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# FRACTIONAL CALCULUS OPERATOR INVOLVING THE PRODUCT OF HYPERGEOMETRIC FUNCTIONS AND POLYNOMIALS OF SEVERAL VARIABLES

By

**V.B.L. Chaurasia**

Department of Mathematics, University of Rajasthan, Jaipur-302004, India

**and**

**Hari Singh Parihar**

Department of Mathematics

Poornima College of Engineering, Jaipur-302022, India

E;mail: harisingh.p@rediffmail.com, harisingh_p@yahoo.co.in

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## ABSTRACT

In the present paper, we derive fractional integration of hypergeometric function and polynomials of several variables.

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**1. Introduction.** We shall define the fractional integrals and derivatives of a funciton $f(x)$[20] as follows :

Let $\alpha$, $\beta$ and $\gamma$ be complex numbers. The fractional integral ($Re(\alpha)>0$) and derivatives ($Re(\alpha)<0$) of a function $f(x)$ defined on $(0,\infty)$ is given by

$$I_{0,x}^{\alpha,\beta,\gamma}\{f(x)\} = \begin{cases} \dfrac{x^{-\alpha-\beta}}{\Gamma\alpha}\displaystyle\int_0^x (x-t)^{\alpha-1} F(\alpha+\beta-\gamma;\alpha;1-t/x) f(t)dt (Re(\alpha)>0) \\ \dfrac{d^q}{dx^q} I_{0,x}^{\alpha+q,\beta-q,\gamma-q} f(x), Re(\alpha)\leq 0, 0<R(\alpha)+q\leq 1, q=1,2,3,.. \end{cases}$$

where $F$ is the Gauss hypergeometric function.

We can obtain several Fractional calculus operators by manipulation in the values of $\alpha$, $\beta$ and $\gamma$.

Also a general class of multivariable polynomials [18] defined as follows

$$S_n^{m_1,\ldots,m_r}[w_1,\ldots,w_r] = \sum_{k_1,\ldots,k_r}^{m_1k_1+\ldots+m_rk_r\leq n} (-n)_{m_1k_1+\ldots+m_rk_r} A(n;k_1,\ldots,k_r)\frac{w_1^{k_1}}{k_1!}\cdots\frac{w_r^{k_r}}{k_r!}$$

where $m_1,\ldots,m_r$ are arbitrary positive integers and the coefficients $A(n;k_1,\ldots,k_r)$

$(n{:}k_i \geq 0,\ i=1,\ldots,r)$ are arbitrary constants, real or complex.

Chandel and Vishwakarma [5] have derived fractional derivatives of confluent forms of Karlsson's multiple hypergeometric function ${}^{(k)}F_{CD}^{(n)}$ [21] while for special interest Chandel and Sharma [9] have also derived fractional derivatives for their hypergeometric functions of four variables ([7],[8]).

This paper is motivatied by the earlier work of Srivastava and Goyal [13], Srivastava, Chandel and Vishwakarma [17] who derived a number of key formulas for fractional calculus operators of the multivaraible $H$-function of Srivastava and Panda [15] and the generalized multiple hypergeometric function of Srivastava and Daoust [14]. The special cases of these results have also been discussed by these authors.

In the present paper, we derive the fractional integration of the product of hypergeometric functions of four variables ([7],[8]) and a general class of multivariable polynomials [18]. We derive some results using multiple hypergeometric functions $F_A^{(n)}$, $F_B^{(n)}$, $F_C^{(n)}$ and $F_D^{(n)}$ of Lauricella [10], ${}^{(k)}_{(1)}E_D^{(n)}$, ${}^{(k)}_{(2)}E_D^{(n)}$ of Exton ([11],[12]), ${}^{(k)}_{(1)}E_C^{(n)}$ of Chandel [1], and intermediate Lauricella functions ${}^{(k)}F_{AC}^{(n)}$, ${}^{(k)}F_{BD}^{(n)}$, ${}^{(k)}F_{AD}^{(n)}$ with their conflluent forms ${}^{(k)}_{(1)}\phi_{AC}^{(n)}$, ${}^{(k)}_{(1)}\phi_{AD}^{(n)}$, ${}^{(k)}_{(1)}\phi_{BD}^{(n)}$ due to Chandel and Gupta [2].

For the sake of brevity, we use here the following notations :

$$A(\phi)=\sum_{k_1,\ldots,k_r=0}^{m_1k_1+\ldots+m_rk_r\leq n}$$

and

$$B(\theta)=A(n:k_1,\ldots,k_r)(-n)_{m_1k_1+\ldots+m_rk_r}\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_r^{k_r}}{k_r!}.$$

**2. Use of One Fractional Integral Operator.** Making an appeal to the formula ([19],p.16, Lemma 1) after a little simplification

$$I_x^{\eta,\upsilon}\{x^\lambda\}=\frac{\Gamma(\lambda+\eta)}{\Gamma(\lambda+\eta+\upsilon)}x^\lambda \qquad \mathrm{Re}(\lambda)>-\eta,$$

we derive the following fractional integrals involving the product of hypergeometric functions of several variables and a general class of polynomials of several variables.

(2.1) $I_n^{\eta,\mu-\eta}\left[H_{A_1}^4(\mu+k_1+k_2+k_3,b,c,d;e,e',e'';z_1x,z_2x,z_3x,z_4)S_n^{m_1,\ldots,m_r}(y_1x,y_2x,y_3x,y_4,\ldots,y_r)\right]$

$$= A(\phi)\frac{\Gamma(\eta+k_1+k_2+k_3)}{\Gamma(\mu+k_1+k_2+k_3)}x^{k_1+k_2+k_3}B(\theta)H_{A_1}^4(\eta+k_1+k_2+k_3,b,c,d;e,e',e'';z_1x,z_2x,z_3x,z_4)$$

$\mathrm{Re}(k_1+k_2+k_3)>-\eta$

(2.2) $I_n^{\eta,\mu-\eta}\left[H_{A_1}^4(a,\mu+k_1+k_4,c,d;e,e',e'';z_1x,z_2x,z_3,z_4x)S_n^{m_1,\dots,m_r}(y_1x,y_2,y_3,y_4x,y_5,\dots,y_r)\right]$

$$= A(\phi)\frac{\Gamma(\eta+k_1+k_4)}{\Gamma(\mu+k_1+k_4)}x^{k_1+k_4}B(\theta)H_{A_1}^4(a,\eta+k_1+k_4,c,d;e,e',e'';z_1x,z_2,z_3,z_4x)$$

$\mathrm{Re}(k_1+k_4)>-\eta$ .

