Jñānābha, Vol. 37, 2007
(Dedicated to Honour Professor S.P. Singh on his 70th Birthday)

N-POLICY FOR *M*/*G*/1 MACHINE REPAIR PROBLEM WITH STANDBY AND COMMON CAUSE FAILURE

By

M. Jain

Department of Mathematics, I.I.T. Roorkee, Uttar Pradesh, India

E-mail : madhufma@iitr.ernet.in, madhujain@sancharnet.in

and

Satya Prakash Gautam

Department of Mathematics, Institute of Basic Science Dr. B.R. Amedkar University, Agra, 282002, Uttar Pradesh, India

E-mail : gautamibs@gmail.com

(Received : April 5, 2007)

ABSTRACT

In this paper, we investigate an optimal *N*-policy for a single repairman machine problem with Poisson arrivals and general service time distribution. We analyze the machining system consisting of M operating machines along with S cold standby machines. For the normal operation, M machines are required; however, the system can also function with at least in $m(<M)$ machines in degraded mode. The machines may fail individually or due to comman cause. The repair times of the failed machines are independent and identically distributed random variables with general distribution. The repair rendered by the repairman is assumed to be imperfect. The governing equations are constructed by introducing the supplementary variable corresponding to remaining repair time. To obtain the steady state probabilities, recursive method is employed. A cost function is developed to calculate the operating policy at minimum cost.

2000 Mathematics Subject Classification : Primary 90B50; Secondary 90B90.

Keywords : Queue, *N*-policy, Cold standby, Recursive method, Supplementary variable technique, Common cause failure, Cost function.

1. Introduction. The modeling of machine repair problems has found increasing attention in recent years due to their practical applications in several areas, such as in manufacturing systems, computer systems, communication networks, etc.. For machining systems, the provosion of spare part support is common to improve the efficiency of the system but it increases the cost of the system. In many realistic situations, the server may start the service after accumulation of the pre-assigned number of the jobs. This policy seems to be cost effective. In the peresent paper, we study an optimal *N*-policy for *M*/*G*/1 machine repair problem with spares under the steady-state conditions.

In a machine repair problem, if at any time a unit fails, it is sent for repair to the repair facility. The repairman can repair only one failed unit at a time. If an operating unit fails, it is immediately replaced by a spare unit if available. In many practical situations, the spare units are needed to operate the system continuously over a long time period. These situations are applicable in different areas such as production lines, airlines, manufacturing organizations, tele communication, etc..

Machine repair problems have been studied by several research worders in different frame-works. Some of them are **Cherian *et al.* (1987), jain *et al.* (2001),** and many more. The policy, in which the repairman will not initiate the repair of failed machines till N failed machines are accumulated to abate idle time of repairman, is termed as N-policy. Several attempts have been made to suggest control policies in this field from time to time.**Choudhury (1997)** developed a queueing system with general setup time and poisson inputes facilitated under N-policy. The queueing system with finite source and warm spares under N-policy was investigated by **Gupta (1999).**

Recently, N-policy for redundant repairable system with additional repairman was investigated by **jain (2003). Wang *et al.*** **(2005)** presented the maximum entropy analysis to an $M/G/1$ queueing system with unreliable server and general startup times. **Kim and Moon (2006)** discussed an $M/G/1$ queueing system under a certain service policy. A two phase batch arrival retrial queueing system with Bernaulli vacation schedule was explored by **Choudhury (2007).**

Numerous queue theorists developed machine repair problem having individual as well as common cause failures. **Hughes(1987)** considered the issue of common cause failure in machining problem. Recently, **jain et al. (2002)** investigated a flexible manufacturing system with common cause failure. **Dhillon and Li (2005)** suggested stochastic analysis of standby systems with common cause failure and human errors. **Jain and Mishra(2006)** studied multi-stage degraded machining system with common cause shock failure and state dependent rates. **Ehsani et al. (2008)** proposed a model for reliability evaluation of deregulated electric power systems with common cause failure for planning applications.

