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MACHINE REPAIR SYSTEM WITH WARM STANDBYS, ADDITIONAL REPAIRMEN DISCOURAGEMENT AND SWITCHING FAILURE

By
Madhu Jain, G.C. Sharma
Institute of Basic Sciences, Khandari, Agra-282002, Uttar Pradesh, India
and
Rakhi Sharma
St. John's College, Agra-282002, Uttar Pradesh, India

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ABSTRACT

The present investigation is concerned with the machining system with warm standbys having R permanent reparimen. When all spares are used, the system begins to work in degraded mode. The provision of r additional repairmen, which turn on according to a threshold rule depending upon the number of failed units in the system to speed up the repair, is made. We have assumed the switching device failure probability q and joining probability of failed units for repair facility as b. Several system characteristics viz. expected number of failed units, the expected number of operative units in the system etc., are established. Numerical illustrations using matrix method is also given to examine the validity of analytical results and to provide the sensitivity analysis.

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1. Introduction. Machining system has pervaded every nook and corner of our lives thus ensuring our utmost dependence on them. As time proceeds, a machine may fail and thus requires a corrective action by the repairmen after which it again starts working properly. If at any time more than one machine needs the repairmen's attention, a queue develops. In case of several repairmen, If the number of machines failing at any given time exceeds the number of repairmen then the excess number of machines will wait until the repairmen are available. This phenomenon needs examination thus machine repair models have captivated the interest of many renowned researchers working in the area of queueing theory.

One can improve system efficiency by providing sufficient spare part support in case of machine failure. In queueing theory a lot of work has been done on machine repair models provided with spares. Sivazalian and Wang ([1] 1989) analyzed the $M/M/R$ machine repair problem with warm standbys. Gupta ([13]

1999) studied machine interference problem with warm spares. Arulmozhi ([1]2002) gave reliability of an M-out-of Nwarm standby system with repair facility. Availability and reliability of k-out-of-$(M+N)$:G warm standby systems was considered by Zhang et al. ([17] 2006).

Sometimes the failed units may decide not to join the queue i.e. balk if sent to the queue or also may renege from the service system due to long queue or insufficient waiting space. Dick ([2], 1970) established some theorems of interest for single server queues with balking. Jain and Premlata ([6], 1994) investigated $M/M/R$ machine repair problem with reneging and spares. Ke and Wang ([9] 1999) made cost analysis of the $M/M/r$ machine repair problem with balking, reneging and server breakdowns. Further more, Ke and Wang ([10],2002) studied the reliability analysis of repairable system with warm standbys, reneging and balking. Jain et al. ([8], 2003) and analyzed $M/M/R$ machine interference model with balking, reneging, spares and two modes of failure. Sharma et al. ([12], 2004) worked on performance modelling of machining system with mixed standby coponents, balking and reneging. Sharma et al. ([13], 2005) considered queueing problems with balking and reneging for limited space.

In many critical situations, the provision of permanent repairmen along with additional removable repairmen is recommended in order to reduce the balking and reneging behaviour of the failed units so that the grade of service can be improved to the reasonable extent. Many mathematicians have contributed significantly towards machine repair problem with additional repairmen. Jain ([14], 1996) analyzed reliability for $M/M/C$ repairable system with spares and additional repairmen. Jian ([5], 2003) studied N-policy for redundant repairable system with additional repairmen. Jain et al. ([7], 2002) studied queueing modeling of machining system with balking, reneging, additional repairmen and two modes of failure. Sharma et al. ([13], 2005) considered loss and delay multi-server queueing model with discouragement and additional servers.

So far as research focused on standby units having switching failure is concened it has not been given much attention by many researchers. Recentaly, Wang et al. ([15],[16], 2006) studied profit analysis of the $M/M/R$ machine repair problem with balking, reneging and standby switching failures. He further in the same year compared reliability and the availability between four systems with warm standby components and standby switching failures.

