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FINITE M/G/1 QUEUEING SYSTEM WITH BERNOULLI FEEDBACK UNDER OPTIMAL *N*-POLICY

By

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ABSTRACT

This paper ivestigates $M/G/1$ queue with Bernoulli feedback under an optimal control policy. According to N-policy, the server renders service only when N customers are accumulated in the system; once he initiates service, continues to provide it till system becomes empty. The system size under steady state conditions are determined by using generating function and supplementary variable technique. The analytical results for average queue length and mean response time are obtained. The optimal N-value is obtained by minimizing the total operation cost of the system.

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1. Introduction. Feedback queueing networks are important in many applications where the customer getting incomplete service may try to seek service repeatedly till his service gets completed. Such situations arise in manufacturing, data communications, telecommunications, packet transmissions, etc. Several authors have investigated the feedback queueing systems in different frameworks; some of the recent works are as given below. Disney et al. (1984) derived stationary queue length and waiting time distribution in single server feedback queues. Vanden et al. (1991) studied the $M/G/1$ queue with processor sharing and its relation to a feedback queue. Sojourn time in vacation queues and polling systems with Bernoulli feedback was discussed by Takine et al. (1991). Rege (1993) discussed the $M/G/1$ queue with Bernoulli feedback. Adve and Nelson (1994) gave the relationship between Bernoulli and fixed feedback policies for the $M/G/1$ queue. Thangaraj and Santhakumaran (1994) analysed Sojourn times in queues with a pair of instantaneous independent Bernoulli feedback. Bhattacharya et al. (1995) studied on adoptive optimization in multiclass $M/GI/1$ queues with Bernoulli feedback. Takagi (1996) had studied the response time in $M/G/1$ queues with service in random order and Bernoulli feedback. Fujian and Yang (1998) discussed queue length performance of some non-exhaustive polling models with Bernoulli feedback. Choi, et al. (2003) discussed an $M/G/1$ queue with multiple types of feedback with gated vacations and *FCFS* policy.

Many researchers have incorporated the concept of N-policy in queueing system in which the server turns on whenever N or more customers are present in the system. Tang (1994) obtained takacs type equation in the $M/G/1$ queue with N-policy adn setup times. Piscataway and Choudhury (1997) considered a Poisson queue under N-policy with a general setup time. Lee and Park (1997) discussed an optimal strategy based on N-policy for production system with early setup. Jain (1997) has introduced an optimal N-policy for single server Markovian queue with breakdown, repair and state dependent arrival rate. Artalejo (1998) gave some results for the $M/G/1$ queue under N-policy. Gupta (1999) considered N-policy queueing system with finite source and warm spares. Hur and Paik, (1999) analysed effect of different arrival rates on the N-policy for $M/G/1$ with server setup. Kumar et al. (2002) considered the N-policy $M/G/1$ feedback queue with varying arrival rates. Jain and Singh (2003) have discussed an optimal N-policy for the state-dependent $M/E_k/1$ queue with server breakdowns. Jain (2003) analysed N-policy for redundant repairable system with additional repairmen. Pearn adn Chang (2004) presented optimal management of the N-policy $M/E_k/1$ queueing system with a removable service station. Berman and Larson (2004) have analysed a queu control model for retail services having back room operations and cross-trained workers.

Several authors used the supplementary variable technique in $M/G/1$ queue. Hokstad (1975) has applied supplimentary variable technique in the $M/G/1$ queue. Wang and Ke (2000) employed a recursive method using the supplementary variable technique to established optimal control policy at a minimum cost involved.

The purpose of this paper is to obtain the mean response time and optimal operating N-policy for finite $M/G/1$ queueing model with Bernoulli feedback. The rest of the paper is organized as follows. We provide notations and terminology related to model in section 2. In section 3, the probability generating function technique to obtain steady state probability distribution of the number of units in the system is discussed. The supplementary variable technique by treating the remaining service time as supplementary variable is used. The explicit results for the average number of customers in the system and the mean response time are determined in section 4. The optimal operating N-policy is also stated to minimize the total expected cost per customer time. In section 5, we deduced some particular cases which match with earlier existing results. Finally conclusion is drawn in seciton 6.

