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N-POLICY OF A TWO-PHASE SERVICE SYSTEM WITH MIXED STANDBYS UNDER UNRELIABLE SERVER

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ABSTRACT

This investigation deals with *N*. policy for a single unreliable server machine repair system with mixed standbys. If any unit fails, it is sent immediately for repair if server is available, otherwise waits in queue. The life tiems of operating units and standbys are assumed to be exponentially distributed. The server is subjected to random breakdown and two-phase repairs. Set up time before reparing the server is incorported in the study to ressemble the realistic situation. The breakdown, repair and setup times also follow eponential distributions. A matrix method for evaluating the transient state probabilities is proposed; furthermore various system performance mesures are obtained. A cost function is developed which can be employed to determine the optimum threshold parameter. A numerical illustration is provided which shows the tactability of matrix method suggested. Sensitivity analysis is also carried out to examine the effect of various parameters on performance indices.

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1. Introduction. Machining system finds large applications in manufacturing and production industries where the high quality products are produced in a most economical way. In a modern context, the failure and repair are common occurrences; in a typical machine repair problem, falied machine is provided with an immediate repair. To ensure the desired efficiency of the machining system, the provision of standbys is recommended. The models dealing with the machine repair system subject to server breakdown have gained significance in many application areas including computer systems, communication sysstems, manufacturing systems, etc. Many researchers have studied such queueing models. Alam et al ([1],1989) used the recursive solution technique for multiserver bilevel queuing system with server breakdown. Repairable single server system with multiple breakdowns was studied by Hsieh et al. ([7], 1995). Grey et al ([3]), 2000) analyzed a vacation queueing model with server breakdown. Ke ([12],2003) established an optimal control of an $M/G/1$ queueing system with server breakdown. Ke et al. ([13], 2004) modified the optimal management policy for heterogeneous arrival queuing system with server breakdown. Sultan et al. ([18], 2005) have computationally analyzed a multiserver bulk queue with two modes of server breakdown.

N-policy refers to the situation wherein the server starts the service when 'N' or more failed units are accumulated in the system. Hresh and Brosh ([5], 1980) analyzed the exponential queueing models operating under N-policy. Medhi and Templeton ([15],1992) investigated the N-policy system with poisson arrivals and general startup times. Jain ([8],1997) gave an optimal N-policy for single server markovian queue with breakdowns Jain and Singh ([11], 2005) explored the optimal N-policy for state dependent $M/E_k/1$ queue with server breakdown. Wang et al. ([20],2005) have done the entropy analysis of N-policy $M/G/1$ queueing system with server breakdown and general startup.

Multi-phase repair/service is quite common in complex embedded systems where in which repair of a failed server is performed in many phases due to complexity involved in such a process. Single server queue with multi-phase service was studied by Neuts and Chakravarthy ([16], 1981). Alfa and Frigui ([2], 1996) proposed a discrete NT policy for single server queue with markovian arrival and phase type service. Van Houdt and Alfa ([19], 2005) considered tandem queue with blocking and gave an expression for the response time of such a queuing system with markovian arrivals and phase type services.

The effectiveness of a machine can be realized by specifying the optimal number of spares used for the maintenance of the machine. Gross et al. ([4], 1977) explored the queueing models with spares provisioning. Jain and Dhyani ([9], 1999) have done the transient analysis of $M/M/C$ machine repair problem with spares. Rao and Gupta ([17], 2000) considered the $M/G/1$ machine repairmen problem with mixed standbys.

The present investigation deals with the matrix method for finding the transient state probabilities, using which the performance measures and reliability indices are determined Jain et al. ([10], 2004) used the matrix method for N-policy machine repair system. Reliability and cost oprimization in distributed computing system was given by Hsieh ([6], 2003) Lam et al. ([14], 2006) obtained some queueing characteristics of system and relibility indices of the service with the same approach.

In this paper, we study a N-policy service system working in multicomponent machining environments with mixed standbys. The paper is organized as follows. In section 2, we give the assumptions being used in formulating the mathematical model. In section 3, the governing equations and the analysis are given. Section 4, provides various perfomance characteristics. The cost function is also constructed. In section 5, we compute the numerical results to show the effect of various input parameters on the system performance characteristics. Section 6, concludes the paper by highlighting the novel features of the investigation done.

2 The Mathematical Model. Consider the N-policy for multicomponent machining system with mixed spares and single server which is subjected to breakdown. To formulate the mathematical model. the following assumptions and notations are used.

