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ON GENERATING RELATIONSHIPS FOR FOX'S *H*-FUNCTION AND MULTIVARIABLE *H*-FUNCTION

By

B.B. Jaimini and Hemlata Saxena
Government Postgraduate College, Kota-324001, Rajasthan, India
E-mail : bbjaimini_67@rediffmail.com
E-mail : hemlata_19_75@yahoo.co.in

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ABSTRACT

In this paper we establish some new results on bilinear, bilateral and multilateral generating relationship for Fox's H-function and multivariable H-function. Some known results for the Fox's H-function and multivariable H-function are also obtained as special cases of our main findings.

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1. Introduction. Chen and Shrivastava [1] gave a family of linear, bilateral and multilateral generating functions involving the sequence $\{\zeta_k^{(\lambda,\rho)}(z)\}_{k=0}^{\infty}$ defined by

$$\zeta_k^{(\lambda,\rho)}(z)={}_uF_{\rho+v}(\alpha_1,...,a_u;\Delta(\rho;1-\lambda-k),\beta_1,...,\beta_v;z) \qquad ...(1)$$

where for convenience, $\Delta(\rho;\lambda)$ abbreviates the array of ρ parameters

$$\frac{\lambda}{\rho},\frac{\lambda+1}{\rho},...,\frac{\lambda+\rho-1}{\rho} \qquad (\rho\in N=N_0\setminus\{0\})$$

and for its multivariable extension defined by ([1], p.172, equation (5.21)).

$$Z_k^{\lambda}(\sigma_1,...,\sigma_r;z_1,...,z_r)=\sum_{k_1,...,k_r=0}^{\infty}\frac{A(k_1,...,k_r)}{(1-\lambda-k)_K}z_1^{k_1}...z_r^{k_r} \qquad ...(2)$$

$$(K=k_1\sigma_1+...+k_r\sigma_r;k_1\in N_0;\lambda,\sigma_j\in C;j=1,...,r)$$

where $\{A(k_1,...,k_r)\}$ is a suitably bounded multiple sequence of complex numbers and $(\lambda)_k$ denotes the Pochhammer symbol.

$$(\lambda)_k=\frac{\Gamma(\lambda+k)}{\Gamma(\lambda)}=\begin{cases}1, & (k=0;\lambda\neq 0)\\ \lambda(\lambda+1)...(\lambda+k-1) & (k\in N;\lambda\in C)\end{cases} \qquad ...(3)$$

Raina[4] derived the following combinatorial identity as a special case of formula in ([4], p. 187,equation (15)).

$$\sum_{k=0}^{\infty}\binom{\lambda+k-1}{k}\binom{\mu+k-1}{k}^{-1}\binom{\alpha+k-1}{k}{}_2F_1\left[\begin{matrix}\lambda+k,\mu-\alpha;\\ \mu+k;\end{matrix}\, z\right]z^k$$

$$=(1-z)^{-\lambda},\quad (|z|<1) \qquad \text{...(4)}$$

where $\binom{n}{k}=\dfrac{\Gamma(n+1)}{\Gamma(k+1)\Gamma(n+k+1)}$ ...(5)

Recently in an earlier paper Jaimini et al. [3] generalized the results of above cited paper [1]. They proved six theorems on the generating function relationships in view of the above results (4).

The Fox's H-function defind and represented in the following manner ([2], p. 408), See also ([5],p 265, equation (1,1))

$$H_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_p,\alpha_p)\\(b_q,\beta_q)\end{matrix}\right.\right]=\frac{1}{2\pi i}\int_L\phi(\xi)z^{\xi}d\xi \qquad \text{...(6)}$$

where

$$\phi(\xi)=\frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\xi)}{\prod_{j=m+1}^{q}\Gamma(1-b_j+\beta_j\xi)\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j\xi)} \qquad \text{...(7)}$$

The multivariable H-function defined and represented in the following manner ([6],pp.251-252, equations (C.1)-(C.3)).

