Jñānābha, Vol. 37, 2007
(Dedicated to Honour Professor S.P. Singh on his 70th Birthday)

ON DECOMPOSITIONS OF PROJECTIVE CURVATURE TENSOR IN CONFORMAL FINSLER SPACE

By

C.K. Mishra and Gautam Lodhi
Deapartment of Mathematics and Statistics, Dr. R.M.L. Avadh University
Faizabad-224001, Uttar Pradesh, India
E-mail :chayankumarmishra@yahoo.com, lodhi_gautam@rediffmail.com

(Received : April 20, 2007)

ABSTRACT

M.S. Knebelman [1] has developed conformal geometry or generalised metric spaces. The projective tensor and curvature tensors in conformal Finisler space were discusssed by Mishra ([2][3]). The decomposition of recurrent curvature tensor in an areal space of submetric class were discussed by M. Gamma [6]. The decomposition of recurrent curvature tensor in Finsler manifold was studied by Sinha and Singh [5]. Singh and Gatoto [9] have also studied the decomposition of curvature tensor in recurrent conformal Finsler space. The purpose of the present paper is to decompose the Projective curvature tensor and study the identities satisfied by projective curvature tensor in conformal Finsler space.

2000 Mathematics Subject Classification : 53B40
Keywords : Finler space, Projective curvature tens, Conformal Finsler Space.

1. Introduction. Let us considered two distinct metric functions $F(x,\dot{x})$ and $\overline{F}(x,\dot{x})$ be defined over an n-dimentional space F_n, both of which satiesfy the requisite conditions for a Finsler space. Th ecorresponding two metric tensor and $\overline{g}^{ij}(x,\dot{x})$ and $\overline{g}^{ij}(x,\dot{x})$ resulting from these functions are called conformal. If there exists a factor of proportionality between two metric tensors, Knebelman has proved that the factor of proportionality between them is at most a point function. Thus we have

$$\overline{g}^{ij}(x,\dot{x}) = e^{-2\sigma} g^{ij}(x,\dot{x}) \quad ...(1)$$

$$\overline{g}^{ij}(x,\dot{x}) = e^{-2\sigma} g^{ij}(x,\dot{x}) \quad ...(2)$$

where

$$\sigma = \sigma(x) \quad ...(3)$$

$$F(x,\dot{x}) = e^{\sigma} F(x,\dot{x}) \quad ...(4)$$

The space equiped with quantities $\overline{F}(x,\dot{x})$ and $\overline{g}(x,\dot{x})$ etc is called a conformal Finsler space, it is denoted by $\overline{F}_n$.

$$B^{ij}(x,\dot{x})=\frac{1}{2}F^2 g^{ij}-\dot{x}^i\dot{x}^j, \qquad \dots(5)$$

where B^{ij} are homogeneous of the second degree in there directional arguements.

The following geometric entities of the conformal Finsler space are given by [7] and [8].

$$\overline{G}^i(x,\dot{x})=G^i(x,\dot{x})-\sigma_m B^{im}(x,\dot{x}) \qquad \dots(6)$$

$$\overline{G}^i_{jk}(x,\dot{x})=G^i_{jk}(x,\dot{x})-\dot{\partial}_k\dot{\partial}_j B^{im}(x,\dot{x})\sigma_m, \qquad \dots(7)$$

$$\overline{G}^i_{jkh}(x,\dot{x})=G^i_{jkh}(x,\dot{x})-\dot{\partial}_h\dot{\partial}_k\dot{\partial}_j B^{im}(x,\dot{x})\sigma_m \qquad \dots(8)$$

$$B^{ij}(x,\dot{x})=\frac{1}{2}F^2 g^{ij}-\dot{x}^i\dot{x}^j, \qquad \dots(9)$$

where $G^i_{jkh}(x,\dot{x})$ are the Berwald's connection coefficients. They satisfy

$$\dot{\partial}_j G^i_k(x,\dot{x})=G^i_{jk}. \qquad \dots(10)$$

and the functions B^{ij} are homogeneous of the second degree in three direcitonal arguments.

2. Identities Satisfied by the Conformally Changed Projective Curvature Tensor. The tensor W^i_h and W^i_{kh} transform under the conformal change as follow [2].

$$\overline{W}^i_h=W^i_h-\sigma_m\Big[2B^{im}_h-\left(\dot{\partial}_h B^{im}\right)_{(r)}\dot{x}^r-\frac{1}{n-1}\delta^i_h\left\{2B^{pm}_{(p)}-\left(\dot{\partial}_p B^{pm}\right)_{(r)}\dot{x}^r\right\}-\frac{\dot{x}^r}{n^2-1}$$

$$\left\{(2n-1)\left(\dot{\partial}_p B^{pm}\right)_{(h)}-(n+1)\left(\dot{\partial}_h B^{pm}\right)_{(p)}+2(n-2)B^{rm}G^p_{rph}-(n-2)\dot{x}^r\left(\dot{\partial}_p\dot{\partial}_h B^{pm}\right)_{(r)}\right\}\Big]$$

$$+\sigma_{m(r)}\dot{x}^r\left\{\dot{\partial}_h B^{im}-\frac{1}{n-1}\delta^i_h\dot{\partial}_p B^{pm}-\frac{n-2}{n^2-1}\dot{x}^i\dot{\partial}_h\dot{\partial}_p B^{pm}\right\}-\sigma_{m(h)}\left\{2B^{im}-\frac{2n-1}{n^2-1}\right.$$

$$\left.\dot{x}^i\dot{\partial}_p B^{pm}\right\}+\sigma_{m(p)}\left\{\frac{2}{n-1}\delta^i_h B^{pm}-\frac{\dot{x}^2}{n-1}\dot{\partial}_h B^{pm}\right\}+\sigma_m\sigma_r\left[2B^{sm}\dot{\partial}_h\dot{\partial}_s B^{ir}-\left(\dot{\partial}_h B^{sm}\right)\dot{\partial}_s B^{ir}\right.$$

$$-\frac{1}{n-1}\delta^i_h\{2B^{sm}\dot{\partial}_p\dot{\partial}_s B^{pr}-(\dot{\partial}_p B^{sm})\dot{\partial}_s B^{pr}\}+\frac{2\dot{x}^s}{n^2-1}\{(n+1)(\dot{\partial}_{|p}B^{sm})\dot{\partial}_{h|}\dot{\partial}_s B^{pr}$$

$$+(n-2)B^{sm}\dot{\partial}_p\dot{\partial}_h\dot{\partial}_s B^{pr}\}\Big], \qquad ...(11)$$

$$\overline{W}^i_{kh}=W^i_{kh}-2\sigma_m\Big[(\dot{\partial}_{[k}B^{im})_{(h)]}-\frac{\dot{x}^i}{n+1}\{(\dot{\partial}_p\dot{\partial}_{[k}B^{pm})_{(h)]}+(\dot{\partial}_{[k}B^{rm})G^p_{h]rp}\}+\frac{1}{n^2-1}\delta^i_{[k}\{(n+1)$$

$$(\dot{\partial}_{h]}B^{pm})_{(p)}-n(\dot{\partial}_p B^{pm})_{(h)]}-(\dot{\partial}_{h]}\dot{\partial}_p B^{pm})_{(r)}\dot{x}^r+2B^{rm}G^p_{h]rp}\}\Big]+2\sigma_{m|(k)}\Big\{\dot{\partial}_{h]}B^{im}-\frac{n}{n^2-1}$$

$$\delta^i_{h]}\dot{\partial}_p B^{pm}-\frac{\dot{x}^s}{n+1}\dot{\partial}_{h]}\dot{\partial}_p B^{pm}\Big\}+\frac{2}{n^2-1}\sigma_{m(p)}\delta^i_{[k}\{\dot{x}^p\dot{\partial}_{h]}\dot{\partial}_r B^{rm}-$$

$$(n+1)\dot{\partial}_{h]}B^{pm}\}+2\sigma_m\sigma_r\Big[\Big(\dot{\partial}_{[k}B^{sm}\Big\{\dot{\partial}_{h]}\dot{\partial}_s B^{ir}-\frac{\dot{x}^s}{n+1}\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}\Big\}\frac{1}{n-1}\delta^i_{[k}\{(\dot{\partial}_{h]}B^{sm})$$

$$\dot{\partial}_p\dot{\partial}_s B^{pr}-(\dot{\partial}_{h]}\dot{\partial}_s B^{pr})\dot{\partial}_p B^{sm}+\frac{2}{n-1}B^{sm}\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}\Big\}\Big] \qquad ...(12)$$

R.B. Mishra [2] have introduced to obtain the conformal transformation or projective curvature tensor W^i_{jkh} by differentiating (12) with respect to $\dot{x}^j$.

$$\overline{W}^i_{jkh}=W^i_{jkh}+2\sigma_m\Big[(\dot{\partial}_{[k}B^{ir})G^m_{h]jr}-\dot{\partial}_j(\dot{\partial}_{[k}B^{im})_{(h)]}+\frac{\dot{x}^s}{n+1}\{\dot{\partial}_j\dot{\partial}_{[k}(\dot{\partial}_p B^{pm})_{(h)]}\}+\frac{\delta^s_j}{n+1}$$

$$\{\dot{\partial}_p(\dot{\partial}_{[k}B^{pm})_{(h)]}-(\dot{\partial}_{[k}B^{pr})G^m_{h]pr}\}-\frac{\delta^i_{[k}}{n^2-1}\{n\dot{\partial}_j(\dot{\partial}_{h]}B^{pm})_{(p)}-n\dot{\partial}_j(\dot{\partial}_p B^{pm})_{(h)]}$$

$$+\dot{\partial}_{h]}(\dot{\partial}_j B^{pm})_{(p)]}-\dot{\partial}_{h]}(\dot{\partial}_p B^{pm})_{(j)]}\}-\frac{\dot{x}^i\delta^i_{[k}}{n^2-1}\dot{\partial}_j\{\dot{\partial}_{h]}(\dot{\partial}_l B^{pm})_{(p)}-\dot{\partial}_{h]}(\dot{\partial}_p B^{pm})_{(l)}\}\Big]+2\sigma_{m|k}\{\dot{\partial}_{h]}(\dot{\partial}_j B^{im})$$

$$-\frac{\dot{x}^i}{n+1}\dot{\partial}_j(\dot{\partial}_{h]}\dot{\partial}_p B^{pm})-\frac{\delta^i_j}{n+1}\dot{\partial}_{h]}(\dot{\partial}_p B^{pm})\}+\frac{2n\,\delta^i_{[k}}{n^2-1}\{\sigma_{m(h)]}(\dot{\partial}_j\dot{\partial}_p B^{pm})-\sigma_{m(p)}(\dot{\partial}_j\dot{\partial}_{h]}B^{pm})\}$$