(2.3) $I_x^{\eta,\mu-\eta}\left[H_{A_1}^4(a,b,\mu+k_3+k_4,d;e,e',e'';z_1,z_2,z_3,z_4x)S_n^{m_1,\dots,m_r}(y_1,y_2,y_3x,y_4x,y_5,\dots,y_r)\right]$

$$= A(\phi)\frac{\Gamma(\eta+k_3+k_4)}{\Gamma(\mu+k_3+k_4)}x^{k_3+k_4}B(\theta)H_{A_1}^4(a,b,\eta+k_3+k_4,d;e,e',e'';z_1,z_2,z_3x,z_4x)$$

$\mathrm{Re}(k_3+k_4)>-\eta$ .

(2.4) $I_x^{\eta,\mu-\eta}\left[H_{A_1}^4(a,b,c,\mu+k_2;e,e',e'';z_1,z_2x,z_3,z_4)S_n^{m_1,\dots,m_r}(y_1,y_2x,y_3,y_4,y_5,\dots,y_r)\right]$

$$= A(\phi)\frac{\Gamma(\eta+k_2)}{\Gamma(\mu+k_2)}x^{k_2}B(\theta)H_{A_1}^4(a,b,c,\eta+k_2;e,e',e'';z_1,z_2x,z_3,z_4),\ Re(k_2)>-\eta .$$

(2.5) $I_x^{\eta,\mu-\eta}\left[H_{A_1}^4(a,b,c,d,\eta+k_1+k_3;e',e'';z_1x,z_2,z_3x,z_4)S_n^{m_1,\dots,m_r}(y_1x,y_2,y_3x,y_4,y_5,\dots,y_r)\right]$

$$= A(\phi)\frac{\Gamma(\eta+k_1+k_3)}{\Gamma(\mu+k_1+k_3)}x^{k_1+k_3}B(\theta)H_{A_1}^4(a,b,c,d;\mu+k_1+k_3;e',e'';z_1x,z_2,z_3x,z_4)$$

$\mathrm{Re}(k_1+k_3)>-\eta$ .

(2.6) $I_x^{\eta,\mu-\eta}\left[H_{A_1}^4(a,b,c,d;e,\eta+k_2;e'';z_1,z_2x,z_3,z_4)S_n^{m_1,\dots,m_r}(y_1x,y_2,y_3x,y_4,y_5,\dots,y_r)\right]$

$$= A(\phi)\frac{\Gamma(\eta+k)_2}{\Gamma(\mu+k_2)}x^{k_2}B(\theta)H_{A_1}^4(a,b,c,d;e,\mu+k_2;e'';z_1,z_2x,z_3,z_4),\ \mathrm{Re}(k_2)>-\eta .$$

(2.7) $I_x^{\eta,\mu-\eta}\left[H_{A_1}^4(a,b,c,d;e,e',\eta+k_4;z_1,z_2,z_3,z_4x)S_n^{m_1,\dots,m_r}(y_1,y_2,y_3,y_4x,y_5,\dots,y_r)\right]$

$$= A(\phi)\frac{\Gamma(\eta+k_4)}{\Gamma(\mu+k_4)}x^{k_4}B(\theta)H_{A_1}^4(a,b,c,d;e,e',\mu+k_4;z_1,z_2,z_3,z_4x),\ \mathrm{Re}(k_4)>-\eta .$$

(2.8) $I_x^{\eta,\mu-\eta}\left[H_{B_1}^4(\mu+k_1+k_2+k_4;b,c,d;e_1,e_2,e_3e_4;z_1x,z_2x,z_3,z_4x)\right.$

$$S_n^{m_1,\ldots,m_r}\left(y_1x,y_2x,y_3,y_4x,y_5,\ldots,y_r\right)\Big]$$

$$=A(\phi)\frac{\Gamma(\eta+k_1+k_2+k_4)}{\Gamma(\mu+k_1+k_2+k_4)}x^{k_1+k_2+k_4}B(\theta)H_{B_1}^4\left(\eta+k_1+k_2+k_4;b,c,d;e_1,e_2,e_3,e_4;z_1x,z_2x,z_3,z_4x\right),$$

$$\mathrm{Re}(k_1+k_2+k_4)>-\eta.$$

(2.9) $$I_x^{\eta,\mu-\eta}\Big[H_{B_1}^4\left(a,\mu+k_1+k_3;c,d;e_1,e_2,e_3,e_4;z_1x,z_2,z_3x,z_4\right)$$

$$S_n^{m_1,\ldots,m_r}\left(y_1x,y_2,y_3x,y_4,y_5,\ldots,y_r\right)\Big]$$

$$=A(\phi)\frac{\Gamma(\eta+k_1+k_3)}{\Gamma(\mu+k_1+k_3)}x^{k_1+k_3}B(\theta)H_{B_1}^4\left(a,\eta+k_1+k_3;c,d;e_1,e_2,e_3,e_4;z_1x,z_2,z_3x,z_4\right),$$

$$\mathrm{Re}(k_1+k_3)>-\eta.$$

(2.10) $$I_x^{\eta,\mu-\eta}\Big[H_{B_1}^4\left(a,b,\mu+k_3+k_4;d;e_1,e_2,e_3e_4;z_1,z_2,z_3x,z_4x\right)$$

$$S_n^{m_1,\ldots,m_r}\left(y_1,y_2,y_3x,y_4x,y_5,\ldots,y_r\right)\Big]$$

$$=A(\phi)\frac{\Gamma\eta+k_3+k_4}{\Gamma\mu+k_3+k_4}x^{k_3+k_4}B(\theta)H_{B_1}^4\left(a,b,\eta+k_3+k_4;d;e_1,e_2,e_3,e_4;z_1,z_2,z_3x,z_4x\right),$$

$$\mathrm{Re}(k_3+k_4)>-\eta.$$

(2.11) $$I_x^{\eta,\mu-\eta}\Big[H_{B_1}^4\left(a,b,c,\mu+k_2;e_1,e_2,e_3,e_4;z_1,z_2x,z_3,z_4\right)S_n^{m_1,\ldots,m_r}\left(y_1,y_2x,y_3,y_4,y_5,\ldots,y_r\right)\Big]$$

$$=A(\phi)\frac{\Gamma(\eta+k_2)}{\Gamma(\mu+k_2)}x^{k_2}B(\theta)H_{B_1}^4\left(a,b,c,\eta+k_2;e_1,e_2,e_3,e_4;z_1,z_2x,z_3,z_4\right),\ \mathrm{Re}(k_2)>-\eta.$$

