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APPLICATIONS OF THE GENERALIZED POLYNOMIALS OF SEVERAL VARIABLES OF SRIVASTAVA AND MULTIVARIABLE *H*-FUNCTION OF SRIVASTAVA-PANDA IN BOUNDARY VALUE PROBLEMS

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ABSTRACT

In the present paper, we shall employ, multivariable H-function of Srivastava and Panda ([14],[15],[16]; also see Srivastava, Gupta and Goyal [17]) and generalized polynomials of several variables of Srivastava [13] in two boundary value problems. First we evaluate an integral involving the product of multivariable H-function of Srivastava and Panda ([14],[15],[16]) and generalized polynomials of several variables of Srivastava [13], and then we make its applications to solve following two boundary value problems:

1. a boundary value problem on heat conduction in a finite bar and to establish an expansion formula involving the product of the above multivariable H-function and the generalized polynomials of several variables.
2. another boundary value problem on electrostatic potential in spherical region.

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Problem-1 A problem on Heat Conduction in a Finite bar.

1. Introduction. Chandel and Yadava [1] have discussed a problem on heat condction involving the multiple hypergeometric function of several variables of Srivastava and Daoust ([8],[9],[10]; also see Srivastva and Karlsson [12]). Srivastava, Gupta and Goyal [17] have discussed a problem on heat conduction in a finite bar using H-function of two variables of Srivastava and Panda ([14],[15],[16]; also see Srivastava, Gupta and Goyal [17]). Further Chandel and Gupta [2] have discussed this problem involving multiple hypergeometric function of Srivastava and Daoust ([8],[9],[10]; also see Srivastava and Karlsson [12]).

Here in the present paper, we discuss the same problem by employing the product of generalized polynomials of several variables of Srivastava [12] and the multivariable H-function of Srivastava and Panda ([14],[15],[16]; also see Srivatava, Gupta and Goyal [17]).

First we evaluate a new integral involving the product of above multivariable H-function of Srivastava and Panda [14],[15],[16] and generalized polynomials of several variables of Srivastava [13, p.185, eqn.(7)] defined by

$$(1.1)\quad S_{N_1,\ldots,N_r}^{M_1,\ldots,M_r}(x_1,\ldots,x_r)=\sum_{s_1=0}^{[N_1/M_1]}\cdots\sum_{s_r=0}^{[N_r/M_r]}A[N_1,s_1;\ldots;N_r,s_r]$$

$$\frac{(-N_1)_{M_1 s_1}}{s_1!}\cdots\frac{(-N_r)_{M_r s_r}}{s_r!}.x_1^{s_1}\ldots x_r^{s_r}$$

where $N_1,\ldots N_r:M_1,\ldots,M_r$ are arbitrary positive integers and the coefficients $A[N_1,s_1;\ldots;N_r,s_r]$ are arbitrary parameters real or complex independent of $x_1,\ldots,x_r$. Then we make its applications to solve the problem on heat conduction in a finite bar and to establish an expansion formula involving the product of above generalized polynomials of several variables [13] and the multivariable H-function of Srivastava and Panda [14],[15],[16].

2. Main Integral. In this section, we avaluate the integral

$$(2.1)\quad \int_{-1}^{1}(1-x)^{\alpha-1}(1+x)^{\beta-1}P_m(x)H_{A,C:[B',D'];\ldots;[B^{(n)},D^{(n)}]}^{0,\lambda:(\mu',\upsilon');\ldots;(\mu^{(n)},\upsilon^{(n)})}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]:\\ [(c):\psi',\ldots,\psi^{(n)}]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'],\ldots,[(b^{(n)}):\phi^{(n)}];\\ [(d'):\delta'],\ldots,[(d^{(n)}):\delta^{(n)}];\end{matrix}\,y_1(1-x)^{\rho_1}(1+x)^{\sigma_1},\ldots,y_n(1-x)^{\rho_n}(1+x)^{\sigma_n}\right)$$

$$S_{N_1,\ldots,N_r}^{M_1,\ldots,M_r}\left[z_1(1-x)^{\xi_1}(1+x)^{\eta_1},\ldots,z_r(1-x)^{\xi_r}(1+x)^{\eta_r}\right]\,dx$$