The purpose of our study is this paper is to provide a recursive method to determie the steady state probability distribution of the number of failed units in the $M/G/1$ system operating under N-policy. In section 2 we outline some assumptions and notations in order to construct the mathematical model of the system. Section 3 covers the analytical expressions if all probabilities for different states. In section 4 some important performance characteristics such as average number of failed machines in the system, the probability of repairman being idle, machine availability etc. are establised. In section 5 we determine the average length of the busy period, average length of the cycle and average length of the idle period. Also we construct the cost function in terms total expected cost per unit time. Finally conclusion is drawn in section 6.

2. **Model Description.** We consider general service system consisting of $K(M$ operating $+$ S cold standby$)$ machines under the supervision of single

repairman to maintain efficient operation for long run. The model is developed by considering the following assumptions:

(i) The life time of operating machines follows exponential distribution with mean $1/\lambda$.

(ii) The system may fail at any moment due to common cause with exponential failure rate λ_c.

(iii) $\alpha(0 \leq \alpha \leq 1)$ is the probability of recovering the failed machine. The renewed machines become as good as new ones.

(iv) The system will be in working state until there are m operating machines. If an operating machine fails, a cold standby machine replaces it with negligible switchover time provided it is a available at that moment.

(v) The repair times of the failed machines are independent and identically distributed (i,i,d) random variables having general distribution function.

(vi) The repairman follows N-failed machines in the system and continues the repair until he repairs all the failed machines.

For modeling purpose, we use the following notations:

N	Threshold level
B	Random variable denoting service time
K	Total number of machines, $K=M+S$.
$B(u)$	Distribution function $(d.f.)$ of B
$b(u)$	Probability density function $(p.d.f)$ of B
$U(t)$	Remaining repair time for the machine at time t
b_1	Mean repair time
α	Probability of recovering of the failed machines
$B^*(\theta)$	Laplace- Stieltijes transform (LST) of B
$B^{*(j)}(\theta)$	jth order derivative of $B^*(\theta)$ with respect to θ
λ_c	Common cause failure rate.
λ_d	Degraded failure rate, when $n \geq S+1$.

Let $Q(t)$ and $L(t)$ be the random variables denoting the status of the repairman at any instant t and the number of failed machines in the system respectivley. Define

$$Q(t)=\begin{cases}0, & \text{if the repairman is in idle state at time } t\\ 1, & \text{if the repaiman is no working state at time } t\end{cases}.$$

Using these notations, we define the following transient probabilities:

$P_{0,0}(t)$ = Prob $\{Q(t)=0, L(t)=0\}$

$P_{0,n}(t)$ = Prob $\{Q(t)=0, L(t)=0, 0 \leq n < N\}$

$P_{1,n}(t)$ = Prob $\{Q(t)=1, L(t)=n, 1 \leq n \leq K\text{-}m\}$.

For the analysis purpose, we follow the supplementary variable technique [cf. Cox, (1995)]. By introducing random variable U corresponding to remaining repair times (u) for the failed machines in repair, we shall construct the equations of different states of the system.

Let us denote

$P_{1,n}(u,t)du$ = Prob $\{Q(t)=1, L(t)=n, u<U(t)\leq u+du, u\geq 0, 1\leq n\leq K\text{-}m\}$

$P_{1,n}(t)=\int_0^\infty P_{1,n}(u,t)du$.

3. The Analysis. Considering the state of the system at time t, we obtain the tansient state equations governing the model as follows :

$$\frac{d}{dt}P_{0,0}(t)=-(M\lambda+\lambda_c)P_{0,0}(t)+P_{1,1}(0,t) \qquad ...(1)$$

$$\frac{d}{dt}P_{0,n}(t)=-(M\lambda+\lambda_c)P_{0,n}(t)+M\lambda P_{0,n-1}(t),\ 1\leq n\leq N-1 \qquad ...(2)$$

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{1,1}(u,t)=-(M\lambda+\lambda_c)P_{1,1}(u,t)+P_{1,2}(0,t)b(u) \qquad ...(3)$$

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{1,n}(u,t)=-(M\lambda+\lambda_c)P_{1,n}(u,t)+\alpha M\lambda P_{1,n-1}(u,t)+P_{1,n+1}(0,t)b(u)$$
$$2\leq n\leq N-1 \qquad ...(4)$$

