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STEADY STATE AVAILABILITY AND BAYESIAN ESTIMATION OF UNKNOWN PARAMETERS FOR TWO UNIT SYSTEM WITH COMMON CAUSE FAILURES

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ABSTRACT

A two-unit system in series and parallel with individual failures as well as common cause skock(*CCS*) failures has been studied. The steady state availability for the Markov model is explored to find the impact of *CCS* failures on the steady state availability with the aid inverse Laplace differential equations governing the model. The Bayesian analysis for special distributions namely exponential, Gamma, Beta and Weibull; has been done for estimating the parameters for establishing the results. To examine the effect of common cause failure, numerical results are given. The sensitively analysis has also been carried out to explore the steady state availability with respect to different parameters.

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Keywords : System availability, Common Cause failure, Two-unit system Bayesian analysis, Parameter extimation.

1. Introduction. The effect of increased availability/reliability due to repair, on two-unit system has been analysed by various research workers. A repairable two-unit parallel redundant system with dependent failures has been analysed by Jack (1986). Prasad and Rao (1989) considered the probabilistic analysis of a two-unit system with dependent components. Tuteja et al. (1991) considered an analysis of two-unit system with partial failures and three types of repairs. The profit evaluation in two unit cold standby system having two types of independent repair facilities was made by Singh et al. (1992). Agrafiotis and Tsoukalas (1994) studied the reliability analysis and optimization applications of a two unit standby redundant system with spares support.

The reliability analysis of two-unit system with common cause shock failures was considered by Jain (1998). Tuteja et al. (2001) studied the reliability and profit analysis of a two-unit cold standby system with partial failure and two types of repairmen. Joorel et al. and Yadavalli et al. (2002) made the reliability analysis of two-unit priority redundant system with multi repair facilities and steady state availability with setup time. The reliability and profit analysis of a two-unit cold standby system with rest period and various types of repair was considered Gupta and Taneja (2003). Castanier et al. (2005) studied a condition based maintenance policy with non-periodic inspections for a two unit series system.

Steady state availability analysis of repairable system is generally performed using stochastic process, including Markovian approach. It is realized that the steady state availabiity of a system decreases as the chance of CCS failures increases. When components of a system fail, they do not necessary fail independently of each other. In many multi-component systems, failures may be synchronized, and as such have a common cause. Apostolakis (1976) studied the effect of a certain class of potential common mode failures on the reliability of redundant system. The availability and frequency of failures of system in the presence of chance common cause failures was studied by Verma and Chari (1991). Dhillon and Anude (1994) and Dhillon-Viswanath (1994) considered the stochastic analysis for common cause failure of a redundant system with repairable units. An estimation techniques for common cause failure data was made by Kvam (1996). The stochastic analysis for an active standby redundant network with two types of common cause failures was made by Dhillon and Yang (1997). The reliability of *k-r-out-of-N: G* system subject to random and common cause failure was investigated by Jain and Ghimire (1997). Kvam and Miller (2002) investigated the common cause failure predication using data mapping.

The powerful technique of the Bayesian analysis developed originally by Kalman in the field of control engineering has recently found extensive applications in the field of economic and business forecasting. It has received attention of the workers in the area of reliability analysis too. The Bayes estimation of the parameter and reliability function of the three-parameter Weibull distribution was studied by Sinha and Sloan (1988). The generalized Gamma function occurring in differaction theory was considered by Kobayshi (1991). Box and Tiao (1992) studied the Bayesian inference in statistical analysis.

The Bayesian group replacement policies were considered by Willson and Benmerzouga (1995). Mazzuchi and Soyer (1996) investigated Bayesian perspective on some replacement strategies. A generalized Gamma distribution and its application in reliability prediction was considered by Agarwal and Kalla (1996). The reliability estimations methods under various probability distribution functions were considered by Pan and Chen (1997). Sheu et al. (1999) analysed Bayesian perspective for age replacement with minimal repairs. The Bayesian approach to an adaptive preventive maintenance model was investigated by Sheu et al. (2001). A Bayesian method on adaptive preventive maintenance problem was investigated by Juang and Anderson (2004). Yadavalli et al. (2005) considered the Bayesian study of a two-component system with common cause shock failures. A Bayesian paired approach for relative accident probability assessment with covariate information was studied by Szwed et al. (2006). Kottas (2006) investigated the nonparameteric Bayesian survival analysis using mixtures of Weibull distributions.