2. The Model Analysis and Description. Consider $M/G/1$ Bernoulli feedback state dependent single server queue with general service time distribution and finite capacity K under N-policy. The customers arrive at the system according to Poisson process with arrival rate λ and the system permits no arrivals when the number of customers in the system is reached to K. The service time of each customer is independently identically distributed (i.i.d.) random variable with distribution function $B(x)(x \geq 0)$. A single server renders the service according to *FCFS* discipline. As soon as the system becomes empty, the server turns off and may not operate until N customers are accumulated in the system. A customer departs from the system after completion of the service with probability σ. If the service is not successfully completed then the customer again joins the system and is cycled back into service with probability $(1-\sigma)$. We apply supplementary variable technique by introducing the new variable X denoting the remaining service time.

The following notations and terminology are used to formulate the mathematical model :

ρ -traffic intensity such that $0 < \sigma \leq 1$.

σ - Bernoulli feedback parameter.

N - Threshold parameter in turn on policy.

X - remaining service time for the customers being served.

W - expected waiting time of the costomers.

pdf - probability density function.

LST - Laplace Stieltjes Transform.

pgf - probability generating function.

B - service time random variable.

$B(x)$ - service time distribution.

$b(x)$ - *pdf* of service time distribution.

$B^*(\theta)$ - *LST* service time distribution.

$B^{*(i)}(\theta)$- i^{th} order derivative of service time distribution with respect to θ.

H - service time random variable in Bernouli feedback queue.

$H(x)$ - service time distribution of H.

$h(x)$ - *pdf* of service time distribution of H.

$H^*(\theta)$ - *LST* service time distribution of H.

$H^{*(i)}(\theta)$- i^{th} order derivative of H with respect to θ.

$p_{0,0}(x)$ = Pr (the system being empty at time t)

$p_{0,n}(x)$ = Pr (there are n customers in the system at time t when the server is turned off, where $n=1,2,\ldots,N\text{-}1.$).

$p_{1,n}(x)$ = Pr(there are n customers in the system at time t when the server is turned on and working, $n=1,2,\ldots$).

$p^*_{1,n}(\theta)$ = LST of $p_{1,n}(x)$.

$$G_0(z)=\sum_{n=0}^{N-1} z^n p_{0,n},\ pgf \text{ of } p_{0,n}(x).$$

$$G_1(z,0)=\sum_{i=1}^{K} z^n p_{1,n}(0), pgf \text{ of } p_{1,n}(x).$$

$$G_1^*(z,\theta)=\sum_{n=1}^{K} z^n p_{1,n}^*(\theta), pgf \text{ of } p_{1,n}^*(\theta).$$

$G(z)$ = pgf of the number of customers in the system.

Let us assume that the customer is feedback the end of the queue and will return again and again till the service is completed. The total service time $H(x)$ of the customer is the time from the initialization of service upto final departure from the system. It is noted that (cf. Kumar et al., 2002)

$$H^*(\theta)=\frac{\sigma B^*(\theta)}{\{1-(1-\sigma)B^*(\theta)\}} \quad \ldots(1)$$

Thus, the mean service time $H^{*(1)}(0)$ and the second moment $H^{*(2)}(0)$ of the service time of customer is given by (cf. Takine et al. 1991)

$$H^{*(1)}(0)=\frac{-B^{*(1)}(0)}{\sigma},\quad H^{*(2)}(0)=\frac{B^{*(2)}(0)}{\sigma}+\frac{2(1-\sigma)}{\sigma^2}\{B^{*(1)}(0)\}^2 \quad \ldots(2)$$

3. The Analysis. We now proceed to analyze M/G/1 queueing system with Bernoulli feedback under N-policy by using gererating function and supplementary variable techniques, treating the remaining service time as supplementary variable X to obta'n steady state results of the model under investigation. Let us define

$$p_{1,n}(x,t)dx=\Pr.\{Z(t)=n, x<X(t)\le x+dx\} \qquad x\ge 0,\ \ n=1,2\ldots$$

$$P_{1,n}(t)=\int_0^{\infty} P_{1,n}(x,t)dx, n=1,2,\ldots \quad \ldots(3.0)$$

The equations that are governing the model are as follows:

$$\frac{d}{dt}p_{0.0}(t) = -\lambda_0 p_{0.0}(t) + p_{1.1}(0,t) \qquad ...(3.1)$$