In this investigation, we consider a machine repair problem with warm standbys. The failed units, which need the attention of service station may balk or renege on finding the long queue of failed units. The provision of permanent along with additionial removable repairmen is made in order to improve the grade of service. The operating units may also start degrading if there are less units than actually required. The rest of the paper is organized as follows. We describe the model along with notations in section 2. Section 3 contains governing global equations used in our paper. The mathematical analysis is done in section 4. In section 5 some performance indices are established. The cost function is evaluated in section 6. In section 7, we provide the numerical results to validate the analytical results. Finally the last section contains conclusion of the investigation.

2. Model Description. We consider a machine repairable system consisting of M operating and S warm standbys. We have also provided R permanent and r additional repairmen in the system. The following assumptions and notations are used for mathematical formulation of our model:

* The life times of oeperating (standby) machines are exponentially distributed with mean rate λ (α).
* The repair time of failed unit is identical exponential distributed with rate μ.
* When all the standbys are used, the failure rate of the machining unit increases from λ to λ_{i-s} as the system works in short mode due to system stress, where $iS \leq i < M+S$ is the number of failed units in the system.
* The switching device has failure probability q during the switching from standby state to operting state.
* If the number of failed units is more than T, the additional removable repairmen turn on one by one with additional load of T failed units so that j^{th}($j=1,2,...,r$) additional repairmen starts repairing when the number of failed units is more than jT but less than or equal to $(j+1)T$ and is removed as the number of failed units decreases to jT. When there are greater than rT units in the system, all permanent and all additional repairmen will provide repair of failed units.
* The repair rate of permanent (j^{th} additional) repairman to restore failed unit is μ_1 (μ_j) where $j=1,2,...,r$.
* When a failed unit is repaired, it joins the standby group if the system is working in normal mode, otherwise works with operating units. After repairing, the failed units are as good as new one.

We further assume that (1-b) is the balking probability of the failed units where b is the probability that a failed unit joins the queue. Further let us assume tht in case of long queue when all repairmen are busy the failed unit may renege with rate υ. Let $P_i(t)$ be the probability that there are i failed units in the system at time t, when $i=0,1,...,M+S$.

The Laplace transform of probabilities are defined as :

$$P_i^*(s)=L\{P_i(t)\}=\int_0^\infty e^{-st}P_i(t)dt.$$

3. Global Equations. The failure and repair rates of the units are state dependent and are given by :

$$\lambda(i)=\begin{cases} M\lambda+S\alpha, & 0\le i<R \\ M\lambda\beta+(S-i)\alpha, & R\le i<S \\ (M+S-i)\lambda_{i-S}\beta, & S\le i<M+S \end{cases} \quad ...(1)$$

$$\mu(i)=\begin{cases} i\mu & 1\le i\le R \\ R\mu+(i-R)v, & R<i\le S \\ R\mu+\mu_1+(i-\overline{R+1})v & S<i\le T \\ R\mu+\sum_{i=1}^{j+1}\mu_i+(i-\overline{R+j+1})v, & jT<i\le (j+1)T \\ R\mu+\sum_{i=1}^{r}\mu_i+(i-\overline{R+r})v, & rT<i\le M+S \end{cases} \quad ...(2)$$

Using (1) and (2), the differential difference equations governing the model are constructed as follows :

$$\frac{d}{dt}P_0(t)=-\lambda(1)P_0(t)+\mu(1)P_0(t) \quad ...(3)$$

$$\frac{d}{dt}P_1(t)=-[M\lambda+S\alpha+\mu(1)]P_1(t)+[M\lambda(1-q)+S\alpha]P_0(t)+\mu(2)P_2(t) \quad ...(4)$$

$$\frac{d}{dt}P_i(t)=-[\lambda(i)+\mu(i)]P_i(t)+\left[M\lambda(1-q)+(S-\overline{i-1})\alpha\right]P_{i-1}(t)+\sum_{n=0}^{i-2}M\lambda q^{i-n-1}(1-q)P_n(t)$$

$$+\mu(i+1)P_{i+1}(t), \qquad 2\le i\le R \quad ...(5)$$

$$\frac{d}{dt}P_i(t)=-[\lambda(i)+\mu(i)]P_i(t)+[M\lambda\beta(1-q)+S\alpha]P_{i-1}(t)+\sum_{n=0}^{R-1}M\lambda q^{i-n-1}(1-q)P_n(t)$$

