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GENERALIZED MULTIDIMENSIONAL LAGUERRE TRANSFORMS

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ABSTRACT

In the present paper, we introduce generalized multidimensional Laguerre transform to present its certain interesting applications to the theory of generalized multiple hypergeometric functions of several variables including multivariable H-function of Srivastava and Panda [12,13]. The various operational formulas thus obtained are believed to be new. These results may be used in deriving new and known properties of special functions involved.

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1. Introduction. Chandel [1] introduced multidimensional Laplacian oprator to give integral representations of Lauricella's multiple hypergeometric functions of several variables [10]. Chandel [2] further used this operator to give integral representations of multiple hypergeometric functions ${}^{(k)}_{(1)}E^{(n)}_{D}$ and ${}^{(k)}_{(2)}E^{(n)}_{D}$ of Exton [8,9]. Further Chandel and Dwivedi [4,5] introduced multidimensional Whittaker transforms of Lauricella's multiple hypergeometric functions [10], Exton [8,9] and generalized multiple hypergeometric function of Srivastava and Daoust [11] (also see Srivastava and Manocha [15, p.64 (18), (19), (20)]). Recently Chandel and Kumar [6] have made applications of above mutidimensional integral transform to derive the results involving Srivastava and Panda H-function of several complex variables [12,13]. Very recently Chandel and Chauhan have introduced two multidimensional Laguerre transforms to present their interesting applications to the theory of generalized multiple hypergeometric functions of several variables including multivariable H-function of Srivastava and Panda [12,13].

In the present paper, making an apppeal to the result due to Erdély etal. [7,p.292]

$$\int_0^\infty z^\rho e^{-z} L_n^{(\alpha)}(z)dz = \frac{(-1)^n \Gamma(\rho-\alpha+1)\Gamma(\rho+1)}{n!\,\Gamma(\rho-\alpha-n+1)}, Re(\rho) > -1,$$

we introduce generalized multidimensional Laguerre transform defined by

$$(1.1)\quad L_{\gamma_1,\dots,\gamma_n}^{(\alpha,\beta,m,K)}\{\ \} = \frac{K(-1)^m m!\,\Gamma(\gamma_1+\dots+\gamma_n)\Gamma(\beta-\alpha-m+\gamma_1+\dots+\gamma_n)}{\Gamma(\beta-\alpha+\gamma_1+\dots+\gamma_n)\Gamma(\beta+\gamma_1+\dots+\gamma_n)\Gamma(\gamma_1)\dots\Gamma(\gamma_n)}$$

$$\int_0^\infty \dots \int_0^\infty e^{-\sum_{j=1}^n (\alpha_1^j x_1+\dots+\alpha_n^j x_n)} \prod_{j=1}^n (\alpha_1^j x_1+\dots+\alpha_n^j x_n)^{\gamma_j-1} \left[\sum_{j=1}^n (\alpha_1^j x_1+\dots+\alpha_n^j x_n)\right]^\beta$$

$$L_m^{(\alpha)}\left(\sum_{j=1}^n (\alpha_1^j x_1+\dots+\alpha_n^j x_n)\right)\{\ \} dx_1 \dots dx_n,$$

where $(\beta-\alpha-m+\gamma_1+\dots+\gamma_n)>0$; m,n are any positive integers $Re(\gamma_j)>0, j=1,\dots,n$;

$$K = \begin{vmatrix} \alpha_1^1 & \alpha_2^1 & \dots & \alpha_n^1 \\ \alpha_1^2 & \alpha_2^2 & \dots & \alpha_n^2 \\ \dots & \dots & \dots & \dots \\ \alpha_1^n & \alpha_2^n & \dots & \alpha_n^n \end{vmatrix} \neq 0.$$

and present its certain interesting applications to the theory of generlized multiple hypergeometric functions of several variables including multivariable *H*-function of Srivastava and Panda [12,13], (also see [14], p.251)). The various operational formulas thus obtained are belived to be new and may be useful in deriving new and known properties of special functions involved.

2. Some Interesting Results. In this section, we esdtablish the following interesting results :

$$(2.1)\quad L_{\gamma_1,\dots,\gamma_n}^{(\alpha,\beta,m,K)}\{1\} = 1,$$

$$(2.2)\quad L_{\gamma_1,\dots,\gamma_n}^{(\alpha,\beta,m,K)}\left\{\prod_{j=1}^n (\alpha_1^j x_1+\dots+\alpha_n^j x_n)^{m_j}\right\}$$

$$= \frac{(\beta-\alpha+\gamma_1+\ldots+\gamma_n)_{m_1+\ldots+m_n}(\beta+\gamma_1+\ldots+\gamma_n)_{m_1+\ldots+m_n}(\gamma_1)_{m_1}\ldots(\gamma_n)_{m_n}}{(\beta-\alpha-m+\gamma_1+\ldots+\gamma_n)_{m_1+\ldots+m_n}(\gamma_1+\ldots+\gamma_n)_{m_1+\ldots+m_n}}$$