(2.12) $$I_x^{\eta,\mu-\eta}\Big[H_{B_1}^4\left(a,b,c,d;\eta+k_1;e_2,e_3,e_4;z_1x,z_2,z_3,z_4\right)S_n^{m_1,\ldots,m_r}\left(y_1x,y_2,y_3,y_4,y_5,\ldots,y_r\right)\Big]$$

$$=A(\phi)\frac{\Gamma(\eta+k_1)}{\Gamma(\mu+k_1)}x^{k_1}B(\theta)H_{B_1}^4\left(a,b,c,d;\mu+k_1;e_2,e_3,e_4;z_1x,z_2,z_3,z_4\right),\ \mathrm{Re}(k_1)>-\eta$$

(2.13) $$I_x^{\eta,\mu-\eta}\Big[H_{B_1}^4\left(a,b,c,d;e_1,\eta+k_2,e_3,e_4;z_1,z_2x,z_3,z_4\right)S_n^{m_1,\ldots,m_r}\left(y_1,y_2x,y_3,y_4,y_5,\ldots,y_r\right)\Big]$$

$$=A(\phi)\frac{\Gamma(\eta+k_2)}{\Gamma(\mu+k_2)}x^{k_2}B(\theta)H_{B_1}^4\left(a,b,c,d;\mu+k_2;e_3,e_4;z_1,z_2x,z_3,z_4\right),\ \mathrm{Re}(k_2)>-\eta.$$

(2.14) $$I_x^{\eta,\mu-\eta}\Big[H_{B_1}^4\left(a,b,c,d;e_1,e_2,\eta+k_3;e_4;z_1,z_2,z_3x,z_4\right)S_n^{m_1,\ldots,m_r}\left(y_1,y_2,y_3x,y_4,y_5,\ldots,y_r\right)\Big]$$

$$= A(\phi)\frac{\Gamma(\eta+k_3)}{\Gamma(\mu+k_3)}x^{k_3}B(\theta)H_{B_1}^4(a,b,c,d;e_1,e_2,\mu+k_3,e_4;z_1,z_2,z_3x,z_4)\,,\ \mathrm{Re}(k_3)>-\eta\,.$$

(2.15) $$I_x^{\eta,\mu-\eta}\left[H_{B_1}^4(a,b,c,d;e_1,e_2,e_3,\eta+k_4;z_1,z_2,z_3,z_4x)S_n^{m_1,\ldots,m_r}(y_1,y_2,y_3,y_4x,y_5,\ldots,y_r)\right]$$

$$= A(\phi)\frac{\Gamma(\eta+k_4)}{\Gamma(\mu+k_4)}x^{k_4}B(\theta)H_{B_1}^4(a,b,c,d;e_1,e_2,e_3,\mu+k_4;z_1,z_2,z_3,z_4x),\ \mathrm{Re}(k_4)>-\eta\,.$$

(2.16) $$I_x^{\eta,\mu-\eta}\left[G_{A_1}^4(a,b,\mu+k_2,d;e,e';z_1,z_2x,z_3,z_4)S_n^{m_1,\ldots,m_r}(y_1,y_2x,y_3,y_4,y_5,\ldots,y_r)\right]$$

$$= A(\phi)\frac{\Gamma(\eta+k_2)}{\Gamma(\mu+k_2)}x^{k_2}B(\theta)G_{A_1}^4(a,b,\eta+k_2;d;e,e';z_1,z_2x,z_3,z_4),\ \mathrm{Re}(k_2)>-\eta.$$

(2.17) $$I_x^{\eta,\mu-\eta}\left[G_{A_1}^4(a,b,c,\mu+k_4;e,e';z_1,z_2,z_3,z_4x)S_n^{m_1,\ldots,m_r}(y_1,y_2,y_3,y_4x,y_5,\ldots,y_r)\right]$$

$$= A(\phi)\frac{\Gamma(\eta+k_4)}{\Gamma(\mu+k_4)}x^{k_4}B(\theta)G_{A_1}^4(a,b,c,\eta+k_4;e,e';z_1,z_2,z_3,z_4x),\ \mathrm{Re}(k_4)>-\eta\,.$$

(2.18) $$I_x^{\eta,\mu-\eta}\left[G_{A_1}^4(a,b,c,d;e,\eta+k_4;z_1,z_2,z_3,z_4x)S_n^{m_1,\ldots,m_r}(y_1,y_2,y_3,y_4x,y_5,\ldots,y_r)\right]$$

$$= A(\phi)\frac{\Gamma(\eta+k_4)}{\Gamma(\mu+k_4)}x^{k_4}B(\theta)G_{A_1}^4(a,b,c,d;e,\mu+k_4;z_1,z_2,z_3,z_4x),Re(k_4)>-\eta.$$

(2.19) $$I_x^{\eta,\mu-\eta}\left[G_{A_2}^4(a,\mu+k_1+k_3+k_4,c,d;e,e';z_1x,z_2,z_3x,z_4x)S_n^{m_1,\ldots,m_r}(y_1x,y_2,y_3x,y_4x,y_5,\ldots,y_r)\right]$$

$$= A(\phi)\frac{\Gamma(\eta+k_1+k_3+k_4)}{\Gamma(\mu+k_1+k_3+k_4)}x^{k_1+k_3+k_4}B(\theta)G_{A_2}^4(a,\eta+k_1+k_3+k_4,c,d;e,e';z_1x,z_2,z_3x,z_4x),$$

$\mathrm{Re}(k_1+k_3+k_4)>-\eta$.

(2.20) $$I_x^{\eta,\mu-\eta}\left[G_{A_2}^4(a,b,\mu+k_2,d;e,e';z_1,z_2x,z_3,z_4)S_n^{m_1,\ldots,m_r}(y_1,y_2x,y_3,y_4,y_5,\ldots,y_r)\right]$$

$$= A(\phi)\frac{\Gamma(\eta+k_2)}{\Gamma(\mu+k_2)}x^{k_2}B(\theta)G_{A_2}^4(a,b,\eta+k_2,d;e,e';z_1,z_2x,z_3,z_4),\ \mathrm{Re}(k_2)>-\eta.$$

(2.21) $$I_x^{\eta,\mu-\eta}\left[G_{A_2}^4(a,b,c,\mu+k_4;e,e';z_1,z_2,z_3,z_4x)S_n^{m_1,\ldots,m_r}(y_1,y_2,y_3,y_4x,y_5,\ldots,y_r)\right]$$

$$= A(\phi)\frac{\Gamma(\eta+k_4)}{\Gamma(\mu+k_4)}x^{k_4}B(\theta)G_{A_2}^4(a,b,c,\eta+k_4;e,e';z_1,z_2,z_3,z_4x),\ \mathrm{Re}(k_4)>-\eta.$$

