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THEORETICAL ANALYSIS OF STEDY BLOOD FLOW ALONG AN INCLINED PLANE INFLUENCED BY THE GRAVITY FORCE

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ABSTRACT

In this paper, we investigate some charcteristics of steady, one-dimensional laminar flow of blood along an inclined plane under action of gravity. A non-Newtonian Herschel Bulkley fluid model of blood has been considered. Analytial expressions for velocity and flow rate have been obtained. The velocity profile for various values of non-Newtonian parameter n, yield stress β and inclination angle have been discussed graphically. Flow rate is also discussed with respect to different parameters. Results are discussed for Newtonian fluid character of blood.

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Keywords : Newtonian fluid, Non-Newtonian Fluid, Non-Newtonican Harschel Bulkley fluid model of blood.

1. Introduction. Blood, from fluid mechanical point of view, can be considered as a neutrally buoyant suspension of erythrocytes in a Newtonian liquid called plasma. The flow of blood through small diameter tube (20-500 μm) is of physiological and clinical importance. A special property of blood is that it exhibits a yields stress. The behaviour of blood flow is principally due to the suspension of cells in an aqueous solution of organic and inorganic substances called plasma. Because of the presence of the suspended particles in plasma, there have been a number of attempts to explain the anomalous behaviour of blood by proposing different theoretical models, e.g. Newtonian and non-Newtonian.

The experimental studies on blood flow (Bugliarello and Sevila (1) and Cokelet [2]) indicate that under certain flow conditions, blood flow may deviate significantly from the Newtonian behaviour.

Whitemore [3], Han and Barnett [4] have been reported that the suspension of red cells in serum, the behaviour of which is similar to the whole blood for shear rates more than 1Sec^{-1}, obey power-law model.

Chaturani and Upadhya [5] have been studied the gravity flow with couple stress along an inclined plane. Verma [6] give a theoretical model to study of power law fluid flow along an inclined plane influenced by the gravity force with application to blood flow.

In our analysis, we have considered the gravity flow of blood through narrow tube satisfying Herschel-Bulkley constitutive equation of motion along an inclined plane. Velocity field, flow rate and pressure distribution are obtained. Results are dicussed for different values of n and inclination angle.

2. Mathematical Formulation and Analysis. Let us consider the steady, one dimensional laminar flow of blood along on inclined plane at an angle θ with horizontal as shown in Fig. 1. It is assumed that blood is incompressible and characterized by the Herschel-Bulkley fluid model. It is also assumed that there is no pressure gradient in x-direction and thickness of blood layer 'a' is uniform throughout the plate. Also assume that the gas phase does not give rise to any shear stress at the free surface i.e. velocity gradient is zero at $y=a$ and only body force is gravity.

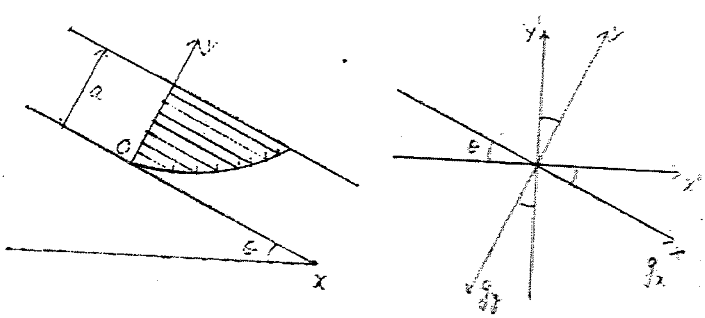

Fig-1

The constitutive equation for incompressible Herschel-Bulkley fluid is given by

$$\tau = \mu\gamma^n + \tau_0 \tag{1}$$

where τ is shear stress, μ is the coefficient of viscosity, γ is velocity gradient, n is non-Newtonian parameter and τ_0 is yield stress.