$$+\sum_{n=R}^{i-2}M\lambda\beta q^{i-n-1}(1-q)P_n(t)+\mu(i+1)P_{i+1}(t), \qquad R+1\le i\le S \quad ...(6)$$

$$\frac{d}{dt}P_{S+1}(t)=-[\lambda(S+1)+\mu(S+1)]P_{S+1}(t)+\lambda(S)P_S(t)+\sum_{n=0}^{R-1}M\lambda q^{S-n}(1-q)P_n(t)$$

$$+\sum_{n=R}^{S-1}M\lambda\beta q^{S-n}(1-q)P_n(t)+\mu(S+2)P_{S+2}(t) \quad ...(7)$$

$$\frac{d}{dt}P_i(t)=-[\lambda(i)+\mu(i)]P_i(t)+\lambda(i-1)P_{i-1}(t)+\mu(i+1)P_{i+1}(t), S+2\le i\le T-1 \qquad ...(8)$$

$$\frac{d}{dt}P_T(t)=-[\lambda(T)+\mu(T)]P_T(t)+\lambda(T-1)P_{i-1}(t)+\mu(T+1)P_{T+1}(t) \qquad ...(9)$$

$$\frac{d}{dt}P_i(t)=-[\lambda(i)+\mu(i)]P_i(t)+\lambda(i-1)P_{i-1}(t)+\mu(i+1)P_{i+1}(t), jT\le i\le (j+1)T \qquad ...(10)$$

$$\frac{d}{dt}P_i(t)=-[\lambda(i)+\mu(i)]P_i(t)+\lambda(i-1)P_{i-1}(t)+\mu(i+1)P_{i+1}(t), rT\le i\le M+S-1 \qquad ...(11)$$

$$\frac{d}{dt}P_L(t)=-\lambda(L-1)P_{L-1}(t)+\mu(L)P_L(t) \qquad ...(12)$$

The initial conditions are $P_0(0)=1$; $P_i(0)=0$, $i=1,2,\ldots,L$ using these and taking Laplace transform of equations (3)-(12), we get

$$1=[\lambda(0)+s]P_0^*(s)-\mu(1)P_1^*(s) \qquad ...(13)$$

$$0=[\lambda(1)+\mu(1)+s]P_1^*(s)-[M\lambda(1-q)+S\alpha]P_0^*(s)-\mu(2)P_2^*(s) \qquad ...(14)$$

$$0=[\lambda(i)+\mu(i)+s]P_i^*(s)-\left[M\lambda(1-q)+\left(S-\overline{i-1}\right)\alpha\right]P_{i-1}^*(s)-\sum_{n=0}^{i-2}M\lambda q^{i-n-1}(1-q)P_n^*(s)$$

$$-\mu(i+1)P_{i+1}^*(s), \qquad 2\le i\le R \qquad ...(15)$$

$$0=[\lambda(i)+\mu(i)+s]P_i^*(s)-\left[M\lambda\beta(1-q)+\left(S-\overline{i-1}\right)\alpha\right]P_{i-1}^*(s)-\sum_{n=0}^{R-1}M\lambda q^{i-n-1}(1-q)P_n^*(s)$$

$$-\sum_{n=R}^{i-2}M\lambda\beta q^{i-n-1}(1-q)P_n^*(s)-\mu(i+1)P_{i+1}^*(s), \qquad R+1\le i\le S \qquad ...(16)$$

$$0=[\lambda(S+1)+\mu(S+1)+s]P_{S+1}^*(s)-\lambda(S)P_S^*(s)-\sum_{n=0}^{R-1}M\lambda q^{i-n-1}(1-q)P_n^*(s)$$

$$-\sum_{n=R}^{S-1}M\lambda\beta q^{i-n-1}(1-q)P_n^*(s)-\mu(S+2)P_{S+2}^*(s) \qquad ...(17)$$

$$0=[\lambda(i)+\mu(i)+s]P_i^*(s)-\lambda(i-1)P_{i-1}^*(s)-\mu(i+1)P_{i+1}^*(s), S+2\le i\le T \qquad ...(18)$$