$$+2\sigma_m\sigma_r\Big[\dot{\partial}_j(\dot{\partial}_{[k}B^{sm})\dot{\partial}_{h]}\dot{\partial}_s B^{ir}-\frac{\dot{x}^i}{n+1}\{\dot{\partial}_j\dot{\partial}_{[k}(\dot{\partial}_p B^{sm})\dot{\partial}_{h]}\dot{\partial}_s B^{pr}+\dot{\partial}_{[k}(\dot{\partial}_p B^{sm})\dot{\partial}_j(\dot{\partial}_{h]}\dot{\partial}_s B^{pr})$$

$$+\dot{\partial}_j(\dot{\partial}_{[k}B^{sm})\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}+(\dot{\partial}_{[k}B^{sm})\dot{\partial}_j\dot{\partial}_{h]}(\dot{\partial}_p\dot{\partial}_s B^{pr})\}-\frac{\delta^i_j}{n+1}\{\dot{\partial}_p(\dot{\partial}_{[h}B^{sm})\dot{\partial}_{k]}\dot{\partial}_s B^{pr}\}$$

$$+\frac{n\delta^i_{[k}}{n^2-1}\{(\dot\partial_j\dot\partial_{h]}B^{sm})\dot\partial_p\dot\partial_s B^{pr}-(\dot\partial_j\dot\partial_p B^{sm})\dot\partial_{h]}\dot\partial_s B^{pr}+(\dot\partial_{h]}B^{sm})\dot\partial_j\dot\partial_p\dot\partial_s B^{pr}$$

$$-(\dot\partial_p B^{sm})\dot\partial_j\dot\partial_{h]}\dot\partial_s B^{pr}\}+\frac{\delta^i_{[k}}{n^2-1}\{\dot\partial_{h]}(\dot\partial_j B^{sm})\dot\partial_p\dot\partial_s B^{pr}-\dot\partial_{h]}(\dot\partial_p B^{sm})\dot\partial_j\dot\partial_s B^{pr}$$

$$+(\dot\partial_j B^{sm})\dot\partial_{h]}\dot\partial_p\dot\partial_s B^{pr}-(\dot\partial_p B^{sm})\dot\partial_{[h}\dot\partial_j\dot\partial_s B^{pr}\}+\frac{\dot x^l\delta^i_{[k}}{n^2-1}[\dot\partial_j(\dot\partial_{h]}\dot\partial_l B^{sm})\dot\partial_p\dot\partial_s B^{pr}$$

$$-\dot\partial_j\dot\partial_{h]}(\dot\partial_p B^{sm})\dot\partial_l\dot\partial_s B^{pr}+\dot\partial_{h]}(\dot\partial_l B^{sm})\dot\partial_j\dot\partial_p\dot\partial_s B^{pr}-\dot\partial_{h]}(\dot\partial_p B^{sm})\dot\partial_j\dot\partial_l\dot\partial_s B^{pr}$$

$$+\dot\partial_j(\dot\partial_l B^{sm})\dot\partial_{h]}(\dot\partial_p\dot\partial_s B^{pr})-\dot\partial_j(\dot\partial_p B^{sm})\dot\partial_{h]}(\dot\partial_l\dot\partial_s B^{pr})+(\dot\partial_l B^{sm})\dot\partial_j\dot\partial_{h]}\dot\partial_p\dot\partial_s B^{pr}$$

$$-(\dot\partial_p B^{sm})\dot\partial_j\dot\partial_{h]}\dot\partial_l\dot\partial_s B^{pr}\}]+\frac{2\delta^i_{[k}}{n^2-1}\{\sigma_{m(j)}\dot\partial_{h]}(\dot\partial_p B^{pm})-\sigma_{m(p)}\dot\partial_{h]}(\dot\partial_j B^{pm})\}$$

$$+\frac{2\dot x^l\delta^i_{[k}}{n^2-1}\{\sigma_{m(l)}\dot\partial_j(\dot\partial_{h]}\dot\partial_p B^{pm})-\sigma_{m(p)}\dot\partial_j\dot\partial_{h]}(\dot\partial_l B^{pm})\}. \tag{13}$$

Multiply (13) by $\bar g_{iu}$, we get

$$\overline{W}_{jkh}\bar g_{iu}=e^{2\sigma}g_{iu}W_{jkh}+2e^{2\sigma}\sigma_m g_{iu}[\dot\partial_{[k}B^{ir})G^m_{h]jr}-\dot\partial_j(\dot\partial_{[k}B^{im})_{(h)]}+\frac{\dot x^i}{n+1}\{\dot\partial_j\dot\partial_{[k}(\dot\partial_p B^{pm})_{(h)]}\}$$

$$+\frac{\delta^i_j}{n+1}\{\dot\partial_p(\dot\partial_{[k}B^{pm})_{(h)]}-(\dot\partial_{[k}B^{pr})G^m_{h]pr}\}-\frac{\delta^i_{[k}}{n^2-1}\{n\dot\partial_j(\dot\partial_{h]}B^{pm})_{(p)}-n\dot\partial_j(\dot\partial_p B^{pm})_{(h)]}$$

$$+\dot\partial_{h]}(\dot\partial_j B^{pm})_{(p)]}-\dot\partial_{h]}(\dot\partial_p B^{pm})_{(j)]}\}-\frac{\dot x^l\delta^i_{[k}}{n^2-1}\dot\partial_j\{\dot\partial_{h]}(\dot\partial_l B^{pm})_{(p)}-\dot\partial_{h]}(\dot\partial_p B^{pm})_{(l)}\}]$$

$$+2e^{2\sigma}g_{iu}[\sigma_{m[k}\{\dot\partial_{h]}(\dot\partial_j B^{im})-\frac{\dot x^i}{n+1}\dot\partial_j(\dot\partial_{h]}\dot\partial_p B^{pm})-\frac{\delta^i_j}{n+1}\dot\partial_{h]}(\dot\partial_p B^{pm})\}$$

$$+\frac{n\delta^i_{[k}}{n^2-1}\{\sigma_{m(h)]}(\dot\partial_j\dot\partial_p B^{pm})-\sigma_{m(p)}(\dot\partial_j\dot\partial_{h]}B^{pm})\}]+2e^{2\sigma}g_{iu}\sigma_m\sigma_r[\dot\partial_j(\dot\partial_{[k}B^{sm})\dot\partial_{h]}\dot\partial_s B^{ir}$$

$$-\frac{\dot x^l}{n+1}\{\dot\partial_j\dot\partial_{[k}(\dot\partial_p B^{sm})\dot\partial_{h]}\dot\partial_s B^{pr}+\dot\partial_{[k}(\dot\partial_p B^{sm})\dot\partial_j(\dot\partial_{h]}\dot\partial_s B^{pr})+\dot\partial_j(\dot\partial_{[k}B^{sm})\dot\partial_{h]}\dot\partial_p\dot\partial_s B^{pr}$$

$$+(\dot\partial_{[k}B^{sm})\dot\partial_j\dot\partial_{h]}(\dot\partial_p\dot\partial_s B^{pr})\}-\frac{\delta^i_j}{n+1}\{\dot\partial_p(\dot\partial_{[h}B^{sm})\dot\partial_{k]}\dot\partial_s B^{pr}\}+\frac{n\delta^i_{[k}}{n^2-1}\{(\dot\partial_j\dot\partial_{h]}B^{sm})\dot\partial_p\dot\partial_s B^{pr}$$

$$-(\dot\partial_j\dot\partial_p B^{sm})\dot\partial_{h]}\dot\partial_s B^{pr}+(\dot\partial_{h]}B^{sm})\dot\partial_j\dot\partial_p\dot\partial_s B^{pr}-(\dot\partial_p B^{sm})\dot\partial_j\dot\partial_{h]}\dot\partial_s B^{pr}\}+$$

$$\frac{\delta^i_{[k}}{n^2-1}\{\dot\partial_{h]}(\dot\partial_j B^{sm})\dot\partial_p\dot\partial_s B^{pr}-\dot\partial_{h]}(\dot\partial_p B^{sm})\dot\partial_j\dot\partial_s B^{pr}+(\dot\partial_j B^{sm})\dot\partial_{h]}\dot\partial_p\dot\partial_s B^{pr}$$

$$-(\dot\partial_p B^{sm})\dot\partial_{[h}\dot\partial_j\dot\partial_s B^{pr}\}+\frac{\dot x^l\delta^i_{[k}}{n^2-1}[\dot\partial_j(\dot\partial_{h]}\dot\partial_l B^{sm})\dot\partial_p\dot\partial_s B^{pr}-\dot\partial_j\dot\partial_{h]}(\dot\partial_p B^{sm})\dot\partial_l\dot\partial_s B^{pr}$$

$$+\dot{\partial}_{h]}\left(\dot{\partial}_l B^{sm}\right)\dot{\partial}_j\dot{\partial}_p\dot{\partial}_s B^{pr}-\dot{\partial}_{h]}\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_j\dot{\partial}_l\dot{\partial}_s B^{pr}+\dot{\partial}_j\left(\dot{\partial}_l B^{sm}\right)\dot{\partial}_{h]}\left(\dot{\partial}_p\dot{\partial}_s B^{pr}\right)$$
$$-\dot{\partial}_j\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_{h]}\left(\dot{\partial}_l\dot{\partial}_s B^{pr}\right)+\left(\dot{\partial}_l B^{sm}\right)\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}-\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_l\dot{\partial}_s B^{pr}\Big\}\Big]$$
$$+\frac{2e^{2\sigma}g_{iu}\delta^i_{[k}}{n^2-1}\Big[\left\{\sigma_{m(j)}\dot{\partial}_{h]}\left(\dot{\partial}_p B^{pm}\right)-\sigma_{m(p)}\dot{\partial}_{h]}\left(\dot{\partial}_j B^{pm}\right)\right\}$$
$$+\left\{\dot{x}^l\sigma_{m(l)}\dot{\partial}_j\left(\dot{\partial}_{h]}\dot{\partial}_p B^{pm}\right)-\dot{x}^l\sigma_{m(p)}\dot{\partial}_j\dot{\partial}_{h]}\left(\dot{\partial}_l B^{pm}\right)\right\}\Big]\tag{14}$$

where

$$\overline{W}_{jukh}=\overline{g}_{iu}\overline{W}^i_{jkh}\ .\tag{15}$$

We have the following indentities.

