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AN INTEGRAL PERTAINING TO CERTAIN PRODUCT OF SPECIAL FUNCTIONS

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ABSTRACT

This paper presents an integral involving the product of the multivariable *H*-funciton, Fox's *H*-funciton and a general class of polynomials of one variable and multivariable with essentially arbitrary coefficients by assigning suitable values to these coefficients, our result can be reduced to integrals involving, for instance the classical polynomials of Jacobi, Laguerre and Hermite. Also since a large variety of functions that occur frequently in problems of analysis, both pure and applied and mathematical physics are only special cases of the *H*-funciton. On the other hand, the multivariable *H*-function occurring in our main result can be reduced under various particular casas, to such simpler functions as the generalized Lauricella' s hypergeometric functions of several complex variables, *G*-function product of Fox's *H*-function, which indeed include a great many of useful functions of hypergeometric type as their special cases.

The results obtained in this paper are of a general character and hence encompass several cases of interests.

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1. Introduction. The general class of polynomials defined by Srivastava [3,p.158, eqn. (1.1)]

$$S_n^m(t)=\sum_{\alpha=0}^{[n/m]}\frac{(-n)_{m\alpha}}{\alpha!}A[n,\alpha]t^{\alpha},\qquad n=0,1,2,\ldots \qquad \ldots(1.1)$$

where m is an arbitrary positive integer, the coefficients $A[n,\alpha]$ $(n,\alpha \geq 0)$ are arbitrary constants, real or complex.

The multivariable polynomials given by Srivastava [4,p.185, eq. (7)] defined and represented in the following form

$$S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}[t_1,\ldots,t_k]=\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_k=0}^{[n_k/m_k]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_k)_{m_k\alpha_k}}{\alpha_k!}A'[n_1,\alpha_1;\ldots;n_k,\alpha_k]t_1^{\alpha_1}\ldots t_k^{\alpha_k}, \quad \ldots(1.2)$$

where $n_i=0,1,2,\ldots,m_i \neq 0$ $(i=1,2,\ldots,k)$, m_i is an arbitrary positive integer, the coefficients $A'[n_1,\alpha_1;\ldots;n_k,\alpha_k]$ being arbitrary constants, real or complex.
The series representation of Fox's H-function [1,p.239-341]

$$H_{P,Q}^{M,N}\left[z\left|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right.\right]=\sum_{G=0}^{\infty}\sum_{g=1}^{M}\frac{(-1)^G}{G!F_g}\phi(\eta_G)z^{\eta_G}, \quad \ldots(1.3)$$

where

$$\phi(\eta_G)=\frac{\prod_{j=1,j\neq g}^{M}\Gamma(f_j-F_j\eta_G)\prod_{j=1}^{N}\Gamma(1-e_j+E_j\eta_G)}{\prod_{j=M+1}^{Q}\Gamma(1-f_j+F_j\eta_G)\prod_{j=N+1}^{P}\Gamma(e_j+E_j\eta_G)} \quad \text{and} \quad \eta_G=\frac{(f_g+G)}{F_g}.$$

The H-function of several complex variables is defined by Srivastava and Panda [5,p.265-274] as:

$$H[z_1,\ldots,z_r]=H_{A,C:[B',D'],\ldots,[B^{(r)},D^{(r)}]}^{0,\lambda:[u',v'],\ldots,[u^{(r)},v^{(r)}]}\left[\begin{matrix}[(a):\theta',\ldots,\theta^r]:[b':\phi'];\ldots;[b^r,\phi^r]\\[(c):\psi',\ldots,\psi^r]:[d':\delta'];\ldots;[d^r,\delta^r]\end{matrix};z_1,\ldots,z_r\right] \quad \ldots(1.4)$$

The H-function of several complex variables in (1.4) converges absolutely if

$$|arg(z_i)|<\pi T_i/2. \quad \ldots(1.5)$$

where

$$T_i=\sum_{j=1}^{\lambda}\theta_j^{(i)}-\sum_{j=\lambda+1}^{A}\theta_j^{(i)}+\sum_{j=1}^{V^{(i)}}\phi_j^{(i)}-\sum_{j=V^{(i)}+1}^{B^{(i)}}\phi_j^{(i)}-\sum_{j=1}^{C}\Psi_j^{(i)}+\sum_{j=1}^{u^{(i)}}\delta_j^{(i)}-\sum_{j=u^{(i)}+1}^{D^{(i)}}\delta_j^{(i)}>0\ \forall i\in(1,\ldots,r) \ldots(1.6)$$

