\rho_1;h_1+k_1,...,h_r+k_r),\left(b_j;\beta'_j,...,\beta_j^{(r)}\right)_{1,q}: \end{matrix}\right.$$

$$\left.\begin{matrix} \left(e_j:u'_jg'_j,...,u_j^{(r)}g_j^{(r)}\right)_{1,IR}:\left(c'_j,\gamma'_j\right)_{1,p_1};...;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r} \\ \left(\ell_j:U'_jf'_j,...,U_j^{(r)}f_j^{(r)}\right)_{1,IR}:\left(d'_j,\delta'_j\right)_{1,q_1};...;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r} \end{matrix}\right]+\frac{2}{\pi}\sum_{P=1}^{\infty}\left\{\sum_{s=0}^{p}\frac{(-P)_s(-P+1/2)_s}{(1/2)_s\,s!}\right\}$$

$$H^{m,n+2:IR':m_1,n_1;...;m_r,n_r}_{p+2,q+1:IR:p_1,q_1;...;p_r,q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix} (1/2+s-P-\rho;h_1,...,h_r),(1/2-s-\rho_1;k_1,...,k_r): \\ (-P-\rho-\rho_1;h_1+k_1,...,h_r+k_r): \end{matrix}\right.$$

$$\left. \begin{matrix} \left(a_j;\alpha'_j,...,\alpha_j^{(r)}\right)_{1,p} : \left(e_j : u'_j g'_j,...,u_j^{(r)} g_j^{(r)}\right)_{1,IR} : \left(c'_j,\gamma'_j\right)_{1,p_1} ;...;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r} \\ \left(b_j;\beta'_j,...,\beta_j^{(r)}\right)_{1,q} : \left(\ell_j : U'_j f'_j,...,U_j^{(r)} f_j^{(r)}\right)_{1,IR} : \left(d'_j,\delta'_j\right)_{1,q_1} ;...;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r} \end{matrix} \right] \quad ...(4.1)$$

with the conditions as in (2.1) and $P=0,1,2,...$

5. Proof. In order to prove (4.1), let us suppose

$$f(\theta) = (cos\theta/2)^{2\rho}(sin\theta/2)^{2\rho_1} \quad ...(5.1)$$

$$H\left[\left\{(cos\theta/2)^{2h_1}(sin\theta/2)^{2k_1} z_1,...,(cos\theta/2)^{2h_r}(sin\theta/2)^{2k_r} z_r\right\}\right]$$

$$= \frac{1}{2}C_0 + \sum_{P=1}^{\infty} C_p \cos P\theta \,. \quad ...(5.2)$$

Integrating (5.1) between the limits 0 to π and using the reult (2.3), we obtain the value of C_0. Again multiplying both the sides of (5.1) by $cos P\theta$ and integrating from 0 to π w.r.t. θ, using (2.2) we get

$$C_p = \frac{2}{\pi}\sum_{s=0}^{\infty} \frac{(-1)_s(-P)_s(-P+1/2)_s}{(1/2)_s S!} H^{m,n+2:IR':m_1,n_1;...;m_r,n_r}_{p+2,q+1:IR:p_1,q_1;...;p_r,q_r}$$

$$\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix} (1/2+s-P-\rho;h_1,...,h_r),(1/2-s-\rho;k_1,...,k_r): \\ (-P-\rho-\rho_1;h_1+k_1,...,h_r+k_r): \end{matrix}\right.$$

$$\left. \begin{matrix} \left(a_j;\alpha'_j,...,\alpha_j^{(r)}\right)_{1,p} : \left(e_j : u'_j g'_j,...,u_j^{(r)} g_j^{(r)}\right)_{1,IR} : \left(c'_j,\gamma'_j\right)_{1,p_1} ;...;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r} \\ \left(b_j;\beta'_j,...,\beta_j^{(r)}\right)_{1,q} : \left(\ell_j : U'_j f'_j,...,U_j^{(r)} f_j^{(r)}\right)_{1,IR} : \left(d'_j,\delta'_j\right)_{1,q_1} ;...;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r} \end{matrix} \right] \quad ...(5.3)$$

with the conditions as in (2.1). Putting the values of C_0 and C_p in (5.2). We obtain (4.1). Fourier sine series may also be obtained similarly.