For steady one dimensional gravity flow velocity field is given by:

$$s\, V_x = V(y),\ V_y = V_z = 0 \tag{2}$$

and

$$\frac{\partial p}{\partial x} = 0. \tag{3}$$

Equation of motion under above assumptions along the x-direction are :

$$\frac{\partial}{\partial y}\left(\tau_{y,x}\right)+\rho g_x=0 \tag{4'}$$

$$-\frac{\partial p}{\partial y}+\rho g_y=0 \tag{5'}$$

$$-\frac{\partial p}{\partial z}=0 \tag{6'}$$

Due to gravity $g=-g\,\hat{j}$, which can be resolved into x and y components as :

$gx=g\ sin\theta$ and $gy=-gCos\theta$. Therefore above equations (4'), (5') and (6') become

$$\frac{\partial}{\partial y}\left(\tau_{yx}\right)+\rho g\, sin\theta=0\ , \tag{4}$$

$$-\frac{\partial p}{\partial y}-\rho g\, cos\theta=0\ , \tag{5}$$

$$-\frac{\partial p}{\partial z}=0\ . \tag{6}$$

where g is acceleration due to gravity, ρ is the density of blood.
The boundary conditions are:

at $y=0$, $V=0$, (7)

at $y=a$, $p=p_0$, $\frac{dV}{dy}=0,$ $\tau_0=0$ (8)

The solution of equation (5) with boundary condition (8) is given by

$$p=p_0+\rho g(a-y)cos\theta \tag{9}$$

where p_0 is atmospheric pressure and p is the absolute pressure distribution in y-direction.

Using equation (1) in (4), we get

$$\frac{d}{dy}\left(\mu\gamma^n+\tau_0\right)+\rho g\, sin\theta=0 \tag{10}$$

The solution of (10) with boundary conditions (7) and (8) is

$$V=\frac{n}{n+1}\left(\frac{\rho g\, sin\theta}{\mu}\right)^{\frac{1}{n}}a^{\frac{(n+1)}{n}}\left[\left(1-\beta cosec\theta\right)^{\frac{(n+1)}{n}}-\left(1-\delta-\beta cosec\,\theta\right)^{\frac{(n+1)}{n}}\right] \tag{11}$$

where non-dimensional yields stress parameter $\beta = \tau_0/(\rho g a)$ and $\delta = y/a$ the flow rate is defined as

$$Q = \int_0^a V dy . \tag{12}$$

Using equation (11) in (12), we obtain

$$Q = \frac{n}{n+1}\left(\frac{\rho g \sin\theta}{\mu}\right)^{\frac{1}{n}} a^{\frac{2n+1}{n}}\left[(1-\beta cose\theta)^{\frac{n+1}{n}} + \frac{n}{2n+1}(-\beta cosec\theta)^{\frac{2n+1}{n}} - (1-\beta cosec\theta)^{\frac{2n+1}{n}}\right]$$

3. Discussion. It is of interest to note that viscosity of normal blood have calculated from the relation given by Gupta (7).

$$\mu = \mu_p\left(1 + 0.025H + 7.35\times10^{-4}H^2\right)$$

where μ_p is the viscosity of plasma, H is the hematocrit of the blood. The plasma viscosity $\mu_p = 0.012 = 1.2\times10^{-3}$ NS/m^2 (Merrill (8)]. We have calculated the viscosity of normal blood μ=0.038 poise=3.8×10^{-3} NS/m^2 for 40% hematocrit. It is also considered that the distance between planes a=200 μm=2×10^{-4} metre for microcirculatory system, density of blood ρ=1060 Kg/m^3, g=9.81m/sec^2 and τ_0=0.005 N/m^2. We have calculated the non-dimensional yield stress parameter β=0.0015.

Using these data the variation of velocity profile with angle of inclination θ are shown in Figs. 2,3,4,5,6 and 7 for different values of non-Newtonian parameter n. From these figures and equation (11) it is clear that the flow is maximum for $\theta=90^0$ and no flow occurs for $\theta=0^0$ i.e. $V=0$. From figures 2 and 4 it is observed that the velocity at any point in the layer increases with θ. Fig. 3 is the variation of velocity profile with θ for Bingham plastic fluid (n=1.0). The trend of figures are same but the numerial value of velocity decreases with increasing non-Newtonian parameter n.