2. The Main Integral. The following integral had been derived in this section :

$$\int_0^{\infty}x^{1-\gamma}(a+bx+cx^2)^{\gamma-3/2}H_{P,Q}^{M,N}\left[z\left(\frac{x}{a+bx+cx^2}\right)^{\sigma}\left|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right.\right]S_n^m\left[w\left(\frac{x}{a+bx+cx^2}\right)^{\beta}\right]$$

$$.S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[w_1\left(\frac{x}{a+bx+cx^2}\right)^{\beta_1},\dots,w_k\left(\frac{x}{a+bx+cx^2}\right)^{\beta_k}\right]$$

$$H\left[z_1\left(\frac{x}{a+bx+cx^2}\right)^{\sigma_1},\dots,z_r\left(\frac{x}{a+bx+cx^2}\right)^{\sigma_r}\right]dx$$

$$=\sqrt{\frac{\pi}{c}}\sum_{G=0}^{\infty}\sum_{g=1}^{M}\sum_{\alpha=0}^{[n/m]}\sum_{\alpha_1=0}^{[n_1/m_1]}\dots\sum_{\alpha_k=0}^{[n_k/m_k]}\frac{(-1)^G}{G!F_g}\frac{(-n)_{m\alpha}}{\alpha!}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\dots\frac{(-n_k)_{m_k\alpha_k}}{\alpha_k!}\phi(\eta_G)z^{\eta_G}$$

$$A[n,\alpha]A'[n_1,\alpha_1;\dots;n_k,\alpha_k]w^{\alpha}w_1^{\alpha_1},\dots,w_k^{\alpha_K}\left(b+2\sqrt{ca}\right)^{\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i-1}$$

$$H_{A+1,C+1[B',D'];\dots;[B^{(r)},D^{(r)}]}^{0,\lambda+1[u',v'];\dots;[u^{(r)},v^{(r)}]}\left[\begin{matrix}z_1\left(b+2\sqrt{ca}\right)^{-\sigma_1}\\ \vdots\\ z_r\left(b+2\sqrt{ca}\right)^{-\sigma_r}\end{matrix}\left|\begin{matrix}\left[\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{K}\alpha_i\beta_i;\sigma_1\dots\sigma_r\right]\\ [(c):\psi',\dots,\psi^{(r)}]\end{matrix}\right.\right.$$

$$\left.\begin{matrix}[(a):\theta',\dots,\theta^{(r)}]:[b',\phi'],\dots,[b^{(r)},\phi^{(r)}]\\ \left[\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i-1/2;\sigma_1,\dots,\sigma_r\right]\ :[d',\delta'],\dots,[d^{(r)},\delta^{(r)}]\end{matrix}\right], \qquad \dots(2.1)$$

provided that $Re(a)>0$, $Re(b)>0$, $c>0$ and

$$\sigma\, min[Re(f_j/F_j)]+\sum_{i'=1}^{r}\sigma_{i'}\, min\left[Re\left(d_{j'}^{(i')}/\delta_{j'}^{(i')}\right)\right]>\alpha-2,\quad j=1,\dots,M \text{ and } j'=1,\dots,u^{(i')},$$

and m and $n_i=0,1,2,\dots,m_i\neq 0$ $(i=1,2,\dots,k)$ and m_i are arbitrary positive integers, the coefficients $A[n,\alpha]$ and A' $[n_1,\alpha_1;\dots;n_k,\alpha_k]$ being arbitrary constants, real or complex.