The present investigation deals with the influence of *CCS* failures on series and parallel configurations of two-units system. The availability expressions for both transient and steady state in case of *CCS* failure as well as individual failures are obtained. A discrete state, Markov chain is used to constitute a set of differential difference equations for transient probabilities governing the model. The rest of the paper is organized as follows. Section 2 deals with model description by stating the requisite assumptions and notations. Section 3 deals with governing equations and Mathematical analysis. The Bayesian analysis is introduced in section 4. In section 5 numerical results are given. Finally section 6 is devoted to conclusion and further research directions related to our study.

2. Model Description. A Markov model to derive steady state availability of series and parallel configuration of the two-units system under the influences of *CCS* failure is developed. The following assumptions are made to formulate the model :

* The system has two statistically independent and identical components. The system is affected by both individual and *CCS* Failures. The time between invididual failures and between shocks failures follows an exponential distribution with paremeters λ, (λ_c) respectively.
* The individual failures and shock failures occur independent of each other. The repair times of failed components follow an exponential distribution with paremeter μ.
* The system must be in any one of the three states.
  State 0, State 1 and State 2; where state 0,1,2 correspond to system with both operating units, state with one operating unit whereas the other one is in failed state, both units in failed state, respectively.
* Let $P_i(t)$ be the probability of system being in $i^{th}(i=0,1,2)$ state at time t.
* Define the failure rates λ_i, $(i=0,1,2)$ in i^{th} state as follows :

$$\lambda_0 = 2\lambda C_1; \lambda_1 = \lambda C_1; \lambda_c = \lambda_c C_2;$$

  where C_1 and C_2 being the probabilities of individual and common cause failures of the components respectively.
* The repair rate in i^{th} $(i=1,2)$ state is given by $\mu_1 = \mu; \mu_2 = 2\mu; \mu_c = \mu_c$

The diagram for depiction of the transition flow in state space $E=\{0,1,2\}$ is shown in figure 1.

Governing Equations and Analysis. The set of differential difference equations governing the model based on transition diagram is given by

$$P'_0(t)=-(\lambda_0+\lambda_c)P_0(t)+\mu_1 P_1(t)+\mu_c P_2(t) \quad \text{... (1)}$$

$$P'_1(t)=\lambda_0 P_0(t)-(\lambda_1+\mu_1)P_1(t)+\mu_2 P_2(t) \quad \text{... (2)}$$

$$P'_2(t)=-(\mu_2+\mu_c)P_2(t)+\lambda_1 P_1(t)+\lambda_c P_0(t). \quad \text{... (3)}$$

Using Laplace transformation, the set of equations (1)-(3) with initial conditions, $P_0(0)=1, P_i(0)=0, i\neq 0$ can be solved. The Laplace transforms of (1)-(3) yield

$$(s+\lambda_0+\lambda_c)\widetilde{P}_0(s)-\mu_1\widetilde{P}_1(s)-\mu_c\widetilde{P}_2(s)=1 \quad \text{...(4)}$$

$$-\lambda_0\widetilde{P}_0(s)+(s+\mu_1+\lambda_1)\widetilde{P}_1(s)-\mu_2\widetilde{P}_2(s)=0 \quad \text{...(5)}$$

$$-\lambda_c\widetilde{P}_0(s)-\lambda_1\widetilde{P}_1(s)+(s+\mu_2+\mu_c)\widetilde{P}_2(s)=0\,. \quad \text{...(6)}$$

Now equations (4)-(6) can be written in matrix form as

$$\text{A(s)P(s)=I}\,, \quad \text{...(7)}$$

where

$$A(s)=\begin{bmatrix}(s+\lambda_0+\lambda_c) & -\mu_1 & -\mu_c\\ -\lambda_0 & (s+\mu_1+\lambda_1) & -\mu_2\\ -\lambda_c & -\lambda_1 & (s+\mu_2+\mu_c)\end{bmatrix}$$

$$\widetilde{P}(s)=\left[\widetilde{P}_0(s),\widetilde{P}_1(s),\widetilde{P}_2(s)\right]$$

$$I=[1,0,0]^T\,.$$

Eq. (7) is also known to have a unique solution. By the method of determinants i.e. Cramer's rule, we obtain