$$0=[\lambda(T)+\mu(T)+s]P_T^*(s)-\lambda(T-1)P_{T-1}^*(s)-\mu(T+1)P_{T+1}^*(s) \qquad ...(19)$$

$$0=[\lambda(i)+\mu(i)+s]P_i^*(s)-\lambda(i-1)P_{i-1}^*(s)-\mu(i+1)P_{i+1}^*(s), jT \le i \le (j+1)T \qquad ...(20)$$

$$0=[\lambda(i)+\mu(i)+s]P_i^*(s)-\lambda(i-1)P_{i-1}^*(s)-\mu(i+1)P_{i+1}^*(s) rT \le i \le M+S-1 \qquad ...(21)$$

$$0=[\mu(L)+s]P_L^*(s)-\lambda(L-1)P_{L-1}^*(s) \qquad ...(22)$$

4. Mathematical Analysis. The equations (13)-(22) can be written in matrix form as;

$$Q(s)P^*(s)=P(0) \qquad ...(23)$$

where matrix $Q(s)=[\ ]_{(L+1)\times(L+1)}$ is shown on the next page.

$P^*(s)=[P_i^*(s)]$ is a column vector of order $(L+1)\times 1$

$P_i^*(s)=[P_0^*(s),P_1^*(s),...,P_L^*(s)]_{(L+1)\times 1}$

$P_0=[1,00,...,0,...0]_{(L+1)\times 1}$ is a initial vector.

To compute probabilities $P_i^*(s)$, we apply Cramer's rule on matrix $Q(s)$ and obtain

$$P_i^*(s)=\frac{\Delta Q'_{i+1}(s)}{\Delta Q(s)},\ 0<i\le L \qquad ...(24)$$

where $\Delta Q(x)$ is the determinant of the $Q(s)$ and matrix $\Delta Q'_{i+1}(s)$ is obtained by replacing $(i+1)$th column of $Q(s)$ with initial vector P_0 for $I=1,2,...,L$.

$$Q(s)=\begin{bmatrix}
\lambda(0)+s & -\mu(1) & 0\ 0\ 0 & 0\ 0\ 0 & 0\ 0\ 0 & 0 & 0 & 0\ 0\ 0\ 0 & 0\\
-[A_0+S\alpha] & \psi_1 & \ldots\ 0\ 0 & 0\ 0\ 0 & 0\ 0\ 0 & 0 & 0 & 0\ 0\ 0\ 0 & 0\\
-A_1 & -[A_0+(S-1)\alpha] & \ldots\ \ldots\ 0 & 0\ 0\ 0 & 0\ 0\ 0 & 0 & 0 & 0\ 0\ 0\ 0 & 0\\
-A_{i-2} & -A_1 & \ldots\ \ldots\ -\mu(i) & 0\ 0\ 0 & 0\ 0\ 0 & 0 & 0 & 0\ 0\ 0\ 0 & 0\\
-A_{i-1} & -A_{i-2} & \ldots\ \ldots\ \psi_i & \ldots\ 0\ 0 & 0\ 0\ 0 & 0 & 0 & 0\ 0\ 0\ 0 & 0\\
\ldots & \ldots & \ldots\ \ldots\ -[A_0+(S-i)\alpha] & \ldots\ \ldots\ 0 & 0\ 0\ 0 & 0 & 0 & 0\ 0\ 0\ 0 & 0\\
\ldots & \ldots & \ldots\ \ldots\ -A_1 & \ldots\ \ldots\ -\mu(R) & 0\ 0\ 0 & 0 & 0 & 0\ 0\ 0\ 0 & 0\\
-A_{R-1} & -A_{R-2} & \ldots\ \ldots\ \ldots & \ldots\ \ldots\ \psi_R & \ldots 0\ 0 & 0 & 0 & 0\ 0\ 0\ 0 & 0\\
-A_R & -A_{R-1} & \ldots\ \ldots\ \ldots & \ldots\ \ldots\ -[bA_0+(S-R)\alpha] & \ldots\ldots 0 & 0 & 0 & 0\ 0\ 0\ 0 & 0\\
\ldots & \ldots & \ldots\ \ldots\ \ldots & \ldots\ \ldots\ -bA_1 & \ldots\ldots -\mu(S-1) & 0 & 0 & 0\ 0\ 0\ 0 & 0\\
\ldots & \ldots & \ldots\ \ldots\ \ldots & \ldots\ \ldots\ \ldots & \ldots\ldots \psi_{S-1} & -\mu(S) & 0 & 0\ 0\ 0\ 0 & 0\\
-A_{S-1} & -A_{S-1} & \ldots\ \ldots\ \ldots & \ldots\ \ldots\ \ldots & \ldots\ldots -[bA_0+\alpha] & \psi_S & -\mu(S+1) & 0\ 0\ 0\ 0 & 0\\
-A_S & -A_{S-1} & \ldots\ \ldots\ -A_{S+1-i} & \ldots\ \ldots\ -A_{S+1-R} & \ldots\ldots -bA_1 & -\lambda(S) & \psi_{S+1} & \ldots 0\ 0\ 0 & 0\\
0 & 0 & 0\ 0\ 0 & 0\ 0\ 0 & 0\ 0\ 0 & 0 & -\lambda(S+1) & \ldots\ldots 0\ 0 & 0\\
0 & 0 & 0\ 0\ 0 & 0\ 0\ 0 & 0\ 0\ 0 & 0 & 0 & \ldots\ldots 0 & 0\\
0 & 0 & 0\ 0\ 0 & 0\ 0\ 0 & 0\ 0\ 0 & 0 & 0 & 0\ \ldots\ldots -\mu(L-1) & 0\\
0 & 0 & 0\ 0\ 0 & 0\ 0\ 0 & 0\ 0\ 0 & 0 & 0 & 0\ 0\ \ldots\ \psi_{L-1} & -\mu(L)\\
0 & 0 & 0\ 0\ 0 & 0\ 0\ 0 & 0\ 0\ 0 & 0 & 0 & 0\ 0\ 0\ -\lambda(L-1) & \mu(L)+S
\end{bmatrix}$$