Proof. To establish (2.1), we first express the Fox's H-function and a general polynomial of one variable and multivariable in the form of series given by (1.3) and (1.1) and (1.2) and the H-function of several complex variables in terms of Mellin-Barnes contour integrals. Now interchanging the order of summations and integrations which is permissible under the stated conditions, we obtain

$$\sum_{G=0}^{\infty}\sum_{g=1}^{M}\sum_{\alpha=0}^{[n/m]}\sum_{\alpha_1=0}^{[n_1/m_1]}\dots\sum_{\alpha_k=0}^{[n_k/m_k]}\frac{(-1)^G}{G!F_g}\frac{(-n)_{m\alpha}}{\alpha!}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\dots\frac{(-n_k)_{m_k\alpha_k}}{\alpha_k!}\phi(\eta_G)z^{\eta_G}$$

$$A[n,\alpha]A'[n_1,\alpha_1;...;n_k,\alpha_k]w^{\alpha}w_1^{\alpha_1},...,w_k^{\alpha_K}\frac{1}{(2\pi i)^r}\int_{L_1}\cdots\int_{L_r}\psi(\xi_1,...,\xi_r)\phi_1(\xi_1)...$$

$$\phi_r(\xi_r)z_1^{\xi_1}...z_r^{\xi_r}\left\{\int_0^\infty x^{1-\left(\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i-\sigma_1\xi_1-...-\sigma_r\xi_r\right)}\right.$$

$$\left.(a+bx+cx^2)^{\left(\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i-\sigma_1\xi_1-...-\sigma_r\xi_r\right)-3/2}dx\right\}d\xi_1...d\xi_r. \qquad ...(2.2)$$

Evaluating the above inner x-integral with the help of a known theorem [Saxena (7)] and reinterpreting the result thus obtained in terms of H-function of r-variables, we arrived at the required result.

3. Applications and Particular Cases.

(I) Putting $\lambda=A$, $u^{(i)}=1$, $v^{(i)}=B^{(i)}$ and $D^{(i)}=D^{(i)}+1$, $\forall i\in(1,...,r)$ then from (2.1), we get

$$\int_0^\infty x^{1-\gamma}(a+bx+cx^2)^{\gamma-\frac{3}{2}}H_{P,Q}^{M,N}\left[z\left(\frac{x}{a+bx+cx^2}\right)^{\sigma}\middle|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right]S_n^m\left[w\left(\frac{x}{a+bx+cx^2}\right)^{\beta}\right]$$

$$.S_{n_1,...,n_k}^{m_1,...,m_k}\left[w_1\left(\frac{x}{a+bx+cx^2}\right)^{\beta_1},...,w_k\left(\frac{x}{a+bx+cx^2}\right)^{\beta_k}\right]$$

$$F_{C:D',...,D^{(r)}}^{A:B',...,B^{(r)}}\left[-z_1\left(\frac{x}{a+bx+cx^2}\right)^{\sigma_1},...,-z_r\left(\frac{x}{a+bx+cx^2}\right)^{\sigma_r}\middle|\begin{matrix}[1-(a):\theta',...\theta^{(r)}]:\\ [1-(c):\psi',...,\psi^{(r)}]:\end{matrix}\right.$$

$$\left.\begin{matrix}[1-(b'):\phi'];...;[1-(b^{(r)}):\phi^{(r)}]\\ [1-(d'):\delta'];...;[1-(d^{(r)}):\delta^{(r)}]\end{matrix}\right]dx$$

$$=\sqrt{\frac{\pi}{c}}\sum_{G=0}^{\infty}\sum_{g=1}^{M}\sum_{\alpha=0}^{[n/m]}\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_k=0}^{[n_k/m_k]}\frac{(-1)^G}{G!F_g}\frac{(-n)_{m\alpha}}{\alpha!}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_k)_{m_k\alpha_k}}{\alpha_k!}\phi(\eta_G)z^{\eta_G}$$

$$A[n,\alpha]A'[n_1,\alpha_1;...;n_k,\alpha_k]w^{\alpha}w_1^{\alpha_1}...w_k^{\alpha_K}(b+2\sqrt{ca})^{\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i-1}$$

$$\frac{\Gamma\left(1-\gamma+\sigma\eta_G+\alpha\beta+\sum_{i=1}^{k}\alpha_i\beta_i\right)}{\Gamma\left(\frac{3}{2}-\gamma+\sigma\eta_G+\alpha\beta+\sum_{i=1}^{k}\alpha_i\beta_i\right)} F^{A+1:B',\ldots,B^{(r)}}_{C+1:D',\ldots,D^{(r)}}\left[-z_1\left(b+2\sqrt{ca}\right)^{-\sigma_1};\ldots;-z_r\left(b+2\sqrt{ca}\right)^{-\sigma_r}\right|$$