$$\widetilde{P}_{n-1}(s)=\frac{\det A_n(s)}{\det A(s)}, n=1,2,3.$$

where $A_n(s)$ is the matrix obtained from $A(s)$ by replacing the n^{th} (n=1,2,3) column of $A(s)$ by the vector I. By inspection of the matrix $A(s)$, it is not difficult to see that determinant of the matrix $A(s)$ must be a polynomial in s of degree 3 with leading coefficient 1, and so we can write

$$\det A(s)=s\prod_{i=1}^{2}(s-s_i)\,,$$

where each s_i is a root of the polynomial. It can be easily verified that these roots must be distinct. By algebraic manipulations, we obtain

$$P_0(t)=[\mu_1\mu_2+\mu_1\mu_c+\mu_c\lambda_1/s_1s_2]+[a_1\exp(s_1t)-a_2\exp(s_2t)]/(s_1-s_2) \quad ...(9)$$

$$P_1(t)=[\lambda_0\mu_2+\mu_c\lambda_0+\mu_2\lambda_c/s_1s_2]+[b_1\exp(s_1t)-b_2\exp(s_2t)]/(s_1-s_2) \quad ...(10)$$

$$P_2(t)=[\lambda_c\mu_1+\lambda_c\lambda_1+\lambda_0\lambda_1/s_1s_2]+[c_1\exp(s_1t)-c_2\exp(s_2t)]/(s_1-s_2) \quad ...(11)$$

where

$$s_1,s_2=\left(-k\pm\sqrt{(k^2-4m)}/2\right)$$

$$k=(\mu_1+\mu_2+\mu_c+\lambda_0+\lambda_1+\lambda_c)$$

$$m=(\mu_1\mu_2+\mu_2\lambda_0+\mu_1\mu_c+\mu_c\lambda_1+\mu_c\lambda_0+\mu_1\lambda_c+\lambda_1\lambda_c+\lambda_1\lambda_0+\mu_2\lambda_c)$$

$$a_1=\left[s_1^2+s_1(\mu_1+\mu_2+\mu_c+\lambda_1)+\mu_1\mu_2+\mu_1\mu_c+\mu_c\lambda_1\right]/s_1$$

$$a_2=\left[s_2^2+s_2(\mu_1+\mu_2+\mu_c+\lambda_1)+\mu_1\mu_2+\mu_1\mu_c+\mu_c\lambda_1\right]/s_2$$

$$b_1=[s_1\lambda_0+\lambda_0\mu_2+\lambda_0\mu_c+\lambda_c\mu_2]/s_1$$

$$b_2=[s_2\lambda_0+\lambda_0\mu_2+\lambda_0\mu_c+\lambda_c\mu_2]/s_2$$

$$c_1=[s_1\lambda_c+\lambda_c\mu_1+\lambda_1\lambda_c+\lambda_0\lambda_1]/s_1$$

$$c_2=[s_2\lambda_c+\lambda_c\mu_1+\lambda_1\lambda_c+\lambda_0\lambda_1]/s_2\,.$$

Steady-state availability .We now derive the steady-state availability for both configurations in the case of CCS failures and individual failures.

(a) Series configuration. As time becomes very large, the teady-state availability of the system can be obtained as

$$A_s(\infty)=Lim_{t\to\infty}A_s(t)$$

$$=Lim_{s\to 0}\left[s\widetilde{P}_0(t)\right]$$

$$=\frac{(2\mu^2+\mu\mu_c)}{[2\mu^2+4\mu\lambda C_1+3\mu\lambda_c C_2+2\lambda\mu_c C_1+\mu\mu_c]} \quad ...(12)$$

(b) Parallel configuration. After a long run usage of the system, the steady-state availability of the parallel configuration is obtained as

$$A_P(\infty)=Lim_{t\to\infty}A_P(t)$$

$$=Lim_{s\to 0}\left[s\widetilde{P}_0(t)+s\widetilde{P}_1(t)\right]$$

$$=\frac{(2\mu^2+\mu\mu_c+4\mu\lambda C_1+2\lambda\mu_c C_1+2\mu\lambda_c C_2)}{[2\mu^2+4\mu\lambda C_1+3\mu\lambda_c C_2+3\lambda\mu_c C_1+\mu\mu_c+\lambda\lambda_c C_1C_2+2\lambda^2C_1^2]} \quad ...(13)$$

4. Bayesian Analysis. In this section the unknown parameters of the models are estimated using Bayesian analysis for exponential distribution, two parameter Gamma distribution and Beta distribution of second kind and Weibull distribution.