* The machining system consists of M operating units. Y warm and S cold standbys.
* The units fail a Poisson fashion with rate λ and are replaced by a standby if available.
* A single server repairs the failed units in accordance to *FCFS* discipline.
* The life time of the operating unit and warm spares are exponentially distributed with parameter λ and v, respectively.
* In case when all standbys are exhausted, the remaining online units fail in a degraded fashion with degraded failure rate λ_d ($\lambda_d \geq \lambda$).
* The server is subject to the random breakdowns; the life time of the server is exponentially distributed with mean $1/\alpha$.
* The server undergoes two-phase repair with probability 'r' and '$1-r$' which indicates that the service is successfully completed in one and the second phase, respectively.
* The server returns to the normal operating mode after any of these stages with rates $r\beta_1$ and β_2, respectively.
* The repair of failed units is govenred by the N-policy; according to which repair starts only when $N(\geq 1)$ failed units are accumulated.
* The setup time taken for reparing the server is exponentially distributed with rate η.

The state of the server at time is denoted by variable $\psi(t)$ as follows

$$\psi(t)=\begin{cases}0, \text{ when server is turned off.}\\ 1, \text{ when server is turned on an busy in rendering repair of failed units.}\\ 2, \text{ when server is turned on and broken down, whereas repairman is under set up.}\\ 3, \text{ when server is in first repair phase.}\\ 4, \text{ when server is in second repair phase.}\end{cases}$$

Define the state probabilities as follow :

$P_{0,n}(t)$: probability of $n(n=0, 1,...,N-1)$ failed units at time t in the system when the server is turned off

$P_{i,n}(t)$: Probability of $n(n=1,2,...,L)$ failed units at time t in the system when server is turned on i.e. $\psi(t)=i$, $i=1,2,3,4$

Laplace transform of probabilities are defined as

$\widetilde{P}_{i,n}(s)=L\{P_{i,n}(t)\}=\int_0^\infty e^{-st}P_{i,n}(t)dt, i=0,1,2,3,4$.

The initial conditions are as follows :

$$P_{0,n}(0)=\begin{cases}1, & n=0\\ 0, & n=1,2,...,N-1\end{cases}$$

and $\quad P_{i,n}(0)=0, \quad n=1,2,...,L$.

The state dependent mean failure rates are defined as

(i) When server is turned off i.e. $\psi(t)=0$

$$\lambda_{0,n}=M\lambda+Sv, \quad 0\le n<N$$

(ii) When server is turned on i.e., $\psi(t)=i$, $(i=1,2,3,4)$

$$\lambda_{i,n}=\begin{cases}M\lambda+Sv & 0\le n<Y\\ M\lambda+(Y+S-n)V & Y\le n<Y+S\\ (M+Y+S-n)\lambda_d & Y+S\le n<L\\ 0, & otherwise\end{cases}$$

3. The Governing Equations and Analysis. The differential difference equations governing the model are as follows

$$P'_{0,0}(t)=-\lambda_{0,0}P_{0,0}(t)+\mu P_{1,1}(t), \quad ...(2)$$

$$P'_{0,n}(t)=-\lambda_{0,n}P_{0,n}(t)+\lambda_{0,n-1}P_{0,n-1}(t), \quad 2\le n\le N \quad ...(3)$$

$$P'_{1,1}(t)=-(\lambda_{1,1}+\mu+\alpha)P_{1,1}(t)+\mu P_{1,2}(t)+\beta_2P_{4,1}(t)+r\beta_1P_{3,1}(t), \quad ...(4)$$

$$P'_{1,n}(t)=-(\lambda_{1,n}+\mu+\alpha)P_{1,n}(t)+\lambda_{1,n-1}P_{1,n-1}(t)+\mu P_{1,n+1}(t)+\beta_2P_{4,n}(t)+r\beta_1P_{3,n}(t),$$
$$2\le n\le L-1 \quad ...(5)$$

$$P'_{1,L}(t)=-(\mu+\alpha)P_{1,L}(t)+\lambda_{1,L-1}P_{1,L-1}(t)+\beta P_{4,L-1}(t)+r\beta_1P_{3,L}(t) \quad ...(6)$$

$$P'_{2,1}(t)=-(\lambda_{2,1}+\eta)P_{2,1}(t)+\alpha P_{1,1}(t), \quad ...(7)$$

$$P'_{2,n}(t)=-(\lambda_{2,n}+\eta)P_{2,n}(t)+\alpha P_{1,n}(t)+\lambda_{2,n-1}P_{2,n-1}(t), \quad 2\le n\le L-1 \quad ...(8)$$

$$P'_{2,L}(t)=-\eta P_{2,L}(t)+\alpha P_{1,L-1}(t)+\lambda_{2,L-1}+P_{2,L-1}(t). \quad ...(9)$$