6. Special Cases.

Case I. If we put $m = IR' = IR = 0$ in (2.1) we get result in terms of multi-variable *H*-function defined by Srivastava and Panda [8] as :

$$\int_0^{\pi} (cos\theta/2)^{2\rho}(sin\theta/2)^{2\rho_1} H\left[(cos\theta/2)^{2h_1}(sin\theta/2)^{2k_1} z_1,...,(cos\theta/2)^{2h_r}(sin\theta/2)^{2k_r} z_r\right] d\theta$$

$$= H^{0,n+2:m_1,n_1;...;m_r,n_r}_{p+2,q+1:p_1,q_1;...;p_r,q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix} (1/2-\rho;h_1,...,h_r),(1/2-\rho_1;k_1,...,k_r)\left(a_j;\alpha'_j,...,\alpha_j^{(r)}\right)_{1,p} : \\ (-\rho-\rho_1;h_1+k_1,...,h_r+k_r), \qquad \left(b_j;\beta'_j,...,\beta_j^{(r)}\right)_{1,q} : \end{matrix}\right.$$

$$\left.\begin{matrix}(c'_j;\gamma'_j)_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\(d'_j,\delta'_j)_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right]$$

provided that

1. $Re\left(2\rho+2\sum_{i=1}^{r}h_i\lambda_i+1\right)>0$,

2. $Re\left(2\rho_1+2\sum_{i=1}^{r}k_i\lambda_i+1\right)>0$ where $\lambda_i=\min_{1\le j\le m_i}\left[Re\left(d_j^{(i)}/\delta_j^{(i)}\right)\right]$ $(i=1,2,\ldots,r)$.

Case II. Similarly if we put $m=IR'=IR=0$ in (4.1) we get the result in terms of multivariable H-function defined by Srivastava and Panda [8] as:

$$(cos\theta/2)^{2\rho}(sin\theta/2)^{2\rho_1}H\left[(cos\theta/2)^{2h_1}(sin\theta/2)^{2k_1}z_1,\ldots,(cos\theta/2)^{2h_r}(sin\theta/2)^{2k_r}z_r\right]$$

$$=\frac{1}{\pi}H^{0,n+1:m_1,n_1;\ldots;m_r,n_r}_{p+1,q+1:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix}z_1\\\vdots\\z_r\end{matrix}\left|\begin{matrix}(1/2-\rho;h_1,\ldots,h_r),\quad(a_j;\alpha'_j,\ldots,\alpha_j^{(r)})_{1,p}:\\(-\rho-\rho_1;h_1+k_1,\ldots,h_r+k_r),(b_j;\beta'_j,\ldots,\beta_j^{(r)})_{1,q}:\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(c'_j,\gamma'_j)_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\(d'_j,\delta'_j)_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right]+\frac{2}{\pi}\sum_{P=1}^{\infty}\left\{\sum_{s=0}^{P}\frac{(-P)_s(-P+1/2)_s}{(1/2)_s\,s!}\right\}$$

$$H^{0,n+2:m_1,n_1;\ldots;m_r,n_r}_{p+2,q+1:p_1q_1;\ldots;p_rq_r}\left[\begin{matrix}z_1\\\vdots\\z_r\end{matrix}\left|\begin{matrix}(1/2+s-P-\rho;h_1,\ldots,h_r),(1/2-s-\rho_1;k_1,\ldots,k_r):\\(-P-\rho-\rho_1;h_1+k_1,\ldots,h_r+k_r):\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(a_j;\alpha'_j,\ldots,\alpha_j^{(r)})_{1,p}:(c'_j,\gamma'_j)_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\(b_j;\beta'_j,\ldots,\beta_j^{(r)})_{1,q}:(d'_j,\delta'_j)_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right],\qquad\ldots(6.2)$$

with the conditions as in (6.1) and $P=0,1,2,\ldots$

Case III. Again in (1.4) if we put $\rho_1=0$, then the hyper-geometric function ${}_3F_2$ reduces to ${}_2F_1$. The resultant is written as:

$$ {}_2F_1(a,b,c;1)=\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}, Re(c-a-b)>0 \; . $$

On further simplification, we get

$$ \int_0^{\pi} cos\,P\theta(cos\,\theta/2)^{2\rho}\,d\theta=\frac{\Gamma(2\rho+1)}{2^{2\rho}\Gamma(\rho\pm P+1)}, \quad ...(6.3) $$

provided that $Re(\rho)>-1/2$ and $P=0,1,2,...$ which is known result given by MacRobert [3].

Case IV. Also if we put $\rho=0$ in (1.4) and proceed similarly as above, we obtain

$$ \int_0^{\pi} cos\,P\theta(sin\,\theta/2)^{2\rho_1}\,d\theta=\frac{\Gamma(2\rho_1+1)\Gamma(1/2\pm P)}{2^{2\rho_1}\Gamma(\rho_1\pm P+1)}, \quad ...(6.4) $$

provided that $Re(\rho_1)>-1/2$ and $P=0,1,2,...$ which is the known result due to Sneddon [7].

Case V. In (2.2) using r=2 and putting $p_1=k_1=k_2=.....=k_r=0$. we obtain a result in terms of H-function of two variables (defined by Mittal and Gupta [4]), given by koul [1,2].

Case VI. Again using $\rho_1=h_1=h_2=.....=h_1=k_1=k_2=...=k_r=0$ and taking r=2 in (2.2) we get the result given by Koul [1.2].

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