(a) Exponnetial distribution. Let $(G_{11},G_{12},...,G_{1}n_{1})$ and $(G_{21},G_{22},...,G_{2}n_{2})$ be random samples of individual failures and common cause shocks failures with samples size n_1 and n_2, respectively. Let $(G_{31},G_{32},...,G_{3}n_{3})$ and $(G_{41},G_{42},...,G_{4}n_{4})$ be the random samples of repair time of components failed individually and due to common cause with samples size n_3 and n_4 respectively. All these samples are drawn from exponential populations. The life time and the repairs times are independently distributed random variables. Then

$L(x)=x\exp(-x\tau)$; where $x=\lambda,\lambda_c,\mu,\mu_c$ and $\tau=\tau_i$.

We assume than $n_i=n;\ i=1,2,3,4$. The likelyhood function is given by

$$L(\lambda,\lambda_c,\mu,\mu_c/\tau_1,\tau_2,\tau_3,\tau_4)=(\lambda\lambda_c\mu\mu_c)^n\exp(-\lambda\tau_1+\lambda_c\tau_2+\mu\tau_3+\mu_c\tau_4)$$

where $\tau_i=\sum_{j=1}^{n}G_{ij};i=1,2,3,4$ is sufficient for $(\lambda,\lambda_c,\mu,\mu_c)$. ...(14)

(b) Two-parameter gamma distribution. If the analysis possesses more detailed information about $\lambda,\lambda_c,\mu,\mu_c$ the prior mean value ω_1 and variance σ_1 through a gamma prior distribution $G(\upsilon_1,\gamma_1)$ for $\lambda,\lambda_c,\mu,\mu_c$ with probability density function,

$$g(x)=\frac{x(x\gamma_1)^{\upsilon-1}\exp(-x\gamma_1)}{\Gamma(\upsilon)};\ \text{where } x=\lambda,\lambda_c,\mu,\mu_c \qquad ...(15)$$

$$g(\lambda)=\frac{\gamma_1(\lambda\gamma_1)^{\upsilon_1-1}\exp(-\lambda\gamma_1)}{\Gamma(\upsilon_1)} \qquad ...(16)$$

$$g(\lambda_c)=\frac{\gamma_2(\lambda_c\gamma_2)^{\upsilon_2-1}\exp(-\lambda_c\gamma_2)}{\Gamma(\upsilon_2)} \qquad ...(17)$$

$$g(\mu)=\frac{\gamma_3(\mu\gamma_3)^{\upsilon_3-1}exp(-\mu\gamma_3)}{\Gamma(\upsilon_3)} \qquad ...(18)$$

$$g(\mu_c)=\frac{\gamma_4(\mu_c\gamma_4)^{\upsilon_4-1}\exp(-\lambda_c\gamma_4)}{\Gamma(\upsilon_4)} \qquad ...(19)$$

The prior mean ω_1 and variance σ_1^2 are given by

$$\omega_1 = \upsilon_1/\gamma_1 ; \sigma_1^2 = \upsilon_1/\gamma_1^2$$

so that the prior information can be easily converted into suitable values of the prior parameters

$$\upsilon_1 = \omega_1^2/\sigma_1^2 ; \gamma_1 = \omega_1/\sigma_1^2$$

The joint posterior distribution according to Bayes theorem, using eqs. (16)-(19) is

$$g(\lambda,\lambda_c,\mu,\mu_c/\tau_1,\tau_2,\tau_3,\tau_4) \propto \lambda^{n+\upsilon_1-1}\lambda_c^{\ n+\upsilon_2-1}\mu^{n+\upsilon_3-1}\mu_c^{\ n+\upsilon_4-1}$$

$$\times exp[-\lambda(\tau_1+\gamma_1)+\lambda_c(\tau_2+\gamma_2)+\mu(\tau_3+\gamma_3)+\mu_c(\tau_4+\gamma_4)] \quad ...(20)$$