Theorem 1. When F_n *and* $\overline{F}_n$ are in conformal correspondence, we have

$$2\overline{W}^i_{[j\cdot k\cdot h]}=2W^i_{[j\cdot k\cdot h]}+2\sigma_m\Big[\dot{\partial}_{[j}\left(\dot{\partial}_{h]}B^{im}\right)_{(k)]}-\dot{\partial}_{[j}\left(\dot{\partial}_k B^{im}\right)_{(h)]}+\frac{\dot{x}^i}{n+1}\Big\{\dot{\partial}_{[j}\dot{\partial}_k\left(\dot{\partial}_p B^{pm}\right)_{(h)]}$$
$$-\dot{\partial}_{[j}\dot{\partial}_{h]}\left(\dot{\partial}_p B^{pm}\right)_{(k)}\Big\}+\frac{\delta^i_{[j}}{n^2-1}\Big\{\dot{\partial}_p\left(\dot{\partial}_k B^{pm}\right)_{(h)]}-\dot{\partial}_p\left(\dot{\partial}_{h]}B^{pm}\right)_{(k)]}-\left(\dot{\partial}_k B^{pr}\right)G^m_{h]pr}$$
$$+\left(\dot{\partial}_{h]}B^{pr}\right)G^m_{kpr}\Big\}-\frac{\delta^i_k}{n^2-1}\Big\{n\dot{\partial}_{[j}\left(\dot{\partial}_{h]}B^{pm}\right)_{(p)}-n\dot{\partial}_{[j}\left(\dot{\partial}_p B^{pm}\right)_{(h])}+\dot{\partial}_{[h}\left(\dot{\partial}_{j]}B^{pm}\right)_{(p)}$$
$$-\dot{\partial}_{[h}\left(\dot{\partial}_p B^{pm}\right)_{(j])}\Big\}+\dot{x}^l\dot{\partial}_{[j}\dot{\partial}_{h]}\left(\dot{\partial}_l B^{pm}\right)_{(p)}-\dot{x}^l\dot{\partial}_{[j}\dot{\partial}_{h]}\left(\dot{\partial}_p B^{pm}\right)_{(l)}\Big\}+\frac{\delta^i_{[h}}{n^2-1}\Big\{n\dot{\partial}_{j]}\left(\dot{\partial}_K B^{pm}\right)_{(p)}$$
$$-n\dot{\partial}_{j]}\left(\dot{\partial}_p B^{pm}\right)_{(k)}+\dot{\partial}_k\left(\dot{\partial}_{j]}B^{pm}\right)_{(p)}+\dot{\partial}_k\left(\dot{\partial}_p B^{pm}\right)_{(j])}+\dot{x}^l\dot{\partial}_{j]}\dot{\partial}_k\left(\dot{\partial}_l B^{pm}\right)_{(p)}$$
$$-\dot{x}^l\dot{\partial}_{j]}\dot{\partial}_k\left(\dot{\partial}_p B^{pm}\right)_{(l)}\Big\}+2\sigma_{m(k)}\Big\{\dot{\partial}_{[h}\left(\dot{\partial}_{j]}B^{im}\right)-\frac{\dot{x}^i}{n+1}\dot{\partial}_{[j}\left(\dot{\partial}_p B^{pm}\right)-\frac{\delta^i_{[j}}{n^2-1}\dot{\partial}_{h]}\left(\dot{\partial}_p B^{pm}\right)$$
$$-n\frac{\delta^i_{[h}}{n^2-1}\dot{\partial}_{j]}\left(\dot{\partial}_p B^{pm}\right)\Big\}-2\sigma_{m[h}\Big\{\dot{\partial}_k\left(\dot{\partial}_{j]}B^{im}\right)-\frac{\dot{x}^i}{n+1}\dot{\partial}_{j]}\left(\dot{\partial}_k\left(\dot{\partial}_p B^{pm}\right)-\frac{\delta^i_{j]}}{n^2-1}\dot{\partial}_k\left(\dot{\partial}_p B^{pm}\right)\right.$$
$$-2\sigma_{m[h}\Big\{\dot{\partial}_k\left(\dot{\partial}_{j]}B^{im}\right)-\frac{\dot{x}^i}{n+1}\dot{\partial}_{j]}\Big(\dot{\partial}_k\left(\dot{\partial}_p B^{pm}\right)-\frac{\delta^i_{j]}}{n^2-1}\dot{\partial}_k\left(\dot{\partial}_p B^{pm}\right)-n\frac{\delta^i_{[h}}{n^2-1}\dot{\partial}_{j]}\left(\dot{\partial}_p B^{pm}\right)\Big)$$
$$+2n\sigma_{m(p)}\Big\{\frac{\delta^i_{[h}}{n^2-1}\dot{\partial}_{k]}\left(\dot{\partial}_k B^{pm}\right)-\frac{\delta^i_{[k}}{n^2-1}\dot{\partial}_{[j}\left(\dot{\partial}_{h]}B^{pm}\right)\Big\}2\sigma_m\sigma_r\Big[\dot{\partial}_{[j}\left(\dot{\partial}_k B^{sm}\right)\dot{\partial}_{h]}\dot{\partial}_s B^{ir}-$$
$$-\frac{\dot{x}^i}{n+1}\Big\{\dot{\partial}_{[j}\dot{\partial}_k\left(\dot{\partial}_p B^{sm}\dot{\partial}_{h]}\dot{\partial}_s B^{pr}-\dot{\partial}_{[h}\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_{j]}\dot{\partial}_k\dot{\partial}_s B^{pr}+\dot{\partial}_{[j}\left(\dot{\partial}_k B^{sm}\right)\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}\right.$$
$$+\left(\dot{\partial}_{[j}B^{sm}\right)\dot{\partial}_{h]}\dot{\partial}_k\left(\dot{\partial}_p B^{pr}\right)\Big\}+\frac{\delta^i_{[j}}{n^2-1}\Big\{\dot{\partial}_p\left(\dot{\partial}_{h]}B^{sm}\right)\dot{\partial}_k\dot{\partial}_s B^{pr}-\dot{\partial}_p\left(\dot{\partial}_k B^{sm}\right)\dot{\partial}_{h]}\dot{\partial}_s B^{pr}\Big\}$$

$$-\frac{\delta^i_{[h}}{n^2-1}\Big\{n\dot\partial_{j]}\dot\partial_k B^{sm}\big)\dot\partial_p\dot\partial_s B^{pr} - n\dot\partial_{j]}\big(\dot\partial_p B^{sm}\big)\dot\partial_p\dot\partial_s B^{pr} + n\big(\dot\partial_k B^{sm}\big)\dot\partial_{j]}\dot\partial_p\dot\partial_s B^{pr}$$

$$-n\big(\dot\partial_{j]}\dot\partial_k B^{sm}\big)\dot\partial_p\dot\partial_s B^{pr} + \dot\partial_k\big(\dot\partial_{j]} B^{sm}\big)\dot\partial_p\dot\partial_s B^{pr} - \dot\partial_k\big(\dot\partial_p B^{sm}\big)\dot\partial_{j]}\dot\partial_s B^{pr}$$

$$+\dot\partial_{j]} B^{sm}\dot\partial_k\dot\partial_p\dot\partial_s B^{pr} - \big(\dot\partial_p B^{sm}\big)\dot\partial_k\dot\partial_{j]}\dot\partial_s B^{pr} - \dot x^l\dot\partial_{j]}\dot\partial_k\big(\dot\partial_l B^{sm}\big)\big(\dot\partial_p\dot\partial_s B^{pr}\big)$$

$$+\dot x^l\dot\partial_{j]}\dot\partial_k\big(\dot\partial_p B^{sm}\big)\big(\dot\partial_l\dot\partial_s B^{pr}\big) + \dot x^l\dot\partial_k\big(\dot\partial_l B^{sm}\big)\dot\partial_{j]}\big(\dot\partial_p\dot\partial_s B^{pr}\big) - \dot x^l\dot\partial_k\big(\dot\partial_p B^{sm}\big)\dot\partial_{j]}\big(\dot\partial_l\dot\partial_s B^{pr}\big)$$

$$+\dot x^l\dot\partial_{j]}\big(\dot\partial_l B^{sm}\big)\dot\partial_k\dot\partial_p\dot\partial_s B^{pr}\big) - \dot x^l\dot\partial_{j]}\big(\dot\partial_p B^{sm}\big)\dot\partial_k\dot\partial_l\dot\partial_s B^{pr}\big) + \dot x^l\big(\dot\partial_l B^{sm}\big)\dot\partial_{j]}\dot\partial_k\dot\partial_p\dot\partial_s B^{pr}$$

$$-\dot x\big(\dot\partial_p B^{sm}\big)\dot\partial_{j]}\dot\partial_k\dot\partial_l\dot\partial_s B^{pr}\Big\} - \frac{\delta^i_k}{n^2-1}\Big\{n\dot\partial_{[j}\big(\dot\partial_p B^{sm}\big)\big(\dot\partial_{h]}\dot\partial_s B^{pr}\big) - n\big(\dot\partial_{[h} B^{sm}\big)\dot\partial_{j]}\dot\partial_p\dot\partial_s B^{pr}$$

$$-\dot\partial_{[h}\big(\dot\partial_p B^{sm}\big)\dot\partial_{j]}\dot\partial_s B^{pr} - \big(\dot\partial_{[j} B^{sm}\big)\dot\partial_{h]}\dot\partial_p\dot\partial_s B^{pr} - \dot x^l\dot\partial_{[h}\big(\dot\partial_l B^{sm}\big)\dot\partial_{j]}\big(\dot\partial_p\dot\partial_s B^{pr}\big) +$$

$$\dot x^l\dot\partial_{[h}\big(\dot\partial_p B^{sm}\big)\dot\partial_{j]}\dot\partial_l\dot\partial_s B^{pr} - \dot x^l\dot\partial_{[j}\dot\partial_{[h}\big(\dot\partial_l B^{sm}\big)\dot\partial_{h]}\dot\partial_p\dot\partial_s B^{pr} - \dot x^l\dot\partial_{[j}\big(\dot\partial_p B^{sm}\big)\dot\partial_{h]}\big(\dot\partial_l\dot\partial_s B^{pr}\big)\Big\}\Big]$$

$$+2\sigma_{m[(j)}\left\{\frac{\delta^i_k}{n^2-1}\dot\partial_{h]}\big(\dot\partial_p B^{pm}\big) - \frac{\delta^i_{[h}}{n^2-1}\dot\partial_k\big(\dot\partial_p B^{pm}\big)\right\} + 2\frac{\delta^i_{[h}}{n^2-1}\sigma_{m(p)}\dot\partial_k\big(\dot\partial_{j]} B^{pm}\big)$$

$$-2\dot x^l\frac{\delta^i_{[h}}{n^2-1}\Big\{\sigma_{m(l)}\dot\partial_{j]}\big(\dot\partial_k\dot\partial_p B^{pm}\big) - \sigma_{m(p)}\dot\partial_{j]}\big(\dot\partial_k\dot\partial_l B^{pm}\big)\Big\} \qquad \ldots(16)$$

Proof. Interchange the indices k and h in (13) and subtracting the equation thus obtained from (13) and using the symmetric property of the function G^i_{jkh}, we get the result (16).

