Jñānābha, Vol. 37, 2007
(Dedicated to Honour Professor S.P. Singh on his 70th Birthday)

A CORRECTION TO AN EXAMPLE OF RENU CHUG AND SANJAY KUMAR FOR WEAKLY COMPATIBLE SELF-MAPS

By

T. Phaneedra

Department of Mathematics, Hyderabad Institute of Technology and Management (HITAM), Gowdavelli Village, Medchal Mandal, R.R. Distt., Hyderabad, Andra Pradesh, India

E-mail : tp_indra@rediffmail.com

(Received : February 15, 2007)

ABSTRACT

A correction is suggested in an example of Renu Chug and Sanjay Kumar, in attempting to disprove the converse of the statement that every compatible pair of self-maps is weakly compatible.

2000 Mathematics Subject Classification : 54H25

Keywords and Phrases : Weakly compatible self-maps.

1. Introduction. Let (X,d) denotes a metric space. As a generalization of commuting self-maps, Gerald Jungck [2] defined self-maps S and A on X to be compatible if $\lim_{n\to\infty} d(SAx_n, ASx_n) = 0$ whenever $\langle x_n \rangle_{n=1}^{\infty}$ is such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$ for some $t \in X$. It is easy to see that if self-maps S and A on X are compatible, then $ASx = SAx$ wheneer $x \in X$ is such that $Ax = Sx$. Self-maps which commute at their coincidence points are known as weakly compatible [3]. In an attempt to disprove the converse of the statement that every compatible pair is weakly compatible, Renu Chug and Sanjay Kumar [1] considered self-maps:

$$Ax = \begin{cases} x & (x = 2 \text{ or } x > 5) \\ 6 & (2 < x \le 5) \end{cases} \quad \text{and} \quad Sx = \begin{cases} x & (x = 2) \\ 12 & (2 < x \le 5) \\ x - 3 & (x > 5). \end{cases} \text{for all } x \in X, \qquad (1)$$

where $X=[2,20]$ with usual metric $d(x,y) = |x-y|$ for all $x, y \in X$. It was claimed that the mappings are not compatible.

The following lines reveals that their claim is not true. The maps A and S are, in fact, compatible. For,

$$d(Sx,Ax)=\begin{cases}0 & (x=2)\\ 6 & (2<x\le 5)\\ 3 & (x>5).\end{cases} \text{ and } d(SAx,ASx)=\begin{cases}0 & (x=2 \text{ or } x>8)\\ 9 & (2<x\le 5)\\ 9-x & (5<x\le 8).\end{cases} \quad (2)$$

so that $d(Sx_n,Ax_n)\to 0$ as $n\to\infty$ whenever $\{x_n\}_{n=1}^{\infty}\subset X$ is such that $x_n=2$ for all n and hence $d(SAx_n,ASx_n)\to 0$ as $n\to\infty$.

However, if we redefine A as $Ax=\begin{cases}2 & (x=2 \text{ or } x>5)\\ 6 & (2<x\le 5)\end{cases}$ a routine computation reveals that A and S are not compatible. In this case, $x=2$ is the only coincidence point for A and S at which they commute and hence (A,S) is weakly compatible.

REFERENCES

[1] Renu Chugh and Sanjay Kumar, Common Fixed Points for Weakly Compatible Maps, *Proc. Indian Acad. Sci. (Math. Sci)*, **111(2)** (2001), 241-247.

[2] Gerald Jungck, Compatible Mapping and Common Fixed Points, *Int. J. Math. and Math. Sci.*, **9(4)** (1986), 771-779.

[3] Gerald Jungeck and B.E. Rhoades, Fixed point for set valued functions without continuity, *Indian J. Pure Appl. Math.* **29** (3) (1998), 227-238.