(c) Beta-distribution of the second order. The *pdf* of Beta-distribution of the second kind $B(m,r)$ for $\lambda,\lambda_c,\mu,\mu_c$ is

$$g(x) = \frac{x^{m-1}}{B(M,r)(1+x)^{m+r}} ; x,m,r > 0 \quad ...(21)$$

where $x = \lambda,\lambda_c,\mu,\mu_c$, Thus

$$g(\lambda) = \frac{\lambda^{m_1-1}}{B(m_1,r_1)(1+\lambda)^{m_1+r_1}} ; \lambda,m_1,r_1 > 0 \quad ...(22)$$

$$g(\lambda_c) = \frac{\lambda_c^{\ m_2-1}}{B(m_2,r_2)(1+\lambda_c)^{m_2+r_2}} ; \lambda_c,m_2,r_2 > 0 \quad ...(23)$$

$$g(\mu) = \frac{\mu^{m_3-1}}{B(m_3,r_3)(1+\mu)^{m_3+r_3}} ; \mu,m_3,r_3 > 0 \quad ...(24)$$

$$g(\mu_c) = \frac{\mu_c^{m_1-1}}{B(m_4,r_4)(1+\mu_c)^{m_4+r_4}} ; \mu_c,m_4,r_4 > 0 \quad ...(25)$$

The suitable values of the hyper parameters are

$$m_i = \omega_i(\omega_i+\omega_i^2+\sigma_i^2)/\sigma_i^2 ; r_j = (\omega_j+\omega_j^2+2\sigma_j^2)/\sigma_j^2 ; i,j = 1,2,3,4 \quad ...(26)$$

where w_i, σ_i^2 are the prior mean and variance, respectively. Assuming independence among the parameters $\lambda,\lambda_c,\mu,\mu_c$ the joint posterior distribution is given by

$$g(\lambda,\lambda_c,\mu,\mu_c/\tau_1,\tau_2,\tau_3,\tau_4)\propto \lambda^{n+m_1-1}\lambda_c^{n+m_2-1}\mu^{n+m_3-1}\mu_c^{n+m_4-1}$$

$$\times\frac{exp[-\{\lambda\tau_1+\lambda_c\tau_2+\mu\tau_3+\mu_c\tau_4\}]}{(1+\lambda)^{m_1+r_1}(1+\lambda_c)^{m_2+r_2}(1+\mu)^{m_3+r_3}(1+\mu_c)^{m_4+r_4}} \quad ...(27)$$

(d) Weibull distribution. At present, it is perhaps the most widely used parameteric family of failure distributions by a proper choice of its shape parameter α. The *pdf* of Weibull distribution is given by

$f(x)=\lambda\alpha x^{\alpha-1}exp(-\lambda x^{\alpha})$, where α is shape parameter. ...(28)

Thus

$$f(\lambda_c/\tau_1)=\alpha\lambda\lambda_c^{n+\alpha-1}exp[-\{\lambda\lambda^{\alpha}+\lambda_c\tau_1\}] \quad ...(29)$$

$$f(\lambda_c/\tau_2)=\alpha\lambda\lambda_c^{n+\alpha-1}exp[-\{\lambda\lambda_c^{\alpha}+\lambda_c\tau_2\}] \quad ...(30)$$

$$f(\mu/\tau_3)=\alpha\lambda\mu^{n+\alpha-1}exp[-\{\lambda\mu^{\alpha}+\mu\tau_3\}] \quad ...(31)$$

$$f(\mu_c/\tau_4)=\alpha\lambda\mu_c^{n+\alpha-1}exp[-\{\lambda\mu_c^{\alpha}+\mu_c\tau_4\}] \quad ...(32)$$

The joint posterior distribution by using Baye's theorem is

$$f(\lambda\lambda_c\mu\mu_c/\tau_1\tau_2\tau_3\tau_4)=(\alpha\lambda)^4(\lambda\lambda_c\mu\mu_c)^{n+\alpha-1}$$

$$\times exp[-\{(\lambda\lambda^{\alpha}+\lambda\tau_1)+(\lambda\lambda_c^{\alpha}+\lambda_c\tau_2)+(\lambda\mu^{\alpha}+\mu\tau_3)+(\lambda\mu_c^{\alpha}+\mu_c\tau_4)\}]$$

$$\propto(\lambda\lambda_c\mu\mu_c)^{n+\alpha-1}$$

$$\times exp[-\{(\lambda\lambda^{\alpha}+\lambda\tau_1)+(\lambda\lambda_c^{\alpha}+\lambda_c\tau_2)+(\lambda\mu^{\alpha}+\mu\tau_3)+(\lambda\mu_c^{\alpha}+\mu_c\tau_4)\}] \quad ...(33)$$

