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(Dedicated to Honour Professor S.P. Singh on his 70^{th} Birthday)

A NOTE ON *MHD* RAYLEIGH FLOW OF A FLUID OF EQUAL KINEMATIC VISCOSITY AND MAGNETIC VISCOSITY PAST A PERFECTLY CONDUCTING POROUS PLATE

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ABSTRACT

The paper deals with the flow of a viscous incompressible fluid of small electrically conductivity past an infinite porous plate started impulsively from rest in presence of a constant transverse magnetic field in fixed relation to the fluid with the imposition of small uniform suction or injection velocity at the plate. Suction or injection velocity at the plate has been calculated using the Laplace transform method. The *MHD* unidirectional flow of a viscous incompressible fluid of small electrical conductivity near an infinite flat plate started impulsively from the rest, which was first studied by Lord Rayleigh [8], has been shown to be self-superposable and an irrotational flow on which it is superposable is determined. Some observations have been made about the vorticity and stream functions of the flow by using the properties of superposability and self-superposability. Vorticity profiles have been plotted and studied for different conditions and for suction and injection.

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1. Introduction. The folw about an infinite flat plate which executes linear harmonic oscillation parallel to itself was studied by Stokes [11] and Rayleigh [8]. The impulsive motion of an inffinite flat plate in a viscous incompressible magnetic fluid in the presence of an external magnetic field was studied by. Rossow [9]. Nath [14] studied the Rayleigh problem in slip flow with transverse magnetic field. In the present paper we have obtained the exact solution for the *MHD* flow of a fluid of equal kinematic viscosity and magnetic viscocity past a perfectly conducting porous plate by using the Laplace transform technique. Vorticity and self superposability of the above *MHD* Rayleigh flow have also been studied.

2. Formulation of the problem. Let us consider the unsteady motion of a semi-infinite mass of incompressible, viscous, perfectly conducting fluid past an infinite plate. Let x-axis be along the plate parallel to the flow direction, y-axis perpendicular to the plate and z-axis perpendicular to both the x-axis and y-axis. The imposed magnetic field $\overline{H}_0$ is applied in the direction of y-axis. Let V_s represents the suction velocity at the plate, then by equation of the continuity

$$\frac{\partial v}{\partial y} = 0 .$$

Also the condition, that at $y=0$, $v=v_s$ leads to every where. Due to motion a magnetic field H_x is introduced in the flow direction and from the symmetry of the problem all physical variables will be functions of y and time t only. Let the plate be started impulsively from rest with a constant velocity U, and subject to the conditions :

at $\quad t=0, \quad u=H_x=0, y>0$

at $\quad y=0, \quad u=U, \quad H_x=0, t>0$

as $\quad y \to \infty, u=0, \quad H_x=0$ . ...(2.1)

The differential equations governing the fluid motion are given by [13]

$$\frac{\partial u_1}{\partial t} + v_s \frac{\partial u_1}{\partial y} = \alpha \frac{\partial u_1}{\partial y} = \upsilon \frac{\partial^2 u_1}{\partial y^2} \qquad ...(2.2)$$

$$\frac{\partial u_2}{\partial t} + v_s \frac{\partial u_2}{\partial y} = -\alpha \frac{\partial u_2}{\partial y} + \upsilon \frac{\partial^2 u_2}{\partial y^2} \qquad ...(2.3)$$

where

$$u_1 = u + \aleph H_x \qquad ...(2.4)$$

$$u_2 = u - \aleph H_x \qquad ...(2.5)$$

u is the x-component of fluid velocity.

λ = Magnetic viscosity of the fluid, $\alpha = \sqrt{\mu H_0 / p}$

η = Coefficient of the viscosity of the fluid, $\aleph = \sqrt{\mu / \rho}$

μ = Permeability of the medium.

Applying Laplace Transform to equations (2.2) and (2.3) we get,

$$\frac{d^2 \bar{u}_1}{dy^2} + \frac{(\alpha - v_s)}{\upsilon} \frac{d\bar{u}_1}{dy} = \frac{p\bar{u}_1}{\upsilon} \qquad ...(2.6)$$

and

$$\frac{d^2\bar{u}_2}{dy^2} - \frac{(\alpha - \upsilon_s)}{\upsilon}\frac{d\bar{u}_2}{dy} = \frac{p\bar{u}_2}{\upsilon}, \qquad ...(2.7)$$

where $\bar{u}_1, \bar{u}_2$ are the Laplace transform of u_1 and u_2 respectively and p is the kernel of the Laplace transform. The solution of equation (2.6) and (2.7) are given by