Figs. 4 and 6 are the variation of velocity profile with θ for power law fluid (β=0) for n=0.5, 1.5 and fig. 5 is for Newtonian fluid (n=1). From these figures it is observed that numerical values of velocity decreases with increasing n.

The variation of flow rate with non-Newtonian parameter n for different angle of inclination θ is shown in Tables 1 and 2.

Table 1. Variation of flow rate Q with $n(\beta=0.0015)$

n \ θ	30^0	45^0	60^0	75^0	90^0
0.5	7.4362×10^{-4}	9.4947×10^{-4}	22.3471×10^{-4}	27.8281×10^{-4}	29.8200×10^{-4}
1.0	0.0363×10^{-4}	0.0514×10^{-4}	0.0630×10^{-4}	0.0702×10^{-4}	0.0728×10^{-4}
1.5	2.5882×10^{-14}	3.2430×10^{-14}	3.7075×10^{-14}	3.0536×10^{-14}	4.0927×10^{-14}

From Table 1, we see that the flow rate increases with angle of inclination θ and from equation (13) there is no flow when $\theta=0^0$. The flow decreases with increasing non-Newtonian parameter n and tends to zero when $n>1$. The values for $n=1$ for Bingham plastic fluid.

Table 2. Variation of flow rate Q with $n(\beta=0.0)$

n \ θ	30^0	45^0	60^0	75^0	90^0
0.5	7.4856×10^{-4}	14.9712×10^{-4}	22.4559×10^{-4}	27.9415×10^{-4}	29.9430×10^{-4}
1.0	0.0355×10^{-4}	0.0516×10^{-4}	0.0632×10^{-4}	0.0705×10^{-4}	0.0730×10^{-4}
1.5	4.9433×10^{-14}	6.2353×10^{-14}	7.1424×10^{-14}	7.6849×10^{-4}	7.8651×10^{-14}

Table 2, is the variation of flow rate for power law and Newtonian fluid with angle of inclination. From this table, the flow rate increases with angle of inclination and decreases with n. The flow rate tends to zero for $n>1$. The value for $n=1$ is for Newtonian fluid.

From equation (9) non-dimensional pressure in obtained as

$\overline{P}=(1-y/a)\cos\theta$, where $\overline{P}=\dfrac{p-p_0}{\rho g a}$

The pressure distribution with y/a for different θ is shown in Table 3.

Table 3. Variation of pressure $\overline{p}$ with y/a for different θ

n \ y/a	0.00	0.20	0.40	0.60	0.80	1.00
0^0	1.000	0.800	0.600	0.400	0.200	0.000
30^0	0.866	0.693	0.520	0.346	0.173	0.000
45^0	0.707	0.566	0.424	0.283	0.141	0.000
60^0	0.500	0.400	0.300	0.200	0.100	0.000
75^0	0.259	0.207	0.155	0.104	0.052	0.000
90^0	0.000	0.000	0.000	0.000	0.000	0.000

From Table 3, we see that pressure decreases with increasing y/a and θ and maximum when $\theta=0^0$ and $y/a=0$ i.e. at lower plane.