5. Numerical results. The numerical results for system performance indices are obtained by developing program in software MALTLAB. The graphical presentation for steady state availability in series and parallel system has been done in figures 2-7. Figures 2 and 3 display availability of series and parallel systems with respect to λ where various parameters are fixed as $\mu_c=1,\ \mu=2,\ \lambda_c=0.9,\ C_2=0.05$. It is observed in figure 2 for series system that availability decreases in the beginning but the rate of decrease show down as λ increases. In figure 3, the availability of parallel system against λ exhibited. The effect of availability is quite significant. The availability with respect to λ decreases sharply in almost linear way. It is noted that the availability is highest for lower values of C_1 for both series and parallel systems.

Figures 4 and 5 are for availability of series and parallel systems, respectively with respect λ_c for different value of C_1 where other parameters are fixed as μ_c=1, μ=2, λ=0.1, C_2=0.03. The availability decreases with the increases in λ_c; for increasing value of C_1 there is a decrement in availability. In figures 6 and 7, the parameters are fixed as μ_c=2, λ=0.01, λ_c=0.5, C_2=0.03. The availability increases sharply with initially up to μ=3.5 but this increasing trend becomes slow then after. The increasing value of C_1 reduces the availability but effect is not much significant. We find that on increasing the value C_1, the availability increases up to μ=3, the effect is not much significance as μ grows. We infer that the availability can be improved to a certain extent by increasing μ.

6. Conclusions. The system availability of a two components in series and parallel systems with common cause failures has been dealt by developing a Markov model. By employing Bayesian analysis, the estimation of unknown parameters has been done. The model developed can be applied for many practical systems, since it takes care of the situations of common cause failure and simultaneous repair. Estimation method for different populations having different parameters has been developed which can be helpful in determining the system descriptors of many real time systems.

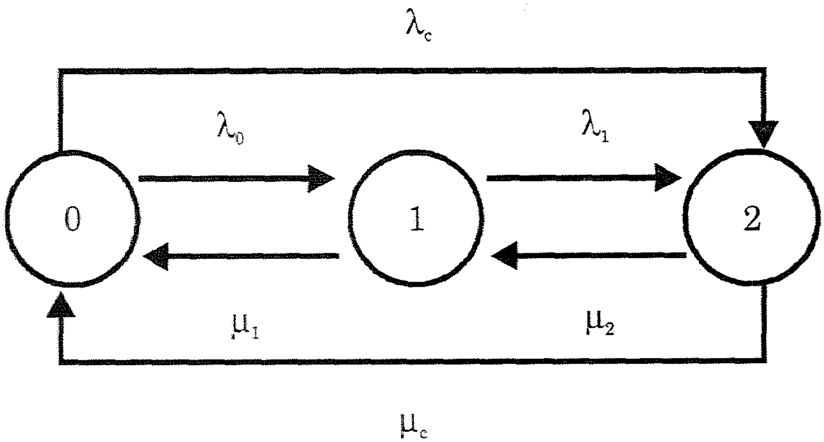

Fig.1 : Transition diagram for two-unit system with individual and *CCS* failures

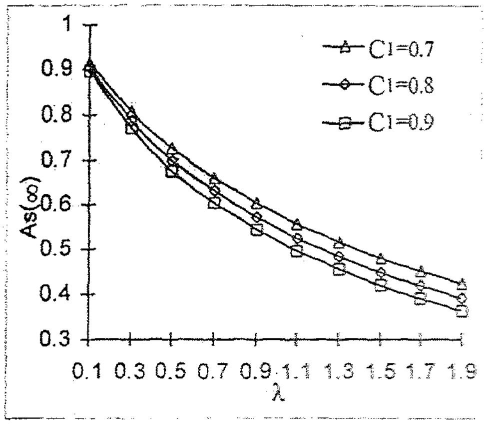

Fig. 2: $A_s(\infty)$ vs. λ for different value of C_1
($\mu = 2$, $\mu c = 1$, $\lambda c = 0.9$, $C_2 = 0.05$)