$$\bar{u}_1 = A\exp\left\{-\frac{(\alpha-\upsilon_s)y}{2\upsilon} + \frac{y}{2}\sqrt{\left(\frac{\alpha-\upsilon_s}{\upsilon}\right)^2 + 4\frac{p}{\upsilon}}\right\} + B\exp\left\{-\frac{(\alpha-\upsilon_s)y}{2\upsilon} - \frac{y}{2}\sqrt{\left(\frac{\alpha-\upsilon_s}{\upsilon}\right)^2 + 4\frac{p}{\upsilon}}\right\}$$

or

$$\bar{u}_1 = \exp\left(-\frac{(\alpha-\upsilon_s)y}{2\upsilon}\right)\left\{A\exp\left(\frac{y}{2}\sqrt{\left(\frac{(\alpha-\upsilon_s)}{\upsilon}\right)^2 + 4\frac{p}{\upsilon}}\right) + B\exp\left\{-\frac{y}{2}\sqrt{\left(\frac{\alpha-\upsilon_s}{\upsilon}\right)^2 + 4\frac{p}{\upsilon}}\right\}\right\} \quad (2.8)$$

and

$$\bar{u}_2 = \exp\left(-\frac{(\alpha-\upsilon_s)y}{2\upsilon}\right)\left\{C\exp\left(\frac{y}{2}\sqrt{\left(\frac{(\alpha+\upsilon_s)}{\upsilon}\right)^2 + 4\frac{p}{\upsilon}}\right) + D\exp\left\{-\frac{y}{2}\sqrt{\left(\frac{\alpha+\upsilon_s}{\upsilon}\right)^2 + 4\frac{p}{\upsilon}}\right\}\right\} \quad (2.9)$$

Applying Laplace Transform to initial conditions we get,

at $\quad y = 0, t > 0 \quad \bar{u}_1 = \frac{U}{P}, \bar{u}_2 = \frac{U}{P}$

and as $\quad y \to \infty, \bar{u} = 0, \quad H_x = 0.$

We have

$A=C=0$ and $B=D=U$.

Equation (2.2) and (2.3) then become

$$\bar{u}_1 = \frac{U}{p}\exp\left\{-\frac{(\alpha-\upsilon_s)y}{2\upsilon} - \frac{y}{2}\sqrt{\left(\frac{(\alpha-\upsilon_s)}{\upsilon}\right)^2 + 4\frac{p}{\upsilon}}\right\} \qquad ...(2.10)$$

$$\bar{u}_2 = \frac{U}{p}\exp\left\{\frac{(\alpha+\upsilon_s)y}{2\upsilon} - \frac{y}{2}\sqrt{\left(\frac{(\alpha-\upsilon_s)}{\upsilon}\right)^2 + 4\frac{p}{\upsilon}}\right\} \qquad ...(2.11)$$

Taking inverse Laplace Transform [7] of equations (2.10) and (2.11) and then substituting the values of u_1 and u_2 in (2.4) and (2.5) finally we get

$$u = \frac{U}{2}\left\{erfc\left(\frac{y+(\alpha - v_s)t}{2\sqrt{vt}}\right) + erfc\left(\frac{y-(\alpha + v_s)t}{2\sqrt{vt}}\right)\right\} \qquad ...(2.12)$$

and

$$H_x = \frac{U}{2}\sqrt{\frac{\rho}{\mu}}\left\{erfc\left(\frac{y+(\alpha - v_s)t}{2\sqrt{vt}}\right) - erfc\left(\frac{y-(\alpha + v_s)t}{2\sqrt{vt}}\right)\right\} \qquad ...(2.13)$$

flow equation indicates the absence of exponential factor which means that there is no Hartmann layer in the ultimate state.

3. Flow Superpsoabile on Rayleigh Flow. Let us suppose that a flow

$$\bar{v} = (v_x, v_y, v_z) \qquad ...(3.1)$$

is superposable on the flow (2.12). Here v_x, v_y, v_z are independent of x and z i.e., these are either functions of y alone or constant.

Applying the conditions of superposability of two flows, laid down by Ballabh [2] i.e., the two flows with velocity $\bar{v}_1$ and $\bar{v}_2$ are mutually superposable to each other if

$$curl[\bar{v}_1 \times curl\bar{v}_2 + \bar{v}_2 \times curl\bar{v}_1] = 0 \qquad ...(3.2)$$

we get,

$$v_y = \frac{A}{(\partial u / \partial y)}, \qquad ...(3.3)$$

where A is constant.