Theorem 2. When F_n and $\overline{F}_n$ are in conformal correspondence, we have

$$\overline{W}_{jukh} - \overline{W}_{jhku} = e^{2\sigma}\big(W_{jukh} - W_{jkhu}\big) + 2e^{2\sigma}\sigma_m g_{i[u}\Big[\Big\{\big(\dot\partial_k B^{ir}\big)G^m_{h]jr} - \big(\dot\partial_{h]} B^{ir}\big)G^m_{kjr}$$

$$-\dot\partial_j\big(\dot\partial_k B^{im}\big)_{(h)} + \dot\partial_j\big(\dot\partial_{h]} B^{im}\big)_{(k)} + \frac{\dot x^i}{n+1}\Big\{\dot\partial_j\dot\partial_k\big(\dot\partial_p B^{pm}\big)_{h]}\Big\} - \dot\partial_j\dot\partial_{h]}\big(\dot\partial_p B^{pm}\big)_k\Big\}$$

$$+\frac{\delta^i_j}{n^2-1}\Big\{\dot\partial_p\big(\dot\partial_k B^{pm}\big)_{(h)]} - \dot\partial_p\big(\dot\partial_{h]} B^{pm}\big)_{(k)} - \dot\partial_k B^{pr}\big)G^m_{h]pr} + \dot\partial_{h]} B^{pr}\big)G^m_{kpr}\Big\}$$

$$+\frac{\delta^i_k}{n^2-1}\Big\{n\dot\partial_j\big(\dot\partial_p B^{pm}\big)_{(h)]} - n\big(\dot\partial_j\dot\partial_{h]} B^{pm}\big)_{(p)} - \dot\partial_{h]}\big(\dot\partial_j B^{pm}\big)_{(p)]} + \dot\partial_{h]}\big(\dot\partial_p B^{pm}\big)_{(j)}$$

$$-\dot x^l\dot\partial_j\big(\dot\partial_{h]}\big(\dot\partial_l B^{pm}\big)_{(p)} + \dot x^l\dot\partial_j\dot\partial_{h]}\big(\dot\partial_p B^{pm}\big)_{(l)}\Big\} + \frac{\delta^i_{h]}}{n^2-1}\Big\{n\dot\partial_j\big(\dot\partial_k B^{pm}\big)_{(p)}$$

$$-n\dot\partial_j\big(\dot\partial_p B^{pm}\big)_{(k)} + \dot\partial_k\big(\dot\partial_j B^{pm}\big)_{(p)]} - \dot\partial_k\big(\dot\partial_p B^{pm}\big)_{(j)} + \dot x^l\dot\partial_j\big(\dot\partial_k\big(\dot\partial_l B^{pm}\big)_{(p)}\big)$$

$$+\dot x^l\dot\partial_j\dot\partial_k\big(\dot\partial_p B^{pm}\big)_{(l)}\Big\} + 2e^{2\sigma}g_{i[u}\Big[\sigma_{m(k)}\Big\{\dot\partial_{h]}\big(\dot\partial_j B^{im}\big) - \frac{\dot x}{n+1}\dot\partial_j\dot\partial_{h]}\big(\dot\partial_p B^{pm}\big)$$

$$-\frac{\delta^i_j}{n^2-1}\dot\partial_{h]}\left(\dot\partial_p B^{pm}\right)-n\frac{\delta^i_{h]}}{n^2-1}\dot\partial_j\left(\dot\partial_p B^{pm}\right)\Big\}-\sigma_{m(h)]}\Big\{\dot\partial_k\left(\dot\partial_j B^{im}\right)$$

$$-\frac{\dot x^i}{n+1}\dot\partial_j\dot\partial_k\left(\dot\partial_p B^{pm}\right)-\frac{\delta^i_j}{n^2-1}\dot\partial_k\left(\dot\partial_p B^{pm}\right)-n\frac{\delta^i_k}{n^2-1}\dot\partial_j\left(\dot\partial_p B^{pm}\right)\Big\}-n\frac{\delta^i_k}{n^2-1}$$

$$\sigma_{m(p)}\Big\{\dot\partial_j\left(\dot\partial_{h]}B^{pm}\right)\Big\}+\Big\{\dot\partial_j\left(\dot\partial_k B^{pm}\right)\Big\}\frac{n\delta^i_{h]}}{n^2-1}\Bigg]+2e^{2\sigma}g_{i[u}\sigma_m\sigma_r\Big[\dot\partial_j\left(\dot\partial_k B^{sm}\right)\dot\partial_{h]}\dot\partial_s B^{ir}$$

$$-\dot\partial_j\left(\dot\partial_{h]}B^{sm}\right)\dot\partial_k\dot\partial_s B^{ir}-\frac{\dot x^i}{n+1}\Big\{\dot\partial_j\dot\partial_k\left(\dot\partial_p B^{sm}\right)\dot\partial_{h]}\dot\partial_s B^{pr}-\dot\partial_j\dot\partial_{h]}\left(\dot\partial_p B^{sm}\right)\dot\partial_k\dot\partial_s B^{pr}$$

$$+\dot\partial_k\left(\dot\partial_p B^{sm}\right)\dot\partial_j\dot\partial_{h]}\left(\dot\partial_s B^{pr}\right)-\dot\partial_{h]}\left(\dot\partial_p B^{sm}\right)\dot\partial_j\left(\dot\partial_k\dot\partial_s B^{pr}\right)+\dot\partial_j\left(\dot\partial_k B^{sm}\right)\dot\partial_{h]}\dot\partial_p\dot\partial_s B^{pr}$$

$$-\dot\partial_j\left(\dot\partial_{h]}B^{sm}\right)\dot\partial_k\dot\partial_p\dot\partial_s B^{pr}+\left(\dot\partial_k B^{sm}\right)\dot\partial_j\dot\partial_{h]}\dot\partial_p\left(\dot\partial_s B^{pr}\right)-\left(\dot\partial_{h]}B^{sm}\right)\dot\partial_j\dot\partial_k\dot\partial_p\left(\dot\partial_s B^{pr}\right)$$

$$+\frac{\dot\delta_j}{n+1}\Big\{\dot\partial_p\left(\dot\partial_{h]}B^{sm}\right)\dot\partial_k\dot\partial_s B^{pr}-\dot\partial_p\left(\dot\partial_k B^{sm}\right)\dot\partial_{h]}\dot\partial_s B^{pr}\Big\}+\frac{\delta^i_k}{n^2-1}\Big\{n\dot\partial_j\left(\dot\partial_{h]}B^{sm}\right)\dot\partial_p\dot\partial_s B^{pr}$$

$$-n\dot\partial_j\left(\dot\partial_p B^{sm}\right)\dot\partial_{h]}\dot\partial_s B^{pr}+n\dot\partial_{h]}\left(\dot\partial_p B^{sm}\right)\dot\partial_j\dot\partial_s B^{pr}-n\left(\dot\partial_p B^{sm}\right)\dot\partial_j\dot\partial_{h]}\dot\partial_s B^{pr}$$

$$+\dot\partial_{h]}\left(\dot\partial_j B^{sm}\right)\dot\partial_p\dot\partial_s B^{pr}-\dot\partial_{h]}\left(\dot\partial_p B^{sm}\right)\dot\partial_j\dot\partial_s B^{pr}+\left(\dot\partial_j B^{sm}\right)\dot\partial_{h]}\dot\partial_p\dot\partial_s B^{pr}$$

$$-\left(\dot\partial_p B^{sm}\right)\dot\partial_{h]}\dot\partial_j\dot\partial_s B^{pr}+\dot x^l\dot\partial_j\dot\partial_{h]}\left(\dot\partial_l B^{sm}\right)\dot\partial_p\dot\partial_s B^{pr}-\dot x^l\dot\partial_j\dot\partial_{h]}\left(\dot\partial_p B^{sm}\right)\dot\partial_l\dot\partial_s B^{pr}$$

$$+\dot x^l\dot\partial_{h]}\left(\left(\dot\partial_l B^{sm}\right)\dot\partial_j\dot\partial_p\dot\partial_s B^{pr}-\dot x^l\dot\partial_{h]}\left(\dot\partial_p B^{sm}\right)\dot\partial_j\dot\partial_l\dot\partial_s B^{pr}+\dot x^l\dot\partial_j\left(\dot\partial_l B^{sm}\right)\dot\partial_{h]}\dot\partial_p\dot\partial_s B^{pr}$$

$$-\dot x^l\dot\partial_j\left(\dot\partial_p B^{sm}\right)\dot\partial_{h]}\dot\partial_l\dot\partial_s B^{pr}+\dot x^l\left(\dot\partial_l B^{sm}\right)\dot\partial_j\dot\partial_{h]}\dot\partial_p\dot\partial_s B^{pr}-\dot x^l\left(\dot\partial_p B^{sm}\right)\dot\partial_j\dot\partial_{h]}\dot\partial_l\dot\partial_s B^{pr}\Big\}$$

$$-\frac{\delta^i_{h]}}{n^2-1}\Big\{n\dot\partial_j\left(\dot\partial_k B^{sm}\right)\dot\partial_p\dot\partial_s B^{pr}-n\dot\partial_j\left(\dot\partial_p B^{sm}\right)\dot\partial_k\dot\partial_s B^{pr}+n\dot\partial_k\left(\dot\partial_p B^{sm}\right)\dot\partial_j\dot\partial_s B^{pr}$$

$$-n\left(\dot\partial_p B^{sm}\right)\dot\partial_j\dot\partial_k\dot\partial_s B^{pr}+\dot\partial_k\left(\dot\partial_j B^{sm}\right)\dot\partial_p\dot\partial_s B^{pr}-\dot\partial_k\left(\dot\partial_p B^{sm}\right)\dot\partial_j\dot\partial_s B^{pr}$$

$$+\left(\dot\partial_j B^{sm}\right)\dot\partial_k\dot\partial_p\dot\partial_s B^{pr}-\left(\dot\partial_p B^{sm}\right)\dot\partial_k\dot\partial_j\dot\partial_s B^{pr}+\dot x\dot\partial_j\dot\partial_k\left(\dot\partial_l B^{sm}\right)\dot\partial_p\dot\partial_s B^{pr}$$

$$-\dot x^l\dot\partial_j\dot\partial_k\left(\dot\partial_p B^{sm}\right)\dot\partial_l\dot\partial_s B^{pr}+\dot x^l\dot\partial_k\left(\left(\dot\partial_l B^{sm}\right)\dot\partial_j\dot\partial_p\dot\partial_s B^{pr}-\dot x^l\dot\partial_k\left(\dot\partial_p B^{sm}\right)\dot\partial_j\dot\partial_l\dot\partial_s B^{pr}$$

$$+\dot x^l\dot\partial_j\left(\dot\partial_l B^{sm}\right)\dot\partial_k\dot\partial_p\dot\partial_s B^{pr}-\dot x^l\dot\partial_j\left(\dot\partial_p B^{sm}\right)\dot\partial_k\dot\partial_l\dot\partial_s B^{pr}+\dot x^l\left(\dot\partial_l B^{sm}\right)\dot\partial_j\dot\partial_k\dot\partial_p\dot\partial_s B^{pr}$$

$$-\dot x^l\left(\dot\partial_p B^{sm}\right)\dot\partial_j\dot\partial_k\dot\partial_l\dot\partial_s B^{pr}\Big\}\Big]+2e^{2\sigma}g_{i[u}\Big[\sigma_{m(j)}\left\{\frac{\delta^i_k}{n^2-1}\dot\partial_{h]}\left(\dot\partial_p B^{pm}\right)-\frac{\delta^i_{h]}}{n^2-1}\dot\partial_k\left(\dot\partial_p B^{pm}\right)\right\}$$

$$+\sigma_{m(p)}\left\{\frac{\delta^i_k}{n^2-1}\dot\partial_{h]}\left(\dot\partial_j B^{pm}\right)-\frac{\delta^i_{h]}}{n^2-1}\dot\partial_k\left(\dot\partial_j B^{pm}\right)\right\}+\dot x^l\sigma_{m(l)}\left\{\frac{\delta^i_k}{n^2-1}\dot\partial_j\dot\partial_{h]}\left(\dot\partial_p B^{pm}\right)\right.$$

$$\left.-\frac{\delta^i_{h]}}{n^2-1}\dot\partial_j\dot\partial_k\left(\dot\partial_p B^{pm}\right)\right\}-\dot x^l\sigma_{m(p)}\left\{\frac{\delta^i_k}{n^2-1}\dot\partial_j\dot\partial_{h]}\left(\dot\partial_l B^{pm}\right)+\frac{\delta^i_{h]}}{n^2-1}\dot\partial_j\dot\partial_k\left(\dot\partial_l B^{pm}\right)\right\}\Bigg]\quad(17)$$

Proof. Interchange the indices u and h in (14) and subtracting the equation thus obtained to (14), we obtain the identity (17).