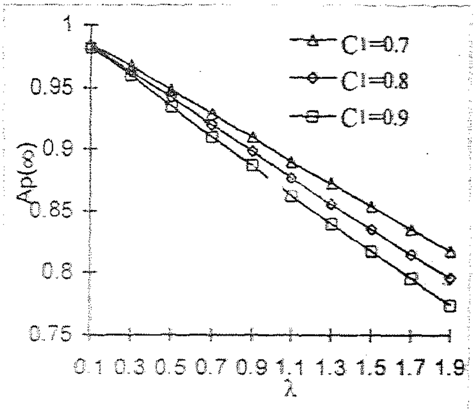

Fig. 3: $A_p(\infty)$ vs. λ for different value of C_1
($\mu = 2$, $\mu c = 1$, $\lambda c = 0.9$, $C_2 = 0.05$)

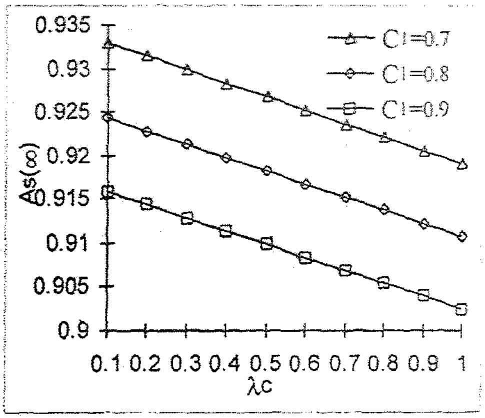

Fig. 4: $A_s(\infty)$ vs. λ_c for different value of C_1
($\mu = 2$, $\mu c = 1$, $\lambda = 0.1$, $C_2 = 0.03$)

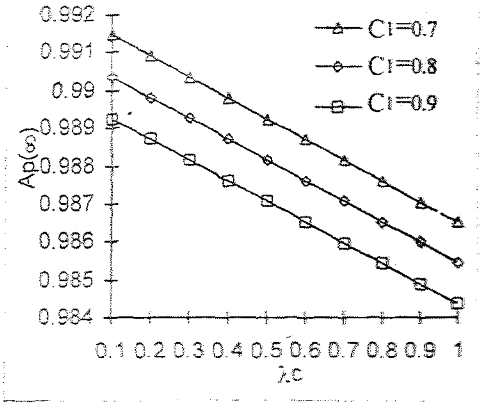

Fig. 5: $A_p(\infty)$ vs. λ_c for different value of C_1
($\mu = 2$, $\mu c = 1$, $\lambda = 0.1$, $C_2 = 0.03$)

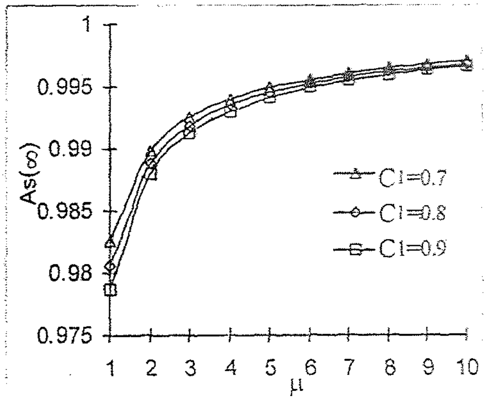

Fig. 6: $A_s(\infty)$ vs. μ for different value of C_1
($\lambda = 0.01$, $\lambda c = 0.5$, $\mu c = 2$, $C_2 = 0.03$)

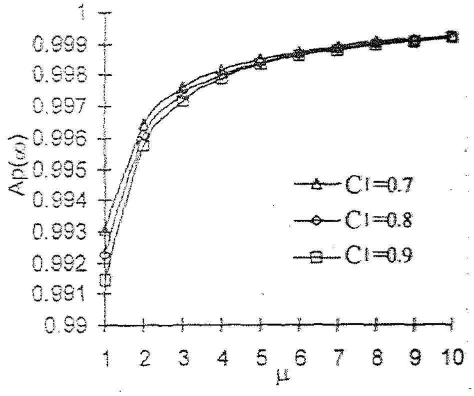

Fig. 7: $A_p(\infty)$ vs. μ for different value of C_1
($\lambda = 0.01$, $\lambda c = 0.5$, $\mu c = 2$, $C_2 = 0.03$)

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