$$P'_{3,1}(t)=-(\lambda_{3,1}+r\beta_1+(1-r)\beta_1)P_{3,1}(t)+\eta P_{2,1}(t), \quad ...(10)$$

$$P'_{3,2}(t)=-(\lambda_{3,2}+r\beta_1+(1-r)\beta_1)P_{3,2}(t)+\eta P_{2,2}(t)+\lambda_{3,1}P_{3,1}(t), \quad ...(11)$$

$$P'_{3,n}(t)=-(\lambda_{3,n}+r\beta_1+(1-r)\beta_1)P_{3,n}(t)+\eta P_{2,n}(t)+\lambda_{3,n-1}P_{3,n-1}(t), \ 2\le n\le L-1 \quad ...(12)$$

$$P'_{4,1}(t)=-(\lambda_{4,1}+\beta_2)P_{4,1}(t)+(1-r)\beta_1 P_{3,1}(t), \quad ...(13)$$

$$P'_{4,2}(t)=-(\lambda_{4,2}+\beta_2)P_{4,2}(t)+(1-r)\beta_1 P_{3,2}(t)+\lambda_{4,1}P_{4,1}(t), \quad ...(14)$$

$$P'_{4,n}(t)=-(\lambda_{4,n}+\beta_2)P_{4,n}(t)+(1-r)\beta_1 P_{3,n}(t)+\lambda_{4,n-1}P_{4,n-1}(t), \quad 2\le n\le L-1 \quad ...(15)$$

$$P'_{4,L}(t)=-\beta_2 P_{4,L}(t)+(1-r)\beta_1 P_{3,L}(t)+\lambda_{4,L-1}P_{4,L-1}(t), \quad ...(16)$$

3.1 The Transient Analysis. Taking the Laplace transform of the equations (2)-(16), the governing equations reduce to

$$(s+\lambda_{0,0})\widetilde{P}_{0,0}(s)-\mu\widetilde{P}_{1,1}(s)=P_{0,0}(0), \quad ...(17)$$

$$(s+\lambda_{0,n})\widetilde{P}_{0,n}(s)-\lambda_{0,n-1}\widetilde{P}_{0,n-1}(s)=P_{0,n}(0), \quad 1\le n\le N-1 \quad ...(18)$$

$$(s+\lambda_{1,1}+\mu+\alpha)\widetilde{P}_{1,1}(s)-\mu\widetilde{P}_{1,2}(s)-\beta_2\widetilde{P}_{4,1}(s)-r\beta_1\widetilde{P}_{3,1}(s)=P_{1,1}(0), \quad ...(19)$$

$$(s+\lambda_{1,n}+\mu+\alpha)\widetilde{P}_{1,n}(s)-\lambda_{1,n-1}\widetilde{P}_{1,n-1}(s)-\mu\widetilde{P}_{1,n+1}(s)-\beta_2\widetilde{P}_{4,n}(s)-r\beta_1\widetilde{P}_{3,n}(s)=P_{1,n}(0), \quad 2\le n\le L-1 \quad ...(20)$$

$$(s+\mu+\alpha)\widetilde{P}_{1,L}(s)-\lambda_{1,L-1}\widetilde{P}_{1,L-1}(s)-\beta_2\widetilde{P}_{4,L}(s)-r\beta_1\widetilde{P}_{3,L}(s)=P_{1,L}(0), \quad ...(21)$$

$$(s+\lambda_{2,1}+\eta)\widetilde{P}_{2,1}(s)-\alpha\widetilde{P}_{1,n}(s)=P_{2,1}(0), \quad ...(22)$$

$$(s+\lambda_{2,n}+\eta)\widetilde{P}_{2,n}(s)-\alpha\widetilde{P}_{1,n}(s)-\lambda_{2,n-1}\widetilde{P}_{2,n-1}(s)=P_{2,n}(0), \quad 2\le n\le L-1 \quad ...(23)$$

$$(s+\eta)\widetilde{P}_{2,L}(s)-\alpha\widetilde{P}_{1,L}(s)-\lambda_{2,L-1}\widetilde{P}_{2,L-1}(s)=P_{2,L}(0), \quad ...(24)$$

$$(s+\lambda_{3,1}+r\beta_1+(1-r)\beta_1)\widetilde{P}_{3,1}(s)-\eta\widetilde{P}_{2,1}(s)=P_{3,1}(0), \quad ...(25)$$

$$(s+\lambda_{3,n-1}+r\beta_1+(1-r)\beta_1)\widetilde{P}_{3,n}(s)-\eta\widetilde{P}_{2,n}(s)-\lambda_{3,n-1}\widetilde{P}_{3,n-1}(s)=P_{3,n}(0), \ 2\le n\le L-1 \quad ...(26)$$