(2.3) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\beta,m,K)}\left\{{}_1F_1\left(\beta-\alpha-m+\gamma_1+\ldots+\gamma_n;\beta-\alpha+\gamma_1+\ldots+\gamma_n;\sum_{j=1}^{n}u_j\left(\alpha_1^j x_1+\ldots+\alpha_n^j x_n\right)\right)\right\}$$

$$= F_D^{(n)}(\beta+\gamma_1+\ldots+\gamma_n,\gamma_1,\ldots,\gamma_n;\gamma_1+\ldots+\gamma_n;u_1,\ldots,u_n),$$

(2.4) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\beta,m,K)}\left\{{}_1F_1\left(\gamma_1+\ldots+\gamma_n;\beta-\alpha+\gamma_1+\ldots+\gamma_n;\sum_{j=1}^{n}u_j\left(\alpha_1^j x_1+\ldots+\alpha_n^j x_n\right)\right)\right\}$$

$$= F_D^{(n)}(\beta+\gamma_1+\ldots+\gamma_n;\gamma_1,\ldots,\gamma_n;\beta-\alpha-m+\gamma_1+\ldots+\gamma_n;u_1,\ldots,u_n),$$

(2.5) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\beta,m,K)}\left\{{}_1F_1\left(\gamma_1+\ldots+\gamma_n;\beta+\gamma_1+\ldots+\gamma_n;\sum_{j=1}^{n}u_j\left(\alpha_1^j x_1+\ldots+\alpha_n^j x_n\right)\right)\right\}$$

$$= F_D^{(n)}(\gamma_1+\ldots+\gamma_n+\beta-\alpha;\gamma_1,\ldots,\gamma_n;\beta-\alpha-m+\gamma_1+\ldots+\gamma_n;u_1,\ldots,u_n),$$

(2.6) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\beta,m,K)}\left\{{}_1F_1\left(\beta-\alpha-m+\gamma_1+\ldots+\gamma_n;\beta+\gamma_1+\ldots+\gamma_n;\sum_{j=1}^{n}u_j\left(\alpha_1^j x_1+\ldots+\alpha_n^j x_n\right)\right)\right\}$$

$$= F_D^{(n)}(\beta-\alpha+\gamma_1+\ldots+\gamma_n;\gamma_1,\ldots,\gamma_n;\gamma_1+\ldots+\gamma_n;u_1,\ldots,u_n),$$

(2.7) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\beta,m,K)}\left\{\left(\sum_{j=1}^{n}\left(\alpha_1^j x_1+\ldots+\alpha_n^j x_n\right)\right)^{m_1+\ldots+m_n}\right\}$$

$$= \frac{(\beta-\alpha+\gamma_1+\ldots+\gamma_n)_{m_1+\ldots+m_n}(\beta+\gamma_1+\ldots+\gamma_n)_{m_1+\ldots+m_n}}{(\beta-\alpha-m+\gamma_1+\ldots+\gamma_n)_{m_1+\ldots+m_n}}$$

which suggests

(2.8) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\beta,m,K)}\left\{\phi_2^{(n)}\left(\beta_1,\ldots,\beta_n;\beta-\alpha+\gamma_1+\ldots+\gamma_n;\right.\right.$$

$$\left.\left.u_1\sum_{j=1}^{n}\left(\alpha_1^j x_1+\ldots+\alpha_n^j x_n\right),\ldots,u_n\sum_{j=1}^{n}\left(\alpha_1^j x_1+\ldots+\alpha_n^j x_n\right)\right)\right\}$$

$$= F_D^{(n)}(\beta + \gamma_1 + \ldots + \gamma_n; \beta_1, \ldots, \beta_n; \beta - \alpha - m + \gamma_1 + \ldots + \gamma_n; u_1, \ldots, u_n).$$

(2.9) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\beta,m,K)}\left\{\phi_2^{(n)}\left(\beta_1,\ldots,\beta_n;\beta+\gamma_1+\ldots+\gamma_n;\right.\right.$$

$$\left.\left. u_1\sum_{j=1}^{n}\left(\alpha_1^j x_1+\ldots+\alpha_n^j x_n\right),\ldots,u_n\sum_{j=1}^{n}\left(\alpha_1^j x_1+\ldots+\alpha_n^j x_n\right)\right)\right\}$$

$$= F_D^{(n)}(\beta - \alpha + \gamma_1 + \ldots + \gamma_n; \beta_1, \ldots, \beta_n; \beta - \alpha - m + \gamma_1, \ldots, \gamma_n; u_1, \ldots, u_n),$$

and

(2.10) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\beta,m,K)}\left\{\psi_2^{(n)}\left(\beta-\alpha-m+\gamma_1+\ldots+\gamma_n,\beta_1,\ldots,\beta_n;\right.\right.$$

$$\left.\left. u_1\sum_{j=1}^{n}\left(\alpha_1^j x_1+\ldots+\alpha_n^j x_n\right),\ldots,u_n\sum_{j=1}^{n}\left(\alpha_1^j x_1+\ldots+\alpha_n^j x_n\right)\right)\right\}$$

$$= F_C^{(n)}(\beta - \alpha + \gamma_1 + \ldots + \gamma_n; \beta + \gamma_1, \ldots, \gamma_n; \beta_1, \ldots, \beta_n; u_1, \ldots, u_n).$$