$H[z_1,...,z_r]$

$$=H_{p,q:p_1,q_1;...;p_r,q_r}^{0,n:m_1,n_1;...;m_r,n_r}\left[\begin{matrix}z_1\\ \vdots\\ z_r\end{matrix}\left|\begin{matrix}(a_j;\alpha_j^{(1)},...,\alpha_j^{(r)})_{1,p}:(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};...;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\(b_j;\beta_j^{(1)},...,\beta_j^{(r)})_{1,q}:(d_j^{(1)},\delta_j^{(1)})_{1,q_1};...;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right.\right]$$

$$=\frac{1}{(2\pi i)^r}\int_{L_1}\cdots\int_{L_r}\phi_1(s_1)...\phi_r(s_r)\psi(s_1,...,s_r)z_1^{s_1}...z_r^{s_r}ds_1....ds_r; \qquad \text{...(8)}$$

where $i=\sqrt{(-1)}$;

$$\phi_i(s_i)=\frac{\prod_{j=1}^{m_i}\Gamma(d_j^{(i)}-\delta_j^{(i)}s_i)\prod_{j=1}^{n_i}\Gamma(1-c_j^{(i)}+\gamma_j^{(i)}s_i)}{\prod_{j=m_1+1}^{q_i}\Gamma(1-d_i^{(i)}+\delta_j^{(i)}s_i)\prod_{j=n_i+1}^{p_i}\Gamma(c_j^{(i)}-\gamma_j^{(i)}s_i)}\ \forall i\in\{1,...,r\} \qquad \text{...(9)}$$

$$\psi(s_1,\ldots,s_r)=\frac{\prod_{j=1}^{n}\Gamma\left(1-a_j+\sum_{j=1}^{r}\alpha_j^{(i)}s_i\right)}{\prod_{j=n+1}^{p}\Gamma\left(a_j-\sum_{j=1}^{r}\alpha_j^{(i)}s_i\right)\prod_{j=1}^{q}\Gamma\left(1-b_j+\sum_{j=1}^{r}\beta_j^{(i)}s_i\right)} \quad \ldots(10)$$

In this paper some generating relations for Fox's H-function and multivariable H-function defined in (6) and (8) respectively are established by following the above cited work of Jamini et al. [3]. The importance of these results lies in the fact that they provide the extensions of the results due to Srivastava and Raina [7] and also provide a wide range of bilinear, bilateral mixed multilateral generating functions for simpler hypergeometric polynomials.

2. Main Bilateral Generating Relationship Involving Fox's *H*-Function

Result -1 . Corresponding to an identically nonvanishing function $\Omega_\vartheta(z_1,\ldots,z_s)$ of s complex variables $z_1,\ldots,z_s (s\in N)$ and of (complex) order ϑ, let.

$$\gamma^{(1)}_{m,\vartheta,\rho,\sigma,\lambda}[y;z_1,\ldots,z_s;t]=\sum_{k=0}^{\infty}\frac{a_k\Omega_{\vartheta+\rho k}(z_1,\ldots,z_s)t^k}{(mk)!}H^{r,s+1}_{u+1,v}\left[y\left|\begin{matrix}(1-\lambda-mk-\sigma mk,\varepsilon), & \{(c_u,\gamma_u)\}\\ & \{(d_v,\delta_v)\}\end{matrix}\right.\right]$$

$$[a_k\neq 0;k\in N_0;\vartheta,\sigma\in C] \quad (11)$$

and

$$M^{\vartheta,\rho,\lambda,\sigma,\mu,\alpha}_{n,m}[y;z_1,\ldots,z_s;\eta]=\sum_{k=0}^{[n/m]}A^{\lambda,\sigma,\mu,\alpha}_{k,n,m}(y;t)\binom{\mu+n+\sigma mk-1}{n-mk}^{-1}\binom{\alpha+n+\sigma mk-1}{n-mk}$$

$$\frac{a_k\Omega_{\vartheta+\rho k}(z_1,\ldots,z_s)\eta^k}{(mk)!(n-mk)!} \quad \ldots(12)$$

where

$$A^{\lambda,\sigma,\mu,\alpha}_{k,n,m}[y;t]=\sum_{l=0}^{\infty}\frac{(\mu-\alpha)_l t^l}{(\mu+n+\sigma mk)_l(l)!}H^{r,s+1}_{u+1,v}\left[y\left|\begin{matrix}(1-\lambda-n-\sigma mk,\varepsilon), & \{(c_u,\gamma_u)\}\\ & \{(d_v,\delta_v)\}\end{matrix}\right.\right] \ldots(13)$$