$$(s+\beta_2+\lambda_{4,1})\widetilde{P}_{4,1}(s)-\alpha\widetilde{P}_{1,L}(s)-\lambda_{2,L-1}\widetilde{P}_{2,L-1}(s)=P_{2,L}(0), \quad ...(27)$$

$$(s+\beta_2+\lambda_{4,1})\widetilde{P}_{4,1}(s)-(1-r)\beta_1\widetilde{P}_{3,1}(s)=P_{4,1}(0), \quad ...(28)$$

$$(s+\beta_2+\lambda_{4,n})\widetilde{P}_{4,n}(s)-(1-r)\beta_1\widetilde{P}_{3,n}(s)-\lambda_{4,n-1}\widetilde{P}_{4,n-1}(s)=P_{4,n}(0), \quad 2\le n\le L-1 \quad ...(29)$$

$$(s+\beta_2)\widetilde{P}_{4,L}(s)-(1-r)\beta_1\widetilde{P}_{3,L}(s)-\lambda_{4,L-1}\widetilde{P}_{4,L-1}(s)=P_{4,L}(0), \quad ...(30)$$

The equations (17)-(30) can be written in the matrix form as

$$B(s)\widetilde{P}(s)=P(0) \quad ...(31)$$

where

$$B(s)=\begin{bmatrix} A_1 & G_1 & 0 & 0 & 0 \\ G_2 & A_2 & 0 & C_1 & C_5 \\ 0 & C_2 & A_3 & 0 & 0 \\ 0 & 0 & C_3 & A_4 & 0 \\ 0 & 0 & 0 & C_4 & A_5 \end{bmatrix}_{(N+4L)\times(N+4L)}$$

$$G_1=\begin{bmatrix} -\mu & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}_{(N\times L)} \qquad G_2=\begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -\lambda_{0,N-1} \\ 0 & 0 & \cdots & 0 \end{bmatrix}_{(L\times N)}$$

$$A_1=\begin{bmatrix} s+\lambda_{0,0} & 0 & 0 & \dots & 0 & 0 \\ -\lambda_{0,0} & s+\lambda_{0,1} & 0 & \dots & 0 & 0 \\ 0 & -\lambda_{0,1} & s+\lambda_{0,2} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & s+\lambda_{0,N-2} & 0 \\ 0 & 0 & 0 & \dots & -\lambda_{0,N-2} & s+\lambda_{0,N-1} \end{bmatrix}_{(N\times N)}$$

$$A_2=\begin{bmatrix} \begin{pmatrix} s+\lambda_{1,1}+ \\ \mu+\alpha \end{pmatrix} & -\mu & 0 & \dots & 0 & 0 \\ -\lambda_{1.1} & \begin{pmatrix} s+\lambda_{1,2}+ \\ \mu+\alpha \end{pmatrix} & -\mu & \dots & 0 & 0 \\ 0 & -\lambda_{1,2} & \begin{pmatrix} s+\lambda_{1,3}+ \\ \mu+\alpha \end{pmatrix} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \begin{pmatrix} s+\lambda_{1,L-1}+ \\ \mu+\alpha \end{pmatrix} & -\mu \\ 0 & 0 & 0 & \dots & -\lambda_{1,L-1} & \begin{pmatrix} s+ \\ \mu+\alpha \end{pmatrix} \end{bmatrix}_{(L\times L)}$$

$$A_3=\begin{bmatrix} s+\lambda_{2,1}+\eta & 0 & 0 & \dots & 0 & 0 \\ -\lambda_{2,1} & s+\lambda_{2,2}+\eta & 0 & \dots & 0 & 0 \\ 0 & -\lambda_{2,2} & s+\lambda_{2,3}+\eta & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & s+\lambda_{2,L-1}+\eta & 0 \\ 0 & 0 & 0 & \dots & -\lambda_{2,L-1} & s+\eta \end{bmatrix}_{(L\times L)}$$

$$A_4 = \begin{bmatrix} s+\lambda_{3,1}+\beta_1 & 0 & 0 & \cdots & 0 & 0 \\ -\lambda_{3,1} & s+\lambda_{3,2}+\beta_1 & 0 & \cdots & 0 & 0 \\ 0 & -\lambda_{3,2} & s+\lambda_{3,3}+\beta_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & s+\lambda_{3,L-1}+\beta_1 & 0 \\ 0 & 0 & 0 & \cdots & -\lambda_{3,L-1} & s+\beta_1 \end{bmatrix}_{(L\times L)}$$

$C_1 = -r\beta I$, $C_2 = -\alpha I$, $C_3 = -\eta I$, $C_4 = -(1-r)\beta_1 I$, $C_5 = -\beta_2 I$ **where I being unit** matrix of order $(L\times L)$.