$$=2^{\alpha+\beta-1}\sum_{p=0}^{m}\frac{(-m)_p(m+1)_p}{(1)_p\Gamma p}2^p\sum_{k_1=0}^{[N_1/M_1]}\cdots\sum_{k_r=0}^{[N_r/M_r]}A[N_1,k_1;\ldots;N_r,k_r]$$

$$\prod_{i=1}^{r}\frac{(-N_i)_{M_i k_i}}{k_i!}2^{(\xi_i+\eta_i)k_i}z^{k_i}H_{A+2,C+1:[B',D'];\ldots;[B^{(n)},D^{(n)}]}^{0,\lambda+2:(\mu',\upsilon');\ldots;(\mu^{(n)},\upsilon^{(n)})}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}],\\ [(c):\psi',\ldots,\psi^{(n)}],\end{matrix}\right.$$

$$\left[1-\alpha-p-\xi_1k_1-\ldots-\xi_rk_r:\rho_1,\ldots,\rho_n\right],\left[1-\beta-\eta_1k_1-\ldots-\eta_rk_r:\sigma_r,\ldots,\sigma_n\right]:$$
$$\left[1-\alpha-\beta-p-(\xi_1+\eta_1)k_1-\ldots-(\xi_r+\eta_r)k_r:\rho_1+\sigma_1,\ldots,\rho_n+\sigma_n\right]:$$

$$\left.\begin{matrix}\left[(b'):\phi'\right];\ldots;\left[\left(b^{(n)}\right):\phi^{(n)}\right];\\ \left[(d'):\delta'\right];\ldots;\left[\left(d^{(n)}\right):\delta^{(n)}\right];\end{matrix}\; 2^{\rho_1+\sigma_1}y_1,\ldots,2^{\rho_n+\sigma_n}y_n\right),$$

where $Re(\alpha)>0$, $Re(\beta)>0$, all ρ_i,σ_i,y_i $(i=1,\ldots,n)$ are positive real numbers; N_j,M_j $(j=1,\ldots,r)$ are positive integers and the coefficients $A[N_1,k_1;\ldots;N_r,k_r]$ are arbitrary parameters real or complex independent of $z_1,\ldots,z_r,x;P_k(x)$ are Legendre polynomials while

$$H^{0,\lambda:(\mu',\upsilon');\ldots;\left(\mu^{(n)},\upsilon^{(n)}\right)}_{A,C:[B',D'];\ldots;\left[B^{(n)},D^{(n)}\right]}$$

is multivariable H-function of Srivastava and Panda ([14],[15],[16]; also see Srivastava, Gupta and Goyal [17]);

$$\left|arg\left(y_i(1-x)^{\rho_i}(1+x)^{\sigma_i}\right)\right|<\frac{\pi}{2}\Delta_i,$$

where

$$\Delta_i=\sum_{j=1}^{\lambda}\theta_j^{(i)}-\sum_{j=\lambda+1}^{A}\theta_j^{(i)}+\sum_{j=1}^{\upsilon^{(i)}}\phi_j^{(i)}-\sum_{j=\upsilon^{(i)}+1}^{B^{(i)}}\phi_j^{(i)}-\sum_{j=1}^{C}\psi_j^{(i)}+\sum_{j=1}^{\mu^{(i)}}\delta_j^{(i)}-\sum_{j=\mu^{(i)}+1}^{D^{(i)}}\delta_j^{(i)}>0,\; i=1,\ldots,n\,.$$

The above integral will be quite useful in our further investigations.
Proof. The left hand side of (2.1)

$$=\sum_{k_1=0}^{[N_1/M_1]}\cdots\sum_{k_r=0}^{[N_r/M_r]}A[N_1,k_1;\ldots;N_r,k_r]\prod_{i=1}^{r}(-N_i)_{M_ik_i}\frac{z^{k_i}}{k!_i},\ldots,\frac{z^{k_r}}{k_r!}\frac{1}{(2\pi\omega)^n}\int_{L_1}\cdots\int_{L_n}$$

$$\prod_{i=1}^{n}\frac{\prod_{j=1}^{\mu^{(i)}}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)\prod_{j=1}^{\upsilon^{(i)}}\Gamma\left(1-b_j^{(i)}+\phi_j^{(i)}s_i\right)}{\prod_{j=\mu^{(i)}+1}^{D^{(i)}}\Gamma\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)\prod_{j=\upsilon^{(i)}+1}^{B^{(i)}}\Gamma\left(b_j^{(i)}-\phi_j^{(i)}s_i\right)}\frac{\prod_{j=1}^{\lambda}\Gamma\left(1-a_j+\sum_{i=1}^{n}\theta_j^{(i)}s_i\right)}{\prod_{j=\lambda+1}^{A}\Gamma\left(a_j-\sum_{i=1}^{n}\theta_j^{(i)}s_i\right)\prod_{j=1}^{C}\Gamma\left(1-c_j+\sum_{i=1}^{n}\psi_j^{(i)}s_i\right)}$$