then

$$\sum_{n=0}^{\infty}M^{\vartheta,\rho,\lambda,\sigma,\mu,\alpha}_{n,m}(y;z_1,\ldots,z_s;\eta)t^n$$

$$=(1-t)^{-\lambda}\gamma^{(l)}_{m,\vartheta,\rho,\sigma,\lambda}\left[\frac{y}{(1-t)^{\varepsilon}};z_1,\ldots,z_s;\frac{\eta t^m}{(1-t)^{(\sigma+1)m}}\right] \quad \ldots(14)$$

Result-2. Let

$$\gamma^{(2)}_{\vartheta,\rho,m}[y;z_1,...,z_s;t]=\sum_{k=0}^{r}\frac{(-1)^{mk}a_k\Omega_{\vartheta+\rho k}(z_1,...,z_s)t^k}{}H^{r,s}_{u,v}\left[y\left|\begin{matrix}\{(c_u,\gamma_u)\}\\\{(d_v,\delta_v)\}\end{matrix}\right.\right] \quad ...(15)$$

and

$$N^{\vartheta,\rho,\lambda,\sigma,\mu,\alpha}_{n,m}[y;z_1,...,z_s;\eta]=\sum_{k=0}^{[n/m]}U^{\lambda,\sigma,\mu,\alpha}_{k,n,m}(y;t)\binom{\mu+n+\sigma mk-1}{n-mk}^{-1}\binom{\alpha+n+\sigma mk-1}{n-mk}$$

$$\frac{(-1)^{mk}a_k\Omega_{\vartheta+\rho k}(z_1,...,z_s)\eta^k}{(mk)!(n-mk)!} \quad ...(16)$$

where

$$U^{\lambda,\sigma,\mu,\alpha}_{k,n,m}(y;t)=\sum_{l=0}^{\infty}\frac{(\mu-\alpha)_l t^l}{(\mu+n+\sigma mk)_l(l)!}H^{r,s+1}_{u+1,v+1}\left[y\left|\begin{matrix}(-\lambda-n-\sigma mk-l,\varepsilon),\{(c_u,\gamma_u)\}\\\{(d_v,\delta_v)\},(-\lambda-mk-\sigma mk,\varepsilon)\end{matrix}\right.\right]. \quad ...(17)$$

Then

$$\sum_{n=0}^{\infty}M^{\vartheta,\rho,\lambda,\sigma,\mu,\alpha}_{n,m}(y;z_1,...,z_s;\eta)t^n=(1-t)^{-(\lambda+1)}\gamma^{(2)}_{\vartheta,\rho,m}\left[\frac{y}{(1-t)^{\varepsilon}};z_1,...,z_s;\frac{\eta t^m}{(1-t)^{(\sigma+1)m}}\right] \quad ...(18)$$

Result-3

Let $\gamma^{(2)}_{\vartheta,\rho,m}[y;z_1,...,z_s;t]$ is defined in (15) and

$$T^{\vartheta,\rho,\lambda,\sigma,\mu,\alpha}_{n,m}[y;z_1,...,z_s;\eta]=\sum_{k=0}^{[n/m]}V^{\lambda,\sigma,\omega,\mu,\alpha}_{k,n,m}(y;t)\binom{\mu+n+\sigma mk-1}{n-mk}^{-1}\binom{\alpha+n+\sigma mk-1}{n-mk}$$

$$\frac{(-1)^{mk}a_k\Omega_{\vartheta+\rho k}(z_1,...,z_s)\eta^k}{(n-mk)!} \quad ...(19)$$

where

$$V^{\lambda,\sigma,\omega,\mu,\alpha}_{k,n,m}(y;t)=\sum_{l=0}^{\infty}\frac{(\mu-\alpha)_l t^l}{(\mu+n+\sigma mk)_l(l)!}H^{r,s+1}_{u+1,v+1}\left[y\left|\begin{matrix}(-\lambda-n-\sigma mk-\omega k-l,\varepsilon),\{(c_u,\gamma_u)\}\\\{(d_v,\delta_v)\},(-\lambda-\sigma mk-(\omega+m)k,\varepsilon)\end{matrix}\right.\right] \quad ...(20)$$