Theorem 3. When F_n and $\overline{F}_n$ are in conformal correspondence, we have

$$\overline{W}_{jukh} + \overline{W}_{ujkh} = e^{2\sigma}\left(W_{jukh} + W_{ujhk}\right) + 4e^{2\sigma}\sigma_m\Big[g_{i(u}\Big\{\left(\dot{\partial}_{[k}B^{ir}\right)G^m_{h]j)r} - \dot{\partial}_{j)}\left(\dot{\partial}_{[k}B^{im}\right)$$

$$+ \frac{g_{i(u}\dot{x}^i}{n+1}\dot{\partial}_{j)}\dot{\partial}_{[k}\left(\dot{\partial}_p B^{pm}\right)_{(h)]} + \frac{g_{i(u}\delta^i_{j)}}{n+1}\Big\{\dot{\partial}_p\left(\dot{\partial}_{[k}B^{pm}\right)_{(h)]} - \left(\dot{\partial}_{[k}B^{pr}\right)G^m_{h]pr}\Big\}$$

$$- \frac{g_{i(u}\delta^i_{[k}}{n^2-1}\Big\{n\dot{\partial}_{j)}\left(\dot{\partial}_{h]}B^{pm}\right)_{(p)} - n\dot{\partial}_{j)}\left(\dot{\partial}_p B^{pm}\right)_{(h)]} + \dot{\partial}_{h]}\left(\dot{\partial}_{j)}B^{pm}\right)_{(p)} - \dot{\partial}_{h]}\left(\dot{\partial}_p B^{pm}\right)_{(j))}$$

$$+ \dot{x}^l\dot{\partial}_{j)}\dot{\partial}_{h]}\left(\dot{\partial}_l B^{pm}\right)_{(p)} - \dot{\partial}_{j)}\dot{\partial}_{h]}\left(\dot{\partial}_p B^{pm}\right)_{(l)}\Big\}\Big] - 4e^{2\sigma}\sigma_{m[(k}g_{i(u)}\Big\{\dot{\partial}_{h]}\left(\dot{\partial}_{j)}B^{im}\right)$$

$$- \frac{\dot{x}^i}{n+1}\dot{\partial}_{j)}\left(\dot{\partial}_{h]}\dot{\partial}_p B^{pm}\right) - \frac{\delta^i_{j)}}{n^2-1}\dot{\partial}_{h]}\left(\dot{\partial}_p B^{pm}\right)\Big\} + 4e^{2\sigma}n\frac{g_{i(u}\delta^i_{[k}}{n^2-1}\Big\{\sigma_{m(h]}\dot{\partial}_{j)}\left(\dot{\partial}_p B^{pm}\right)$$

$$- \sigma_{m(p)}\left(\dot{\partial}_{j)}\dot{\partial}_{h]}B^{pm}\right)\Big\} + 4e^{2\sigma}\sigma_m\sigma_r g_{i(u}\Big[\Big\{\dot{\partial}_{j)}\left(\dot{\partial}_{[k}B^{sm}\right)\dot{\partial}_{h]}\dot{\partial}_s B^{ir} - \frac{\dot{x}^i}{n+1}\dot{\partial}_{j)}\dot{\partial}_{[k}$$

$$\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_{h]}\dot{\partial}_s B^{pr} + \dot{\partial}_{[k}\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_{j)}\left(\dot{\partial}_{h]}\dot{\partial}_s B^{pr}\right) + \dot{\partial}_{j)}\left(\dot{\partial}_{[k}B^{sm}\right)\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}$$

$$+ \left(\dot{\partial}_{[k}B^{sm}\right)\dot{\partial}_{j)}\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr} + \frac{\delta^i_{j)}}{n^2-1}\dot{\partial}_p\left(\dot{\partial}_{[k}B^{sm}\right)\dot{\partial}_{k]}\dot{\partial}_s B^{pr} + \frac{\delta^i_{[k}}{n^2-1}\Big\{n\dot{\partial}_{j)}\left(\dot{\partial}_{h]}B^{sm}\right)$$

$$\dot{\partial}_p\dot{\partial}_s B^{pr} - n\dot{\partial}_{j)}\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_{h]}\dot{\partial}_s B^{pr} + n\left(\dot{\partial}_{h]}B^{sm}\right)\dot{\partial}_{j)}\dot{\partial}_p\dot{\partial}_s B^{pr} - n\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_{j)}\dot{\partial}_{h]}\dot{\partial}_s B^{pr}\Big)$$

$$+ \dot{\partial}_{h]}\left(\dot{\partial}_{j)}B^{sm}\right)\dot{\partial}_p\dot{\partial}_s B^{pr} - \dot{\partial}_{h]}\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_{j)}\dot{\partial}_s B^{pr} + \left(\dot{\partial}_{j)}B^{sm}\right)\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}$$

$$- \left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_{h]}\dot{\partial}_{j)}\dot{\partial}_s B^{pr} + \dot{x}^l\dot{\partial}_{j)}\dot{\partial}_{h]}\left(\dot{\partial}_l B^{sm}\right)\dot{\partial}_p\dot{\partial}_s B^{pr} - \dot{\partial}_{j)}\dot{\partial}_{h]}\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_l\dot{\partial}_s B^{pr}$$

$$+ \dot{\partial}_{h]}\left(\dot{\partial}_l B^{sm}\right)\dot{\partial}_{j)}\dot{\partial}_p\dot{\partial}_s B^{pr} - \dot{\partial}_{h]}\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_{j)}\dot{\partial}_l\dot{\partial}_s B^{pr} + \dot{\partial}_{j)}\left(\dot{\partial}_l B^{sm}\right)\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}$$

$$- \dot{\partial}_{j)}\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_{h]}\dot{\partial}_l\dot{\partial}_s B^{pr} + \left(\dot{\partial}_l B^{sm}\right)\dot{\partial}_{j)}\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr} - \left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_{j)}\dot{\partial}_{h]}\dot{\partial}_l\dot{\partial}_s B^{pr}\Big)\Big\}\Big]$$

$$+ 4e^{2\sigma}\frac{g_{i(u}\delta^i_{[k}}{n-1}\Big[\Big\{\sigma_{m(j)}\dot{\partial}_{h]}\left(\dot{\partial}_p B^{pm}\right) - \sigma_{m(p)}\dot{\partial}_{h]}\left(\dot{\partial}_{j)}B^{pm}\right)\Big\}$$

$$+ \dot{x}^l\Big\{\sigma_{m(l)}\dot{\partial}_{j)}\left(\dot{\partial}_{h]}\dot{\partial}_p B^{sm}\right) - \sigma_{m(p)}\dot{\partial}_{j)}\left(\dot{\partial}_{h]}\dot{\partial}_l B^{pm}\right)\Big\}\Big]. \qquad \ldots(18)$$

Proof. Interchanging the indices in each pair (j,u) and (k,h) in equation (14) and adding the equations thus obtained from (14), we obtain the identity (18).

Theorem 4. When F_n and $\overline{F}_n$ are in conformal correspondence, we have

$$\overline{W}_{jukh} + \overline{W}_{jhku} + \overline{W}_{khju} + \overline{W}_{kujh} = e^{2\sigma}\left(W_{jhku} + W_{jukh} + W_{kujh} + W_{khju}\right) + 4e^{2\sigma}\sigma_m g_{i(u}$$

$$\left(\dot{\partial}_{[k}B^{ir}\right)G^{m}_{h)j|r} - \dot{\partial}_{[j}\left(\dot{\partial}_{k]}B^{im}\right)_{(h))} + \dot{\partial}_{[j}\left(\dot{\partial}_{h)}B^{im}\right)_{(k)]} + \frac{\dot{x}^{i}}{n+1}\left\{\dot{\partial}_{[j}\dot{\partial}_{k]}\left(\dot{\partial}_{p}B^{pm}\right)_{(h))}\right.$$

$$-\frac{\dot{x}^{i}}{n+1}\left\{\dot{\partial}_{[j}\dot{\partial}_{h)}\left(\dot{\partial}_{p}B^{pm}\right)_{(k)]} + \frac{\delta_{[j}}{n+1}\left\{\dot{\partial}_{p}\left(\dot{\partial}_{k]}B^{pm}\right)_{(h))} - \dot{\partial}_{[j}\left(\dot{\partial}_{k]}B^{im}\right)_{(h))}\right.\right.$$

$$+\dot{\partial}_{[j}\left(\dot{\partial}_{h)}B^{im}\right)_{(k)]} + \frac{\dot{x}^{i}}{n+1}\left\{\dot{\partial}_{[j}\dot{\partial}_{k]}\left(\dot{\partial}_{p}B^{pm}\right)_{(h))} - \frac{\dot{x}^{i}}{n+1}\left\{\dot{\partial}_{[j}\dot{\partial}_{h)}\left(\dot{\partial}_{p}B^{pm}\right)_{(k)]}\right.\right.$$

$$+\frac{\delta^{i}_{[j}}{n+1}\left\{\dot{\partial}_{p}\left(\dot{\partial}_{k]}B^{pm}\right)_{(h))} - \dot{\partial}_{p}\left(\dot{\partial}_{h)}B^{pm}\right)_{(k)]} - \left(\dot{\partial}_{k]}B^{pr}\right)G^{m}_{h)pr} + \left(\dot{\partial}_{h)}B^{pr}\right)G^{m}_{k]pr}\right\}$$

$$-\frac{\delta^{i}_{[k}}{n^{2}-1}\left\{n\dot{\partial}_{j]}\left(\dot{\partial}_{h)}B^{pm}\right)_{(p)} - n\dot{\partial}_{j]}\left(\dot{\partial}_{p}B^{pm}\right)_{(h))} + \dot{\partial}_{h)}\left(\dot{\partial}_{j]}B^{pm}\right)_{(p)}\right.$$