(2.22) $$I_x^{\eta,\mu-\eta}\left[G_{A_2}^4(a,b,c,d;e,\eta+k_4;z_1,z_2,z_3,z_4x)S_n^{m_1,\ldots,m_r}(y_1,y_2,y_3,y_4x,y_5,\ldots,y_r)\right]$$

$$=A(\phi)\frac{\Gamma(\eta+k_4)}{\Gamma(\mu+k_4)}x^{k_4}B(\theta)G_{A_2}^4(a,b,c,d;e,\mu+k_4;z_1,z_2,z_3,z_4x),\ \mathrm{Re}(k_4)>-\eta.$$

(2.23) $$I_x^{\eta,\mu-\eta}\left[G_{A_3}^4(a,\mu+k_1+k_3+k_4,c,d;e,e';z_1x,z_2,z_3x,z_4x)S_n^{m_1,\ldots,m_r}(y_1x,y_2,y_3x,y_4x,y_5,\ldots,y_r)\right]$$

$$=A(\phi)\frac{\Gamma(\eta+k_1+k_3+k_4)}{\Gamma(\mu+k_1+k_3+k_4)}x^{k_1+k_3+k_4}B(\theta)G_{A_3}^4(a,\eta+k_1+k_3+k_4,c,d;e,e';z_1x,z_2,z_3x,z_4x),$$

$\mathrm{Re}(k_1+k_3+k_4)>-\eta.$

(2.24) $$I_x^{\eta,\mu-\eta}\left[G_{A_3}^4(a,b,\mu+k_2+k_4,d;e,e';z_1,z_2x,z_3,z_4x)S_n^{m_1,\ldots,m_r}(y_1,y_2x,y_3,y_4x,y_5,\ldots,y_r)\right]$$

$$=A(\phi)\frac{\Gamma(\eta+k_2+k_4)}{\Gamma(\mu+k_2+k_4)}x^{k_2+k_4}B(\theta)G_{A_3}^4(a,b,\eta+k_2+k_4,d;e,e';z_1,z_2x,z_3,z_4x),$$

$\mathrm{Re}(k_2+k_4)>-\eta.$

(2.25) $$I_x^{\eta,\mu-\eta}\left[G_{A_3}^4(a,b,c,\mu+k_4;e,e';z_1,z_2,z_3,z_4x)S_n^{m_1,\ldots,m_r}(y_1,y_2,y_3,y_4x,y_5,\ldots,y_r)\right]$$

$$=A(\phi)\frac{\Gamma(\eta+k_4)}{\Gamma(\mu+k_4)}x^{k_4}B(\theta)G_{A_3}^4(a,b,c,\eta+k_4;e,e';z_1,z_2,z_3,z_4x),\ \mathrm{Re}(k_4)>-\eta.$$

(2.26) $$I_x^{\eta,\mu-\eta}\left[G_{A_3}^4(a,b,c,d;e,\eta+k_4;z_1,z_2,z_3,z_4x)S_n^{m_1,\ldots,m_r}(y_1,y_2,y_3,y_4x,y_5,\ldots,y_r)\right]$$

$$=A(\phi)\frac{\Gamma(\eta+k_4)}{\Gamma(\mu+k_4)}x^{k_4}B(\theta)G_{A_3}^4(a,b,c,d;e,\mu+k_4;z_1,z_2,z_3,z_4x),$$

(2.27) $$I_x^{\eta,\mu-\eta}\left[G_{B_1}^4(a,\mu+k_1,b_2,b_3,b_4;e,e';z_1x,z_2,z_3,z_4)S_n^{m_1,\ldots,m_r}(y_1x,y_2,y_3,y_4,y_5,\ldots,y_r)\right]$$

$$=A(\phi)\frac{\Gamma(\eta+k_1)}{\Gamma(\mu+k_1)}x^{k_1}B(\theta)G_{B_1}^4(a,\eta+k_1,b_2,b_3,b_4;e,e';z_1x,z_2,z_3,z_4),\ Re(k_1)>-\eta.$$

(2.28) $$I_x^{\eta,\mu-\eta}\left[G_{B_1}^4(a,b_1,\mu+k_2,b_3,b_4;e,e';z_1,z_2x,z_3,z_4)S_n^{m_1,\ldots,m_r}(y_1,y_2x,y_3,y_4,y_5,\ldots,y_r)\right]$$

$$=A(\phi)\frac{\Gamma(\eta+k_2)}{\Gamma(\mu+k_2)}x^{k_2}B(\theta)G_{B_1}^4(a,b_1,\eta+k_2,b_3,b_4;e,e';z_1,z_2x,z_3,z_4),\ \mathrm{Re}(k_2)>-\eta.$$

(2.29) $$I_x^{\eta,\mu-\eta}\left[G_{B_1}^4(a,b_1,b_2,\mu+k_3,b_4;e,e';z_1,z_2,z_3x,z_4)S_n^{m_1,\ldots,m_r}(y_1,y_2,y_3x,y_4,y_5,\ldots,y_r)\right]$$

$$= A(\phi)\frac{(\Gamma\eta + k_3)}{(\Gamma\mu + k_3)} x^{k_3} B(\theta) G_{B_1}^4 (a, b_1, b_2, \eta + k_3, b_4; e, e'; z_1, z_2, z_3 x, z_4), \quad \mathrm{Re}(k_3) > -\eta.$$

(2.30) $$I_x^{\eta,\mu-\eta}\left[G_{B_1}^4 (a, b_1, b_2, b_3, \mu + k_4; e, e'; z_1, z_2, z_3, z_4 x) S_n^{m_1,\ldots,m_r}(y_1, y_2, y_3, y_4 x, y_5, \ldots, y_r)\right]$$

$$= A(\phi)\frac{\Gamma(\eta + k_4)}{\Gamma(\mu + k_4)} x^{k_4} B(\theta) G_{B_1}^4 (a, b_1, b_2, b_3, \eta + k_4; e, e'; z_1, z_2, z_3, z_4 x), \quad \mathrm{Re}(k_4) > -\eta.$$

(2.31) $$I_x^{\eta,\mu-\eta}\left[G_{B_1}^4 (a, b_1, b_2, b_3, b_4; e, \eta + k_4; z_1, z_2, z_3, z_4 x) S_n^{m_1,\ldots,m_r}(y_1, y_2, y_3, y_4 x, y_5, \ldots, y_r)\right]$$

$$= A(\phi)\frac{\Gamma(\eta + k_4)}{\Gamma(\mu + k_4)} x^{k_4} B(\theta) G_{B_1}^4 (a, b_1, b_2, b_3, b_4; e, \mu + k_4; z_1, z_2, z_3, z_4 x), \quad \mathrm{Re}(k_4) > -\eta.$$