then

$$\sum_{n=0}^{\infty}T^{\vartheta,\rho,\lambda,\sigma,\omega,\mu,\alpha}_{n,m}[y;z_1,...,z_s;\eta]t^n=(1-t)^{-(\lambda+1)}\gamma^{(2)}_{\vartheta,\rho,m}\left[\frac{y}{(1-t)^{\varepsilon}};z_1,...,z_s;\frac{\eta t^m}{(1-t)^{(\sigma+1)m+\omega}}\right] \quad ...(21)$$

Result-4

Let

$$\gamma^{(,)}_{m,\sigma,\vartheta,\rho,\omega,\lambda}[y;z_1,...,z_s;t]=\sum_{k=0}^{\infty}(-1)^{mk}a_k\Omega_{\vartheta+\rho k}(z_1,...,z_s)t^s$$

$$H_{u+1,v+1}^{r,s+1}\left[y\left|\begin{matrix}(1-\lambda-mk-\sigma mk-\omega k,\varepsilon),\{(c_u,\gamma_u)\}\\ \{(d_v,\delta_v)\},(-\lambda-\sigma mk-(\omega+m)k,\varepsilon)\end{matrix}\right.\right] \qquad ...(22)$$

and

$$\theta_{n,m}^{\vartheta,\rho,\lambda,\sigma,\omega,\mu,\alpha}[y;z_1,...,z_s;\eta]=\sum_{k=0}^{[n/m]}W_{k,n,m}^{\lambda,\sigma,\omega,\mu,\alpha}(y;t)$$

$$\binom{\mu+n+\sigma mk-1}{n-mk}^{-1}\binom{\alpha+n+\sigma mk-1}{n-mk}\frac{(-1)^{mk}a_k\Omega_{\vartheta+\rho k}(z_1,...,z_s)\eta^k}{(n-mk)!} \qquad ...(23)$$

where

$$W_{k,n,m}^{\lambda,\sigma,\omega,\mu,\alpha}(y;t)=\sum_{l=0}^{\infty}\frac{(\mu-\alpha)_l t^l}{(\mu+n+\sigma mk)_l(l)!}H_{u+1,v+1}^{r,s+1}\left[y\left|\begin{matrix}(1-\lambda-n-\sigma mk-\omega k-l,\varepsilon),\{(c_u,\gamma_u)\}\\ \{(d_v,\delta_v)\},(-\lambda-\sigma mk-(\omega+m)k,\varepsilon)\end{matrix}\right.\right] (24)$$

Then

$$\sum_{n=0}^{\infty}\theta_{n,m}^{\vartheta,\rho,\lambda,\sigma,\omega,\mu,\alpha}[y;z_1,...,z_s;\eta]=(1-t)^{-\lambda}\gamma_{m,\sigma,\vartheta,\rho,\omega,\lambda}^{(4)}\left[\frac{y}{(1-t)^{\varepsilon}};z_1,...,z_s;\frac{\eta t^m}{(1-t)^{(\sigma+1)m+\omega}}\right] \qquad ...(25)$$

Proof of Result-1

We denote the left hand side of the assertion (14) of Result-1 by $H[x,y,t]$ then we use to the definitons in (12) and (13) we have :

$$H[x,y,t]=\sum_{n,l=0}^{\infty}\sum_{k=0}^{[n/m]}\frac{(\mu-\alpha)_l t^l}{(\mu+n+\sigma mk)_l(l)!}\binom{\mu+n+\sigma mk-1}{n-mk}^{-1}\binom{\alpha+n+\sigma mk-1}{n-mk}$$

$$\frac{a_k\Omega_{\vartheta+\rho k}(z_1,...,z_s)\eta^k}{(mk)!(n-mk)!}H_{u+1,v}^{r,s+1}\left[y\left|\begin{matrix}(1-\lambda-n-\sigma mk-l,\varepsilon),\{(c_u,\gamma_u)\}\\ \{(d_v,\delta_v)\}\end{matrix}\right.\right]t^n .$$