$$y_1^{s_i}\ldots y_n^{s_n}ds_1\ldots ds_n\int_{-1}^{1}P_m(x)(1-x)^{\alpha-1+\rho_1s_1+\ldots+\rho_ns_n+\xi_1k_1+\ldots+\xi_rk_r}(1+x)^{\beta-1+\sigma_1s_1+\ldots+\sigma_ns_n+\eta_1k_1+\ldots+\eta_rk_r}dx$$

which by making an appeal to Erdélyi [7,p. 276(6)]

$$\int_{-1}^{1}(1-x)^{\alpha-1}(1+x)^{\beta-1}P_n(x)dx = 2^{\alpha+\beta-1}B(\alpha,\beta)\,{}_3F_2\left[\begin{matrix}-n,n+1,\alpha; \\ 1,1+\beta;\end{matrix}\ 1\right]$$

$$=\sum_{k=0}^{n}\frac{(-n)_k(n+1)_k\Gamma(\alpha+k)}{k!(1)_k\Gamma(\alpha+\beta+k)}, Re(\alpha)>0, Re(\beta)>0\,,$$

gives right hand side of (2.1)

3 Problem. In this section, we consider the problem of determinig a function $u(x,t)$ representing the temperature in a nonhomogeneous bar with ends at x=-1 and x=1 in which the thermal conductivity is proportional to $(1-x^2)$. Let the lateral surface of the bar be insulated. Thus our problem reduces to solve the equation of heat conduction in one dimension.

(3.1) $$\frac{\partial u}{\partial t} = k\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial u}{\partial x}\right],$$

where k and thermal coefficients both are constants.

4. Solution of the Problem. Boundry conditions of the problem are that both ends of the bar at $x=\pm 1$ are insulated so that conductivity vanishes there and the initial condition is

(4.1) $u(x,0)=f(x)$.

Here we may assume the solution of the problem (3.1) in the form:

(4.2) $$u(x,t)=\sum_{n=0}^{\infty}A_n e^{-kn(n+1)t}P_n(x)$$

which is quite justified and for t=0, reduces to

(4.3) $$u(x,0)=f(x)=\sum_{n=0}^{\infty}A_nP_n(x).$$

Now in (4.3), we may choose

(4.4) $$f(x)=(1-x)^{\alpha-1}(1+x)^{\beta-1}H^{0,\lambda:(\mu',\upsilon');\ldots;(\mu^{(n)},\upsilon^{(n)})}_{A,C:[B',D'];\ldots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]: \\ [(c):\psi',\ldots,\psi^{(n)}]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\ y_1(1-x_1)^{\rho_1}(1+x)^{\sigma_1},\ldots,y_n(1-x)^{\rho_n}(1+x)^{\sigma_n}\right)$$

$$S_{N_1,\ldots,N_r}^{M_1,\ldots,M_r}\left[z_1(1-x)^{\xi_1}(1+x)^{\eta_1},\ldots,z_r(1-x)^{\xi_r}(1+x)^{\eta_r}\right]\,dx\,.$$

Therefore, by making an appeal to (4.3) and (4.4), we have

$$f(x)=(1-x)^{\alpha-1}(1+x)^{\beta-1}H^{0,\lambda:(\mu',\upsilon');\ldots;(\mu^{(n)},\upsilon^{(n)})}_{A,C:[B',D'];\ldots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]:\\ [(c):\psi',\ldots,\psi^{(n)}]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}];\\ [(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\;y_1(1-x_1)^{\rho_1}(1+x)^{\sigma_1},\ldots,y_n(1-x)^{\rho_n}(1+x)^{\sigma_n}\right)$$

$$S_{N_1,\ldots,N_r}^{M_1,\ldots,M_r}\left[z_1(1-x)^{\xi_1}(1+x)^{\eta_1},\ldots,z_r(1-x)^{\xi_r}(1+x)^{\eta_r}\right]$$