(2.32) $$I_x^{\eta,\mu-\eta}\left[G_{B_2}^4 (a, \mu + k_1 + k_4, b_2, b_3, b_4; e, e'; z_1 x, z_2, z_3, z_4 x) S_n^{m_1,\ldots,m_r}(y_1 x, y_2, y_3, y_4 x, y_5, \ldots, y_r)\right]$$

$$= A(\phi)\frac{\Gamma(\eta + k_1 + k_4)}{\Gamma(\mu + k_1 + k_4)} x^{k_1 + k_4} B(\theta) G_{B_2}^4 (a, \eta + k_1 + k_4, b_2, b_3, b_4; e, e'; z_1 x, z_2, z_3, z_4 x),$$

$\mathrm{Re}(k_1 + k_4) > -\eta$.

(2.33) $$I_x^{\eta,\mu-\eta}\left[G_{B_2}^4 (a, b_1, \mu + k_2, b_3, b_4; e, e'; z_1, z_2 x, z_3, z_4) S_n^{m_1,\ldots,m_r}(y_1, y_2 x, y_3, y_4, y_5, \ldots, y_r)\right]$$

$$= A(\phi)\frac{\Gamma(\eta + k_2)}{\Gamma(\mu + k_2)} x^{k_2} B(\theta) G_{B_2}^4 (a, b_1, \eta + k_2, b_3, b_4; e, e'; z_1, z_2 x, z_3, z_4), \quad \mathrm{Re}(k_2) > -\eta.$$

(2.34) $$I_x^{\eta,\mu-\eta}\left[G_{B_2}^4 (a, b_1, b_2, \mu + k_3, b_4; e, e'; z_1, z_2, z_3 x, z_4) S_n^{m_1,\ldots,m_r}(y_1, y_2, y_3 x, y_4, y_5, \ldots, y_r)\right]$$

$$= A(\phi)\frac{\Gamma(\eta + k_3)}{\Gamma(\mu + k_3)} x^{k_3} B(\theta) G_{B_2}^4 (a, b_1, b_2, \eta + k_3, b_4; e, e'; z_1, z_2, z_3 x, z_4), \quad \mathrm{Re}(k_3) > -\eta.$$

(2.35) $$I_x^{\eta,\mu-\eta}\left[G_{B_2}^4 (a, b_1, b_2, b_3, \mu + k_4; e, e'; z_1, z_2, z_3, z_4 x) S_n^{m_1,\ldots,m_r}(y_1, y_2, y_3, y_4, y_5 x, \ldots, y_r)\right]$$

$$= A(\phi)\frac{\Gamma(\eta + k_4)}{\Gamma(\mu + k_4)} x^{k_4} B(\theta) G_{B_2}^4 (a, b_1, b_2, b_3, \eta + k_4; e, e'; z_1, z_2, z_3, z_4 x), \quad \mathrm{Re}(k_4) > -\eta.$$

(2.36) $$I_x^{\eta,\mu-\eta}\left[G_{B_2}^4 (a, b_1, b_2, b_3, b_4; e, \eta + k_4; z_1, z_2, z_3, z_4 x) S_n^{m_1,\ldots,m_r}(y_1, y_2, y_3, y_4 x, y_5, \ldots, y_r)\right]$$

$$= A(\phi)\frac{\Gamma(\eta + k_4)}{\Gamma(\mu + k_4)} x^{k_4} B(\theta) G_{B_2}^4 (a, b_1, b_2, b_3, b_4; e, \mu + k_4; z_1, z_2, z_3, z_4 x), \quad \mathrm{Re}(k_4) > -\eta.$$

(2.37) $I_x^{\eta,\mu-\eta}\left[G_{B_3}^4\left(a,b_1,b_2,\mu+k_3,b_4;e,e';z_1,z_2,z_3x,z_4\right)S_n^{m_1,\ldots,m_r}\left(y_1,y_2,y_3x,y_4,y_5,\ldots,y_r\right)\right]$

$$=A(\phi)\frac{\Gamma(\eta+k_3)}{\Gamma(\mu+k_3)}x^{k_3}B(\theta)G_{B_3}^4\left(a,b_1,b_2,\eta+k_3,b_4;e,e';z_1,z_2,z_3x,z_4\right),\ \mathrm{Re}(k_3)>-\eta.$$

(2.38) $I_x^{\eta,\mu-\eta}\left[G_{B_3}^4\left(a,\mu+k_2+k_4,b_2,b_3,b_4;e,e';z_1,z_2x,z_3,z_4x\right)S_n^{m_1,\ldots,m_r}\left(y_1,y_2x,y_3,y_4x,y_5,\ldots,y_r\right)\right]$

$$=A(\phi)\frac{\Gamma(\eta+k_2+k_4)}{\Gamma(\mu+k_2+k_4)}x^{k_2+k_4}B(\theta)G_{B_3}^4\left(a,\eta+k_2+k_4,b_2,b_3,b_4;e,e';z_1,z_2x,z_3,z_4x\right),$$

$\mathrm{Re}(k_2+k_4)>-\eta$.

(2.39) $I_x^{\eta,\mu-\eta}\left[G_{B_3}^4\left(a,b_1,\mu+k_1,b_3,b_4;e,e';z_1x,z_2,z_3,z_4\right)S_n^{m_1,\ldots,m_r}\left(y_1x,y_2,y_3,y_4,y_5,\ldots,y_r\right)\right]$

$$=A(\phi)\frac{\Gamma(\eta+k_1)}{\Gamma(\mu+k_1)}x^{k_1}B(\theta)G_{B_3}^4\left(a,b_1,\mu+k_1,b_3,b_4;e,e';z_1x,z_2,z_3,z_4\right),\ \mathrm{Re}(k_1)>-\eta.$$

(2.40) $I_x^{\eta,\mu-\eta}\left[G_{B_3}^4\left(a,b_1,b_2,b_3,\mu+k_4;e,e';z_1,z_2,z_3,z_4x\right)S_n^{m_1,\ldots,m_r}\left(y_1,y_2,y_3,y_4x,y_5,\ldots,y_r\right)\right]$

$$=A(\phi)\frac{\Gamma(\eta+k_4)}{\Gamma(\mu+k_4)}x^{k_4}B(\theta)G_{B_3}^4\left(a,b_1,b_2,b_3,\eta+k_4;e,e';z_1,z_2,z_3,z_4x\right),\ \mathrm{Re}(k_4)>-\eta.$$