$$\left.\begin{matrix}\left[1-\gamma+\sigma\eta_G+\alpha\beta+\sum_{i=1}^{k}\alpha_i\beta_i:\sigma_1,\ldots,\sigma_r\right],\left[1-(a):\theta',\ldots,\theta^{(r)}\right]:\left[1-(b'):\phi'\right];\ldots;\left[1-\left(b^{(r)}\right):\phi^{(r)}\right]\\ \left[1-(c):\psi'\ldots\psi^{(r)}\right],\left[\frac{3}{2}-\gamma+\sigma\eta_G+\alpha\beta+\sum_{i=1}^{k}\alpha_i\beta_i:\sigma_1,\ldots,\sigma_r\right]:\left[1-(d'):\delta'\right];\ldots;\left[1-\left(d^{(r)}\right):\delta^{(r)}\right]\end{matrix}\right] \quad ...(3.1)$$

provided that $Re\ (a)>0$, $Re(b)>0$, $c>0$ and the series on the right side exists.

(II) Putting $\theta',\ldots,\theta^{(r)}=\phi',\ldots,\phi^{(r)}=\psi',\ldots,\psi^{(r)}=\delta',\ldots,\delta^{(r)}=\sigma_1,\ldots,\sigma_r=\alpha',\ldots,\alpha^{(r)}$ in (2.1), then we get the following integral transformation :

$$\int_0^\infty x^{1-\gamma}\left(a+bx+cx^2\right)^{\gamma-3/2} H^{M,N}_{P,Q}\left[z\left(\frac{x}{a+bx+cx^2}\right)^{\sigma}\middle|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right] S^m_n\left[w\left(\frac{x}{a+bx+cx^2}\right)^{\beta}\right]$$

$$.\,S^{m_1,\ldots,m_k}_{n_1,\ldots,n_k}\left[w_1\left(\frac{x}{a+bx+cx^2}\right)^{\beta_1},\ldots,w_k\left(\frac{x}{a+bx+cx^2}\right)^{\beta_k}\right]$$

$$G^{0,\lambda:(u',v'),\ldots,(u^{(r)},v^{(r)})}_{A,C:(B',D'),\ldots,(B^{(r)},D^{(r)})}\left[z_1^{1/\alpha'}\left(\frac{x}{a+bx+cx^2}\right),\ldots,z_r^{1/\alpha^r}\left(\frac{x}{a+bx+cx^2}\right)\middle|\begin{matrix}(a):(b'),\ldots,(b^{(r)})\\(c):(d'),\ldots,(d^{(r)})\end{matrix}\right]dx$$

$$=\sqrt{\frac{\pi}{c}}\sum_{G=0}^{\infty}\sum_{g=1}^{M}\sum_{\alpha=0}^{[n/m]}\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_k=0}^{[n_k/m_k]}\frac{(-1)^G}{G!\,F_g}\frac{(-n)_{m\alpha}}{\alpha!}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_k)_{m_k\alpha_k}}{\alpha_k!}\phi(\eta_G)z^{\eta_G}$$

$$A[n,\alpha]A'[n_1,\alpha_1;\ldots;n_k,\alpha_k]w^{\alpha}w_1^{\alpha_1},\ldots,w_k^{\alpha_K}\left(b+2\sqrt{ca}\right)^{\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i-1}$$

$$G^{0,\lambda+1:(u',v'),\ldots,(u^{(r)},v^{(r)})}_{A+1,C+1:(B',D'),\ldots,(B^{(r)},D^{(r)})}\left[-z_1^{1/\alpha'}\left(b+2\sqrt{ca}\right)^{-1};\ldots;-z_r^{1/\alpha^r}\left(b+2\sqrt{ca}\right)^{-1}\right.$$

$$\left|\begin{matrix}\left[\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i\right],(a):(b');\ldots;(b^{(r)})\\(c),\left[\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i-1/2\right]:(d');\ldots;(d^{(r)})\end{matrix}\right], \quad ...(3.2)$$

provided that $Re\ (a)>0$, $Re(b)>0$, $c>0$; $\alpha^{(i)}>0(i=1,\ldots,r)$,

$2\left(u^{(i)}+v^{(i)}\right)>\left(A+C+B^{(i)}+D^{(i)}\right),\ \left|arg(z_i)\right|<\left[u^{(i)}+v^{(i)}-A/2-C/2-B^{(i)}/2-D^{(i)}/2\right]\pi$ and

$$\sigma\left[\min_{i\le j\le M}\left[Re\left(f_j/F_j\right)\right]\right]+\sum_{i=1}^{r}\left[\min_{i\le j\le u^{(i)}}\left[Re\left(d_j^{(i)}\right)\right]\right]>\alpha-2.$$