$$=\sum_{n=0}^{\infty}A_nP_n(x).$$

Now making an appeal to the orthogonal property of Legendre polynomials Erdélyi (7, p. 277 (13)); we derive

$$\text{(4.5)}\quad A_m=(2m+1)2^{\alpha+\beta-2}\sum_{p=0}^{m}\frac{(-m)_p(m+1)_p}{(p!)^2}2^{\rho}\sum_{k_1=0}^{[N_1/M_1]}\cdots\sum_{k_r=0}^{[N_r/M_r]}A[N_1,k_1;\ldots;N_r,k_r]$$

$$\prod_{i=1}^{r}(-N_i)_{M_ik_i}\frac{2^{(\xi_i+\eta_i)k_i}z_i^{k_i}}{k_i!}H^{0,\lambda+2:(\mu',\upsilon');\ldots;(\mu^{(n)},\upsilon^{(n)})}_{A+2,C+1:[B',D'];\ldots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}],\\ [(c):\psi',\ldots,\psi^{(n)}],\end{matrix}\right.$$

$$\begin{matrix}[1-\alpha-p-\xi_1k_1-\ldots-\xi_rk_r:\rho_1,\ldots,\rho_n],[1-\beta-\eta_1k_1-\ldots-\eta_rk_r:\sigma_r,\ldots,\sigma_n]:\\ [1-\alpha-\beta-p-(\xi_1+\eta_1)k_1-\ldots-(\xi_r+\eta_r)k_r:\rho_1+\sigma_1,\ldots,\rho_n+\sigma_n]:\end{matrix}$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}];\\ [(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\;2^{\rho_1+\sigma_1}y_1,\ldots,2^{\rho_n+\sigma_n}y_n\right),$$

which is valid if all conditions of (2.1) are satisfied.
Now substituting the value of A_n from (4.5) in (4.2), we obtain the following required solution of the problem :

$$\text{(4.6)}\quad u(x,t)=2^{\alpha+\beta-2}\sum_{m=0}^{\infty}(2m+1)e^{-m(m+1)kt}P_m(x)\sum_{p=0}^{m}\sum_{k_1=0}^{[N_1/M_1]}\cdots\sum_{k_r=0}^{[N_r/M_r]}A[N_1,k_1;\ldots;N_r,k_r]$$

$$\prod_{i=1}^{r}(-N_i)_{M_i k_i}\frac{2^{(\xi_i+\eta_i)^{k_i}}z_i^{k_i}}{k_i!}H^{0,\lambda+2:(\mu',\upsilon');\ldots;(\mu^{(n)},\upsilon^{(n)})}_{A+2,C+1:[B',D'];\ldots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}],\\ [(c):\psi',\ldots,\psi^{(n)}],\end{matrix}\right.$$

$$\begin{matrix}[1-\alpha-p-\xi_1k_1-\ldots-\xi_rk_r:\rho_1,\ldots,\rho_n],[1-\beta-\eta_1k_1-\ldots-\eta_rk_r:\sigma_r,\ldots,\sigma_n]:\\ [1-\alpha-\beta-p-(\xi_1+\eta_1)k_1-\ldots-(\xi_r+\eta_r)k_r:\rho_1+\sigma_1,\ldots,\rho_n+\sigma_n]:\end{matrix}$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}];\\ [(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}2^{\rho_1+\sigma_1}y_1,\ldots,2^{\rho_n+\sigma_n}y_n\right),$$

valid if all the conditions of (2.1) are satisfied.

5. Expansion Formula. Now making an appeal to (4.3), (4.4) and (4.5), we derive the following expansion formula:

(5.1)
$$(1-x)^{\alpha-1}(1+x)^{\beta-1}H^{0,\lambda:(\mu'\upsilon');\ldots;(\mu^{(n)},\upsilon^{(n)})}_{A,C:[B',D'];\ldots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta^1,\ldots,\theta^{(n)}]:\\ [(c):\psi',\ldots,\psi^{(n)}]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}];\\ [(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}y_1(1-x_1)^{\rho_1}(1+x)^{\sigma_1},\ldots,y_n(1-x)^{\rho_n}(1+x)^{\sigma_n}\right)$$

$$S^{M_1,\ldots,M_r}_{N_1,\ldots,N_r}\left[z_1(1-x)^{\xi_1}(1+x)^{\eta_1},\ldots,z_r(1-x)^{\xi_r}(1+x)^{\eta_r}\right]\,dx$$