3. Special Cases.

Case I. When $\beta = \alpha$, we derive

(3.1) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\alpha,m,K)}\left\{\prod_{j=1}^{n}\left(\alpha_1^j x_1+\ldots+\alpha_n^j x_n\right)^{m_j}\right\}$$

$$= \frac{(\alpha+\gamma_1+\ldots+\gamma_n)_{m_1+\ldots+m_n}(\gamma_1)_{m_1}\cdots(\gamma_n)_{m_n}}{(\gamma_1+\ldots+\gamma_n-m)_{m_1+\ldots+m_n}}$$

(3.2) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\alpha,m,K)}\left\{exp\left(\sum_{j=1}^{n}\left(\alpha_1^j x_1+\ldots+\alpha_n^j x_n\right)u_j\right)\right\}$$

$$= F_D^{(n)}(\alpha + \gamma_1 + \ldots + \gamma_n, \gamma_1, \ldots, \gamma_n; \gamma_1 + \ldots + \gamma_n - m; u_1, \ldots, u_n),$$

(3.3) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\alpha,m,K)}\left\{\psi_2^{(n)}\left(\gamma_1+\ldots+\gamma_n-m;\beta_1,\ldots,\beta_n;u_1\left(\alpha_1^1 x_1+\ldots+x_n^1 x_n\right),\ldots,u_n\left(\alpha_1^n x_1+\ldots+\alpha_n^n x_n\right)\right)\right\}$$

$$= F_A^{(n)}(\alpha + \gamma_1 + \ldots + \gamma_n, \gamma_1, \ldots, \gamma_n; \beta_1, \ldots, \beta_n; u_1, \ldots, u_n),$$

(3.4) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\alpha,m,K)}\left\{\psi_2^{(n)}\left(a;\gamma_1,\ldots,\gamma_n;u_1\left(\alpha_1^1 x_1+\ldots+\alpha_n^1 x_n\right),\ldots,u_n\left(\alpha_1^n x_1+\ldots+\alpha_n^n x_n\right)\right)\right\}$$

$$= {}_2F_1(a, \alpha + \gamma_1 + \ldots + \gamma_n; \gamma_1 + \ldots + \gamma_n - m; u_1, \ldots, u_n),$$

$$(3.5)\quad L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\alpha,m,K)}\left\{\psi_2^{(n)}\left(\gamma_1,\ldots,\gamma_n-m;\gamma_1,\ldots,\gamma_n;u_1\left(\alpha_1^1x_1+\ldots+x_n^1x_n\right),\ldots,u_n\left(\alpha_1^nx_1+\ldots+\alpha_n^nx_n\right)\right)\right\}$$

$$=\left[1-\left(u_1+\ldots+u_n\right)\right]^{-(\alpha+\gamma_1+\ldots+\gamma_n)},$$

$$(3.6)\quad L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\alpha,m,K)}\left\{\phi_2^{(n)}\left(\beta_1,\ldots,\beta_n;\alpha+\gamma_1+\ldots+\gamma_n;u_1\left(\alpha_1^1x_1+\ldots+\alpha_n^1x_n\right),\ldots,u_n\left(\alpha_1^nx_1+\ldots+\alpha_n^nx_n\right)\right)\right\}$$

$$=F_B^{(n)}\left(\beta_1,\ldots,\beta_n,\gamma_1,\ldots,\gamma_n;\gamma_1+\ldots+\gamma_n-m;u_1,\ldots,u_n\right),$$

$$(3.7)\quad L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\alpha,m,K)}\left\{\psi_2\left(\gamma_1+\ldots+\gamma_n-m,\beta,\beta';\right.\right.$$

$$\left.\left.\sum_{j=1}^{k}u_j\left(\alpha_1^jx_1+\ldots+\alpha_n^jx_n\right),\ \sum_{j=k+1}^{n}u_j\left(\alpha_1^jx_1+\ldots+\alpha_n^jx_n\right)\right)\right\}$$

$$=\binom{k}{1}E_D^{(n)}\left(\alpha+\gamma_1+\ldots+\gamma_n,\gamma_1,\ldots,\gamma_n;\beta,\beta';u_1,\ldots,u_n\right),$$