Matrix $Q(s)$

$\Lambda_i = M\lambda(1-q)q^i$ and $\psi_i=\lambda(i)+\mu(i)+s$

Now we proceed to calculate characteristic roots of matrix $Q(s)$ and note that $s=0$ is one of the roots. Let $s=(-d)$ so that we obtain $Q(-d)=(Q-dI)$.
Now equation (23) becomes
$Q(-d)P^*(s)=(Q\text{-}dI)P^*(s)=P_0$.
Other (L) roots in which real roots are denoted by
$d_1, d_2, ..., d_L$ respectively. Thus we have

$$\Delta Q(s) = s\left[\prod_{k=1}^{L}(s+d_k)\right]. \qquad ...(25)$$

Equations (24) and (25) yield

$$P_i^*(s) = \frac{\Delta Q'_{i+1}(s)}{s\left[\prod_{k=1}^{L}(s+d_k)\right]}. \qquad ...(26)$$

We expand $P_i^*(s)$ by partial fractions i.e. in the form of

$$P_i^*(s) = \frac{a_0}{s} + \frac{a_1}{s+d_1} + ... + \frac{a_L}{s+d_L} \qquad ...(27)$$

Here a_0 and a_m $(m=1,2,...,L)$ are real numbers and calculated as:

$$a_0 = \frac{\Delta Q'_{i+1}(0)}{\left[\prod_{k=1}^{L} d_k\right]}, \qquad ...(28)$$

$$a_m = \frac{\Delta Q'_{i+1}(-d_m)}{(-d_m)\left[\prod_{k=1}^{L}(d_k - d_m)\right]}, m = 1,2,...,L. \qquad ...(29)$$

On taking inverse Laplace transform of equation (27), we get

$$P_i = a_0 + \sum_{m=1}^{L} a_m e^{-d_m t} \qquad ...(31)$$

5. Performance Measures. After determining the probabilities, we derive various performance measures as follows:

* The expected number of failed units in the system is givey by

$$E(N) = \sum_{i=1}^{M+S} iP(i). \qquad ...(32)$$

* The expected numberr of operative units in the system is

$$E(O) = M - \sum_{i=S+1}^{M+S} (i-S)P(i). \qquad ...(33)$$

* The expected number of idle permanent repairmen in the system is

$$E(I) = \sum_{i=0}^{R-1} (R-i)P(i). \qquad ...(34)$$

* The expected number of busy permanent repairmen in the system is

$$E(B) = R - E(I). \qquad ...(35)$$

* The expected number of busy additional repairmen in the system is

$$E(A) = \sum_{j=1}^{r-1} j \sum_{i=jT+1}^{M+S} P(i) + r \sum_{i=rT+1}^{M+S} P(i). \qquad ...(36)$$