$$\frac{d}{dt}p_{0.n}(t) = -\lambda_0\beta_0 p_{0.n}(t) + \lambda_0\beta_0 p_{1.1}(0,t), \qquad 1 \le n \le N-1 \qquad ...(3.2)$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial x}\right)p_{1.1}(x,t) = -\lambda p_{1.1}(x,t) + p_{1.2}(0,t)h(x) \qquad ...(3.3)$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial x}\right)p_{1.n}(x,t) = -\lambda p_{1.n}(x,t) + \lambda p(x,t) + p_{1,n+1}(0,t)h(x), \qquad 2 \le n \le N-1 \qquad ...(3.4)$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial x}\right)p_{1.N}(x,t) = -\lambda p_{1.N}(x,t) + \lambda p_{1,N-1}(x,t) + \lambda p_{0,N-1}(x,t)p_{1,N+1}(0,t)h(x) \qquad ...(3.5)$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial x}\right)p_{1.n}(x,t) = -\lambda p_{1.n}(x,t) + \lambda p_{1.n-1}(x,t) + p_{1,n+1}(0,t)h(x), N+1 \le n \le K-1 \qquad ...(3.6)$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial x}\right)p_{1.K}(x,t) = -\lambda p_{1.K-1}(x,t) \,. \qquad ...(3.7)$$

For steady state, we use the following notations

$$p_{0.n} = \lim_{t\to\infty} p_{0.n}(t), \qquad n = 0,1,2,\ldots,N-1$$

$$p_{1.n} = \lim_{t\to\infty} p_{1,n}(t), \qquad n = 1,2,\ldots,K$$

$$p_{1.n}(x) = \lim_{t\to\infty} p_{1,n}(x,t), \qquad n = 1,2,\ldots,K.$$

Further let us define

$$p_{0.N-1}(x) = p_{0.N-1}h(x) \,. \qquad ...(4.0)$$

In steady state, the equations (3.1)–(3.7) reduce to the following:

$$0 = -\lambda_0 p_{0.0} + p_{1.1}(0) \qquad ...(4.1)$$

$$0 = -\lambda_0\beta_0 p_{0.n} + \lambda_0\beta_0 p_{1.1}, \qquad 1 \le n \le N-1 \qquad ...(4.2)$$

$$-\frac{d}{dx}p_{1.1}(x) = \lambda p_{1.1}(x) + p_{1.2}(0,t)h(x) \qquad ...(4.3)$$

$$-\frac{d}{dx}p_{1.n}(x) = \lambda p_{1.n}(x) + \lambda p_{1.n-1}(x) + p_{1.n+1}(0)h(x), \qquad 2 \le n \le N-1 \qquad ...(4.4)$$

$$-\frac{d}{dx}p_{1.N}(x)=-\lambda p_{1.N}(x)+\lambda p_{1.N-1}(x)+\lambda p_{0.N-1}(x)+p_{1.N+1}(0)h(x), \quad ...(4.5)$$

$$-\frac{d}{dx}p_{1.n}(x)=-\lambda p_{1.n}(x)+\lambda p_{1.n-1}(x)+p_{1.n-1}(0)h(x), \quad N+1\le n\le K-1 \quad ...(4.6)$$

$$-\frac{d}{dx}p_{1.K}(x)=\lambda p_{1.n-1}(x)\ . \quad ...(4.7)$$

Using equations (4.1)–(4.2), we obtain

$$p_{1.1}(0)=\lambda_0 p_{0.0}=\lambda_0 p_{0.n}, \qquad 1\le n\le N-1 \quad ...(5.0)$$