$$(3.8)\quad L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\alpha,m,K)}\left\{\phi_2\left(\beta,\beta';\alpha+\gamma_1+\ldots+\gamma_n;\right.\right.$$

$$\left.\left.\sum_{j=1}^{k}u_j\left(\alpha_1^jx_1+\ldots+\alpha_n^jx_n\right),\ \sum_{j=k+1}^{n}u_j\left(\alpha_1^jx_1+\ldots+\alpha_n^jx_n\right)\right)\right\}$$

$$=\binom{k}{2}E_D^{(n)}\left(\beta,\beta';\gamma_1,\ldots,\gamma_n;\gamma_1+\ldots+\gamma_n-m;u_1,\ldots,u_n\right),$$

where $\binom{k}{1}E_D^{(n)}$ and $\binom{k}{2}E_D^{(n)}$ are Exton's multiple hypergeometric functions [7] related to Lauricella's $F_D^{(n)}$.

$$(3.9)\quad L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\alpha,m,K)}\left\{\prod_{i=1}^{n}L_{n_i}^{(\gamma_i-1)}\left[u_i\left(\alpha_1^ix_1+\ldots+\alpha_n^ix_n\right)\right]\right\}$$

$$=\prod_{i=1}^{n}\frac{n_i!}{(r_i)_{n_i}}F_D^{(n)}\left(\alpha+\gamma_1+\ldots+\gamma_n,-n_1,\ldots,-n_n;\gamma_1+\ldots+\gamma_n-m;u_1,\ldots,u_n\right).$$

Case II When $\beta=0$. For $\beta=0$, (2.2) gives

$$(3.10)\quad L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,0,m,K)}\left\{\prod_{j=1}^{n}\left(\alpha_1^jx_1+\ldots+\alpha_n^jx_n\right)^{m_j}\right\}$$

$$=\frac{\left(-\alpha+\gamma_1+\ldots+\gamma_n\right)_{m_1+\ldots+m_n}\left(\gamma_1\right)_{m_1}\cdots\left(\gamma_n\right)_{m_n}}{\left(-\alpha-m+\gamma_1+\ldots+\gamma_n\right)_{m_1+\ldots+m_n}}.$$

Therefore

(3.11) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,0,m,K)}\left\{\phi_2^{(n)}\left(\beta_1,\ldots,\beta_n;\gamma_1+\ldots+\gamma_n-\alpha;u_1\left(\alpha_1^1x_1+\ldots+\alpha_n^1x_n\right),\ldots,u_n\left(\alpha_1^nx_1+\ldots+\alpha_n^nx_n\right)\right)\right\}$$

$$=F_B^{(n)}\left(\beta_1,\ldots,\beta_n,\gamma_1,\ldots,\gamma_n;\gamma_1+\ldots+\gamma_n-\alpha-m;u_1,\ldots,u_n\right),$$

(3.12) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,0,m,K)}\left\{\psi_2^{(n)}\left(\gamma_1+\ldots+\gamma_n-\alpha-m;\beta_1,\ldots,\beta_n;\right.\right.$$

$$\left.\left.u_1\left(\alpha_1^1x_1+\ldots+\alpha_n^1x_n\right),\ldots,u_n\left(\alpha_1^nx_1+\ldots+\alpha_n^nx_n\right)\right)\right\}$$

$$=F_A^{(n)}\left(-\alpha+\gamma_1+\ldots+\gamma_n;\gamma_1,\ldots,\gamma_n;\beta_1,\ldots,\beta_n;u_1,\ldots,u_n\right),$$

(3.13) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,0,m,K)}\left\{\psi_2^{(n)}\left(\gamma_1+\ldots+\gamma_n-\alpha-m;\gamma_1,\ldots,\gamma_n;\right.\right.$$

$$\left.\left.u_1\left(\alpha_1^1x_1+\ldots+\alpha_n^1x_n\right),\ldots,u_n\left(\alpha_1^nx_1+\ldots+\alpha_n^nx_n\right)\right)\right\}$$

$$=\left[1-\left(u_1+\ldots+u_n\right)\right]^{-(\gamma_1+\ldots+\gamma_n-\alpha)},$$

(3.14) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,0,m,K)}\left\{{}_0F_1\left(-;\gamma_1+\ldots+\gamma_n-\alpha;\sum_{j=1}^{n}\left[u_j\left(\alpha_1^jx_1+\ldots+\alpha_n^jx_n\right)\right]\right)\right\}$$

$$=\phi_2^{(n)}\left(\gamma_1,\ldots,\gamma_n;\gamma_1+\ldots+\gamma_n-\alpha-m;u_1,\ldots,u_n\right).$$

(3.15) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,0,m,K)}\left\{exp\left(\sum_{j=1}^{n}u_j\left(\alpha_1^jx_1+\ldots+\alpha_n^jx_n\right)\right)\right\}$$

$$=F_D^{(n)}\left(\gamma_1+\ldots+\gamma_n-\alpha,\gamma_1,\ldots,\gamma_n;\gamma_1+\ldots+\gamma_n-\alpha-m;u_1,\ldots,u_n\right),$$