$$=2^{\alpha+\beta-2}\sum_{m=0}^{\infty}\sum_{p=0}^{m}(2m+1)\rho_m(x)\frac{(-m)_p(m+1)_p}{(p!)^2}\sum_{k_1=0}^{[N_1/M_1]}\cdots\sum_{k_r=0}^{[N_r/M_r]}A[N_1,k_1;\ldots;N_r,k_r]$$

$$\prod_{i=1}^{r}\frac{(-N_i)_{M_i k_i}}{k_i!}2^{(\xi_i+\eta_i)^{k_i}}z_i^{k_i}H^{0,\lambda+2:(\mu',\upsilon');\ldots;(\mu^{(n)},\upsilon^{(n)})}_{A+2,C+1:[B',D'];\ldots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}],\\ [(c):\psi',\ldots,\psi^{(n)}],\end{matrix}\right.$$

$$\begin{matrix}[1-\alpha-p-\xi_1k_1-\ldots-\xi_rk_r:\rho_1,\ldots,\rho_n],[1-\beta-\eta_1k_1-\ldots-\eta_rk_r:\sigma_r,\ldots,\sigma_n]:\\ [1-\alpha-\beta-p-(\xi_1+\eta_1)k_1-\ldots-(\xi_r+\eta_r)k_r:\rho_1+\sigma_1,\ldots,\rho_n+\sigma_n]:\end{matrix}$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}];\\ [(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}2^{\rho_1+\sigma_1}y_1,\ldots,2^{\rho_n+\sigma_n}y_n\right),$$

provided that all the conditions of (2.1) are satisfied.

Problem 2.

6. A Problem on Electrostatic Potential in Spherical Region. Chandel, Agrawal and Kumar [3] have discussed a problem involving multivariable *H*-funciton of Srivastava and Panda ([14],[15],[16]; also see Srivastava, Gupta and Goyal [17]). Recently Chandel and Sengar [3] have employed the multivariable *H*-function of Srivastava-Panda and the general class of polynomials of Srivastava [11] in two boundary value problems on

I. Heat conduction in a rod.

2. Homogeneous wave problem.

Very recently, Chandel and Singh [4] evaluated an integral involving multivariable polynomials of Srivastava [13] and multivariable *H*-function of Srivartava-Panda ([14],[15],[16]) then made its applications to solve two boundary value problems on

I Heat conduction in a rod

II Deflection of vibrating string under certain conditions.

Here in this section, we shall make applications of the multivariable *H*-function of Srivastava-Panda ([14],[15],[16]) and the generalized polynomials of several variables of Srivastava [13] to obtain the harmonic function V reprresenting the electrostatic potential in the domain $R<c$ such that V assumes a prescribed value $F(\theta)$ on the spherical surface $R=c$, where R,θ,ϕ are the spherical polar coordinates and V is independent of ϕ. Thus V satisfies Laplace equaiton

$$(6.1) \quad R\frac{\partial^2(rV)}{\partial R^2}+\frac{1}{sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial V}{\partial\theta}\right)=0$$

in the domain $R<c$, $0\leq\theta<\pi$ and under the condition

$$(6.2) \quad \lim_{R\to c} V=F(\theta),(0\leq\theta<\pi,R=c)$$

where V and its derivatives of first and second orders are assumed to be continuous throughout the interior of the sphere :

$$0\leq R<c, 0\leq\theta<\pi.$$

Physically, the function V may represent steady temperature in a solid sphere $R\leq c$, whose surface temperature depends only on θ i.e. the surface temperature is uniform over each circle $\theta=\theta_0$, $R=c$. Here V also denote electrostatic potential in the surface $R<c$ free of charges, for $V=F(\theta)$ on the boundary $R=c$.

If we take $cos\theta=x$ ($0\leq\theta<\pi$), the equation (6.1) reduces to

(6.3) $$R\frac{\partial^2(RV)}{\partial R^2}+\frac{\partial}{\partial x}\left[\left(1-x^2\right)\frac{\partial V}{\partial x}\right]=0, \quad R<c,-1<x<1.$$

If we further take $F(\theta)=f(\cos\theta)=f(x)$, then $V(R,x)$ satisfies the transformed equation (6.3), with the boundary conditions

(6.4) $$\lim_{R\to c} V(R,x)=f(x) \qquad (R<c,-1<x<1),$$

where V is continuous every where interior to the sphere and bounded when $0\le R<R_0<c$.