Also $\widetilde{P}(s) = \left[\widetilde{P}_{0,n}(s), \widetilde{P}_{1,n}(s), \widetilde{P}_{2,n}(s),\right]^T$ is a column vector of order $(N+4L)$,

where $\widetilde{P}_{0,n}(s) = \left[\widetilde{P}_{0,0}(s), \widetilde{P}_{0,1}(s), \ldots \widetilde{P}_{0,N-1}(s)\right]_{N\times 1}$

and $\widetilde{P}_{i,n}(s) = \left[\widetilde{P}_{i,1}(s), \widetilde{P}_{i,2}(s), \ldots, \widetilde{P}_{i,N-1}(s), \widetilde{P}_{i,N}(s), \widetilde{P}_{i,N+1}(s), \ldots, \widetilde{P}_{i,L-1}(s), \widetilde{P}_{i,L}(s)\right]^T_{L\times 1}$ for $i=1,2,3,4$.

$P(0) = [1,0,0,\ldots,0,0,0,0,\ldots,0,0,0,0\ldots 0]_{(N+2L)\times 1}$ is initial vector.

To compute probabilities $\widetilde{P}_{i,n}(s), (i=0,1,2,3,4)$, we apply Cramer's rule on matrix $B(s)$ and obtain

$$\widetilde{P}_{i,n}(s) = \frac{\Delta B'_{n+1}(s)}{\Delta B(s)},$$

where $\Delta B(s)$ is the determinant of the matrix $B(s)$. Determinant $\Delta B'_{n+1}(s)$ is obtained by replacing

(i) $(n+1)^{th}$ column of $B(s)$ with initial vector $P(0)$ for $i=0$; $n=0,1,\ldots,N-1$

(ii) $(N+n)^{th}$ column of $B(s)$ with initial vector $P(0)$ for $i=1$; $n=1,2,\ldots,L$

(iii) $(N+L+n)^{th}$ column of $B(s)$ with initial vector $P(0)$ for $i=2; n=1,2,\ldots L$.

Now we calculate characteristic roots of tridiagonal matrix $B(s)$, $s=0$ is one of the roots. Let $s=(-d)$, so that obtain $B(-d)=(B-dI)$. ...(33)

Now equation (31) becomes

$$B(-d)\widetilde{P}(s) = (B-dI)\widetilde{P}(s) = P(0). \quad \ldots(34)$$

Other $(N+4L-1)$ roots in which x real and y complex roots in pairs are denoted by : $d_1, d_2, \ldots, d_x$ and $(d_{x+1}, \bar{d}_{x+1}), (d_{x+2}, \bar{d}_{x+2}), \ldots, (d_{x+y}, \bar{d}_{x+y})$, respectively. Thus we have

$$\Delta B(s) = s\left[\prod_{k=1}^{x}(s+d_k)\right]\left[\prod_{k=1}^{y}(s+d_{x+k})(s+\bar{d}_{x+k})\right] \quad \ldots(35)$$

Equations (32) and (35) yield

$$\widetilde{P}_{i,n}(s) = \frac{\Delta B'_{n+1}(s)}{s\left[\prod_{k=1}^{x}(s+d_k)\right]\left[\prod_{k=1}^{y}(s+d_{x+k})(s+\bar{d}_{x+k})\right]} \quad \ldots(36)$$

On expanding by partial fractions, we get

$$\widetilde{P}_{i,n}(s)=\frac{a_0}{s}+\frac{a_1}{s+d_1}+\ldots+\frac{a_x}{s+d_x}+\frac{b_1 s+c_1}{(s+d_{x+1})(s+\bar{d}_{x+1})}+\ldots+\frac{b_y s+c_y}{(s+d_{x+y})(s+d_{x+y})}.$$

Here a_0 and a_m $(m=1,2,\ldots,x)$ are real numbers calculated as :

$$a_0=\frac{\Delta B'_{n+1}(0)}{\left(\prod_{k=1}^{x} d_k\right)\left(\prod_{k=1}^{y} d_{x+k}\bar{d}_{x+k}\right)} \qquad \ldots(38)$$

$$a_m=\frac{\Delta B'_{n+1}(-d_m)}{(-d_m)\left[\prod_{\substack{k=1\\k\neq m}}^{x}(d_k-d_m)\right]\left[\prod_{\substack{k=1\\k\neq m}}^{y}(-d_m+d_{x+k})(-d_m+\bar{d}_{x+k})\right]}. \qquad \ldots(39)$$