When we take LST of equations (4.3)-(4.7), we get the following:

$$(\lambda-\theta)p_{1.1}^{*}(\theta)=p_{1.2}(0)H^{*}(\theta)-p_{1.1}(0) \quad ...(5.1)$$

$$(\lambda-\theta)p_{1.n}^{*}(\theta)=\lambda p_{1.n-1}^{*}(\theta)+p_{1.n+1}(0)H^{*}(\theta)-p_{1.n}(0), \quad 2\le n\le N-1 \quad ...(5.2)$$

$$(\lambda-\theta)p_{1.N}^{*}(\theta)=\lambda p_{1.N-1}^{*}(\theta)+p_{1.N+1}(0)H^{*}(\theta)+\lambda p_{0.N-1}(0)H^{*}(\theta)-p_{1.N}(0), \quad ...(5.3)$$

$$(\lambda-\theta)p_{1.n}^{*}(\theta)=\lambda p_{1.n-1}^{*}(\theta)+p_{1.n+1}(0)H^{*}(\theta)-p_{1.n}(0), \qquad N+1\le n\le K-1 \quad ...(5.4)$$

$$-\theta p_{1.K}^{*}(\theta)=\lambda p_{1.K-1}^{*}(\theta)-p_{1.K}(0) \quad ...(5.5)$$

Using (5.0),we get

$$\lambda_0 p_{1.N-1}H^{*}(\theta)=p_{1.1}(0)H^{*}(\theta)\ . \quad ...(6)$$

Now using this result equation (5.3) reduces to

$$(\lambda-\theta)p_{1.N}^{*}(\theta)=\lambda p_{1.N-1}^{*}(\theta)+p_{1.N+1}(0)H^{*}(\theta)+p_{1.1}(0)H^{*}(\theta)-p_{1.N}(0) \quad ...(7)$$

Now, adding equations (5.1)–(5.5), we get

$$\sum_{n=1}^{K}p_{1.n}^{*}(\theta)=\frac{\lambda(1-\beta)}{\theta}p_{1.K-1}(\theta)+\left\{\frac{1-H^{*}(\theta)}{\theta}\right\}\sum_{n=1}^{K}p_{1.n}(0)\ . \quad ...(8)$$

Applying L'Hospital' s rule in (8), we get

$$\sum_{n=1}^{K}p_{1.n}^{*}(\theta)=\frac{b_1}{\sigma}\sum_{n=1}^{K}p_{1.n}(0) \quad ...(9)$$

where $b_1=B^{*(1)}(0)$

3.1 Probability Generating Function Technique. The equations (5.1) and (5.2) are diffcult to be solved recursively. In view of this we proceed to obtain analytic solution for $p_{1,n}, p_{1,n}^{*}(0)$ $(n=1,2...)$ in closed-form by using probability generating function technique.

Using $p_{0,0} = p_{0,n}(n = 0,1,2,...,N-1)$, we have

$$G_0(z) = \frac{1-z^N}{1-z} p_{0,0} \qquad ...(10)$$

Multiplying (5.0) and (5.5) by appropriate power of z and summing, we get the following equation

$$(\lambda-\theta-\lambda z)G_1^*(z,\theta) = \left\{\frac{H^*(\theta)}{z}-1\right\}G_1(z,0)+\lambda z^N H^*(\theta)p_{0,0} - \lambda H^*(\theta)p_{0,0} + \lambda z^K p_{1,K}^*(\theta) \qquad ...(11)$$

Now putting $\theta = \lambda - \lambda z$ in (1.1), we have

$$G_1(z,0) = \frac{\lambda(1-z^N)H^*(\lambda-\lambda z)}{\{H^*(\lambda-\lambda z)/z\}-1\}} p_{0,0} + \frac{\lambda z^K p_{1,K}^*(\lambda-\lambda z)}{\{H^*(\lambda-\lambda z)/z\}-1\}} \qquad ...(12)$$

Substituting the value of $G_1(z,0)$ from (12) into (11), we find

$$G_1^*(z,0) = \frac{\lambda(1-Z^N)}{(\lambda-\theta-\lambda z)}\left[\left\{\frac{H^*(\theta)}{z}-1\right\}\left\{\frac{zH^*(\lambda-\lambda z)}{H^*(\lambda-\lambda z)-z}\right\} - H^*(\theta)\right]p_{0,0} + \lambda z^K p_{1,K}^*(\lambda-\lambda z) \qquad ...(13)$$

By Putting $\theta = 0$ in (13), we obtain $G_1^*(z,0)$ as

$$G_1^*(z,0) = \left[\frac{(1-Z^N)H^*(\lambda-\lambda z)}{H^*(\lambda-\lambda z)-z} - \frac{(1-Z^N)}{(1-Z)}\right]p_{0,0} + \lambda z^K p_{1,K}^*(\lambda-\lambda z) = \sum_{n=1}^{\infty} z^n p_{1,n}^*(0) \qquad (14)$$