If $\alpha = 0$, $v_s = 0$ i.e., when there is no suction or jnjection and magnetic field, we get

$$v_y = -A\sqrt{\pi vt}\exp(y^2/4vt) \qquad ...(3.4)$$

If we consider the motion in z–x plane only and v_z= constant, then from (3.1) we have

$$v = (0, -A\sqrt{\pi vt}\exp(y^2/4vt), v_z) \qquad ...(3.5)$$

From (3.5), we readily have

$Curl\,\bar{v} = 0.$

This means that the motion denoted by (3.5) is irrotational. Hence we can say that an irrotational flow denoted by (3.5) is superposable on the flow (2.12) under the condition $\alpha = 0, v_s = 0$.

It was shown by Ballabh [3] that an irrotational flow is superposable on a rotational one if and only if the vorticity of the latter is constant along the stream lines of the former.

The equation of the stream lines of the motion (1.12) can be deduced as

$x=$ *constant* and $z=Derf(y)$. (3.6)

Hence the vorticity of the Rayleigh flow is constant along the curve (3.6).

4. Self Superposability of the Flow. from equation (2.12) we have

$$curl[\overline{u} \times curl\overline{u}] = 0. \qquad ...(4.1)$$

This is in accordance with the condition of self-superposability laid down by Ballabh [3]. Hence the flow of viscous incompressible fluid of equal kinematics viscosity and magnetic viscosity past a perfectly conducting porous flat plate, started impulsiively from rest in the presence of transverse magnetic field is self-superposable.

It was found by Ballabh [3] that, if the axis of the symmetry in the axially symmetrical flow be x axis and the axis perpendicular to it be R-axis, The condition for self-superposability of the flow will be

$$\zeta = Rf(\Psi), \qquad ...(4.2)$$

where ζ is the vorticity of flow, $f(\Psi)$ is any function of the stream function ψ. Condition (4.2) i our case reduce to

$$\zeta = yf(\Psi). \qquad ...(4.3)$$

Since the axis perpendicular to the flow in this case has been taken as the y-axis.

Now if $\alpha = 0$ and $v_s=0$, equation (4.3) yields for the flow as

$$f(\Psi) = \frac{U}{y\sqrt{\pi v t}} \exp\left(-\frac{y^2}{4vt}\right). \qquad ...(4.4)$$

It is now evident that for the flow (2.12) under the condition $\alpha = 0$ and $v_s=0$, the right hand side of equation (4.4) is a function of y at any instant. It means that at any particular time the stream function of the flow can be denoted as

$$\Psi = \Psi(y). \qquad ...(4.5)$$

Thus the stream function ψ of the flow is a function of y and is in the direction of z axis i.e., in the direction perpendicular to the axis of flow and the direction of the magnetic field both.

5. Vorticity of flow. From equation (2.12) we get the vorticity of flow as

$$\zeta = \frac{U}{2\sqrt{\pi v t}}\left\{\exp\left(-\left(\frac{y+(\alpha - v_s)t}{2\sqrt{vt}}\right)^2\right) + \exp\left(-\left(\frac{y+(\alpha + v_s)t}{2\sqrt{vt}}\right)^2\right)\right\} \qquad ...(5.1)$$

Let the motion is such that

$v_s = v$ and $\alpha = 2v_s = 2v$

then,

$$\zeta = \frac{U}{2\sqrt{\pi v t}}\left\{\exp\left(-\left(\frac{y}{2\sqrt{vt}}+\frac{1}{2}\sqrt{vt}\right)^2\right)+\exp\left(-\left(\frac{y}{2\sqrt{vt}}+\frac{3}{2}\sqrt{vt}\right)^2\right)\right\} \qquad ...(5.2)$$

6. For Injection or Suction.

Case I. When $t=1/v$ we have

$$\frac{\zeta}{U} = \frac{U}{2\sqrt{\pi}}\left\{\exp\left(-\frac{(y+1)^2}{4}\right)+\exp\left(-\frac{(y+3)^2}{4}\right)\right\}. \qquad ...(6.1)$$

Case II . When $t=2/v$, we have

$$\frac{\zeta}{U} = \frac{U}{2\sqrt{2\pi}}\left\{\exp\left(-\frac{(y+2)^2}{8}\right)+\exp\left(-\frac{(y+6)^2}{8}\right)\right\}. \qquad ...(6.2)$$