(3.16) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,0,m,K)}\left\{\psi_2\left(\gamma_1+\ldots+\gamma_n-\alpha-m;\beta,\beta';\sum_{i=1}^{k}u_i\left(\alpha_1^ix_1+\ldots+\alpha_n^ix_n\right),\sum_{i=k+1}^{n}u_i\left(\alpha_1^ix_1+\ldots+\alpha_n^ix_n\right)\right)\right\}$$

$$={}^{(k)}_{(1)}E_D^{(n)}\left(\gamma_1+\ldots+\gamma_n-\alpha,\gamma_1,\ldots,\gamma_n;\beta,\beta';u_1,\ldots,u_n\right),$$

(3.17) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,0,m,K)}\left\{\phi_2\left(\beta,\beta';\gamma_1+\ldots+\gamma_n-\alpha;\sum_{j=1}^{n}u_j\left(\alpha_1^jx_1+\ldots+\alpha_n^jx_n\right),\sum_{j=k+1}^{n}u_i\left(\alpha_1^jx_1+\ldots+\alpha_n^jx_n\right)\right)\right\}$$

$$={}^{(k)}_{(2)}E_D^{(n)}\left(\beta,\beta';\gamma_1,\ldots,\gamma_n;\gamma_1+\ldots+\gamma_n-\alpha-m;u_1,\ldots,u_n\right),$$

(3.18) $$L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,0,m,K)}\left\{\prod_{i=1}^{n}L_{n_1}^{(\gamma_1-1)}u_i\left(\alpha_1^ix_1+\ldots+\alpha_n^ix_n\right)\right\}$$

$$= \prod_{i=1}^{n} \frac{n_i!}{(\gamma_i)_{n_i}} F_D^{(n)}(\gamma_1 + \ldots + \gamma_n - \alpha, -n_1, \ldots, -n_n; \gamma + \ldots + \gamma_n - \alpha - m; u_1, \ldots, u_n),$$

4. Results involving Multiple Hypergeometric Function of Srivastava and Daoust. In this section we derive generalized multidimensional Laguerre Transforms of generalized multiple hypergeometric function of Srivastava and Daoust [11,15] :

$$(4.1)\quad L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\beta,m,K)}\left\{ F_{C:D';\ldots;D^{(n)}}^{A:B';\ldots;B^{(n)}}\left(\begin{matrix} [(a):\theta',\ldots,\theta^{(n)}],[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}]; \\ [(c):\psi',\ldots\psi^{(n)}],[(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}]; \end{matrix} \right.\right.$$

$$\left.\left. u_1(\alpha_1^1 x_1 + \ldots + \alpha_n^1 x_n)^{\xi_1}, \ldots, u_n(\alpha_1^n x_1 + \ldots + \alpha_n^n x_n)^{\xi_n} \right)\right\}$$

$$= F_{C+2:D';\ldots;D^{(n)}}^{A+2:B'+1;\ldots;B^{(n)}+1}\left(\begin{matrix} [(a):\theta',\ldots,\theta^{(n)}],[\beta-\alpha+\gamma_1+\ldots+\gamma_n:\xi_1,\ldots,\xi_n], \\ [(c):\psi',\ldots,\psi^{(n)}],[\beta-\alpha-m+\gamma_1+\ldots+\gamma_n:\xi_1,\ldots,\xi_n]; \end{matrix} \right.$$

$$\left. \begin{matrix} [\beta+\gamma_1+\ldots+\gamma_n:\xi_1,\ldots,\xi_n]:[(b'):\phi'],[\gamma_1:\xi_1];\ldots;[(b^{(n)}):\phi^{(n)}],[\gamma_n:\xi_n]; \\ [\gamma_1+\ldots+\gamma_n;\xi_1,\ldots,\xi_n]:[(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}]; \end{matrix} u_1,\ldots,u_n \right),$$

where $Re(\beta-\alpha-m+\gamma_1+\ldots+\gamma_n)>0, m, n$ are any positive integers,

$$Re(\gamma_i)>0, 1-\xi_i+\sum_{j=1}^{C}\psi_j^{(i)}+\sum_{j=1}^{D^{(i)}}\delta_j^{(i)}-\sum_{j=1}^{D^{(i)}}\delta_j^{(i)}-\sum_{j=1}^{A}\theta_j^{(i)}-\sum_{j=1}^{B^{(i)}}\phi_j^{(i)}>0; i=1,\ldots,n, \text{ and}$$

$$\begin{vmatrix} \alpha_1^1 & \alpha_2^1 & \ldots & \alpha_n^1 \\ \alpha_1^2 & \alpha_2^2 & \ldots & \alpha_n^2 \\ \ldots & \ldots & \ldots & \ldots \\ \alpha_1^n & \alpha_2^n & \ldots & \alpha_n^n \end{vmatrix} = K \neq 0.$$