Let complex characteristic root d_{x+m} is a combination of real part u_m and imaginary part v_m. Then

$$b_m(-d_{x+m})+c_m=\frac{\Delta B'_{n+1}(-d_{x+m})}{(-d_{x+m})\left[\prod_{\substack{k=1\\k\neq m}}^{x}(d_k-d_{x+m})\right]\left[\prod_{\substack{k=1\\k\neq m}}^{y}(-d_{x+m}+d_{x+k})(-d_{x+m}+\bar{d}_{x+k})\right]}$$

$$(m=1,2,\ldots,y) \qquad \ldots(40)$$

On taking inverse Laplace transform of equation (37), we get

$$P_{i,n}(t)=a_0+\sum_{m=1}^{x}a_m e^{-d_m t}+\sum_{m=1}^{y}\left[b_m e^{-u_m t}\cos(v_m t)+\frac{c_m-b_m u_m}{v_m}e^{-u_m t}\sin(v_m t)\right] \qquad \ldots(41)$$

where a_0, a_m, b_m, c_m, u_m and v_m all are real numbers.

4 Performance Measures. After determining the probabilities for different system states, now we provide expressions for various performance indices as follows :

(i) Expected number of cold standbys units in the system

$$E(Y)=\sum_{i=0}^{4}\sum_{n=0}^{Y}(Y-n)P_{i,n}(t).$$

(ii) Expected number of warm standbys in the system

$$E(S)=S\sum_{i=0}^{4}\sum_{n=0}^{Y}P_{i,n}(t)+\sum_{i=0}^{4}\sum_{n=Y+1}^{Y+S}(Y+S-n)P_{i,n}(t).$$

(iii) Expected number of short units when all standbys are exhausted

$$E(X)=\sum_{i=1}^{4}\sum_{n=Y+S+1}^{L}\{n-(Y+S)\}P_{i,n}(t).$$

(iv) Expected number of failed units in the queue

$$E(N_q)=\sum_{n=0}^{N-1} nP_{0,n}(t)+\sum_{i=1}^{4}\sum_{n=1}^{L}(n-1)P_{i,n}(t).$$

(v) Expected number of failed units in the system

$$E(N)=\sum_{n=0}^{N-1} nP_{0,n}(t)+\sum_{i=1}^{4}\sum_{n=1}^{L} nP_{i,n}(t).$$

(vi) The probability of the server being idle

$$P(I)=\sum_{n=0}^{N-1} P_{0,n}(t).$$

(vii) The probability of the server being busy

$$P(B)=\sum_{n=1}^{L} P_{1,n}(t).$$

(viii) The probability of the repairman is under setup state is

$$P(s)=\sum_{n=1}^{L} P_{2,n}(t).$$

(ix) The probability of the server being broken down and under i^{th} phase repair

$$P_i(R)=\sum_{n=1}^{L} P_{i,2}(t).$$

(x) The system reliability is given by

$$R(t)=\sum_{n=0}^{N-1} P_{0,n}(t)+\sum_{n=1}^{L-1} P_{1,n}(t).$$

(xi) $MTTF$ is obtained as:

$$\int_0^\infty R(t)dt = Lt_{s\to 0}\sum_{n=0}^{N-1}\widetilde{P}_{0,n}(s)+\sum_{n=1}^{L-1}\widetilde{P}_{1,n}(s).$$

In order to determine the optimal threshold parameters, we construct a cost function as follows:

Average total cost per unit time is calculated as :

$$TC(N)=C_N E(N)+C_Y E(Y)+C_S E(S)+C_1 P(I)+C_B P(B)+C_R P(R),$$

where

C_N=Cost of repairing failed unit per unit time,

C_Y= Cost of providing each cold standby per unit time when it is not used,

C_S= Cost of providing each warm standby per unit time when it is not used,

C_I= Cost of server per unit time in idle state

C_B= Cost of server per unit time in busy state

C_R= Cost of repairing of the server per unit time.

5. Numerical Illustrations. In this section, some numerical examples and sensitivity analysis to explore the effect of parameters on the performance characteristics are presented. For illustration, we consider that the service system has 6 operating units, 4 cold spares, and 2 warm spares. We set other default parameters as r=0.4, υ=0.5, η=0.5, λ'=1.5.

In Fig. 1, we plot the variation of throughput of the system. It is noted that throughput first increases sharply and then decreases before attaining almost constant with time grows. Fig. 1(b) depicts the increasing trend of throughput with the service rate μ. It can be seen that on increasing the service rate, the throughput first increases and then becomes constant. It agrees with the paracticl situation as if when better service is being provided then there is an increase in the throughput of the system. Fig. 1 (c) depicts the same increasing pattern of throughput with time, but it is being observed that throughput decrease on increasing the breakdwon rate α. Thus it shows that the system is being affected by the server breakdown and hence decreases the throughput of the system.