(2.41) $I_x^{\eta,\mu-\eta}\left[G_{B_3}^4\left(a,b_1,b_2,b_3,b_4;e,\eta+k_4;z_1,z_2,z_3x,z_4x\right)S_n^{m_1,\ldots,m_r}\left(y_1,y_2,y_3,y_4x,y_5,\ldots,y_r\right)\right]$

$$=A(\phi)\frac{\Gamma(\eta+k_4)}{\Gamma(\mu+k_4)}x^{k_4}B(\theta)G_{B_3}^4\left(a,b_1,b_2,b_3,b_4;e,\mu+k_4;z_1,z_2,z_3,z_4x\right),\ \mathrm{Re}(k_4)>-\eta.$$

(2.42) $I_x^{\eta,\mu-\eta}\left[G_{C_1}^4\left(\mu+k_1+k_3+k_4,b,c,d;e,e';z_1x,z_2,z_3x,z_4x\right)S_n^{m_1,\ldots,m_r}\left(y_1x,y_2,y_3,y_4x,y_5,\ldots,y_r\right)\right]$

$$=A(\phi)\frac{\Gamma(\eta+k_1+k_3+k_4)}{\Gamma(\mu+k_1+k_3+k_4)}x^{k_1+k_3+k_4}B(\theta)G_{C_1}^4\left(\eta+k_1+k_3+k_4,b,c,d;e,e';z_1x,z_2,z_3x,z_4x\right)$$

$\mathrm{Re}(k_1+k_3+k_4)>-\eta$.

(2.43) $I_x^{\eta,\mu-\eta}\left[G_{C_1}^4\left(a,\mu+k_1+k_2,c,d;e,e';z_1x,z_2x,z_3,z_4\right)S_n^{m_1,\ldots,m_r}\left(y_1x,y_2x,y_3,y_4,y_5,\ldots,y_r\right)\right]$

$$=A(\phi)\frac{\Gamma(\eta+k_1+k_2)}{\Gamma(\mu+k_1+k_2)}x^{k_1+k_2}B(\theta)G_{C_1}^4\left(a,\eta+k_1+k_2,c,d;e,e';z_1x,z_2x,z_3,z_4\right),$$

$\mathrm{Re}(k_1+k_2)>-\eta$.

(2.44) $$I_x^{\eta,\mu-\eta}\left[G_{C_1}^4(a,b,c,\mu+k_4,;e,e';z_1,z_2,z_3,z_4x)S_n^{m_1,\dots,m_r}(y_1,y_2,y_3,y_4x,y_5,\dots,y_r)\right]$$

$$=A(\phi)\frac{\Gamma(\eta+k_4)}{\Gamma(\mu+k_4)}x^{k_4}B(\theta)G_{C_1}^4(a,b,c,\eta+k_4;e,e';z_1,z_2,z_3,z_4x),\quad \mathrm{Re}(k_4)>-\eta.$$

(2.45) $$I_x^{\eta,\mu-\eta}\left[G_{C_1}^4(a,b,c,d;e,\eta+k_4;z_1,z_2,z_3,z_4x)S_n^{m_1,\dots,m_r}(y_1,y_2,y_3,y_4x,y_5,\dots,y_r)\right]$$

$$=A(\phi)\frac{\Gamma(\eta+k_4)}{\Gamma(\mu+k_4)}x^{k_4}B(\theta)G_{C_1}^4(a,b,c,d;e,\mu+k_4;z_1,z_2,z_3,z_4x),\quad \mathrm{Re}(k_4)>-\eta.$$

(2.46) $$I_x^{\eta,\mu-\eta}\left[G_{C_2}^4(\mu+k_1+k_2+k_4,b,c,d;e,e';z_1x,z_2x,z_3,z_4x)S_n^{m_1,\dots,m_r}(y_1x,y_2x,y_3,y_4x,y_5,\dots,y_r)\right]$$

$$=A(\phi)\frac{\Gamma(\eta+k_1+k_2+k_4)}{\Gamma(\mu+k_1+k_2+k_4)}x^{k_1+k_2+k_4}B(\theta)G_{C_2}^4(\eta+k_1+k_2+k_4,b,c,d;e,e';z_1x,z_2x,z_3,z_4x),$$

$$\mathrm{Re}(k_1+k_2+k_4)>-\eta\,.$$

(2.47) $$I_x^{\eta,\mu-\eta}\left[G_{C_2}^4(a,\mu+k_1+k_3,c,d;e,e';z_1x,z_2,z_3x,z_4)S_n^{m_1,\dots,m_r}(y_1x,y_2,y_3x,y_4,y_5,\dots,y_r)\right]$$

$$=A(\phi)\frac{\Gamma(\eta+k_1+k_3)}{\Gamma(\mu+k_1+k_3)}x^{k_1+k_3}B(\theta)G_{C_2}^4(a,\eta+k_1+k_3,c,d;e,e';z_1x,z_2,z_3x,z_4),$$

$$\mathrm{Re}(k_1+k_3)>-\eta\,.$$

(2.48) $$I_x^{\eta,\mu-\eta}\left[G_{C_2}^4(a,b,c,\mu+k_4;e,e';z_1,z_2,z_3,z_4x)S_n^{m_1,\dots,m_r}(y_1,y_2,y_3,y_4x,y_5,\dots,y_r)\right]$$

$$=A(\phi)\frac{\Gamma(\eta+k_4)}{\Gamma(\mu+k_4)}x^{k_4}B(\theta)G_{C_2}^4(a,b,c,\eta+k_4;e,e';z_1,z_2,z_3,z_4x),\quad \mathrm{Re}(k_4)>-\eta.$$

(2.49) $$I_x^{\eta,\mu-\eta}\left[G_{C_2}^4(a,b,c,d;e,\eta+k_4;z_1,z_2,z_3,z_4x)S_n^{m_1,\dots,m_r}(y_1,y_2,y_3,y_4x,y_5,\dots,y_r)\right]$$

$$=A(\phi)\frac{\Gamma(\eta+k_4)}{\Gamma(\mu+k_4)}x^{k_4}B(\theta)G_{C_2}^4(a,b,c,d;e,\mu+k_4;z_1,z_2,z_3,z_4x),\quad \mathrm{Re}(k_4)>-\eta.$$