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{1,N}(u,t)=-(M\lambda+\lambda_c)P_{1,N}(u,t)+\alpha M\lambda P_{1,N-1}(u,t)+P_{1,N+1}(0,t)b(u)$$
$$+M\lambda P_{0,N-1}(u) \qquad ...(5)$$

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{1,n}(u,t)=-(M\lambda+\lambda_c)P_{1,n}(u,t)+\alpha M\lambda P_{1,n-1}(u,t)+P_{1,n+1}(0,t)b(u),$$
$$N+1\leq n\leq S \qquad ...(6)$$

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{1,n}(u,t)=-((K-n)\lambda_d+\lambda_c)P_{1,n}(u,t)+\alpha(K-n+1)\lambda_d P_{1,n-1}(u,t)$$
$$+P_{1,n+1}(0,t)b(u),\qquad S+1\leq n\leq K-m-1 \qquad ...(7)$$

$$\frac{d}{dt}P_{1,K-m}(t)=-(m\lambda_d+\lambda_c)P_{1,K-m}(t)+\alpha(m+1)\lambda_d P_{1,K-m-1}(t) \qquad ...(8)$$

$$\frac{d}{dt}P_F(t)=\alpha(K-m)\lambda_d P_{1,K-m}(t)+\sum_{i=1}^{S}\{(1-\alpha)M\lambda+\lambda_c\}P_{1,i}(t)$$
$$+\sum_{i=S+1}^{K-m-1}\{(1-\alpha)(K-i)\lambda_d+\lambda_c\}P_{1,i}(t)+\sum_{i=0}^{N-1}\lambda_c P_{0,i}(t) \qquad ...(9)$$

For steady state probabilities, we deonote

$$P_F=\lim_{t\to\infty}P_F(t);\qquad P_{0,n}=\lim_{t\to\infty}P_{0,n}(t);\qquad 1\leq n\leq N-1$$

$P_{1,n} = \lim_{t\to\infty} P_{1,n}(t)$; $\qquad P_{1,n}(u) = \lim_{t\to\infty} P_{1,n}(u,t)$; $\; 1 \le n \le K-m$

$P_{0,N-1}(u) = P_{0,N-1} b(u)$.

From equations (1)-(9), the steady state equations are obtained as

$$-(M\lambda + \lambda_c)P_{0,0} + P_{1,1}(0) = 0 \,, \qquad ...(10)$$

$$-(M\lambda + \lambda_c)P_{0,n} + M\lambda P_{0,n-1}(0) = 0, \qquad 1 \le n \le N-1 \qquad ...(11)$$

$$-\frac{d}{du}P_{1,1}(u) = -(M\lambda + \lambda_c)P_{1,1}(u) + P_{1,2}(0)b(u) \qquad ...(12)$$

$$-\frac{d}{du}P_{1,n}(u) = -(M\lambda + \lambda_c)P_{1,n}(u) + \alpha M\lambda P_{1,n-1}(u) + P_{1,n-1}(0)b(u), \; 2 \le n \le N-1 \qquad ...(13)$$

$$-\frac{d}{du}P_{1,N}(u) = -(M\lambda + \lambda_c)P_{1,N}(u) + \alpha M\lambda P_{1,N-1}(u) + M\lambda P_{0,N-1}(u) + P_{1,N+1}0b(u) \qquad ...(14)$$

$$-\frac{d}{du}P_{1,n}(u) = -(M\lambda + \lambda_c)P_{1,n}(u) + \alpha M\lambda P_{1,n-1}(u) + P_{1,n+1}(0)b(u), N+1 \le n \le S \qquad ...(15)$$

$$-\frac{d}{du}P_{1,n}(u) = -[(K-n)\lambda_d + \lambda_c]P_{1,n}(u) + \alpha(K-n+1)\lambda_d P_{1,n-1}(u) + P_{1,n+1}(0)b(u),$$

$$S+1 \le n \le K-m-1 \qquad ...(16)$$

$$-(m\lambda_d + \lambda_c)P_{1,K-m} + \alpha(m+1)\lambda_d P_{1,K-m-1} = 0 \qquad (17)$$