Fig.2 depicts the graphs for the system reliability for different values of λ, μ and α. It has been observed that the reliability of the system decreases with a given span. Fig. 2 (a) shows a decreasing trend of reliability with time. It is noted from fig. 2(b) that the system relibility is not much affected by the service rate, but fig. 2(c) shows a tremendously decreasing trend of system reliability with an increased rate of server breakdwon. This indicates the poor performance of the system with the increase in the server breakdown, and which is quite obvious.

Finally fig. 3 shows the profile of expected number of the failed unite $E(N)$ in the system with the variation of time. Fig. 3 exhibits the almost same linear pattern of $E(N)$ with the varying parameters. Fig. 3(a)-(c) shows an increase in $E(N)$ with an increasing arrival rate λ and the server breakdown rate α. It is worth noting that only a marginal decrease of $E(N)$ is there with an increase of service rate μ.

6 Conclusions. In this paper we have investigated the two-phase service system with server breakdown and setup. The concept of two-phase service and spares is incorporated into the model in order to make it closer to the real life situation. We have obtained the transient state probabilities, using which various performance characteristics are also established. Cost function is also formulated. A sensitivity analysis has also performed in order to visualize the effect of different parameters on the performance measures and the reliability indices. The graphs so obtained indicate that the system reliability is not much affected by the service, but server breakdown greatly affects the reliability of the system, which leads to the poor performance of the system. So, in order to increase the efficiency of the system, the server breakdown should be reduced i.e. repair policy should ensure the server breakdown occurs, it should be repaired immediately.

Further, the setup time taken by the server for providing service also play an important role in system performance. Hence the incorporation of the setup time in our model make it more feasible with the real time application.

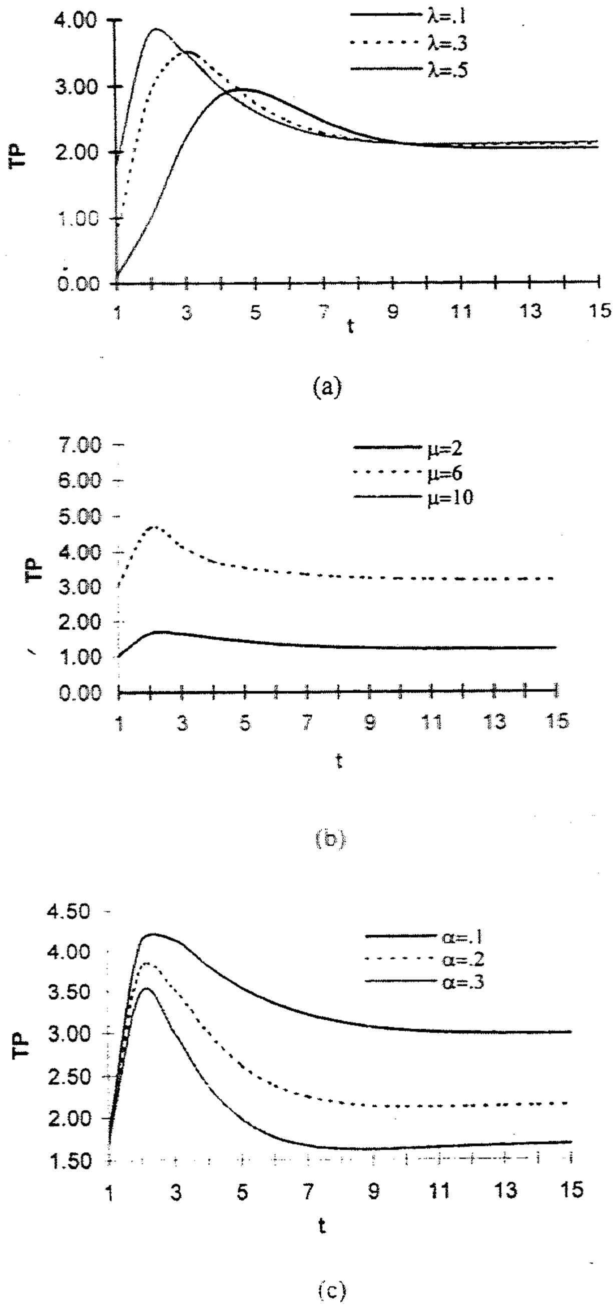
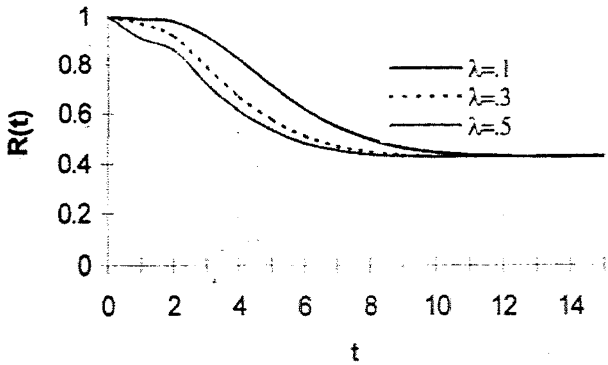
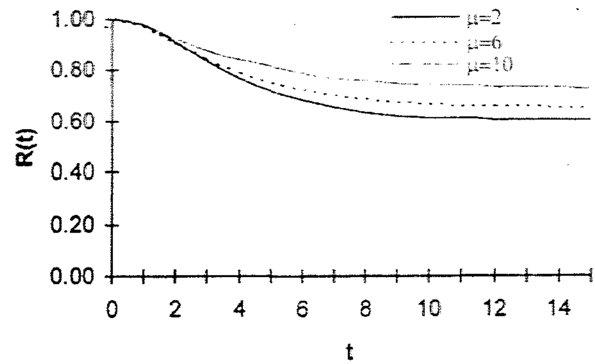