Now using the definition of Fox's H-function from(6) and changing the order of summation and integration and then on making series rearrangement therein, it takes the following from:

$$H[x,y,t]=\frac{1}{2\pi i}\int_L\phi(\xi)y^{\xi}\left[\sum_{n,l=0}^{\infty}\sum_{k=0}^{\infty}\frac{(\mu-\alpha)_l t^l}{(\mu+n+mk+\sigma mk)_l(l)!}\binom{\mu+n+mk+\sigma mk-1}{n}^{-1}\right.$$

$$\left.\binom{\alpha+n+mk+\sigma mk-1}{n}\frac{a_k\Omega_{\vartheta+\rho k}(z_1,...,z_s)}{(mk)!(n)!}\Gamma(\lambda+n+mk+\sigma mk+l+\varepsilon\xi)\eta^k t^{n+mk}\right]d\xi$$

Now in view of the relation

$$\frac{\Gamma(\rho+n+l)}{n!}=(\rho+n)_l\binom{\rho+n-1}{n}\Gamma(\rho) \qquad ...(26)$$

and then interpreting the inner series into Gauss' hypergeometric function ${}_2F_1$ we have:

$$H[x,y,t]=\frac{1}{2\pi i}\int_L \phi(\xi)y^{\xi}\left[\sum_{k=0}^{\infty}\left\{\sum_{n=0}^{\infty}\binom{\lambda+n+mk+\sigma mk+\varepsilon\xi-1}{n}\binom{\mu+n+mk+\sigma mk-1}{n}^{-1}\right.\right.$$

$$\left.\binom{\alpha+n+mk+\sigma mk-1}{n}{}_2F_1\left[\begin{matrix}\lambda+n+mk+\sigma mk+\varepsilon\xi,\mu-\alpha;\\ \mu+n+mk+\sigma mk;\end{matrix}\ t\right]t^n\right\}$$

$$\left.\frac{a_k\Omega_{\vartheta+\rho k}(z_1,...,z_s)\eta^k t^{mk}}{(mk)!}\Gamma(\lambda+mk+\sigma mk+\varepsilon\xi)\right]d\xi.$$

Now using the combinatorial identity (4) and then on interpreting the resulting contour into H-function with the helf of (6), we atonce arrive at the desired result in (14).

Similarly the proof of Results-2,3,4,would run parallel to that of Result-1, which we have already detailed above fairly adequately.

3. Some generating relationship involving *H*-function of seceral variables. The Results-5,6,7,8 given below are established for the multivariable H-function defined in (8) by following the corresponding result proved in section-2.

Result-5

$$\text{Let } \gamma^{(5)}_{\vartheta,\rho,m,\sigma,\lambda}[y_1,...,y_r;z_1,...,z_s;t]=\sum_{k=0}^{\infty}\frac{a_k\Omega_{\vartheta+\rho k}(z_1,...,z_s)t^k}{(mk)!}H^{0,v+1:u_1,v_1;...;u_r,v_r}_{p+1,q:p_1,q_1;...;p_r,q_r}\left[\begin{matrix}y_1(1-t)^{-\varepsilon_1}\\ \vdots\\ y_r(1-t)^{-\varepsilon_r}\end{matrix}\right.$$

$$\left|\begin{matrix}(1-\lambda-\sigma mk-mk;\varepsilon_1,...,\varepsilon_r),(a_j;\alpha_j^{(1)},...,\alpha_j^{(r)})_{1,p}:(c_j^{(1)},\gamma_j^{(1)})_{1,p_1},...,(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta_j^{(1)},...,\beta_j^{(r)})_{1,q}:(d_j^{(1)},\delta_j^{(1)})_{1,q_1},...,(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right] \qquad ...(27)$$

and

$$R^{\vartheta,\rho,\lambda,\sigma,\mu,\alpha}_{n,m}[y_1,...,y_r;z_1,...,z_s;\eta]=\sum_{k=0}^{[n/m]}B^{\lambda,\sigma,\mu,\alpha}_{k,n,m}[y_1,...,y_r;z_1,...,z_s;t]\binom{\mu+n+\sigma mk-1}{n-mk}^{-1}$$