$$\alpha(K-m)\lambda_d P_{1,K-m} + \sum_{i=1}^{S}\{(1-\alpha)M\lambda + \lambda_c\}P_{1,i} + \sum_{i=S+1}^{K-m-1}\{(1-\alpha)(K-i)\lambda_d + \lambda_c\}P_{1,i}$$

$$+ \sum_{i=0}^{N-1}\lambda_c P_{0,i} = 0 \qquad ...(18)$$

Equation (10) yields

$$P_{1,1}(0) = (M\lambda + \lambda_c)P_{0,0} \,. \qquad ...(19)$$

Putting $n=1$, equation (11) provides

$$P_{0,1} = \frac{M\lambda}{M\lambda + \lambda_c}P_{0,0} \,. \qquad ...(20)$$

Now putting $n=2$, equation (11) yields

$$P_{0,2} = \left(\frac{M\lambda}{M\lambda + \lambda_c}\right)^2 P_{0,0} \,. \qquad ...(21)$$

Similarly by using rcursive approach, we obtain

$$P_{0,n}=\left(\frac{M\lambda}{M\lambda+\lambda_c}\right)^n P_{0,0} \qquad 1\le n\le N-1 \; . \qquad \text{...(22)}$$

Further let us define Laplace-Stieltjes transform as

$$B^*(\theta)=\int_0^\infty e^{-\theta u}dB(u)=\int_0^\infty e^{-\theta u}b(u)du \; . \qquad \text{...(23)}$$

$$P_{1,n}^*(\theta)=\int_0^\infty e^{-\theta u}P_{1,n}(u)du, \qquad 1<n\le K-m \qquad \text{...(24)}$$

so that, we have

$$P_{1,n}=P_{1,n}^*(0)=\int_0^\infty P_{1,n}(u)du, \quad 1\le n\le N-m \; , \qquad \text{...(25)}$$

$$\int_0^\infty e^{-\theta u}\frac{\partial}{\partial u}P_{1,n}(u)du=\theta P_{1,n}^*(\theta)-P_{1,n}(0), \qquad \text{...(26)}$$

$$P_{1,N-1}(u)=P_{0,N-1}b(u) \; . \qquad \text{...(27)}$$

Taking Laplace-Stieltjes tansforms (LST) on both sides of equations (12) to (16), we have

$$(M\lambda+\lambda_c-\theta)P_{1,1}^*(\theta)=P_{1,2}(0)B^*(\theta)-P_{1,1}(0) \qquad \text{...(28)}$$

$$(M\lambda+\lambda_c-\theta)P_{1,n}^*(\theta)=\alpha M\lambda P_{1,n-1}^*(\theta)+P_{1,n+1}(0)B^*(\theta)-P_{1,n}(0), \; 2\le n\le N-1 \qquad \text{...(29)}$$

$$(M\lambda+\lambda_c-\theta)P_{1,N}^*(\theta)=\alpha M\lambda P_{1,N-1}^*(\theta)+M\lambda P_{0,N-1}B^*(\theta)+P_{1,N-1}(0)B^*(\theta)-P_{1,N}(0) \qquad \text{...(30)}$$

$$(M\lambda+\lambda_c-\theta)P_{1,n}^*(\theta)=\alpha M\lambda P_{1,n-1}^*(\theta)+P_{1,n+1}(0)B^*(\theta)-P_{1,n}(0) \; , \; N+1\le n\le S \qquad \text{...(31)}$$

$$[(K-n)\lambda_d+\lambda_c-\theta]P_{1,n}^*(\theta)=\alpha(K-n+1)\lambda_d P_{1,n-1}^*(\theta)+P_{1,n+1}(0)B^*(\theta)-P_{1,n}(0),$$

$$S+1\le n\le K-m-1 \cdot \qquad \text{...(32)}$$

From equations (19) and (22), we get

$$P_{0,n}=\left(\frac{1}{M\lambda+\lambda_c}\right)\left(\frac{M\lambda}{M\lambda+\lambda_c}\right)^n P_{1,1}(0), \; 1\le n\le N-1 \; . \qquad \text{...(33)}$$