(2.50) $$I_x^{\eta,\mu-\eta}\left[F_A^{(n)}(\mu+k_1+k_2+\dots+k_r;b_1,\dots,b_n;c_1,\dots,c_n;z_1x,\dots,z_nx)S_n^{m_1,\dots,m_r}(y_1x,y_2x,\dots,y_r)\right]$$

$$=A(\phi)\frac{\Gamma(\eta+k_1+\dots+k_r)}{\Gamma(\mu+k_1+\dots+k_r)}x^{k_1+\dots+k_r}B(\theta)F_A^{(n)}(\eta+k_1+\dots+k_r,b_1,\dots,b_n;c_1,\dots,c_n;z_1x,\dots,z_nx),$$

$$\mathrm{Re}(k_1+\dots+k_r)>-\eta.$$

(2.51) $$I_x^{\eta,\mu-\eta}\left[F_B^{(n)}(a_1,\dots,a_n;b_1,\dots,b_n;\eta+k_1+\dots+k_r;z_1x,\dots,z_nx)S_n^{m_1,\dots,m_r}(y_1x,\dots,y_rx)\right]$$

$$= A(\phi)\frac{\Gamma(\eta+k_1+\ldots+k_r)}{\Gamma(\mu+k_1+\ldots+k_r)}x^{k_1+\ldots+k_r}B(\theta)F_B^{(n)}(a_1,\ldots,a_n,b_1,\ldots,b_n;\mu+k_1+\ldots+k_r;z_1x,\ldots,z_nx),$$

$$\mathrm{Re}(k_1+\ldots+k_r)>-\eta.$$

(2.52) $$I_x^{\eta,\mu-\eta}\left[F_C^{(n)}(\mu+k_1+\ldots+k_r;b;c_1,\ldots,c_n;z_1x,\ldots,z_nx)S_n^{m_1,\ldots,m_r}(\nu_1x,y_2x,\ldots,y_r)\right]$$

$$= A(\phi)\frac{\Gamma(\eta+k_1+\ldots+k_r)}{\Gamma(\mu+k_1+\ldots+k_r)}x^{k_1+\ldots+k_r}B(\theta)F_C^{(n)}(\eta+k_1+\ldots+k_r,b;c_1,\ldots,c_n;z_1x,\ldots,z_nx)$$

$$\mathrm{Re}(k_1+\ldots+k_r)>-\eta.$$

(2.53) $$I_x^{\eta,\mu-\eta}\left[F_C^{(n)}(a,\mu+k_1+\ldots+k_r;c_1,\ldots,c_n;z_1x,\ldots,z_nx)S_n^{m_1,\ldots,m_r}(y_1x,y_2x,\ldots,y_rx)\right]$$

$$= A(\phi)\frac{\Gamma(\eta+k_1+\ldots+k_r)}{\Gamma(\mu+k_1+\ldots+k_r)}x^{k_1+\ldots+k_r}B(\theta)F_C^{(n)}(a,\eta+k_1+\ldots+k_r;c_1,\ldots,c_n;z_1x,\ldots,z_nx),$$

$$\mathrm{Re}(k_1+\ldots+k_r)>-\eta.$$

(2.54) $$I_x^{\eta,\mu-\eta}\left[F_D^{(n)}(\mu+k_1+\ldots+k_r;b_1,\ldots,b_n;c;z_1x,\ldots,z_nx)S_n^{m_1,\ldots,m_r}(y_1x,\ldots,y_rx)\right]$$

$$= A(\phi)\frac{\Gamma(\eta+k_1+\ldots+k_r)}{\Gamma(\mu+k_1+\ldots+k_r)}x^{k_1+\ldots+k_r}B(\theta)F_D^{(n)}(\eta+k_1+\ldots+k_r,b_1,\ldots,b_n;c;z_1x,\ldots,z_nx),$$

$$\mathrm{Re}(k_1+\ldots+k_r)>-\eta.$$

(2.55) $$I_x^{\eta,\mu-\eta}\left[F_D^{(n)}(a,b_1,\ldots,b_n;\eta+k_1+\ldots+k_r;z_1x,\ldots,z_nx)S_n^{m_1,\ldots,m_r}(y_1x,y_2x,\ldots,y_rx)\right]$$

$$= A(\phi)\frac{\Gamma(\eta+k_1+\ldots+k_r)}{\Gamma(\mu+k_1+\ldots+k_r)}x^{k_1+\ldots+k_r}B(\theta)F_D^{(n)}(a,b_1,\ldots,b_n;\mu+k_1+\ldots+k_r;z_1x,\ldots,z_nx),$$

$$\mathrm{Re}(k_1+\ldots+k_r)>-\eta.$$

(2.56) $$I_x^{\eta,\mu-\eta}\left[{}^{(k)}_{(1)}E_D^{(n)}(\mu+k_1+\ldots+k_r;b_1,b_2,\ldots,b_n;c,c';z_1x,\ldots,z_nx)S_n^{m_1,\ldots,m_r}(y_1x,\ldots,y_rx)\right]$$

$$= A(\phi)\frac{\Gamma(\eta+k_1+\ldots+k_r)}{\Gamma(\mu+k_1+\ldots+k_r)}x^{k_1+\ldots+k_r}B(\theta)\,{}^{(k)}_{(1)}E_D^{(n)}(\eta+k_1+\ldots+k_r,b_1,\ldots,b_n;c,c';z_1x,\ldots,z_nx),$$

$$\mathrm{Re}(k_1+\ldots+k_r)>-\eta.$$

(2.57) $$I_x^{\eta,\mu-\eta}\left\{{}^{(k)}_{(2)}E_D^{(n)}(a,a';b_1,\ldots,b_n;\eta+k_1+\ldots+k_r;z_1x,\ldots,z_nx)S_n^{m_1,\ldots,m_r}(y_1x,\ldots,y_rx)\right\}$$

$$= A(\phi)\frac{\Gamma(\eta+k_1+\ldots+k_r)}{\Gamma(\mu+k_1+\ldots+k_r)}x^{k_1+\ldots+k_r}B(\theta)\,{}^{(k)}_{(2)}E_D^{(n)}(a,a';b_1,\ldots,b_n;\mu+k_1+\ldots+k_r;z_1x,\ldots,z_nx),$$

$$\mathrm{Re}(k_1+\ldots+k_r)>-\eta\,.$$

(2.58) $$I_x^{\eta,\mu-\eta}\left\{{}^{(k)}_{(1)}E_C^{(n)}(a,a';\mu+k_1+\ldots+k_r;c_1,\ldots,c_n;z_1x,\ldots,z_nx)S_n^{m_1,\ldots,m_r}(y_1x,\ldots,y_rx)\right\}$$