$$\binom{\alpha+n+\sigma mk-1}{n-mk}\frac{a_k\Omega_{\vartheta+\rho k}(z_1,\ldots,z_s)}{(mk)!(n-mk)!}\eta^k\ , \qquad \ldots(28)$$

where

$$B_{k,n,m}^{\lambda,\sigma,\mu,\alpha}(y_1,\ldots,y_r;t)=\sum_{l=0}^{\infty}\frac{(\mu-\alpha)_l t^l}{(\mu+n+\sigma mk)_l(l)!}H_{p+1,q:p_1,q_1;\ldots;p_r,q_r}^{0,v+1:u_1,v_1;\ldots;u_r,v_r}\left[\begin{array}{c}y_1\\ \vdots\\ y_r\end{array}\middle|\begin{array}{l}(1-\lambda-n-\sigma mk-l;\varepsilon_1,\ldots,\varepsilon_r),\\ \end{array}\right.$$

$$\left.\begin{array}{l}\left(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)}\right)_{1,p}:\left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,p_1};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r}\\ \left(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)}\right)_{1,q}:\left(d_j^{(1)},\delta_j^{(1)}\right)_{1,q_1};\ldots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r}\end{array}\right] \qquad \ldots(29)$$

Then

$$\sum_{n=0}^{\infty}R_{n,m}^{\vartheta,\rho,\lambda,\sigma,\mu,\alpha}[y_1,\ldots,y_r;z_1,\ldots,z_s;\eta]t^n$$

$$=(1-t)^{-\lambda}\gamma_{\vartheta,\rho,m,\sigma,\lambda}\left[\frac{y_1}{(1-t)^{\varepsilon_1}},\ldots,\frac{y_r}{(1-t)^{\varepsilon_r}};z_1,\ldots,z_s;\frac{\eta t^m}{(1-t)^{(\sigma+1)m}}\right] \qquad \ldots(30)$$

Result-6

Let $\gamma_{m,\vartheta,\rho}^{(6)}[y_1,\ldots,y_r;z_1,\ldots,z_s;t]=\sum_{k=0}^{\infty}(-1)^{mk}\alpha_k\Omega_{\vartheta+\rho k}(z_1,\ldots,z_s)t^k H_{p,q:p_1,q_1;\ldots;p_r,q_r}^{0,v:u_1,v_1;\ldots;u_r,v_r}$

$$\left[\begin{array}{c}y_1\\ \vdots\\ y_r\end{array}\middle|\begin{array}{l}\left(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)}\right)_{1,p}:\left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,p_1},\ldots,\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r}\\ \left(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)}\right)_{1,q}:\left(d_j^{(1)},\delta_j^{(1)}\right)_{1,q_1},\ldots,\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r}\end{array}\right] \qquad \ldots(31)$$

and

$$S_{n,m}^{\vartheta,\rho,\lambda,\sigma,\mu,\alpha}[y_1,\ldots,y_r;z_1,\ldots,z_s;\eta]=\sum_{k=0}^{[n/m]}E_{k,n,m}^{\lambda,\sigma,\mu,\alpha}[y_1,\ldots,y_r;z_1,\ldots,z_s;\eta]\binom{\mu+n+\sigma mk-1}{n-mk}^{-1}$$

$$\binom{\alpha+n+\sigma mk-1}{n-mk}\frac{(-1)^{mk}a_k\Omega_{\vartheta+\rho k}(z_1,\ldots,z_s)}{(n-mk)!}\eta^k \qquad \ldots(32)$$

where

$$E_{k,n,m}^{\lambda,\sigma,\mu,\alpha}(y_1,\ldots,y_r;t)=\sum_{l=0}^{\infty}\frac{(\mu-\alpha)_l t^l}{(\mu+n+\sigma mk)_l(l)!}H_{p+1,q+1:p_1,q_1;\ldots;p_r,q_r}^{0,v+1:u_1,v_1;\ldots;u_r,v_r}\left[\begin{array}{c}y_1\\ \vdots\\ y_r\end{array}\middle|\begin{array}{l}(-\lambda-n-\sigma mk-l;\varepsilon_1,\ldots,\varepsilon_r),\\ (-\lambda-\sigma mk-mk;\varepsilon_1,\ldots,\varepsilon_r),\end{array}\right.$$

$$\left.\begin{matrix}(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,p}:(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,q}:(d_j^{(1)},\delta_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right], \qquad \ldots(33)$$