$$(4.2)\quad L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\beta,m,K)}\left\{ F_{C:D';\ldots;D^{(n)}}^{A:B';\ldots;B^{(n)}}\left(\begin{matrix} [(a):\theta',\ldots,\theta^{(n)}],[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}]; \\ [(c):\psi',\ldots\psi^{(n)}],[(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}]; \end{matrix} \right.\right.$$

$$\left.\left. u_1\left(\sum_{i=1}^{n}(\alpha_1^i x_1 + \ldots + \alpha_n^i x_n)\right)^{\eta_1}, \ldots, u_n\left(\sum_{i=1}^{n}(\alpha_1^i x_1 + \ldots + \alpha_n^i x_n)\right)^{\eta_n} \right)\right\}$$

$$= F^{A+2:B';\dots;B^{(n)}}_{C+1:D';\dots;D^{(n)}}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]\ ,[\beta-\alpha+\gamma_1+\dots+\gamma_n:\eta_1,\dots,\eta_n],\\ [(c):\psi',\dots\psi^{(n)}]\ ,[\beta-\alpha-m+\gamma_1+\dots+\gamma_n:\eta_1,\dots,\eta_n]:\end{matrix}\right.$$

$$\left.\begin{matrix}[\beta+\gamma_1+\dots+\gamma_n:\eta_1,\dots,\eta_n][(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}];\\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\ u_1,\dots,u_n\right),$$

provided that $Re(\beta-\alpha-m+\gamma_1+\dots+\gamma_n)>0, m,n$ are any positive integers,

$Re(\gamma_i)>0, 1-\eta_i+\sum_{j=1}^{C}\psi_j^{(i)}+\sum_{j=1}^{D^{(i)}}\delta_j^{(i)}-\sum_{j=1}^{A}\theta_j^{(i)}-\sum_{j=1}^{B^{(i)}}\phi_j^{(i)}>0; i=1,\dots n,$ and $K\neq 0$ (as given in (1.1.)).

(4.3) $$L^{(\alpha,\beta,m,K)}_{\gamma_1,\dots,\gamma_n}\left\{\prod_{i=1}^{n}\left(\alpha_1^i x_1+\dots+\alpha_n^i x_n\right)^{m_1\xi_1}\left[\sum_{i=1}^{n}\left(\alpha_1^i x_1+\dots+\alpha_n^i x_n\right)\right]^{m_1\eta_1+\dots+m_n\eta_n}\right\}$$

$$=\frac{(\beta+\gamma_1+\dots+\gamma_n-\alpha)_{m_1(\xi_1+\eta_1)+\dots+m_n(\xi_n+\eta_n)}(\beta+\gamma_1+\dots+\gamma_n)_{m_1(\xi_1+\eta_1)+\dots+m_n(\xi_n+\eta_n)}(\gamma_1)_{m_1\xi_1}\dots(\gamma_n)_{m_n\xi_n}}{(\beta+\gamma_1+\dots+\gamma_n-\alpha-m)_{m_1(\xi_1+\eta_1)+\dots+m_n(\xi_n+\eta_n)}(\gamma_1+\dots+\gamma_n)_{m_1\xi_1+\dots+m_n\xi_n}}$$

(4.4) $$L^{(\alpha,\beta,m,K)}_{\gamma_1,\dots,\gamma_n}\left\{F^{A:B';\dots;B^{(n)}}_{C:D';\dots;D^{(n)}}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]:[(b'):\phi']\ ;\dots;[(b^{(n)}):\phi^{(n)}]\ ;\\ [(c):\psi',\dots\psi^{(n)}]:[(d'):\delta']\ ;\dots;[(d^{(n)}):\delta^{(n)}]\ ;\end{matrix}\right.\right.$$

$$u_1\left(\alpha_1^1 x_1+\dots+\alpha_n^1 x_n\right)^{\xi_1}\left(\sum_{i=1}^{n}\left(\alpha_1^i x_1+\dots+\alpha_n^i x_n\right)\right)^{\eta_1}$$

$$\left.\left.,\dots,u_n\left(\alpha_1^n x_1+\dots+\alpha_n^n x_n\right)^{\xi_n}\left(\sum_{i=1}^{n}\left(\alpha_1^i x_1+\dots+\alpha_n^i x_n\right)\right)^{\eta_n}\right)\right\}$$

$$= F^{A+2:B'+1;\dots;B^{(n)}+1}_{C+2:D';\dots;D^{(n)}}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}],[\beta+\gamma_1+\dots+\gamma_n-\alpha:\xi_1+\eta_1,\dots,\xi_n+\eta_n],\\ [(c):\psi',\dots\psi^{(n)}],[\beta+\gamma_1+\dots+\gamma_n-\alpha-m:\xi_1+\eta_1,\dots,\xi_n+\eta_n],\end{matrix}\right.$$