$$=A(\phi)\frac{\Gamma(\eta+k_1+\ldots+k_r)}{\Gamma(\mu+k_1+\ldots+k_r)}x^{k_1+\ldots+k_r}B(\theta)\,{}^{(k)}_{(1)}E_C^{(n)}(a,a';\eta+k_1+\ldots+k_r;c_1,\ldots,c_n;z_1x,\ldots,z_nx),$$

$$\mathrm{Re}(k_1+\ldots+k_r)>-\eta\,.$$

(2.59) $$I_x^{\eta,\mu-\eta}\left\{{}^{(k)}F_{AC}^{(n)}(\mu+k_1+\ldots+k_r;b,b_{k+1},\ldots,b_n;c_1,\ldots,c_n;z_1x,\ldots,z_nx)S_n^{m_1,\ldots,m_r}(y_1x,\ldots,y_rx)\right\}$$

$$=A(\phi)\frac{\Gamma(\eta+k_1+\ldots+k_r)}{\Gamma(\mu+k_1+\ldots+k_r)}x^{k_1+\ldots+k_r}B(\theta)\,{}^{(k)}F_{AC}^{(n)}(\eta+k_1+\ldots+k_r;b,b_{k+1},\ldots,b_n;c_1,\ldots,c_n;z_1x,\ldots,z_nx)$$

$$\mathrm{Re}(k_1+\ldots+k_r)>-\eta$$

(2.60) $$I_x^{\eta,\mu-\eta}\left\{{}^{(k)}F_{AD}^{(n)}(\mu+k_1+\ldots+k_r;b_1,\ldots,b_n;c,c_{k+1},\ldots,c_n;z_1x,\ldots,z_nx)S_n^{m_1,\ldots,m_r}(y_1x,\ldots,y_rx)\right\}$$

$$=A(\phi)\frac{\Gamma(\eta+k_1+\ldots+k_r)}{\Gamma(\mu+k_1+\ldots+k_r)}x^{k_1+\ldots+k_r}B(\theta)\,{}^{(k)}F_{AD}^{(n)}(\eta+k_1+\ldots+k_r;b_1,\ldots,b_n;c_1,c_{k+1},\ldots,c_n;z_1x,\ldots,z_nx)$$

$$\mathrm{Re}(k_1+\ldots+k_r)>-\eta\,.$$

(2.61) $$I_x^{\eta,\mu-\eta}\left\{{}^{(k)}F_{BD}^{(n)}(a,a_{k+1},\ldots,a_n;b_1,\ldots,b_n;\eta+k_1+\ldots+k_r;z_1x,\ldots,z_nx)S_n^{m_1,\ldots,m_r}(y_1x,\ldots,y_rx)\right\}$$

$$=A(\phi)\frac{\Gamma(\eta+k_1+\ldots+k_r)}{\Gamma(\mu+k_1+\ldots+k_r)}x^{k_1+\ldots+k_r}B(\theta)\,{}^{(k)}F_{BD}^{(n)}(a,a_{k+1},\ldots,a_n;b_1,\ldots b_n;\mu+k_1+\ldots+k_r;z_1x,\ldots,z_nx)$$

$$\mathrm{Re}(k_1+\ldots+k_r)>-\eta\,.$$

(2.62) $$I_x^{\eta,\mu-\eta}\left\{{}^{(k)}_{(1)}\phi_{AC}^{(n)}(\mu+k_1+\ldots+k_r;b;c_1,\ldots,c_n;z_1x,\ldots,z_nx)S_n^{m_1,\ldots,m_r}(y_1x,\ldots,y_rx)\right\}$$

$$=A(\phi)\frac{\Gamma(\eta+k_1+\ldots+k_r)}{\Gamma(\mu+k_1+\ldots+k_r)}x^{k_1+\ldots+k_r}B(\theta)\,{}^{(k)}_{(1)}\phi_{AC}^{(n)}(\eta+k_1+\ldots+k_r;b;c_1,\ldots,c_n;z_1x,\ldots,z_nx),$$

$$\mathrm{Re}(k_1+\ldots+k_r)>-\eta$$

(2.63) $$I_x^{\eta,\mu-\eta}\left\{{}^{(k)}_{(1)}\phi_{AC}^{(n)}(a;\mu+k_1+\ldots+k_r;c_1,\ldots,c_n;z_1x,\ldots,z_nx)S_n^{m_1,\ldots,m_r}(y_1x,\ldots,y_rx)\right\}$$

$$=A(\phi)\frac{\Gamma(\eta+k_1+\ldots+k_r)}{\Gamma(\mu+k_1+\ldots+k_r)}x^{k_1+\ldots+k_r}B(\theta)\,{}^{(k)}_{(1)}\phi_{AC}^{(n)}(a,\eta+k_1+\ldots+k_r;c_1,\ldots,c_n;z_1x,\ldots,z_nx),$$

$$\mathrm{Re}(k_1+\ldots+k_r)>-\eta\,.$$

(2.64) $I_x^{\eta,\mu-\eta}\left\{{}^{(k)}_{(1)}\phi_{AD}^{(n)}(\mu+k_1+\ldots+k_r;b_1,b_2,\ldots,b_n;c;z_1x,\ldots,z_nx)S_n^{m_1,\ldots,m_r}(y_1x,\ldots,y_rx)\right\}$

$$=A(\phi)\frac{\Gamma(\eta+k_1+\ldots+k_r)}{\Gamma(\mu+k_1+\ldots+k_r)}x^{k_1+\ldots+k_r}B(\theta)\,{}^{(k)}_{(1)}\phi_{AD}^{(n)}(\eta+k_1+\ldots+k_r;b_1,\ldots,b_n;c;z_1x,\ldots,z_nx),$$

$$\mathrm{Re}(k_1+\ldots+k_r)>-\eta.$$

(2.65) $I_x^{\eta,\mu-\eta}\left\{{}^{(k)}_{(1)}\phi_{BD}^{(n)}(a;b_1,\ldots,b_n;\eta+k_1+\ldots+k_r;z_1x,\ldots,z_nx)S_n^{m_1,\ldots,m_r}(y_1x,\ldots,y_rx)\right\}$

$$=A(\phi)\frac{\Gamma(\eta+k_1+\ldots+k_r)}{\Gamma(\mu+k_1+\ldots+k_r)}x^{k_1+\ldots+k_r}B(\theta)\,{}^{(k)}_{(1)}\phi_{BD}^{(n)}(a;b_1,\ldots,b_n;\mu+k_1+\ldots+k_r;z_1x,\ldots,z_nx),$$

$$\mathrm{Re}(k_1+\ldots+k_r)>-\eta.$$

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