Then

$$\sum_{n=0}^{\infty} S_{n,m}^{\vartheta,\rho,\lambda,\sigma,\mu,\alpha}[y_1,\ldots,y_r;z_1,\ldots,z_r;\eta]t^n$$

$$=(1-t)^{-(\lambda+1)}\gamma_{m,\vartheta,\rho}^{(b)}\left[\frac{y_1}{(1-t)^{\varepsilon_1}}\cdots\frac{y_r}{(1-t)^{\varepsilon_r}};z_1,\ldots,z_r;\frac{\eta t^m}{(1-t)^{(\sigma+1)/n}}\right] \qquad \ldots(34)$$

Result-7. Let $\gamma_{m,\vartheta,\rho}^{(6)}[y_1,\ldots,y_r;z_1,\ldots,z_r;t]$ is defined in (31)

and

$$U_{n,m}^{\vartheta,\rho,\lambda,\sigma,\omega,\mu,\alpha}[y_1,\ldots,y_r;z_1,\ldots,z_s;\eta]=\sum_{k=0}^{[n/m]}F_{k,n,m}^{\lambda,\sigma,\omega,\mu,\alpha}[y_1,\ldots,y_r:t]$$

$$\binom{\mu+n+\sigma mk-1}{n-mk}^{-1}\binom{\mu+n+\sigma mk-1}{n-mk}\frac{(-1)^{mk}a_k\Omega_{\vartheta+\rho k}(z_1,\ldots,z_s)}{(n-mk)!}\eta^k \qquad \ldots(35)$$

where

$$F_{k,n,m}^{\lambda,\sigma,\omega,\mu,\alpha}[y_1,\ldots,y_r;t]=\sum_{l=0}^{\infty}\frac{(\mu-\alpha)_l t^l}{(\mu+n+\sigma mk)_l (l)!}$$

$$H_{p+1,q+1:p_1,q_1,\ldots,p_r,q_r}^{o,v+1:u_1,v_1,\ldots,u_r,v_r}\left[\begin{matrix}y_1\\ \vdots\\ y_r\end{matrix}\middle|\begin{matrix}(-\lambda-n-\sigma mk-\omega k-l;\varepsilon_1,\ldots\varepsilon_r),(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,p}\\ \\(-\lambda-wk-\sigma mk-mk;\varepsilon_1,\ldots\varepsilon_r),(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,p}\end{matrix}\right]$$

...(36)

Then

$$\sum_{n=0}^{\infty} U_{n,m}^{\vartheta,p,\lambda,\omega,\mu,\alpha}[y_1,\ldots,y_r;z_1,\ldots,z_s;\eta]t^n$$

$$=(1-t)^{-(\lambda+1)}\gamma_{m,\vartheta,p}^{(6)}\left[\frac{y_1}{(1-t)^{\varepsilon_1}},\ldots,\frac{y_r}{(1-t)^{\varepsilon_r}};z_1,\ldots,z_s;\frac{\eta t^m}{(1-t)^{(\sigma+1)m+\omega}}\right]. \qquad \ldots(37)$$

Result-8. Let $\gamma_{m,\vartheta,p,\lambda,\sigma,\omega}^{(8)}[y_1,\ldots,y_r;z_1,\ldots,z_s;t]=\sum_{k=0}^{\infty}(-1)^{mk}a_k\Omega_{\vartheta+pk}(z_1,\ldots,z_s)t^k$

$$H^{0,v+1:u_1,\ldots,u_r,v_r}_{p+1,q+1:p_1,q_1,\ldots,p_r,q_r}\left[\begin{matrix}y_1\\ \vdots\\ y_r\end{matrix}\middle|\begin{matrix}(1-\lambda-mk-\omega k-\sigma mk;\varepsilon_1,\ldots,\varepsilon_r),(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,p}:\\ (-\lambda-mk-\omega k-\sigma mk;\varepsilon_1,\ldots,\varepsilon_r),(b_j;\ldots,\beta_j^{(r)})_{1,q}:\end{matrix}\right.$$