$$\left.-\dot{\partial}_{h)}\left(\dot{\partial}_{p}B^{pm}\right)_{(j)]}\right\} + \frac{\delta^{i}_{h)}}{n^{2}-1}\left\{n\dot{\partial}_{[j}\left(\dot{\partial}_{k]}B^{pm}\right)_{(p)} - n\dot{\partial}_{[j}\left(\dot{\partial}_{p}B^{pm}\right)_{(k)]}\right.$$

$$\left.+\dot{\partial}_{[k}\left(\dot{\partial}_{j]}B^{pm}\right)_{(p)} - \dot{\partial}_{[k}\left(\dot{\partial}_{p}B^{pm}\right)_{(j)]}\right\} - \frac{\dot{x}^{l}\delta^{i}_{[k}}{n^{2}-1}\dot{\partial}_{j]}\left\{\dot{\partial}_{h)}\left(\dot{\partial}_{l}B^{pm}\right)_{(p)}\right.$$

$$\left.\left.-\dot{\partial}_{h)}\left(\dot{\partial}_{p}B^{pm}\right)_{(j)]}\right\} + \frac{\dot{x}^{l}\delta^{i}_{h)}}{n^{2}-1}\dot{\partial}_{[j}\left(\dot{\partial}_{l}B^{pm}\right)_{(p)} - \dot{\partial}_{[k}\left(\dot{\partial}_{p}B^{pm}\right)_{(j)]}\right\}\Bigg]$$

$$+4e^{2\sigma}g_{i(u}\left[\sigma_{m[k}\left\{\dot{\partial}_{h)}\left(\dot{\partial}_{j]}B^{im}\right) - \frac{\dot{x}^{l}}{n+1}\dot{\partial}_{j]}\left(\dot{\partial}_{h)}\dot{\partial}_{p}B^{pm}\right) - \frac{\delta^{i}_{j]}}{n+1}\dot{\partial}_{h)}\left(\dot{\partial}_{p}B^{pm}\right)\right\}\right.$$

$$-\sigma_{m(h))}\left\{\dot{\partial}_{[k}\left(\dot{\partial}_{j]}B^{im}\right) + \frac{\dot{x}^{i}}{n+1}\dot{\partial}_{[j}\left(\dot{\partial}_{k]}\dot{\partial}_{p}B^{pm}\right) + \frac{\delta^{i}_{[j}}{n+1}\dot{\partial}_{k]}\left(\dot{\partial}_{p}B^{pm}\right)\right\}$$

$$+\frac{n\delta^{i}_{[k}}{n^{2}-1}\left\{\sigma_{m(h))}\left(\dot{\partial}_{j]}\dot{\partial}_{p}B^{pm}\right) + \frac{n\delta^{i}_{[k}}{n^{2}-1}\left\{\sigma_{m(h))}\left(\dot{\partial}_{j]}\dot{\partial}_{p}B^{pm}\right)\right.\right.$$

$$\left.-\sigma_{m(p)}\left(\dot{\partial}_{j]}\dot{\partial}_{h)}B^{pm}\right)\right\} - \frac{n\delta^{i}_{h)}}{n^{2}-1}\left\{\sigma_{m[(k)}\left(\dot{\partial}_{j]}\dot{\partial}_{p}B^{pm}\right) - \sigma_{m(p)}\left(\dot{\partial}_{[j}\dot{\partial}_{k]}B^{pm}\right)\right\}$$

$$+4e^{2\sigma}\sigma_{m}\sigma_{r}g_{i(u}\left[\dot{\partial}_{[j}\left(\dot{\partial}_{k]}B^{sm}\right)\dot{\partial}_{h)}\dot{\partial}_{s}B^{ir} - \dot{\partial}_{[j}\left(\dot{\partial}_{h)}B^{sm}\right)\dot{\partial}_{k]}\dot{\partial}_{s}B^{ir}\right.$$

$$-\frac{\dot{x}^{i}}{n+1}\left\{\dot{\partial}_{[j}\dot{\partial}_{k]}\left(\dot{\partial}_{p}B^{sm}\right)\dot{\partial}_{h)}\dot{\partial}_{s}B^{pr} - \dot{\partial}_{[j}\dot{\partial}_{h)}\left(\dot{\partial}_{p}B^{sm}\right)\dot{\partial}_{k]}\dot{\partial}_{s}B^{pr}\right.$$

$$-\dot{\partial}_{[j}\dot{\partial}_{h)}\left(\dot{\partial}_{p}B^{sm}\right)\dot{\partial}_{k]}\dot{\partial}_{s}B^{pr} + \dot{\partial}_{[k}\left(\dot{\partial}_{p}B^{sm}\right)\dot{\partial}_{j]}\left(\dot{\partial}_{h)}\dot{\partial}_{s}B^{pr}\right) - \dot{\partial}_{h)}\left(\dot{\partial}_{p}B^{sm}\right)$$

$$\dot{\partial}_{[j}\left(\dot{\partial}_{k]}\dot{\partial}_{s}B^{pr}\right) + \dot{\partial}_{[j}\left(\dot{\partial}_{k]}B^{sm}\right)\dot{\partial}_{h)}\dot{\partial}_{p}\dot{\partial}_{s}B^{pr} - \dot{\partial}_{[j}\left(\dot{\partial}_{k]}B^{sm}\right)\dot{\partial}_{h)}\dot{\partial}_{p}\dot{\partial}_{s}B^{pr}$$

$$\left.-\left(\dot{\partial}_{[k}B^{sm}\right)\dot{\partial}_{j]}\dot{\partial}_{h)}\left(\dot{\partial}_{p}\dot{\partial}_{s}B^{pr}\right) + \left(\dot{\partial}_{h)}B^{sm}\right)\dot{\partial}_{[j}\dot{\partial}_{k]}\left(\dot{\partial}_{p}\dot{\partial}_{s}B^{pr}\right)\right\}$$

$$+\frac{\delta^{i}_{[j}}{n+1}\left\{\dot{\partial}_{p}\left(\dot{\partial}_{h)}B^{sm}\right)\dot{\partial}_{k]}\dot{\partial}_{s}B^{pr} - \dot{\partial}_{p}\left(\dot{\partial}_{k]}B^{sm}\right)\dot{\partial}_{h)}\dot{\partial}_{s}B^{pr}\right\}$$

$$+\frac{\delta^i_{[k}}{n^2-1}\Big\{n\big(\dot\partial_{j]}\dot\partial_{h)}B^{sm}\big)\dot\partial_p\dot\partial_s B^{pr}-n\big(\dot\partial_{j]}\dot\partial_p B^{sm}\big)\dot\partial_{h)}\dot\partial_s B^{pr}$$

$$+n\big(\dot\partial_{h)}B^{sm}\big)\dot\partial_{j]}\dot\partial_p\dot\partial_s B^{pr}-n\big(\dot\partial_p B^{sm}\big)\dot\partial_{j]}\dot\partial_{h)}\dot\partial_s B^{pr}$$

$$+\dot\partial_{h)}\big(\dot\partial_{j]}B^{sm}\big)\dot\partial_p\dot\partial_s B^{pr}-\dot\partial_{h)}\big(\dot\partial_p B^{sm}\big)\dot\partial_{j]}\dot\partial_s B^{pr}+\big(\dot\partial_{j]}B^{sm}\big)\dot\partial_{h)}\dot\partial_p\dot\partial_s B^{pr}$$

$$-\big(\dot\partial_p B^{sm}\big)\dot\partial_{h)}\dot\partial_{j]}\dot\partial_s B^{pr}\Big\}-\frac{\delta^i_{h)}}{n^2-1}\Big\{n\big(\dot\partial_{[j}\dot\partial_{k]}B^{sm}\big)\dot\partial_p\dot\partial_s B^{pr}$$

$$-n\big(\dot\partial_{[j}\dot\partial_p B^{sm}\big)\dot\partial_{k]}\dot\partial_s B^{pr}+n\big(\dot\partial_{[k}B^{sm}\big)\dot\partial_{j]}\dot\partial_p\dot\partial_s B^{pr}-n\big(\dot\partial_p B^{sm}\big)$$

$$\dot\partial_{[j}\dot\partial_{k]}\dot\partial_s B^{pr}+\dot\partial_{[k}\big(\dot\partial_{j]}B^{sm}\big)\dot\partial_p\dot\partial_s B^{pr}+\dot\partial_{[k}\big(\dot\partial_p B^{sm}\big)\dot\partial_{j]}\dot\partial_s B^{pr}+\big(\dot\partial_{[j}B^{sm}\big)\dot\partial_{k]}\dot\partial_p\dot\partial_s B^{pr}$$

$$-\big(\dot\partial_p B^{sm}\big)\dot\partial_{[k}\dot\partial_{j]}\dot\partial_s B^{pr}\Big\}+\frac{\dot x^l\delta^i_{[k}}{n^2-1}\Big\{\dot\partial_{j]}\big(\dot\partial_{h)}\dot\partial_l B^{sm}\big)\dot\partial_p\dot\partial_s B^{pr}-\dot\partial_{[j}\dot\partial_{h)}\big(\dot\partial_p B^{sm}\big)\dot\partial_l\dot\partial_s B^{pr}$$

$$+\dot\partial_{h)}\big(\dot\partial_l B^{sm}\big)\dot\partial_{j]}\dot\partial_p\dot\partial_s B^{pr}-\dot\partial_{h)}\big(\dot\partial_p B^{sm}\big)\dot\partial_{j]}\dot\partial_l\dot\partial_s B^{pr}+\dot\partial_{j]}\big(\dot\partial_l B^{sm}\big)\dot\partial_{h)}\big(\dot\partial_p\dot\partial_s B^{pr}\big)$$

$$-\dot\partial_{j]}\big(\dot\partial_p B^{sm}\big)\dot\partial_{h)}\big(\dot\partial_l\dot\partial_s B^{pr}\big)+\big(\dot\partial_l B^{sm}\big)\dot\partial_{j]}\dot\partial_{h)}\dot\partial_p\dot\partial_s B^{pr}-\big(\dot\partial_p B^{sm}\big)\dot\partial_{j]}\dot\partial_{h)}\dot\partial_l\dot\partial_s B^{pr}\Big\}$$

$$-\frac{\dot x^l\delta^i_{h)}}{n^2-1}\Big\{\dot\partial_{[j}\big(\dot\partial_{k]}\dot\partial_l B^{sm}\big)\dot\partial_p\dot\partial_s B^{pr}-\dot\partial_{[j}\dot\partial_{k]}\big(\dot\partial_p B^{sm}\big)\dot\partial_l\dot\partial_s B^{pr}+\dot\partial_{[k}\big(\dot\partial_l B^{sm}\big)\dot\partial_{j]}\dot\partial_p\dot\partial_s B^{pr}$$