$$\left.\begin{matrix}[\beta+\gamma_1+\dots+\gamma_n:\xi_1+\eta_1,\dots,\xi_n+\eta_n]:[(b'):\phi'],[\gamma_1:\xi_1]\ ;\dots;[(b^{(n)}):\phi^{(n)}],[\gamma_n:\xi_n];\\ [\gamma_1+\dots+\gamma_n;\xi_1,\dots,\xi_n]:[(d'):\delta']\ ;\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\ u_1,\dots,u_n\right),$$

provided that $Re(\beta-\alpha-m+\gamma_1+...+\gamma_n)>0, m,n$ are any positive integers,

$Re(\gamma_i)>0$ and $1-(\xi_i+\eta_i)+\sum_{j=1}^{C}\psi_j^{(i)}+\sum_{j=1}^{D^{(i)}}\delta_j^{(i)}-\sum_{j=1}^{A}\theta_j^{(i)}-\sum_{j=1}^{B^{(i)}}\phi_j^{(i)}>0; i=1,...,n.$

5. Results Involving Srivastava and Panda's *H*-function of Several Complex Variables. Making an appeal to (4.3), we derive, the following multidimensional generalized Laguerre Transform of Srivastava and Panda's *H*-function of several complex variables [12,13].

$$(5.1)\quad L_{\gamma_1,...,\gamma_n}^{(\alpha,\beta,m,K)}\left\{H_{A,C:[B',D'];...;[B^{(n)},D^{(n)}]}^{0,\gamma:[\mu',\nu'];...;[\mu^{(n)},\upsilon^{(n)}]}\left(\begin{matrix}[(a):\theta',...,\theta^{(n)}],[(b'):\phi'];...;[(b^{(n)}):\phi^{(n)}];\\ [(c):\psi',...\psi^{(n)}],[(d'):\delta'];...;[(d^{(n)}):\delta^{(n)}];\end{matrix}\right.\right.$$

$$u_1\left(\alpha_1^1x_1+...+\alpha_n^1x_n\right)^{\xi_1}\left(\sum_{i=1}^{n}\left(\alpha_1^ix_1+...+\alpha_n^ix_n\right)\right)^{\eta_1}$$

$$\left.\left.,...,u_n\left(\alpha_1^nx_1+...+\alpha_n^nx_n\right)^{\xi_n}\left(\sum_{i=1}^{n}\left(\alpha_1^ix_1+...+\alpha_n^ix_n\right)\right)^{\eta_n}\right)\right\}$$

$$=\frac{\Gamma\left(\beta-\alpha-m+\sum_{i=1}^{n}\gamma_i\right)\Gamma\left(\sum_{i=1}^{n}\gamma_i\right)}{\Gamma\left(\beta-\alpha+\sum_{i=1}^{n}(\gamma_i)\right)\Gamma\left(\beta+\sum_{i=1}^{n}(\gamma_i)\right)\prod_{i=1}^{n}\Gamma(\gamma_i)}H_{A+2,C+2:[d',\delta'];...;[d^{(n)},\delta^{(n)}]}^{0,\gamma+2:(\mu',\nu'+1);...;(\mu^{(n)},\nu^{(n)}+1)}$$

$$\left(\begin{matrix}[(a):\theta',...,\theta^{(n)}],[1+\alpha-\beta-\gamma_1-...-\gamma_n:\xi_1+\eta_1,...,\xi_n+\eta_n],\\ [(c):\psi',...\psi^{(n)}],[1+\alpha+m-\beta-\gamma_1-...-\gamma_n:\xi_1+\eta_1,...,\xi_n+\eta_n],\end{matrix}\right.$$

$$\left.\begin{matrix}[1-\beta-\gamma_1-...-\gamma_n:\xi_1+\eta_1,...\xi_n+\eta_n]:[(b'):\phi'],[1-\gamma_1:\xi_1];...;[(b^{(n)}):\phi^{(n)}],[1-\gamma_n:\xi_n];\\ [1-\gamma_1-...-\gamma_n:\xi_1,...\xi_n]:[(d'):\delta'];...;[(d^{(n)}):\delta^{(n)}];\end{matrix}\;u_1,...u_n\right),$$

provided that $Re(\beta-\alpha-m+\gamma_1+...+\gamma_n)>0, Re(\gamma_1)>0, \xi_i,\eta_i$ are any positive numbers, m,n are positive integers, and $|arg\, ui|<\frac{\pi}{2}\Delta_i$, where

$$\Delta_i = -\sum_{j=\gamma+1}^{A}\theta_j^{(i)} + \sum_{j=1}^{\upsilon^{(i)}}\phi_j^{(i)} - \sum_{j=\upsilon^{(i)}+1}^{B^{(i)}}\phi_j^{(i)} - \sum_{j=1}^{C}\psi_j^{(i)} + \sum_{j=1}^{\mu^{(i)}}\delta_j^{(i)} - \sum_{j=\mu^{(i)}+1}^{D^{(i)}}\delta_j^{(i)} > 0, i = 1,\dots,n.$$

and $K \neq 0$ (as given in (1.1)).