Using equation (33) in equation (30) and adding equations (28)-(32), we get

$$(M\lambda+\lambda_c-\theta)\sum_{n=1}^{S}P_{1,n}^*(\theta)+\sum_{n=S+1}^{K-m-1}[(K+n)\lambda_d+\lambda_c-\theta]P_{1,n}^*(\theta)$$

$$=\alpha M\lambda\sum_{n=2}^{S}P_{1,n-1}^*(\theta)+\alpha\lambda_d\sum_{n=S+1}^{K-m-1}(K-n+1)P_{1,n-1}^*(\theta)$$

$$+\sum_{n=1}^{K-m-1}P_{1,n+1}(0)B^*(\theta)+\left[\left(\frac{M\lambda}{M\lambda+\lambda_c}\right)^N\right]P_{1,1}(0)-1 \; . \qquad \text{...(34)}$$

Now using equation (33) in equation (30) and setting $\theta = M\lambda + \lambda_c$ in equations (28)-(31), we obtain,

$$P_{1,n+1}(0) = \frac{P_{1,n}(0) - \phi P_{1,1}(0) - \alpha M\lambda P^{*}_{1,n-1}(M\lambda + \lambda_c)}{B^{*}(M\lambda + \lambda_c)}, \qquad 1 \le n \le S \qquad ...(35)$$

where

$$\phi = \begin{cases} \left(\dfrac{M\lambda}{M\lambda + \lambda_c}\right)^{n}, & n = N \\ 0, & \text{otherwise} \end{cases} \quad \text{and } P^{*}_{1,n}(;) = 0 \text{ for } n < 1.$$

Now substituting $\theta = (K-n)\lambda_d + \lambda_c$ in equation (32) we get,

$$P_{1,n+1}(0) = \frac{P_{1,n}(0) - \alpha[(K-n+1)\lambda_d]P^{*}_{1,n-1}[(K-n)\lambda_d + \lambda_c]}{B^{*}[(K-n)\lambda_d + \lambda_c]},$$

$$S+1 \le n \le K-m-1. \qquad ...(36)$$

Differentiating equations (28)-(31) j times w.r.t. 'θ' and setting $\theta = M\lambda + \lambda_c$ we have

$$P^{*(j-1)}_{1,n}(M\lambda + \lambda_c) = -\frac{1}{j}\Big[P_{1,n+1}(0)B^{*(j)}(M\lambda + \lambda_c) + \phi P_{1,1}(0)B^{*(j)}(M\lambda + \lambda_c) +$$

$$\alpha M\lambda P^{*(j)}_{1,n-1}(M\lambda + \lambda_c)\Big], \quad 2 \le n \le S-1, \quad 1 \le n \le S-n-1 \qquad ...(37)$$

where $P^{*(0)}_{1,n}(\theta) = P^{*}_{1,n}(\theta)$ & $B^{*(j)}(\theta) = \dfrac{d^j}{d\theta^j}B^{*}(\theta)$ is the j^{th} derivative of $B^{*}(\theta)$.

Again setting $\theta = (M-r)\lambda + \lambda_c$, in equations (28)-(31), we get

$$P^{*}_{1,n}[(M-r)\lambda_d + \lambda_c] = \frac{1}{r\lambda}\Big[\alpha M\lambda P^{*}_{1,n-1}\{(M-r)\lambda + \lambda_c\} + \phi P_{1,1}(0)B^{*}\{(M-r)\lambda + \lambda_c\}$$

$$+ P^{*}_{1,n+1}(0)B^{*}\{(M-r)\lambda + \lambda_c\} - P_{1,n}(0)\Big], \; 1 \le n \le S, \; 1 \le r \le M-m-1. \qquad ...(38)$$

Again setting $\theta = (M-r)\lambda_d + \lambda_c$, equation (32) yields

$$P^{*}_{1,n}[(M-r)\lambda_d + \lambda_c] = \frac{1}{(S-n+r)\lambda_d}\Big[\alpha(K-n+1)\lambda_d P^{*}_{1,n-1}\{(M-r)\lambda_d + \lambda_c\} + P_{1,n+1}(0)B^{*}$$

$$\{(M-r)\lambda_d + \lambda_c\} - P_{1,n}(0)\Big], \; S+1 \le n \le S+r-1, \; 2 \le r \le M-m-1. \qquad ...(39)$$

Now $P_{1,2}(0), P_{1,3}(0), \ldots, P_{1,k-m}(0)$, can be obtained recursively with the help of equations (35)-(39) in terms of P_{00}.