* The expected number of spare units available in the system is

$$E(S) = \sum_{i=0}^{S} (S-i)P(i). \qquad ...(37)$$

* Machine availability is given by

$$M(A) = 1 - \frac{E(N)}{M+S}. \qquad ...(38)$$

* Average reneging rate is

$$R.R. = \sum_{i=R+1}^{M+S} (i-R)rP(i). \qquad ...(39)$$

* Average switching failure rate is

$$S.R. = \sum_{i=1}^{S} M\lambda q P(i-1). \qquad ...(40)$$

6. Cost Analysis. In order to determine the optimal threshold parameters, we suggest a cost function. The various cost factors used to construct cost function are as follows :

C_1 = Cost per unit time of failed units when all standbys are being used.
C_2 = Cost per unit time of a permanent repairman when he is idle.
C_3 = Cost per unit time of a permanent repairmen when he is providing service
C_4 = Cost per unit time of providing a standby unit.
C_5 = Cost per unit time of an additional repairman when he is providing repair.

The expected total cost per unit time is given by :

$$E(C) = C_1 \sum_{i=S+1}^{M+S} (i-S)P(i) + C_2E(I) + C_3E(B) + C_4E(S) + C_5E(A) \qquad ...(41)$$

7. Numerical Results. To obtain numerical results, we develop computer program in software *MATLAB*. In table 1-3, we summarize numerical results for the expected number of operative unit $E(O)$, expected number of permanent repairmen $E(I)$, machine availability $M(A)$, average reneging rate (*R.R.*) and average switching failure rate (*S.R.*) by varying λ and b, λ and q and λ and α respectively, for different values of time (t).

In Table 1, with the increase in t all the performance measures except *R.R.* decrease but the decrease in $E(O)$ is more remarkable for $t=1$ for different values of joining probability (b). For greater value of t there is small decrease in $E(I)$ and *S.R.* whereas *R.R.* slightly increases for different value of λ. We further observe that for $\lambda=0.1$ and $b=0.4$, $E(I)$, R.R. and S.R. become almost constant for greater value of t. For $\lambda=0.3$, we notice that for smaller value of b, $E(O)$ shows less change than for greater value of b.

In Table 2, $E(O)$ decreases with the increment in t and λ as expected, since with increasing the time and failure rate, the number of operative unit decreases. $E(I)$ is 2 at $t=0$ i.e. all the permanent repairmen are in idle state but as time increases $E(I)$ decreases but for $\lambda=0.3$, $E(I)$ decreases very sharply. Similarly in Table 3, the results follow the same trend by increasing the value of λ and α as time grows. There is very slight difference in all the performance measures for different value of α and same value of λ but the difference is noticeable for different value of λ.

Fig. 1 displays the effect of various parameters on $E(N)$ by varying t. Fig. 1(a) exhibits that $E(N)$ is more for greater value of λ, however the difference in the value of E (N) is more prominent for smaller value of λ. In figs. 1(b)–1(c) we see that, $E(N)$ is greater for greater value of switching failure rate (q) and joining probabilitiy (b) but the difference of $E(N)$ for different values of the two parameters is remarkable in fig. 1 (c).

In figs. 1 (d)-1(f), we observe that $E(N)$ decreases as repair rate (μ), reneging parameter (ν) and the number of warm standbys (S) increase. The change in the value of $E(N)$ is almost same for different value of μ but in fig. 1(e) this difference is more perceptible for $\upsilon=0.1$ Fig. 1 (f) shows hardly any difference for different value of S and as time increases the value of $E(N)$ becomes constant for different value of S.