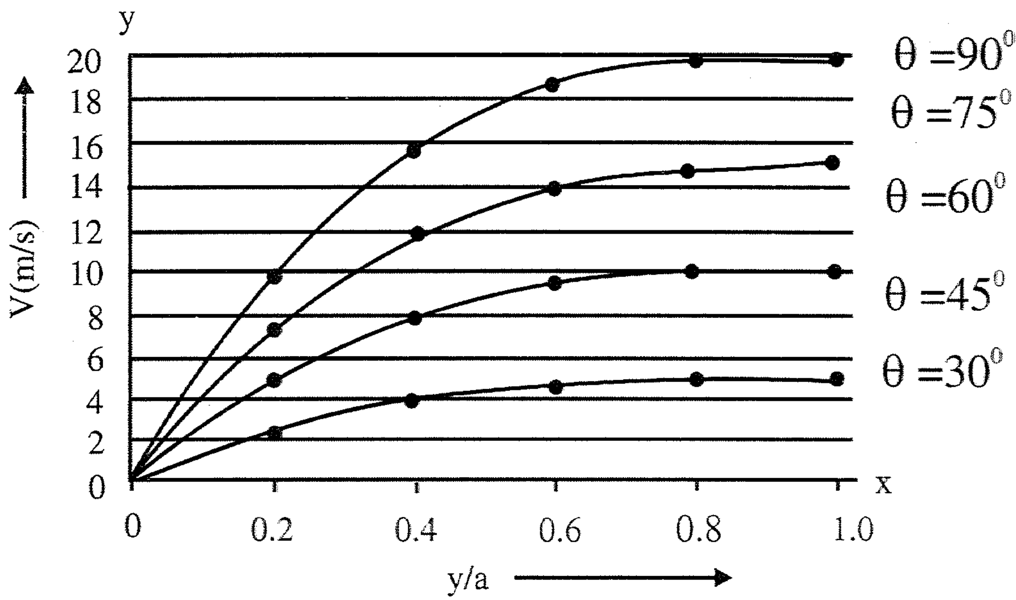

Fig. 2 Variation of velocity with y/a for different angle of inclination $n=0.5$ $\beta=0.0015$

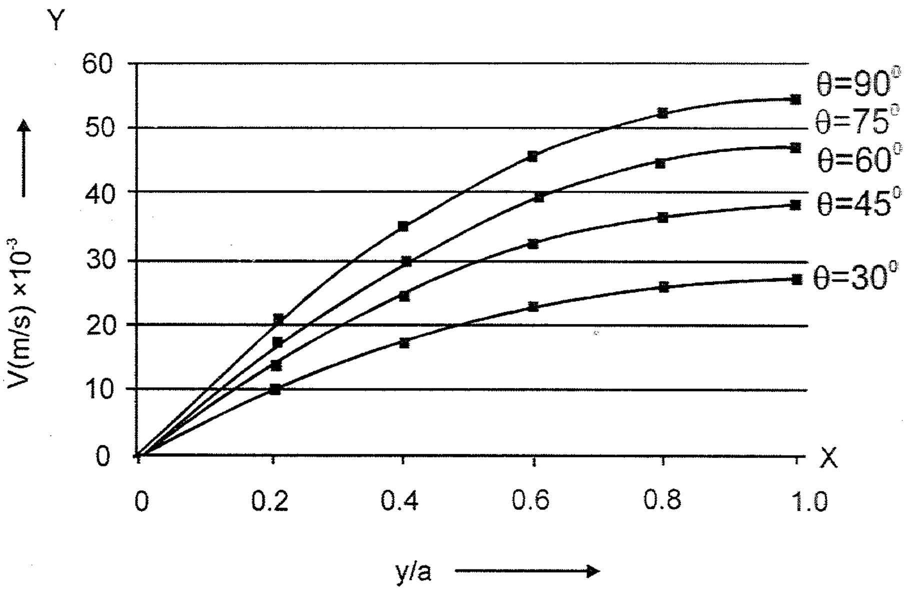

Fig. 3 Variation of velocity with y/a for different angle of inclination $n=1.0$ $\beta=0.0015$ (Bingham plastioc fluid)