Again setting $\theta = 0$ in equations (28)-(32), we have

$$P_{1,n}^{*}(0)=\frac{\alpha M\lambda P_{1,n-1}^{*}(0)+P_{1,n+1}(0)-P_{1,n}(0)}{M\lambda-\lambda_c},\quad 1\le n\le N-1 \qquad ...(40)$$

where $B^{*}(0)=1$.
Also

$$P_{1,n}^{*}(0)=\frac{\alpha M\lambda P_{1,n-1}^{*}(0)+P_{1,n+1}(0)+\phi_{1,1}(0)-P_{1,n}(0)}{M\lambda+\lambda_c},\qquad N\le n\le S \qquad ...(41)$$

and

$$P_{1,n}^{*}(0)=\frac{\alpha(K-n+1)\lambda_d P_{1,n-1}^{*}(0)+P_{1,n+1}(0)-P_{1,n}(0)}{(K-n)\lambda_d+\lambda_c},\ S+1\le n\le K-m-1 \qquad ...(42)$$

Now $P_{1,n}(0),(1\le n\le K-m)$, can be determined respectively by using equations (40)-(42) in te~rms of $P_{0,0}$.
From equation (4.17), we obtain

$$\therefore\ P_{1,K-m}=\frac{\alpha(m+1)\lambda_d P_{1,K-m-1}}{(m\lambda_d+\lambda_c)} \qquad ...(43)$$

The value of $P_{0,0}$ can be determined by using normalizing condition as

$$\sum_{n=0}^{N-1}P_{0,n}+\sum_{n=1}^{K-m}P_{1,n}^{*}(0)=1\ . \qquad ...(44)$$

The failure probability of the system can be obtained by

$$P_f=1-\sum_{n=0}^{N-1}P_{0,n}-\sum_{n=1}^{K-m}P_{1,n}^{*}(0) \qquad ...(45)$$

4. Some Performance Indices. In this section, we present some performance indices so that the system designer may ensure efficiency and effectiveness of the system for future design and development. Now, we formulate the performance indices in terms of queue size distribution obtained in previous section.

Let us denote the performance metrics as follows :

$E(N)$ Average number of failed machines in the system.
$E(N_q)$ Average number faild of machines that are waiting for repair in the queue.
$E(O)$ Average number of operating machines in the system.
$E(S)$ Average number of standby machines in the system.
$P(I)$ The probability of repaiman being idle.
$P(B)$ The probability of repaiman being busy.
MA Machine availability.

The above performance indices can be given in terms of queue size as follows :

(i) $$E(N)=\sum_{n=0}^{N-1}nP_{0,n}+\sum_{n=1}^{K-m}nP_{1,n} \qquad ...(46)$$

(ii) $E(N_q)=\sum_{n=1}^{N-1}(n-1)P_{0,n}+\sum_{n=1}^{K-m}(n-1)P_{1,n}$ ...(47)

(iii) $E(O)=M-\sum_{n=S+1}^{K-m}(n-S)P_{1,n}$ ...(48)

(iv) $E(S)=\sum_{n=0}^{S}(S-n)P_{1,n}$ ...(49)

(v) $P(I)=\sum_{n=0}^{N-1}P_{0,n}=NP_{0,0}$ ...(50)

(vi) $P(B)=1-P(I)$ ...(51)

(vii) $MA=\frac{1}{K-m}\left[\sum_{n=0}^{N-1}(K-m-n)P_{0,n}+\sum_{n=1}^{K-m}(K-m-n)P_{1,n}\right].$ ...(52)