(III) Putting λ=A=C=0 in (2.1), we get the following integral

$$\int_0^\infty x^{1-\gamma}\left(a+bx+cx^2\right)^{\gamma-3/2}H_{P,Q}^{M,N}\left[z\left(\frac{x}{a+bx+cx^2}\right)^{\sigma}\left|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right.\right]S_n^m\left[w\left(\frac{x}{a+bx+cx^2}\right)^{\beta}\right]$$

$$.S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[w_1\left(\frac{x}{a+bx+cx^2}\right)^{\beta_1},\ldots,w_k\left(\frac{x}{a+bx+cx^2}\right)^{\beta_k}\right]$$

$$\prod_{i=1}^{r}H_{B^{(i)},D^{(i)}}^{u^{(i)},v^{(i)}}\left[z_i\left(\frac{x}{a+bx+cx^2}\right)^{\sigma_i}\left|\begin{matrix}\left(b^{(i)}\right):\phi^{(i)}\\\left(d^{(i)}\right):\delta^{(i)}\end{matrix}\right.\right]dx$$

$$=\sqrt{\frac{\pi}{c}}\sum_{G=0}^{\infty}\sum_{g=1}^{M}\sum_{\alpha=0}^{[n/m]}\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_k=0}^{[n_k/m_k]}\frac{(-1)^G}{G!F_g}\frac{(-n)_{m\alpha}}{\alpha!}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_k)_{m_k\alpha_k}}{\alpha_k!}\phi(\eta_G)z^{\eta_G}$$

$$A[n,\alpha]A'[n_1,\alpha_1;\ldots;n_k,\alpha_k]w^{\alpha}w_1^{\alpha_1},\ldots,w_k^{\alpha_K}\left(b+2\sqrt{ca}\right)^{\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i-1}$$

$$H_{1,1[B',D'];\ldots;[B^{(r)},D^{(r)}]}^{0,1[u',v'];\ldots;[u^{(r)},v^{(r)}]}\left[\begin{matrix}z_1\left(b+2\sqrt{ca}\right)^{-\sigma_1}\\\vdots\\z_r\left(b+2\sqrt{ca}\right)^{-\sigma_r}\end{matrix}\left|\begin{matrix}\left[\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i;\sigma_1;\ldots;\sigma_r\right]:\\\left[\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{K}\alpha_i\beta_i-1/2;\sigma_1;\ldots;\sigma_r\right]:\end{matrix}\right.\right.$$

$$\left.\begin{matrix}\left[(b'),\phi'\right],\ldots,\left[\left(b^{(r)}\right),\phi^{(r)}\right]\\\left[(d'),\delta'\right]..\left[\left(d^{(r)}\right),\delta^{(r)}\right]\end{matrix}\right], \qquad \ldots(3.3)$$

valid under the same conditions as obtainable from (2.1).

(IV) By applying our results given in (2.1) to the case of Hermite polynomials [2,p.106, Eq. (5.5;4)] by setting

$$S_n^2(t)\to t^{n/2}H_n\left(1/2\sqrt{t}\right)$$

in which case $m=2$, $A[n,\alpha]=(-1)^{\alpha}$.

We have the following interesting consequences of the main result

$$\int_0^\infty x^{1-\gamma}\left(a+bx+cx^2\right)^{\gamma-3/2} H_{P,Q}^{M,N}\left[z\left(\frac{x}{a+bx+cx^2}\right)^\sigma \middle| \begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right] w^{n/2}\left[\left(\frac{x}{a+bx+cx^2}\right)^{\beta n/2}\right]$$

$$H_n\left\{\frac{1}{2\sqrt{w\left(\frac{x}{a+bx+cx^2}\right)^\beta}}\right\} S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[w_1\left(\frac{x}{a+bx+cx^2}\right)^{\beta_1},\dots,w_k\left(\frac{x}{a+bx+cx^2}\right)^{\beta_k}\right]$$