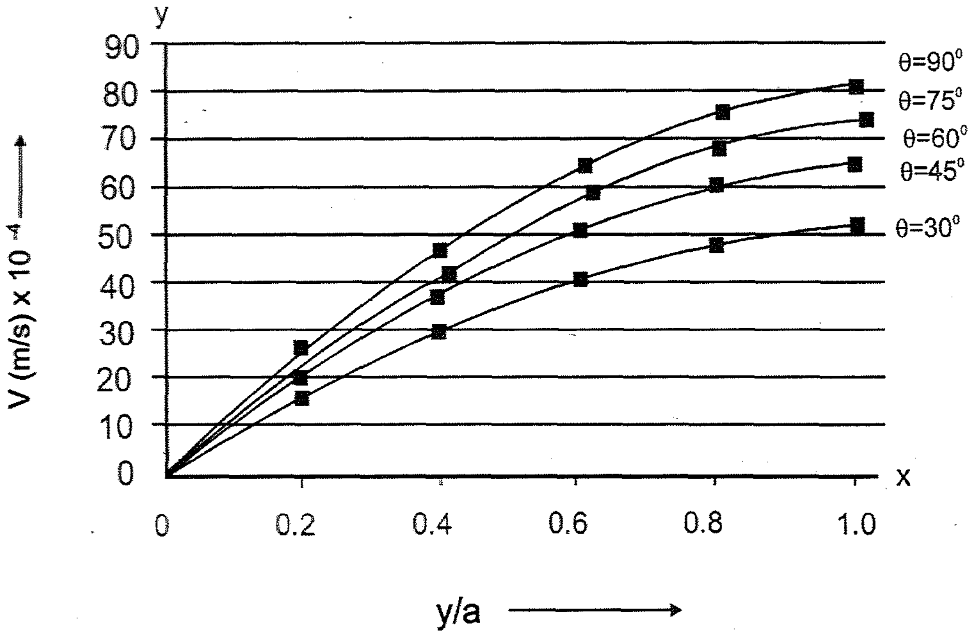

Fig. 4 variation of velocity with y/a for different angle of inclination , n=15, β=0.0015

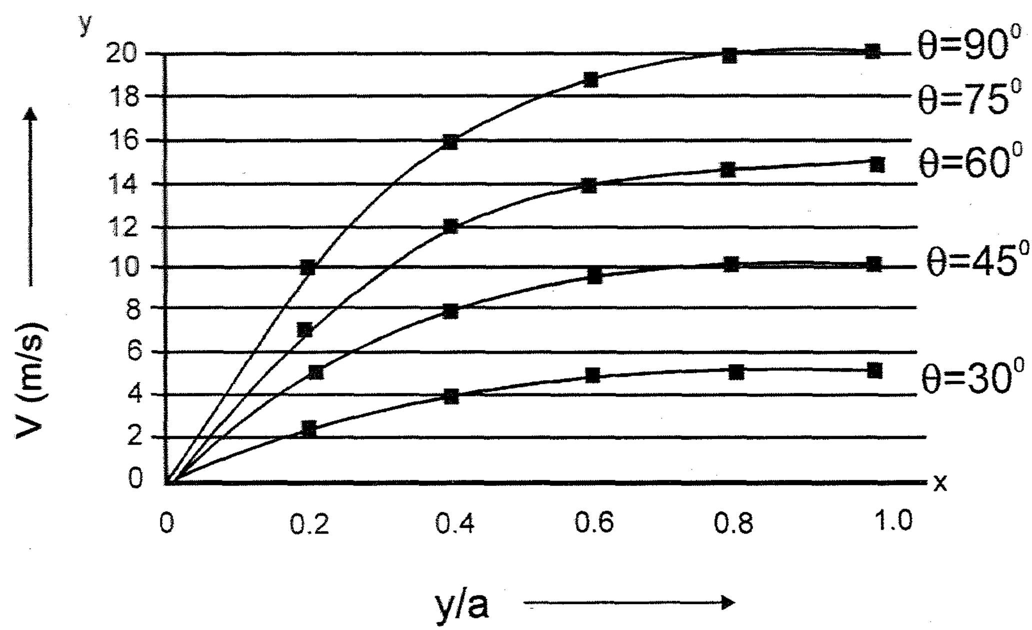

Fig. 5 variation of velocity with y/a for different angle of inclinatin *n=0* (power law fluid)