$$-\dot\partial_{[k}\big(\dot\partial_p B^{sm}\big)\dot\partial_{j]}\dot\partial_l\dot\partial_s B^{pr}+\dot\partial_{ij}\big(\dot\partial_l B^{sm}\big)\dot\partial_{k]}\big(\dot\partial_p\dot\partial_s B^{pr}\big)-\dot\partial_{[j}\big(\dot\partial_p B^{sm}\big)\dot\partial_{k]}\big(\dot\partial_l\dot\partial_s B^{pr}\big)$$

$$+\big(\dot\partial_l B^{sm}\big)\dot\partial_{[j}\dot\partial_{k]}\dot\partial_p\dot\partial_s B^{pr}-\big(\dot\partial_p B^{sm}\big)\dot\partial_{[j}\dot\partial_{k]}\dot\partial_l\dot\partial_s B^{pr}\Big\}\Big]+4e^{2\sigma}g_{iu}\Bigg[\frac{\delta^i_{[k}}{n^2-1}\Big\{\sigma_{m(j)]}\dot\partial_{h)}\big(\dot\partial_p B^{pm}\big)$$

$$-\sigma_{m(p)}\dot\partial_{h)}\big(\dot\partial_{j]}B^{pm}\big)+\dot x^l\sigma_{m(l)}\dot\partial_{j]}\big(\dot\partial_{h)}\dot\partial_p B^{pm}\big)-\dot x^l\sigma_{m(p)}\dot\partial_{j]}\dot\partial_{h)}\big(\dot\partial_l B^{pm}\big)\Big\}$$

$$-\frac{\delta^i_{h)}}{n^2-1}\Big\{\sigma_{m(j)}\dot\partial_{k]}\big(\dot\partial_p B^{pm}\big)-\sigma_{m(p)}\dot\partial_{[k}\big(\dot\partial_{j]}B^{pm}\big)+\dot x^l\sigma_{m(l)}\dot\partial_{[j}\big(\dot\partial_{k]}\dot\partial_p B^{pm}\big)$$

$$-\dot x^l\sigma_{m(p)}\dot\partial_{[j}\dot\partial_{k]}\big(\dot\partial_l B^{pm}\big)\Big\}-\frac{\dot x^l\delta^i_{[k}}{n^2-1}\dot\partial_j\Big\{\dot\partial_{h]}\big(\dot\partial_l B^{pm}\big)_{(p)}-\dot\partial_{h]}\big(\dot\partial_p B^{pm}\big)_{(l)}\Big\}\Bigg]$$

$$+2e^{2\sigma}g_{iu}\sigma_{m[k}\left\{\dot\partial_{h]}\big(\dot\partial_j B^{im}\big)-\frac{\dot x^i}{n+1}\dot\partial_j\big(\dot\partial_{h]}\dot\partial_p B^{pm}\big)-\frac{\delta^i_j}{n+1}\dot\partial_{h]}\big(\dot\partial_p B^{pm}\big)\right\}$$

$$+\frac{2n\delta^i_{[k}}{n^2-1}e^{2\sigma}g_{iu}\Big\{\sigma_{m(h)]}\big(\dot\partial_j\dot\partial_p B^{pm}\big)-\sigma_{m(p)}\big(\dot\partial_j\dot\partial_{h]}B^{pm}\big)\Big\}+2e^{2\sigma}g_{iu}\sigma_m\sigma_r\Big[\dot\partial_j\big(\dot\partial_{[k}B^{sm}\big)$$

$$\dot\partial_{h]}\dot\partial_s B^{ir}-\frac{\dot x^i}{n+1}\Big\{\dot\partial_j\dot\partial_{[k}\big(\dot\partial_p B^{sm}\big)\dot\partial_{h]}\dot\partial_s B^{pr}+\dot\partial_{[k}\big(\dot\partial_p B^{sm}\big)\dot\partial_j\big(\dot\partial_{h]}\dot\partial_s B^{pr}\big)$$

$$+\dot\partial_j\big(\dot\partial_{[k}B^{sm}\big)\dot\partial_{h]}\dot\partial_p\dot\partial_s B^{pr}-\big(\dot\partial_{[k}B^{sm}\big)\dot\partial_j\dot\partial_{h]}\big(\dot\partial_p\dot\partial_s B^{pr}\big)\Big\}-\frac{\delta^i_j}{n+1}\Big\{\dot\partial_p\big(\dot\partial_{[h}B^{sm}\big)\dot\partial_{k]}\dot\partial_s B^{pr}\Big\}$$

$$+\frac{\delta^i_{[k}}{n^2-1}\Big\{n\big(\dot\partial_j\dot\partial_{h]}B^{sm}\big)\dot\partial_p\dot\partial_s B^{pr}-n\big(\dot\partial_j\dot\partial_p B^{sm}\big)\dot\partial_{h]}\dot\partial_s B^{pr}+n\big(\dot\partial_{h]}B^{sm}\big)\dot\partial_j\dot\partial_p\dot\partial_s B^{pr}$$

$$-n\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_s B^{pr} + \left\{\dot{\partial}_{h]}\left(\dot{\partial}_j B^{sm}\right)\dot{\partial}_p\dot{\partial}_s B^{pr} - \dot{\partial}_{h]}\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_j\dot{\partial}_s B^{pr} +\right.$$

$$\left. + \left(\dot{\partial}_j B^{sm}\right)\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr} - \left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_{[h}\dot{\partial}_j\dot{\partial}_s B^{pr}\right\} + \frac{\dot{x}^l \delta^i_{[k}}{n^2-1}\left\{\dot{\partial}_j\left(\dot{\partial}_{h]}\dot{\partial}_l B^{sm}\right)\dot{\partial}_p\dot{\partial}_s B^{pr}\right.$$

$$-\dot{\partial}_j\dot{\partial}_{h]}\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_l\dot{\partial}_s B^{pr} + \dot{\partial}_{h]}\left(\dot{\partial}_l B^{sm}\right)\dot{\partial}_j\dot{\partial}_p\dot{\partial}_s B^{pr} - \dot{\partial}_{h]}\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_j\dot{\partial}_l\dot{\partial}_s B^{pr}$$

$$+\dot{\partial}_j\left(\dot{\partial}_l B^{sm}\right)\dot{\partial}_{h]}\left(\dot{\partial}_p\dot{\partial}_s B^{pr}\right) - \dot{\partial}_j\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_{h]}\left(\dot{\partial}_l\dot{\partial}_s B^{pr}\right) + \left(\dot{\partial}_l B^{sm}\right)\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}$$

$$\left.-\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_l\dot{\partial}_s B^{pr}\right\}\Big] + \frac{2e^{2\sigma} g_{iu}\delta^i_{[k}}{n^2-1}\Big[\left\{\sigma_{m(j)}\dot{\partial}_{h]}\left(\dot{\partial}_p B^{pm}\right) - \sigma_{m(p)}\dot{\partial}_{h]}\left(\dot{\partial}_j B^{pm}\right)\right\}$$

$$+\dot{x}^l\left\{\sigma_{m(l)}\dot{\partial}_j\left(\dot{\partial}_{h]}\dot{\partial}_p B^{pm}\right) - \sigma_{m(p)}\dot{\partial}_j\dot{\partial}_{h]}\left(\dot{\partial}_l B^{pm}\right)\right\}\Big] \quad ...(19)$$

Proof. The proof follows in consequence of (14) anad (17).

3. Decomposition of Conformal Projective Curvature Tensor. We considered the decomposition of conformal projective curvature tensor in the form

$$\overline{W}^i_{jkh} = \overline{X}^i \overline{\phi}_{jkh} \quad (20)$$

where $\overline{\phi}_{jkh}$ is a homogeneous conformal decomposition tensor and $\overline{X}^i$ is a non zoro conformal vector such

$$\overline{X}^i \overline{V}_i = 1 \quad (21)$$

Similar manner the decomosition of projective curvature tensor W^i_{jkh} in the form

$$W^i_{jkh} = X^i \phi_{jkh} \quad (22)$$

where the decomposition vector X^i also satisfies the condition

$$X^i V_i = 1 \quad (23)$$

Transvecting (22) by $\dot{x}^i$, we get

$$W^i_{kh} = X^i \phi_{kh} \quad (24)$$

where

$$\phi_{kh} = -\phi_{jhk}\dot{x}^j \quad (25)$$

The decomposition tensor ϕ_{jkh} satisfies the identity

$$\phi_{kh} = -\phi_{hk} \quad (26)$$

We notice that the decomposition vector $\overline{X}^i$ and the recurrence vector V_i

are transformed conformally as under :

$$\overline{X}^i = e^{-\sigma} X^i \tag{27}$$

and

$$\overline{V}_i = e^{\sigma} V_i \tag{28}$$

respectively.

Using equation (20) and (22) in equation (13), the obtained equation transvercted by $\overline{V}_i$ and using the equation (21), (23), (27) and (28), we get

$$\overline{\phi}_{jkh} = e^{\sigma}\phi_{jkh} + V_i 2\sigma_m [\dot\partial_{[k} B^{ir}) G^m_{h]jr} - \dot\partial_j (\dot\partial_{[k} B^{im})_{(h)} + \frac{\dot x^i}{n+1} \{\dot\partial_j \dot\partial_{[k} (\dot\partial_p B^{pm})_{(h)]}$$

$$- \dot\partial_j \dot\partial_{[k} (\dot\partial_{h]} B^{pm})_{(p)}\} + \frac{\delta^i_j}{n+1} \{\dot\partial_p (\dot\partial_{[k} B^{pm})_{(h)]} - (\dot\partial_{[k} B^{pr}) G^m_{h]pr}\} - \frac{\delta^i_{[k}}{n^2-1} \{n\dot\partial_j (\dot\partial_{h]} B^{pm})_{(p)}$$

$$- n\dot\partial_j (\dot\partial_p B^{pm})_{(h)]} + \dot\partial_{h]} (\dot\partial_j B^{pm})_{(p)]} - \dot\partial_{h]} (\dot\partial_p B^{pm})_{(j)]}\} - \frac{\dot x^l \delta^i_{[k}}{n^2-1} \dot\partial_j \{\dot\partial_{h]} (\dot\partial_l B^{pm})_{(p)}$$

$$- \dot\partial_{h]} (\dot\partial_p B^{pm})_{(l)}\}] + V_i 2\sigma_{m|k} \{\dot\partial_{h]} (\dot\partial_j B^{im}) - \frac{\dot x^i}{n+1} \dot\partial_j (\dot\partial_{h]} \dot\partial_p B^{pm})$$

$$- \frac{\delta^i_j}{n+1} \dot\partial_{h]} (\dot\partial_p B^{pm})\} + \frac{V_i n \delta^i_{[k}}{n^2-1} 2\{\sigma_{m(h)]} (\dot\partial_j \dot\partial_p B^{pm}) - \sigma_{m(p)} (\dot\partial_j \dot\partial_{h]} B^{pm})\}$$