Special Cases of (5.1)

For $\xi_1 = \dots = \xi_n = 0$. (5.1) gives

$$(5.2)\quad L_{\gamma_1,\dots,\gamma_n}^{(\alpha,\beta,m,K)}\left\{ H_{A,C:[B',D'];\dots;[B^{(n)},D^{(n)}]}^{0,\gamma:[\mu',\upsilon'];\dots;[\mu^{(n)},\upsilon^{(n)}]}\left(\begin{matrix} [(a):\theta',\dots,\theta^{(n)}],[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]; \\ [(c):\psi',\dots\psi^{(n)}],[(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}]; \end{matrix} \right.\right.$$

$$\left.\left. u_1\left(\sum_{i=1}^{n}\left(\alpha_1^i x_1 + \dots + \alpha_n^i x_n\right)\right)^{\eta_1},\dots,u_n\left(\sum_{i=1}^{n}\left(\alpha_1^i x_1 + \dots + \alpha_n^i x_n\right)\right)^{\eta_n} \right)\right\}$$

$$= \frac{\Gamma(\beta-\alpha-m+\gamma_1+\dots+\gamma_n)}{\Gamma(\beta-\alpha+\gamma_1+\dots+\gamma_n)\Gamma(\beta+\gamma_1+\dots+\gamma_n)} H_{A+2,C+1:[B',D'];\dots;[B^{(n)},D^{(n)}]}^{0,\gamma+2:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})}$$

$$\left(\begin{matrix} [(a):\theta',\dots,\theta^{(n)}],[1+\alpha-\beta-\gamma_1-\dots-\gamma_n:\eta_1,\dots,\eta_n], \\ [(c):\psi',\dots\psi^{(n)}],[1+\alpha+m-\beta-\gamma_1-\dots-\gamma_n:\eta_1,\dots,\eta_n], \end{matrix} \right.$$

$$\left. \begin{matrix} [1-\beta-\gamma_1-\dots-\gamma_n:\eta_1,\dots,\eta_n]:[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]; \\ :[(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}]; \end{matrix} u_1,\dots,u_n \right).$$

For $\eta_1 = \dots = \eta_n = 0$, (f.1) further gives

$$(5.3)\quad L_{\gamma_1,\dots,\gamma_n}^{(\alpha,\beta,m,K)}\left\{ H_{A,C:[B',D'];\dots;[B^{(n)},D^{(n)}]}^{0,\gamma:[\mu',\upsilon'];\dots;[\mu^{(n)},\upsilon^{(n)}]}\left(\begin{matrix} [(a):\theta',\dots,\theta^{(n)}]:[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]; \\ [(c):\psi',\dots\psi^{(n)}]:[(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}]; \end{matrix} \right.\right.$$

$$\left.\left. u_1\left(\alpha_1^1 x_1 + \dots + \alpha_n^1 x_n\right)^{\xi_1},\dots,u_n\left(\alpha_1^n x_1 + \dots + \alpha_n^n x_n\right)^{\xi_n} \right)\right\}$$

$$= \frac{\Gamma(\beta-\alpha-m+\gamma_1+\dots+\gamma_n)\Gamma(\gamma_1+\dots+\gamma_n)}{\Gamma(\beta-\alpha+\gamma_1+\dots+\gamma_n)\Gamma(\beta+\gamma_1+\dots+\gamma_n)\Gamma(\gamma_1)\dots\Gamma(\gamma_n)} H_{A+2,C+2:[B',D'];\dots;[B^{(n)},D^{(n)}]}^{0,\gamma+2:(\mu',\nu'+1);\dots;(\mu^{(n)},\nu^{(n)}+1)}$$

$$\left(\begin{matrix} [(a):\theta',\dots,\theta^{(n)}],[1+\alpha-\beta-\gamma_1-\dots-\gamma_n:\xi_1,\dots,\xi_n], \\ [(c):\psi',\dots\psi^{(n)}],[1+\alpha+m-\beta-\gamma_1-\dots-\gamma_n:\xi_1,\dots,\xi_n], \end{matrix} \right.$$

$$\left.\begin{matrix}[1-\beta-\gamma_1-\ldots-\gamma_n:\xi_1,\ldots,\xi_n]:[(b'):\phi'],[1-\gamma_1:\xi_1];\ldots;[(b^{(n)}):\phi^{(n)}],[1-\gamma_n:\xi_n];\\ [1-\gamma_1-\ldots-\gamma_n:\xi_1,\ldots,\xi_n]:[(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\; u_1,\ldots,u_n\right).$$

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