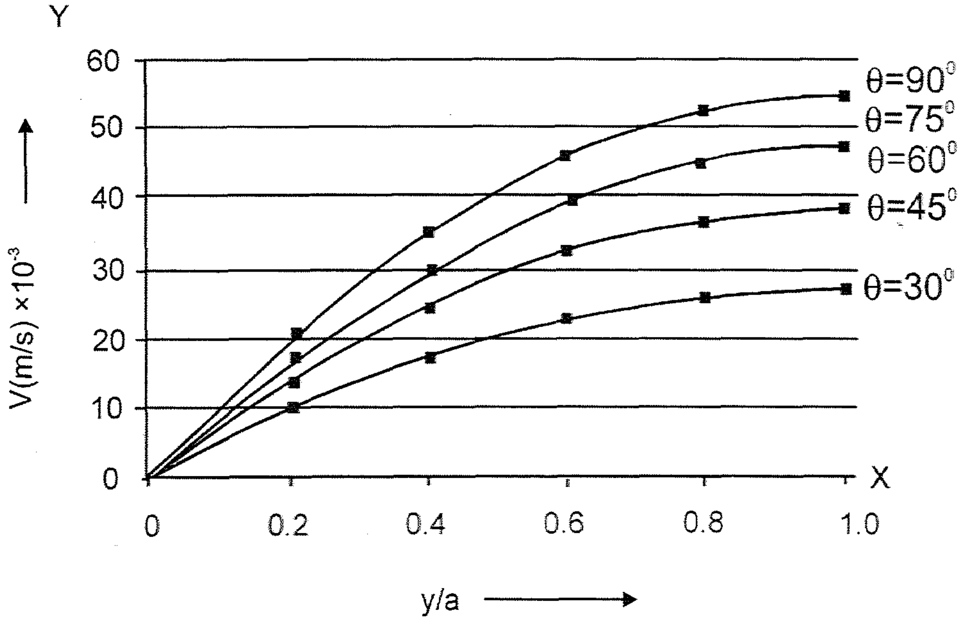

Fig. 6 Variation of velocity with y/a for differnt angle of inclination $n=1.0$, $\beta=0.00$ (Newtonian fluid)

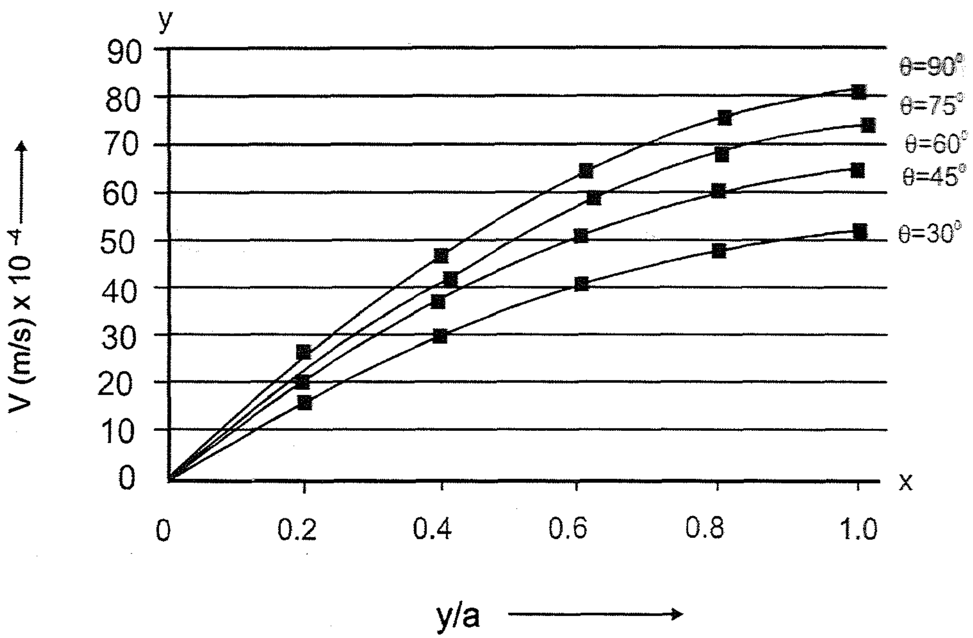

Fig. 7 Variation of velocity with y/a for differnt angle of inclination $n=1.5$, $\beta=0.00$ (Power law fluid)

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