$$+ V_i 2\sigma_m \sigma_r [\dot\partial_j (\dot\partial_{[k} B^{sm}) \dot\partial_{h]} \dot\partial_s B^{ir} + \frac{\dot x^i}{n+1} \{\dot\partial_j \dot\partial_{[k} (\dot\partial_{h]} B^{sm}) \dot\partial_p \dot\partial_s B^{pr}$$

$$- \dot\partial_j \dot\partial_{[k} (\dot\partial_p B^{sm}) \dot\partial_j \dot\partial_{[k} (\dot\partial_p B^{sm}) \dot\partial_j \dot\partial_{[k} (\dot\partial_p B^{sm}) \dot\partial_{h]} \dot\partial_s B^{pr} + \dot\partial_{[k} (\dot\partial_{h]} B^{sm}) \dot\partial_j (\dot\partial_p \dot\partial_s B^{pr})$$

$$- \dot\partial_{[k} (\dot\partial_p B^{sm}.) \dot\partial_j (\dot\partial_{h]} \dot\partial_s B^{pr}) - \dot\partial_j (\dot\partial_{[k} B^{sm}) \dot\partial_{h]} \dot\partial_p \dot\partial_s B^{pr} - \dot\partial_j (\dot\partial_p B^{sm}) \dot\partial_{[k} (\dot\partial_{h]} \dot\partial_s B^{pr}$$

$$- (\dot\partial_{[k} B^{sm}) \dot\partial_j \dot\partial_{h]} (\dot\partial_p \dot\partial_s B^{pr}) - (\dot\partial_p B^{sm}) \dot\partial_j \dot\partial_{[k} (\dot\partial_{h]} \dot\partial_s B^{pr})\} - \frac{\delta^i_j}{n+1} \{\dot\partial_p (\dot\partial_{[h} B^{sm}) \dot\partial_{k]} \dot\partial_s B^{pr}\}$$

$$+ \frac{n\delta^i_{[k}}{n^2-1} \{(\dot\partial_j \dot\partial_{h]} B^{sm}) \dot\partial_p \dot\partial_s B^{pr} - (\dot\partial_j \dot\partial_p B^{sm}) \dot\partial_{h]} \dot\partial_s B^{pr} + (\dot\partial_{h]} B^{sm}) \dot\partial_j \dot\partial_p \dot\partial_s B^{pr}$$

$$- (\dot\partial_p B^{sn}) \dot\partial_j \dot\partial_{h]} \dot\partial_s B^{pr}\} + \frac{\delta^i_{[k}}{n^2-1} \{\dot\partial_{h]} (\dot\partial_j B^{sm}) \dot\partial_p \dot\partial_s B^{pr} - \dot\partial_{h]} (\dot\partial_p B^{sm}) \dot\partial_j \dot\partial_s B^{pr}$$

$$- (\dot\partial_j B^{sm}) \dot\partial_{h]} \dot\partial_p \dot\partial_s B^{pr} - (\dot\partial_p B^{sm}) \dot\partial_{[h} \dot\partial_j \dot\partial_s B^{pr}\}] + \frac{V_i \dot x^l \delta^i_{[k}}{n^2-1} [\dot\partial_j (\dot\partial_{h]} \dot\partial_l B^{sm}) \dot\partial_p \dot\partial_s B^{pr}$$

$$- \dot\partial_j \dot\partial_{h]} (\dot\partial_p B^{sm}) \dot\partial_l \dot\partial_s B^{pr} + \dot\partial_{h]} (\dot\partial_l B^{sm}) \dot\partial_j \dot\partial_p \dot\partial_s B^{pr} - \dot\partial_{h]} (\dot\partial_p B^{sm}) \dot\partial_j \dot\partial_l \dot\partial_s B^{pr}$$

$$+\dot{\partial}_j\left(\dot{\partial}_l B^{sm}\right)\dot{\partial}_{h]}\left(\dot{\partial}_p\dot{\partial}_s B^{pr}\right)-\dot{\partial}_j\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_{h]}\left(\dot{\partial}_l\dot{\partial}_s B^{pr}\right)+\left(\dot{\partial}_l B^{sm}\right)\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}$$

$$-\left(\dot{\partial}_p B^{sm}\right)\dot{\partial}_j\dot{\partial}_{h]}\dot{\partial}_l\dot{\partial}_s B^{pr}\Big\}\Big]-V_i 2\sigma_{m(p)}\left\{\frac{\dot{x}^i}{n+1}\dot{\partial}_j\left(\dot{\partial}_{[k}\dot{\partial}_{h]}B^{pm}\right)+\frac{\delta^i_{[k}}{n^2-1}\dot{\partial}_{h]}\dot{\partial}_l B^{pm}\right.$$

$$\left.+\frac{\dot{x}^l\delta^i_{[k}}{n^2-1}\dot{\partial}_j\left(\dot{\partial}_{h]}\dot{\partial}_l B^{pm}\right)\right\}+\frac{V_i\delta^i_{[k}}{n^2-1}2\left\{\sigma_{m(j)}\dot{\partial}_{h]}\left(\dot{\partial}_p B^{pm}\right)\right\}+\frac{V_i\dot{x}^l\delta^i_{[k}}{n^2k-1}2\left\{\sigma_{m(l)}\dot{\partial}_j\left(\dot{\partial}_{h]}\dot{\partial}_p B^{pm}\right)\right\}$$

...(29)

which represents the conformal transformation of the decomposition tensor under the change (1).

Thus we state

Theorem 5. Under the decomposition (20), the conformal decomposition tensor ϕ_{jkh} is expressed in the form (29).

Transvecting equation (20) by $\dot{x}^j$, we have

$$W^i_{kh}=X^i\phi_{kh} \tag{30}$$

where

$$\bar{\phi}_{kh}=\phi_{jkh}\dot{x}^j \tag{31}$$

Using the equation (23), (28) and (30) in the equation (12), we obtain

$$\bar{\phi}_{kh}=e^\sigma V_i\left(W^i_{kh}\right)-e^\sigma V_i 2\sigma_m\left[\left(\dot{\partial}_{[k}B^{im}\right)_{(h)]}-\frac{\dot{x}^i}{n+1}\left\{\left(\dot{\partial}_p\dot{\partial}_{[k}B^{pm}\right)_{(h)]}+\left(\dot{\partial}_{[k}B^{rm}\right)G^p_{h]rp}\right\}\right.$$

$$+\frac{1}{n^2-1}\delta^i_{[k}\left\{(n+1)\left(\dot{\partial}_{h]}B^{pm}\right)_{(p)}-n\left(\dot{\partial}_p B^{pm}\right)_{(h)]}-\left(\dot{\partial}_{h]}\dot{\partial}_p B^{pm}\right)_{(r)}\dot{x}^r+2B^{rm}G^p_{h]rp}\right\}\Big]$$

$$+e^\sigma V_i 2\sigma_{m[(k)}\left\{\dot{\partial}_{h]}B^{im}-\frac{n}{n^2-1}\delta^i_{h]}\dot{\partial}_p B^{pm}-\frac{\dot{x}^i}{n+1}\dot{\partial}_{h]}\dot{\partial}_p B^{pm}\right\}$$

$$+\frac{e^\sigma V_i 2}{n^2-1}\sigma_{m(p)}\delta^i_{[k}\left\{\dot{x}^p\dot{\partial}_{h]}\dot{\partial}_r B^{rm}-(n+1)\dot{\partial}_{h]}B^{pm}\right\}+e^\sigma V_i 2\sigma_m\sigma_r\Big[\left(\dot{\partial}_{[k}B^{sm}\right)\Big\{\dot{\partial}_{h]}\dot{\partial}_s B^{ir}$$

$$-\frac{\dot{x}^i}{n+1}\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}\Big\}+\frac{1}{n-1}\delta^i_{[k}\left\{\left(\dot{\partial}_{h]}B^{sm}\right)\dot{\partial}_p\dot{\partial}_s B^{pr}-\left(\dot{\partial}_{h]}\dot{\partial}_s B^{pr}\right)\dot{\partial}_p B^{sm}\right.$$

$$\left.+\frac{2}{n-1}B^{sm}\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}\right\}\Big]$$

(32)

Interchange the indices k and h in the equation (32), we get

$$\bar{\phi}_{kh}=-\bar{\phi}_{hk} \tag{33}$$

by vritue or relation $W^i_{kh} = -W^i_{hk}$ [4].

In the veiw of the equations (23) and (24), the equation (32), becomes

$$\begin{aligned}\bar{\phi}_{kh} = e^{\sigma}\phi_{kh} - e^{\sigma}V_i 2\sigma_m \Bigg[\left(\dot{\partial}_{[k} B^{im}\right)_{(h)]} - \frac{\dot{x}^i}{n+1}\left\{\left(\dot{\partial}_p \dot{\partial}_{[k} B^{pm}\right)_{(h)]} + \left(\dot{\partial}_{[k} B^{rm}\right) G^p_{h]rp}\right\} \\ + \frac{1}{n^2-1}\delta^i_{[k}\left\{(n+1)\left(\dot{\partial}_{h]} B^{pm}\right)_{(p)} - n\left(\dot{\partial}_p B^{pm}\right)_{(h)]} - \left(\dot{\partial}_{h]}\dot{\partial}_p B^{pm}\right)_{(r)}\dot{x}^r + 2B^{rm}G^p_{h]rp}\right\}\Bigg] \\ + e^{\sigma}V_i 2\sigma_{m|(k)}\left\{\dot{\partial}_{h]} B^{im} - \frac{n}{n^2-1}\delta^i_{h]}\dot{\partial}_p B^{pm} - \frac{\dot{x}^i}{n+1}\dot{\partial}_{h]}\dot{\partial}_p B^{pm}\right\} \\ + \frac{e^{\sigma}V_i 2}{n^2-1}\sigma_{m(p)}\delta^i_{[k}\left\{\dot{x}^p \dot{\partial}_{h]}\dot{\partial}_r B^{rm} - (n+1)\dot{\partial}_{h]} B^{pm}\right\} - e^{\sigma}V_i 2\sigma_m\sigma_r \Bigg[\left(\dot{\partial}_{[k} B^{sm}\right)\left\{\dot{\partial}_{h]}\dot{\partial}_s B^{ir} \right. \\ \left. - \frac{\dot{x}^i}{n+1}\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}\right\} \frac{1}{n-1}\delta^i_{[k}\left\{\left(\dot{\partial}_{h]} B^{sm}\right)\dot{\partial}_p\dot{\partial}_s B^{pr} - \left(\dot{\partial}_{h]}\dot{\partial}_s B^{pr}\right)\dot{\partial}_p B^{sm} \right. \\ \left. + \frac{2}{n-1}B^{sm}\dot{\partial}_{h]}\dot{\partial}_p\dot{\partial}_s B^{pr}\right\}\Bigg] \end{aligned} \tag{34}$$

Which gives the conformal transformation of decomposition tensor $\bar{\phi}_{kh}$ under the conformal change.

Theorem 6. Under the decomposition (20) and (30) the conformal decomposition tensor $\bar{\phi}_{kh}$ is expressed in the form (34).

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