Overall we conclude that

* $E(O)$, $E(I)$, $M(A)$ and *SR* decrease and *R.R.* increases as time (t) and failure rate (λ) increase.
* The difference in various performance measures for different value of b,q and λ is not much significant.
* As we expect, $E(N)$ is more for greater value of λ,q and b, on the contrary for smaller value of μ, ν and S, it is higher.

8. Conclusion. It is an important issue of system designer of repairable machining system to provide additional repairmen, which would help in upgrading the service of multicomponent systems and reducing the congestion of failed units. In case the failed unit does not require additional repairmen, these can be removed thus ensuring economic feasibility of the model. The warm standbys are provided to replace the system for smooth running of the system. To reduce the balking and reneging behavior of the units as well as for improving the reliability and availability of the system, the additional repairmen are provided according to threshold rule so that the system may produce a desired quantity of production in a long run with reasonable economic constraints. By using degraded failure rate our model deals with more practical application for machining system. We have also incorporated the switching device failure, which makes the system realistic since there is possibility of failure of switching device which may effect the replacement of the failed units with the standbys.

	E(0)		E(I)		MA		RR		SR	
λ=.1	b=.2	b=.4	b=.2	b=.4	b=.2	b=.4	b=.2	b=.4	b=.2	b=.4
t=0	20	20	2	2	1	1	0	0	0.4	0.4
t=1	18.5	18.2	0.816	0.792	0.937	0.925	0.005	0.010	0.341	0.317
t=2	18.0	17.1	0.601	0.517	0.916	0.876	0.010	0.027	0.301	0.242
t=3	17.8	16.1	0.535	0.392	0.905	0.833	0.014	0.044	0.279	0.190
t=4	17.6	15.3	0.502	0.392	0.897	0.796	0.017	0.044	0.265	0.190
λ=.3	b=.2	b=.4	b=.2	b=.4	b=.2	b=.4	b=.2	b=.4	b=.2	b=.4
t=0	20	20	2	2	1	1	0	0	1.2	1.2
t=1	17.3	16.3	0.195	0.141	0.885	0.842	0.016	0.035	0.662	0.426
t=2	16.8	14.7	0.133	0.054	0.861	0.770	0.026	0.066	0.501	0.188
t=3	16.5	13.5	0.114	0.030	0.849	0.720	0.031	0.089	0.440	0.108
t=4	16.3	12	0.106	0.02	0.841	0.686	0.035	0.104	0.410	0.073

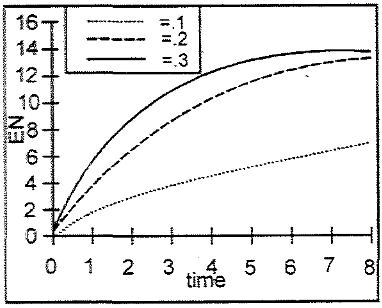
Table 1:' Various performance measures for different value of failure rate of operating machine (λ) and joining probability (b).

	E(O)		E (I)		MA		RR		SR	
λ=.1	q=.1	q=.2	q=.1	q=.2	q=.1	q=.2	q=.1	q=.2	q=.1	q=.2
t=0	20	20	2	2	1	1	0	0	0.2	0.4
t=1	18.5	18.2	1.073	1.000	0.937	0.923	0.010	0.015	0.168	0.314
t=2	17.2	16.6	0.838	0.732	0.881	0.854	0.031	0.041	0.136	0.238
t=3	15.9	15.1	0.681	0.560	0.826	0.769	0.053	0.067	0.111	0.183
t=4	14.8	13.8	0.559	0.433	0.775	0.734	0.074	0.090	0.091	0.142
λ=.3	q=.1	q=.2	q=.1	q=.2	q=.1	q=.2	q=.1	q=.2	q=.1	q=.2
t=0	20	20	2	2	1	1.00	0	0	0.6	1.2
t=1	15.7	15.3	0.239	0.199	0.813	0.797	0.050	0.057	0.198	0.330
t=2	12.9	12.5	0.079	0.059	0.691	0.676	0.103	0.110	0.070	0.107
t=3	11.1	10.9	0.033	0.023	0.616	0.605	0.136	0.141	0.030	0.045
t=4	10.1	10.0	0.016	0.011	0.573	0.566	0.156	0.159	0.015	0.023