$$H\left[z_1\left(\frac{x}{a+bx+cx^2}\right)^{\sigma_1},\dots,z_r\left(\frac{x}{a+bx+cx^2}\right)^{\sigma_r}\right]dx$$

$$=\sqrt{\frac{\pi}{c}}\sum_{G=0}^{\infty}\sum_{g=1}^{M}\sum_{\alpha=0}^{[n/m]}\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_k=0}^{[n_k/m_k]}\frac{(-1)^G}{G!\,F_g}\frac{(-n)_{m\alpha}}{\alpha!}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_k)_{m_k\alpha_k}}{\alpha_k!}\phi(\eta_G)z^{\eta_G}$$

$$A'[n_1,\alpha_1;\dots;n_k,\alpha_k](-w)^\alpha w_1^{\alpha_1},\dots,w_k^{\alpha_k}\left(b+2\sqrt{ca}\right)^{\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i-1}$$

$$H_{A+1,C+1[B',D'];\dots;[B^{(r)},D^{(r)}]}^{0,\lambda+1[u',v'];\dots;[u^{(r)},v^{(r)}]}\left[\begin{matrix}z_1\left(b+2\sqrt{ca}\right)^{-\sigma_1}\\ \vdots\\ z_r\left(b+2\sqrt{ca}\right)^{-\sigma_r}\end{matrix}\middle|\begin{matrix}\left[\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i;\sigma_1\dots\sigma_r\right]\\ [(c):\psi',\dots,\psi^{(r)}]\end{matrix}\right.$$

$$\left.\begin{matrix}[(a):\theta',\dots,\theta^{(r)}]:[(b'),\phi'];\dots;[(b^{(r)}),\phi^{(r)}]\\ \left[\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i-\frac{1}{2};\sigma_1,\dots,\sigma_r\right]:[(d'),\delta'];\dots;[(d^{(r)}),\delta^{(r)}]\end{matrix}\right], \qquad \dots(3.4)$$

which holds true under the same conditions as needed for (2.1).

(V) For the Laguerre polynomials [2,p.101, Eq. (15),(16)], setting $S_n^1(t)\to L_n^{(\alpha')}(t)$ in which case $m=1, A[n,\alpha]=\binom{n+\alpha'}{n}\frac{1}{(\alpha'+1)_\alpha}$ the result in (2.1) reduces to the following formulae

$$\int_0^\infty x^{1-\gamma}\left(a+bx+cx^2\right)^{\gamma-3/2} H_{P,Q}^{M,N}\left[z\left(\frac{x}{a+bx+cx^2}\right)^\sigma \middle| \begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right] L_n^{(\alpha')}\left[w\left(\frac{x}{a+bx+cx^2}\right)^\beta\right]$$

$$.S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[w_1\left(\frac{x}{a+bx+cx^2}\right)^{\beta_1},\ldots,w_k\left(\frac{x}{a+bx+cx^2}\right)^{\beta_k}\right]$$

$$H\left[z_1\left(\frac{x}{a+bx+cx^2}\right)^{\sigma_1},\ldots,z_r\left(\frac{x}{a+bx+cx^2}\right)^{\sigma_r}\right]dx$$

$$=\sqrt{\frac{\pi}{c}}\sum_{G=0}^{\infty}\sum_{g=1}^{M}\sum_{\alpha=0}^{[n]}\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_k=0}^{[n_k/m_k]}\frac{(-1)^G}{G!\,F_g}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_k)_{m_k\alpha_k}}{\alpha_k!}\phi(\eta_G)z^{\eta_G}$$

$$\binom{n+\alpha'}{n-\alpha}\frac{(-w)^{\alpha}}{\alpha!}A'[n_1,\alpha_1;\ldots;n_k,\alpha_k]w_1^{\alpha_1}\ldots w_k^{\alpha_K}\left(b+2\sqrt{ca}\right)^{\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i-1}$$

$$H_{A+1,C+1:[B',D'];\ldots;[B^{(r)},D^{(r)}]}^{0,\lambda+1:[u',v'];\ldots;[u^{(r)},v^{(r)}]}\left[\begin{matrix}z_1\left(b+2\sqrt{ca}\right)^{-\sigma_1}\\ \vdots\\ z_r\left(b+2\sqrt{ca}\right)^{-\sigma_r}\end{matrix}\middle|\begin{matrix}\left[\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i;\sigma_1,\ldots,\sigma_r\right]\\ [(c):\psi',\ldots,\psi^{(r)}],\end{matrix}\right.$$

$$\left.\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]:[(b'),\phi'];\ldots;[(b^{(r)}),\phi^{(r)}]\\ \left[\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i-1/2;\sigma_1,\ldots,\sigma_r\right]:[(d'),\delta'];\ldots;[(d^{(r)}),\delta^{(r)}]\end{matrix}\right] \qquad \ldots(3.5)$$

which holds true under the same condition as needed for (2.1).