$$\left.\begin{matrix}(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (d_j^{(1)},\delta_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right] \quad \ldots(38)$$

and

$$V^{\vartheta,\rho,\lambda,\sigma,\omega,\mu,\alpha}_{n,m}[y_1,\ldots,y_r;z_1,\ldots,z_s;\eta]=\sum_{k=0}^{[n/m]}G^{\lambda,\sigma,\omega,\mu,\alpha}_{k,n,m}[y_1,\ldots,y_r;t]$$

$$\binom{\mu+n+\sigma mk-1}{n-mk}^{-1}\binom{\alpha+n+\sigma mk-1}{n-mk}(-1)^{mk}a_k\Omega_{\vartheta+\rho k}(z_1,\ldots,z_s)\eta^k \quad \ldots(39)$$

where

$$G^{\lambda,\sigma,\omega,\mu,\alpha}_{k,n,m}[y_1,\ldots,y_r;t]=\sum_{l=0}^{\infty}\frac{(\mu-\alpha)_l t^l}{(\mu+n+\sigma mk)_l(l)!}$$

$$H^{0,v+1:u_1,v_1,\ldots,u_r,v_r}_{p+1,q+1:p_1,q_1,\ldots,p_r,q_r}\left[\begin{matrix}y_1\\ \vdots\\ y_r\end{matrix}\middle|\begin{matrix}(1-\lambda-n-\omega k-\sigma mk-l;\varepsilon_1,\ldots,\varepsilon_r),(a_j;\alpha_j^{(1)},\alpha_j^{(r)})_{1,p}:\\ (-\lambda-\sigma mk-\omega k-mk;\varepsilon_1,\ldots,\varepsilon_r),(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,q}:\end{matrix}\right.$$

$$\left.\begin{matrix}(c_j^{(1)},\gamma_j^{1})_{1,p_1},\ldots,(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (d_j^{(1)},\delta_j^{(1)})_{1,q_1},\ldots,(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right]. \quad \ldots(40)$$

Then

$$\sum_{n=0}^{\infty}V^{\vartheta,\rho,\lambda,\sigma,\omega,\mu,\alpha}_{n,m}[y_1,\ldots,y_r;z_1,\ldots,z_s:\eta]t^n$$

$$=(1-t)^{-\lambda}\gamma^{(8)}_{m,\vartheta,\rho,\lambda,\sigma,\omega}\left[\frac{y_1}{(1-t)^{\varepsilon_1}},\ldots,\frac{y_r}{(1-t)^{\varepsilon_r}};z_1,\ldots,z_s;\frac{\eta t^m}{(1-t)^{(\sigma+1)m+\omega}}\right]. \quad \ldots(41)$$

4. Special Cases. If Results- 1 to 5 and in Result- 8 we take $\Omega_{\vartheta+\rho k}(z_1,\ldots,z_r)\to 1$, $\sigma=0$ and $\mu=\alpha$ these results reduce to the respective known result in ([7],pp.37-44, equations (1.10), (1.14), (3.3), (5.3), (6.9), (6.6) at $\beta=0$).

If in the result of sections-2, 3 we take $\sigma=0$, and $\Omega_{\vartheta+\rho k}(z_1,\ldots,z_s)\to 1$ then these result are reduced into certain families of new generating functions associated with the Fox's H-function and multivariable H-function, but we skip the results here.

All the results of sections 2 and 3, the product of the essentially arbitrary cofficients

$$a_k \neq 0 (k \in N_0)$$

and the identically nonvanishing function

$$\Omega_{\vartheta+\rho k}(z_1,\ldots,z_s)(k \in N_0; \rho, s \in N; \vartheta \in C)$$

can indeed be notationally into one set of essentially arbitrary (and identically nonvanishing) coefficients depending on the order ϑ and one, two or more variables. In view to applying such results as in section 2 above to derive bilateral generating relationship involving Fox's H-function and as in section 3 to derive mixed multilateral generating relationships involving multivariable H-function. We find it to be convenient to specialize a_k and $\Omega_k(z_1,\ldots,z_s)$ individually as well as separately. Our general results asserted by sections 2 and 3 can be shown to yield various families of bilateral and mixed multilateral generating relations for the specific function generated in these families but there are not recorded due to lack of space.

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