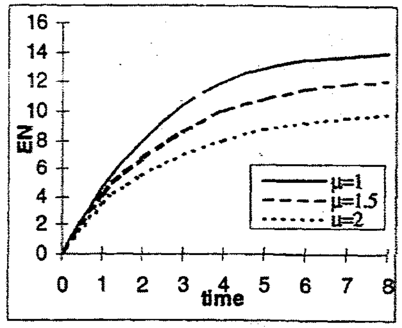
Table 2: Various performance measures for different value of failure rate of operating machine (λ) and switching device failure probability (q).

	E(O)		E (I)		MA		RR		SR	
λ=.2	α =.05	α =.1	α =.05	α =.1	α =.05	α =.1	α =.05	α =.1	α =.05	α =.1
t=0	20	20	2	2	1	1	0	0	0.4	0.4
t=1	17.4	17.3	0.636	0.614	0.888	0.886	0.023	0.024	0.252	0.248
t=2	15.7	15.6	0.400	0.382	0.813	0.809	0.053	0.055	0.165	0.160
t=3	14.4	14.3	0.281	0.266	0.758	0.755	0.076	0.077	0.118	0.114
t=4	13.6	13.5	0.211	0.199	0.721	0.718	0.092	0.093	0.069	0.066
λ=.3	α =.05	α =.1	α =.05	α =.1	α =.05	α =.1	α =.05	α =.1	α =.05	α =.1
t=0	20	20	2	2	1	1	0	0	0.4	0.4
t=1	16.3	16.2	0.326	0.315	0.840	0.838	0.040	0.040	0.247	0.242
t=2	14.2	14.2	0.155	0.149	0.751	0.750	0.077	0.077	0.125	0.121
t=3	13.1	13.0	0.094	0.090	0.700	0.698	0.099	0.100	0.078	0.076
t=4	12.4	12.4	0.066	0.063	0.671	0.670	0.112	0.113	0.056	0.055

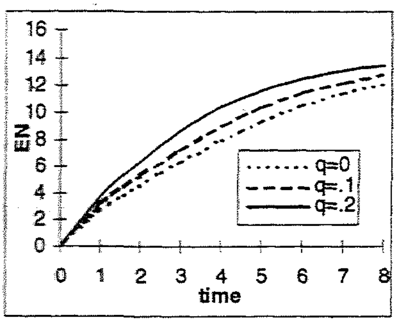
Table 3: Various performance measures for different value of failure rate of operating machine (λ) failure rate of warm standbys (α



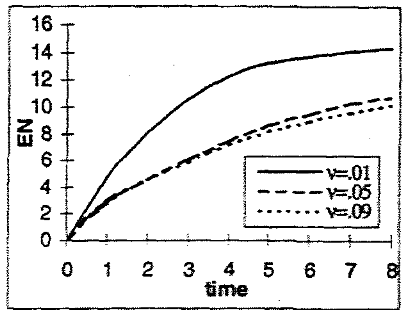
(a)



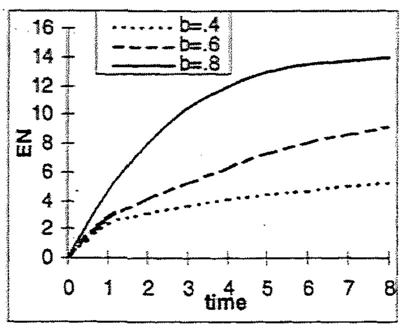
(b)



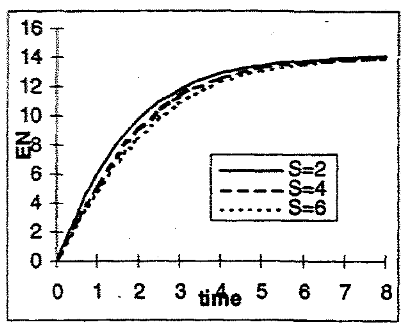
(b)



(d)



(e)



(f)

Figure 1: Expected number of failed units vs. time for different values of
(a) λ (b) μ (c) q (d) ν (e) b (f) S.

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