(6.5) $$\lim_{R\to\infty} W(R,x)=0,$$

where W is harmonic function in the bounded domain $R>c$, exterior to the spherical surface and RV is bounded for large value of R and for all $x(-1\le x\le 1)$.

7. Formal Solution of the Problem. We shall obtain formal solution of the above boundary value problem for

(7.1) $$(1-x)^{\alpha-1}(1+x)^{\beta-1}H^{0,\lambda:(\mu',\upsilon');\dots;(\mu^{(n)},\upsilon^{(n)})}_{A,C:[B',D'];\dots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]: \\ [(c):\psi',\dots,\psi^{(n)}]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix} y_1(1-x_1)^{\rho_1}(1+x)^{\sigma_1},\dots,y_n(1-x)^{\rho_n}(1+x)^{\sigma_n}\right)$$

$$S^{M_1,\dots,M_r}_{N_1,\dots,N_r}\left[z_1(1-x)^{\xi_1}(1+x)^{\eta_1},\dots,z_r(1-x)^{\xi_r}(1+x)^{\eta_r}\right],$$

where $H^{0,\lambda:(\mu',\upsilon');\dots;(\mu^{(n)},\upsilon^{(n)})}_{A,C:[B',D'];\dots;[B^{(n)},D^{(n)}]}$ is multivariable H-function of Srivastava and Panda ([14,15,16]) and $S^{M_1,\dots,M_r}_{N_1,\dots,N_r}(z_1,\dots,z_r)$ are Srivvastava's generzliazed polynomials of several variables [13] and $Re(\alpha)=0$, $Re(\beta)=0$, all $\lambda_1,\dots,\lambda_n$; $\mu_1,\dots,\mu_n$, $y_1,\dots,y_n$ are positive real numbers; $f(x)$ and $f'(x)$ both are assumed to be sectionally continuous over the interval $(-1,1)$.

Case 1. Solution for $V(R,x)$ when $R<c$. (Interior to the sphere)

In this case solution of the problem is given by

(7.2) $V(R_1x)=\sum_{m=0}^{\infty}A_m(R/c)^m P_m(x) \qquad (R<c),$

which by appeal to (6.4) reduces to

(7.3) $f(x)=\sum_{m=0}^{\infty}A_m P_m(x) \qquad (R<c),$

where A_m is given by (4.5) .

Substituting the value of A_m in (6.2), we derive the following required solution of the problem

$$(7.4) \quad V(R,x)=2^{\alpha+\beta-2}\sum_{m=0}^{\infty}(2m+1)(R/c)^m P_m(x)\sum_{p=0}^{m}\sum_{s_1=0}^{[N_1/M_1]}\cdots\sum_{s_r=0}^{[N_r/M_r]}\frac{(-m)_p(m+1)_p}{(1)_p\, p!}2^p$$

$$\prod_{i=1}^{r}(-N_i)_{M_i s_i}\frac{z_i^{s_i}2^{(\xi_i+\eta_i)s_i}}{s_i!}H^{0,\lambda+2:(\mu',\upsilon');\ldots;(\mu^{(n)},\upsilon^{(n)})}_{A+2,C+1:[B',D'];\ldots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}],\\ [(c):\psi',\ldots,\psi^{(n)}],\end{matrix}\right.$$

$$\begin{matrix}[1-\alpha-p-\xi_1k_1-\ldots-\xi_rk_r:\rho_1,\ldots,\rho_n],[1-\beta-\eta_1k_1-\ldots-\eta_rk_r:\sigma_r,\ldots,\sigma_n]:\\ [1-\alpha-\beta-p-(\xi_1+\eta_1)k_1-\ldots-(\xi_r+\eta_r)k_r:\rho_1+\sigma_1,\ldots,\rho_n+\sigma_n]:\end{matrix}$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}]\\ [(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}]\end{matrix}\; 2^{\rho_1+\sigma_1}y_1,\ldots,2^{\rho_n+\sigma_n}y_n\right),$$

provided that all conditions of (2.1) are satisfied and $R<c$.