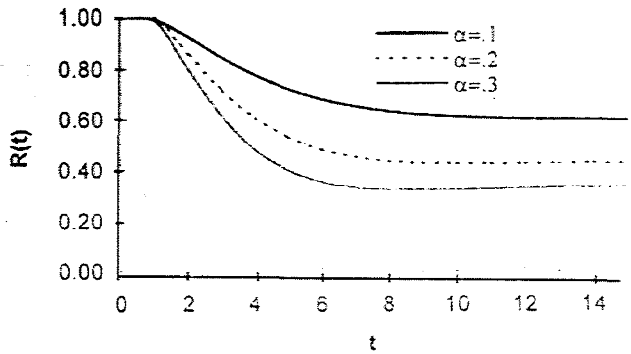
Fig. 1: Throughput profiles for different values of (a) λ (b) μ (c) α



(a)

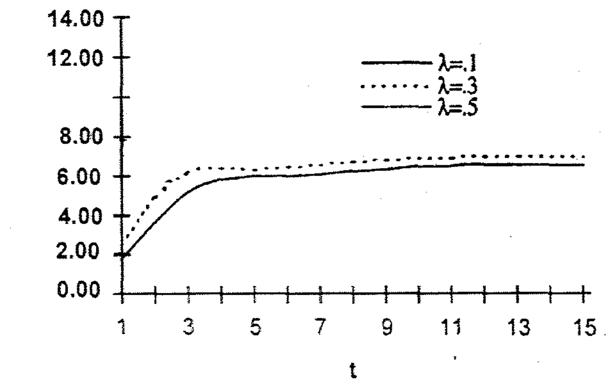


(b)

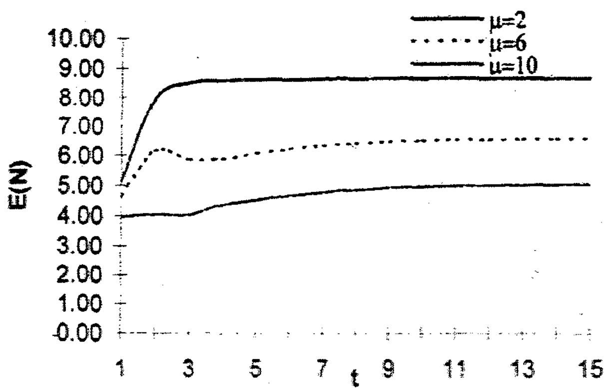


(c)

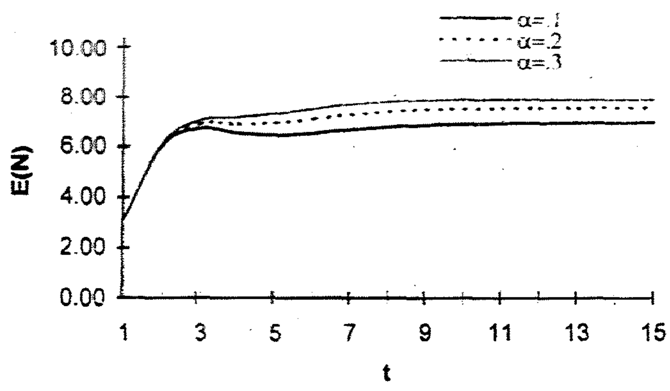
Fig. 2: Reliability profiles for different values of (a) λ (b) μ (c) α



(a)



(b)



(c)

Fig. 3: Profiles for Expected number of failed units for different values of (a) λ (b) μ (c) α

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