Now the probability generating function of the number of customers in the system is given by

$$G(z) = G_0(z) + G_1^*(z,0) = \left[\frac{(1-z^N)H^*(\lambda-\lambda z)}{H^*(\lambda-\lambda z)-z}\right]p_{0,0} + \lambda z^K p_{1,K}^*(\lambda-\lambda z). \qquad ...(15)$$

The probability $p_{0,0}$ can be determined using the normalizing condition

$$Np_{0,0} + \sum_{i=1}^{K} p_{1,i}^* = 1 \qquad ...(16)$$

To determine $p_{1,K}^*(0)$, we use recursive approach and obtain

$$p_{1,K}^*(0) = -\left(H^{*(1)}(0)\sum_{i=1}^{K} p_{1,i} + \sum_{i=1}^{K} p_{1,i}^*\right)$$

which gives $\quad p_{1,K}^*(0) = -\left(\sum_{i=1}^{K-1} p_{1,i}\right). \qquad ...(17)$

4. The Expressions for performance Measures. We now, propose to find the following performance characteristics in exlicit form by using generating functions tecnique as follows:

(i) Expected queue length. The expected number of coustomers in the queue is obtained by using (17) as

$$L = G'(z)\big|_{z=1}$$

$$\left[\frac{N(N-1)\sigma(\sigma-\rho)+2N\rho(\sigma-\rho)+N\lambda^2\sigma^2 H^{*(2)}(0)}{2(\sigma-\rho)^2}\right]p_{0,0}$$

$$+\lambda K(K-1)\, p_{1,K}^{*}(0)-2\lambda^2 Kp_{1,K}^{*(')}(0)+\lambda^3 Kp_{1,K}^{*('')}(0) \qquad \text{...(18)}$$

where $p_{1,K}^{*(')}(0)$ and $p_{1,K}^{*('')}(0)$ are determined by using (17)

and $H^{*(2)} = \left[\frac{B^{*(2)}(0)}{\sigma}+\frac{2(1-\sigma)}{\sigma^2}\{B^{*(1)}(0)\}^2\right]$,

where $B^{*(2)}(0)$ stands for the second moment of the service time.

(ii) Mean response time.

The mean response time is given by

$$E(R) = W + B^{*(1)}(0) = \frac{L}{\lambda} + B^{*(1)}(0) \qquad \text{...(19)}$$

where the value of L is given by (18)

(iii) Optimal N-Policy

Let $E[I], E[B]$ and $E[C]$ denote the expected length of the idle period. the busy period and busy cycle, repectively, such that $E[C] = E[I], E[B]$. As the length of the idle period is the sum of N exponetial random variables each having a mean of $1/\lambda$ so that we get

$$E[I] = \frac{N}{\lambda}.$$

The long-run fractions of time of idle and busy server are given as follows :

$$\frac{E[I]}{E[C]} = G_0(1) = Np_{0,0} \qquad \text{...(20)}$$

$$\frac{E[B]}{E[C]} = C(1,0) = \left(\frac{N\rho}{\sigma-\rho}\right)p_{0,0} + \lambda Kp_{1,K}^{*}(0) - \lambda p_{1,k}^{*(')}(0) \qquad \text{...(21)}$$

The number of busy cycles per unit time is obtained as

$$\frac{1}{E[C]} = \lambda p_{0,0} \qquad ...(22)$$

we now consider a cost function in order to get optimal value of threshold parameter N as given below:

$$E\{F(N)\} = C_h L + C_f \frac{E[I]}{E[C]} + C_0 \frac{E[B]}{E[C]} + (C_s + C_d)\frac{1}{E[C]} \qquad ...(23)$$

where, C_h = Holding cost per unit time per customer present in the system.

C_f = Cost incurred per unit time for keeping the server off.

C_o = Cost incurred per unit time for keeping the server on.

C_s = Start-up cost per unit time for turning the server on.

C_d = Shutdown cost per unit time for turning the server off.