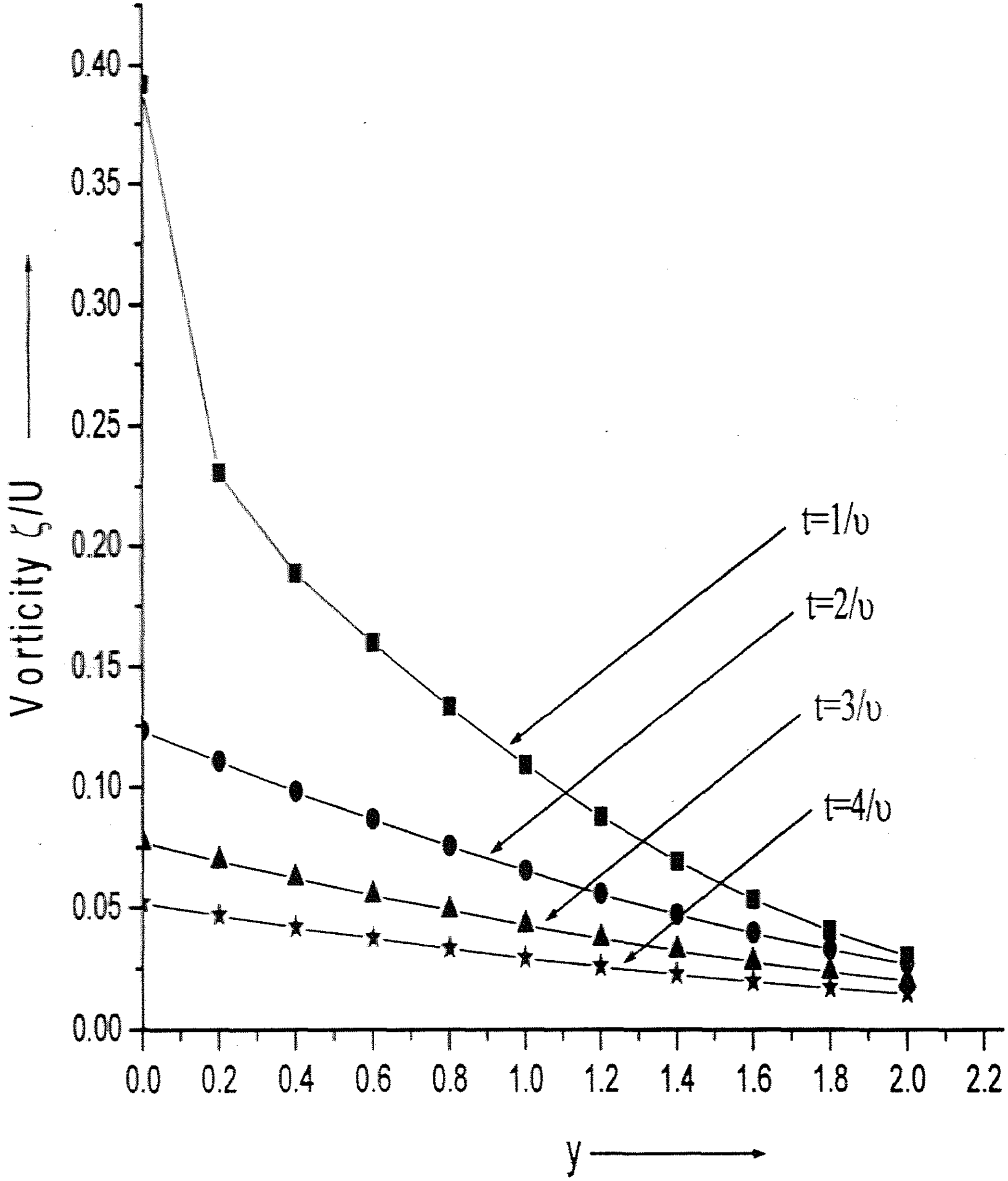
Case III . When $t=3/v$, we have

$$\frac{\zeta}{U} = \frac{U}{2\sqrt{3\pi}}\left\{\exp\left(-\frac{(y+3)^2}{12}\right)+\exp\left(-\frac{(y+9)^2}{12}\right)\right\}. \qquad ...(6.3)$$

Case IV . When $t= 4/v$, we have

$$\frac{\zeta}{U} = \frac{U}{4\sqrt{\pi}}\left\{\exp\left(-\frac{(y+4)^2}{16}\right)+\exp\left(-\frac{(y+12)^2}{12}\right)\right\}. \qquad ...(6.4)$$

7. Results and Discussion. To observe the quantitative effects on vorticity field numerical results have been calculated and plotted for above four cases. It is clear from the graph that the vorticity is maximum at the plate and it decreases as we move away from plate. At small times the vorticity near the plate falls abruptly and then it decreases and become steadier as we move far from the plate. As time increases the fall in vorticity becomes less sharp in comparison to that $t=1/v$. Thus as time increases the vorticity tends to become zero throughout. Thus after large time we may expect an almost irrotational flow.



Vorticity Profile for Suction or Injection

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