5. Cost Analysis. We assume that $E(B)$, $E(C)$ and $E(I)$ represent the average length of the busy period, average length of the cycle and average length of the idle period, respectively. Then average length of idle period is obtained as

$$E(I)=\frac{N}{M\lambda}. \quad ...(53)$$

The long run fraction of time for which repairman remains idle and busy, are given by

$$\frac{E(I)}{E(C)}=\sum_{n=1}^{N}nP_{0,0}. \quad ...(54)$$

and

$$\frac{E(B)}{E(C)}=1-\sum_{n=1}^{N}nP_{0,0}. \quad ...(55)$$

To determine the optimal value of N in the system, we construct the cost function denoting the total expected cost per unit time as given below:

$$TC(N,S)=Co\frac{E(B)}{E(C)}+C_r\frac{E(I)}{E(C)}+C_hE(N)+(C_b+C_S)\frac{1}{E(C)}+(M\lambda+\lambda_c)P^*_{1,K-m}(0) \quad ...(56)$$

where different cost elements used are as follows :

C_o = Cost per unit time for keeping the repairman on

C_f = Cost per unit time for keeping the repairman off.

C_h = Holding cost per unit time per machine present in the system.

C_b = Break-down cost per unit time for turning the repairman off.

C_s =Start-up cost per unit time for turning the repairman on.

6. Conclusion. In the present paper we have obtained the steady state solution for the N-policy $M/G/1$ machine repair problem with cold spares. The concept of the common cause failure along with individual failures is incorporated, which is common in many real time machining systems. The suggested recursive method using supplementary variable technique to determine the steady-state probabilities can be easily implementable to quantify various performance indices as well as cost function. Our study provides good tradeoff for gaining initial insight of system's performance and cost incurred so as to determine the optimal service parameters. The concerned model is more sophisticated and versatile in our real life situations as it includes common cause failure and imperfect repair performed by a single repairman.

REFERENCE

[1] J., Cherian, M. Jain and G.C. Sharma, Reliability of a standby system with repair, *Indian J. Pure Appl. Math.*, **18** No. **12**, (1987) 1061-1068.

[2] G. Choudhury, A poisson queue under *N* policy with a general setup time, *Indian J. Pure Appl. Math.*, **28** (1997), 1595-1608.

[3] G. Choudhury, A two phase batch arrival retrial queueing system with Bernoulli vacation schedule, *Appl. Math. Comput.*, **188 No.2** (2007), 1455.

[4] B.S. Dhillon and Z. Li, Stochastic analysis of a system containing *N*-redundant robots and *M*-redundant built-in safety units, *Int. J. Perform. Engg.*, **1 No.2** (2005), 179-185.

[5] A. Ehsani, A.M. Ranjbar, A. Jafari and M. Fotuhi-Firuxabad, Reliability evaluation of deregulated electric power systems for planning applications, *Reliab. Engg. Syst. Safety*, **93**, No. **10**, **(2008)**, 1473-1484.

[6] S.M. Gupta, N-policy queueing system with finite source and warm spares, *OPSEARCH*, **36** No. 3, (1999), 189-217.

[7] R.P. Hughes, A new approach to common cause failure, *Reliab. Engg. System Safety*, **17** (1987) 211-236.

[8] M. Jain, *N*-policy for redundant repairable system with additioal repairmen, *OPSEARCH*, **40, No.2**, (2003), 97-114.

[9] M. Jain and A. Mishra, Multi-stage degraded machining system with common cause shock failure and state dependent rates, *Raj. Acad. Phy. Sci.*, **5 No.3**, 251-262.

[10] M. Jain, K.P.S. Baghel and M. Jadown, A repairable system with spares, state dependent rates and additional repairman, *Proc. of Thirty Fourth Annual Convention of ORSI*, (2001), 134-136.

[11] M. Jain, G.C. Sharma and M. Singh, A flexible manufacturing system with common cause failure, *Int. J. Engg.*, **1,15, No.1** (2002) 49-56.

[12] D.J. Kim and S.A. Moon, Randomized control of *T* policy for an *M/G/*1 system, *Comput. Ind. Engg.*, **51 No.4** (2006), 684-692.

[13] K.H. Wang, T.Y. Wang and W.L. Pearn, Maximum entropy analysis to the *N*-policy *M/G/*1 queueing system with server breakdowns and general startup times, *Appl. Math. Comput.*, **165 No. 1**, (2005), 45-61.