Since $\frac{1}{E[C]}$ is independent of the decision variable N, neglecting fourth term of (23), we get a new cost function per unit time to be minimized as follows:

$$E\{C(N)\} = C_h\left\{\frac{N(N-1)}{2}\right\} + C_f N p_{0,0} + C_0 N\left(\frac{N}{\sigma-\rho}\right)p_{0,0}. \qquad ...(24)$$

To find the optimal value of threshold parameter N (say N^*), we put $\frac{dE\{C(N)\}}{dN} = 0$, yielding to

$$N^* = \frac{1}{C_h}\left[\frac{C_h}{2} - \left\{C_f + \frac{C_0\rho}{(\sigma-\rho)}\right\}p_{0,0}\right] \qquad ...(25)$$

We observe that N^* is not an integer, the best positive integeral value of N is obtained by rounding off the value of N^*.

5. Particular Cases. In this section , some special cases by taking appropriate parameter, will be obtained as follows

Case1: The *M/G/*1 Model with *N*-Policy and Bernoulli Feedback. Putting $\lambda_0 = \lambda$ and $k \to \infty$ i.e. the case is of infinite capacity system, we get results as follows:

The average numbers of customers in the system and mean response times are

$$L = \frac{N-1}{2} + \frac{\rho}{\sigma} + \frac{\lambda^2\sigma}{2(\sigma-\rho)}\left[\frac{B^{*(2)}(0)}{\sigma} + \frac{2(1-\sigma)}{\sigma^2}\{B^{*(1)}(0)\}^2\right] \qquad ...(26)$$

and $$E(R)=\frac{N-1}{2\lambda}+\frac{2b_1}{\sigma}+\frac{\lambda B^{*(2)}(0)}{2(\sigma-\rho)}+\frac{\lambda(1-\sigma)}{\sigma(\sigma-\rho)}\left[B^{*(1)}(0)\right]^2 \quad ...(27)$$

By changing the specific distributiobn for service time, the expressions for the average number of customers in the system and mean response time reduces to

$$L=\begin{cases}\frac{N-1}{2}+\frac{\rho}{\sigma}+\frac{\rho^2}{\sigma(\sigma-\rho)}, & \text{exponential}\\ \frac{N-1}{2}+\frac{\rho}{\sigma}+\frac{\rho^2(2-\sigma)}{2\sigma(\sigma-\rho)}, & \text{deterministic}\\ \frac{N-1}{2}+\frac{\rho}{\sigma}+\frac{\rho^2(3-\sigma)}{3\sigma(\sigma-\rho)}, & \text{3-stage Erlang}\end{cases} \quad ...(28)$$

$$E(R)=\begin{cases}\frac{N-1}{2\lambda}+\frac{2b_1}{\sigma}+\frac{\rho}{\mu(\sigma-\rho)}+\frac{\rho(1-\sigma)}{\sigma\mu(\sigma-\rho)}, & \text{exponential}\\ \frac{N-1}{2\lambda}+\frac{2b_1}{\sigma}+\frac{\rho}{2\mu(\sigma-\rho)}+\frac{\rho(1-\sigma)}{\sigma\mu(\sigma-\rho)}, & \text{determinisitc}\\ \frac{N-1}{2}+\frac{2b_1}{\sigma}+\frac{2\rho}{3\mu(\sigma-\rho)}+\frac{\rho(1-\sigma)}{\sigma\mu(\sigma-\rho)}, & \text{3-stage Erlang}\end{cases} \quad ...(29)$$

Case 2 : The $M/G/1$ Model with state Dependent Arrival Rate. If $H^*(\theta)\equiv B^*(\theta)$ and $N\to 1$, then our model converts to $M/G/1$ model with state dependent arrival rate which was studied by Gupta and Srinivasa Rao (1996a). Their results are derived directly by putting above variable.

Conclusion. In this study, we have considered an optimal N-policy for $M/G/1$ finite queue with Bernoulli feedback. The analytical expressions for the average number of units in the queueing system are established by using generating function and supplementary variable technique in steady state probabilities. The cost analysis is helpful for the system designer in determining the optimal value of threshold parameter so as to minmize total expected cost. The analysis of queueing system operating under N-policy and Bernoulli feedback mechanism, provides unified treatment in many real life congestion problems encountered in computer, communication systems, manufacturing, production and distribution process.

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