Case II. Solution for $\omega(R,x)$ when $R \geq c$. (exterior to the spherical surface)

In this case, in view of Churchil [6, p. 219, eqn. (12)], we can write the following solution of the problem:

(7.5) $\omega(R,x)=\sum_{m=0}^{\infty}A_m(c/R)^{m+1}P_m(x),(R\geq c).$

Substituting the value of A_m in (7.5), we derive the following solution of the problem

$$(7.6) \quad \omega(R,x)=2^{\alpha+\beta-2}\sum_{m=0}^{\infty}(2m+1)(c/R)^{(m+1)}P_m(x)\sum_{p=0}^{m}\frac{(-m)_p(m+1)_p 2^p}{(p!)^2}$$

$$\sum_{s_1=0}^{[N_1/M_1]}\cdots\sum_{s_r=0}^{[N_r/M_r]}A(N_1,s_i;\ldots;N_r,s_r)\prod_{i=1}^{r}(-N_i)_{M_i s_i}\frac{z_i^{s_i}2^{(\xi_i+\eta_i)s_i}}{s_i!}H^{0,\lambda+2:(\mu',\upsilon');\ldots;(\mu^{(n)},\upsilon^{(n)})}_{A+2,C+1:[B',D'];\ldots;[B^{(n)},D^{(n)}]}$$

$$\left(\begin{matrix} [(a):\theta',\ldots,\theta^{(n)}],[1-\alpha-p-\xi_1 k_1-\ldots-\xi_r k_r:\rho_1,\ldots,\rho_n], \\ [(c):\psi',\ldots,\psi^{(n)}],[1-\alpha-\beta-p-(\xi_1+\eta_1)k_1-\ldots-(\xi_r+\eta_r)k_r:\rho_1+\sigma_1,\ldots,\rho_n+\sigma_n]: \end{matrix} \right.$$

$$\left. \begin{matrix} [1-\beta-\eta_1 k_1-\ldots-\eta_r k_r:\sigma_r,\ldots,\sigma_n]:[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}]; \end{matrix} 2^{\rho_1+\sigma_1}y_1,\ldots,2^{\rho_n+\sigma_n}y_n \right),$$

where all conditions of (2.1) are satisfied and $R \geq c$.

REFERENCES

[1] R.C.S. Chandel and H.C. Yadava, Heat conduction and multiple hypergeometric functions of Srivastava and Daoust, *Indian J. Pure Appl. Math.*, 15(4) (1984), 371-376.

[2] R.C.S. Chandel and A.K. Gutpa, A problem on heat conduction in a finite bar., *Jour. Maulana Azad College Tech.* **19** (1986), 91-95.

[3] R.C.S. Chandel, R.D. Agrawal and H. Kumar, The multivariable H-function of Srivastava and Panda and its applications in a problem on a electrostatic potential in spherical region, *Jour. Maulana Azad College Tech.*, **23** (1990), 39-46.

[4] R.C.S. Chandel and S. Sengar, On two boundary value problems, *Jn̄ānābha* , **31/32** (2002), 89-104.

[5] R.C.S. Chandel and Y. Singh, Two boundary value problems, *Indian J. of Theoretical Physics*, **53** (4) (2005), 339-350.

[6] R.V. Churchil, *Fourier Series and Boundary Value Problems,* McGraw Hill, NewYork 1961.

[7] A. Erdélyi, *Tables of Integral Transforms,* 2, Mc Graw Hill Book Company, 1954.

[8] H.M. Srivsatava and M.C. Daoust, On Eulerian integrals associated with Kampé de Fériet's function, *Publ. Inst. Math. (Beograd) (N.S.)* **9 23** (1969), 199-202.

[9] H.M. Srivastava and M.C. Daoust, Certain generalized Neumann expansions associated with the Kampé de Fériet function. *Nederl. Akad. Wetensch, Proc. Ser., A* **72** Indag Math., **31** (1969), 449.

[10] H.M. Srivastava and M.C. Daoust, A note on the convergence of Kampé de Fériet's double hypergeometric series, *Math. Nachr,* **53** (1972), 151-159.

[11] H.M. Srivastava, A contour integral involving fox's H-function, *Indian J. Math.,* **14** (1972), 1-6.

[12] H.M. Srivastava and P.W. Karlsson, *Multiple Gaussain Hypergeometric Series.* John Wiley and Sons, New York, 1985.

[13] H.M. Srivastava, A multilinear generating function for the Konhauser sets of biorthogonal polynomials suggested by Laguerre polynomails, *Pacific J. Math.,* **117** (1985), 183-191.