(VI) For the Jacobi polynomials [2, p.68, Eq. (4.3.2)], setting

$S_n'(t)\to P_n^{(\alpha',\beta')}(1-2t)$ in which case $m=1$,

$A[n,\alpha]=\binom{n+\alpha'}{n}\frac{(\alpha'+\beta'+n+1)_\alpha}{(\alpha'+1)_\alpha}$ from equation (2.1), we obtain the following result

$$\int_0^\infty x^{1-\gamma}\left(a+bx+cx^2\right)^{\gamma-\frac{3}{2}}H_{P,Q}^{M,N}\left[z\left(\frac{x}{a+bx+cx^2}\right)^{\sigma}\middle|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right]P_n^{(\alpha',\beta')}\left[1-2\left\{w\left(\frac{x}{a+bx+cx^2}\right)\right\}\right]$$

$$.S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[w_1\left(x/\left(a+bx+cx^2\right)\right)^{\beta_1},\ldots,w_k\left(x/\left(a+bx+cx^2\right)\right)^{\beta_k}\right]$$

$$H\left[z_1\left(x/\left(a+bx+cx^2\right)\right)^{\sigma_1},\ldots,z_r\left(x/\left(a+bx+cx^2\right)\right)^{\sigma_r}\right]dx$$

$$=\sqrt{\frac{\pi}{c}}\sum_{G=0}^{\infty}\sum_{g=1}^{M}\sum_{\alpha=0}^{[n/m]}\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_k=0}^{[n_k/m_k]}\frac{(-1)^G}{G!\,F_g}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_k)_{m_k\alpha_k}}{\alpha_k!}\phi(\eta_G)z^{\eta_G}$$

$$\binom{n+\alpha'}{n}\frac{\alpha'+\beta'+n+\alpha}{\alpha}(-w)^{\alpha}A'[n_1,\alpha_1;\ldots;n_k,\alpha_k]w_1^{\alpha_1},\ldots,w_k^{\alpha_K}\left(b+2\sqrt{ca}\right)^{\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i-1}$$

$$H^{0,\lambda+1:[u',v'];\ldots;[u^{(r)},v^{(r)}]}_{A+1,C+1:[B',D'];\ldots;[B^{(r)},D^{(r)}]}\left[\begin{matrix} z_1\left(b+2\sqrt{ca}\right)^{-\sigma_1}\\ \vdots \\ z_r\left(b+2\sqrt{ca}\right)^{-\sigma_r}\end{matrix}\left|\begin{matrix}\left[\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i:\sigma_1,\ldots,\sigma_r\right]\\ [(c):\psi',\ldots,\psi^{(r)}],\end{matrix}\right.\right.$$

$$\left.\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]:[(b'),\phi'];\ldots;[(b^{(r)}),\phi^{(r)}]\\ \left[\gamma-\sigma\eta_G-\alpha\beta-\sum_{i=1}^{k}\alpha_i\beta_i-1/2;\sigma_1,\ldots,\sigma_r\right]:[(d'),\delta'];\ldots;[(d^{(r)}),\delta^{(r)}]\end{matrix}\right], \qquad \ldots(3.6)$$

valid under the same conditions as obtainable from (2.1).

(VII) Putting $n\to 0$, the result obtained in (2.1) reduces to a known result recently obtained by Chaurasia and Shekhawat in [9].

(VIII) Taking $n_i\to 0$ $(i=1,\ldots,k)$, $a=0$, $c=1, n\to 0$, the result in (2.1) reduces to a known result after a slight simplification obtained by Goyal and Mathur [8].

(IX) If $\gamma=1$ and $m_i,n_i\to 0$ $(i=2,\ldots,k)$, $n\to 0$, the result in (2.1) reduces to a known result with a slight modification derived by Gupta and Jain [6].

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