Jñānābha, Vol. 35, 2005

A QUEUEING NETWORK APPROACH TO ANALYZE THE EXISTING CAREER ADVANCEMENT POLICY

By

Madhu Jain

Department of Mathematics, Institute of Basic Science,
Khandari, Agra- 2852002, India

and

Anamika Jain

Department of Mathematics, St. John's College, Agra-282002, India

(Received : April 06, 2005)

ABSTRACT

The purpose of this investigation is to identify the factors of various situations dealing with the appointment and career advancement policy in the universities and colleges for teachers in India. The model proposed takes care of different options that are available to a teacher at various points of time and also different channels through which he may be appointed/promoted to the next higher position. The model is based on a queueing netwok of service channels in series for different phases of services. Also the availability of alternative parallel avenues is also taken care in the study. The idea is to point out various decision factors in the career advancement schme.Probabilities have been calculated for different stages and the same have been used to find he mean queue length at a point of time. This provides insight into the delay process of the career advancement scheme. This study may be useul for the Government sector while revision of scales of pay along with promotional scheme takes place

AMS Subject Classification (2000) :Primary 90B10; Secondary 90B22

Keywords: Queue network, Career advancement, Poisson process, Service channels, Queue length.

1. Introduction. Queueing network models can be employed to explore and improve the congestion problems of real life situations. In teaching career long waiting time to reach the next higher position is of immense importance in the academic circles. A person having entered in the teaching stream expects that he avails due position in his career without any delay. No precise information is available in the literature regarding the queue length and delay probability. An effort in this regard has made by Sharma and Ahuja (2004). However with the revision of scales of pay and career advancement policies their model needs some important changes and up gradation. We have attempted to incorporate the new changes in our model to predict the delay in promotion of faculty who have started their career in a college/ university. In this investigation we use the concept of queueing network to find the desired results. Some important works, which are of relevant to this study, are as given below.

Grinold (1976) developed a model for manpower planning with uncertain recruitment. Weber (1978) discussed an optimal assignment of customers to parallel servers. Nakagawa (1984) obtained the optimal number of units for a parallel system. Noble and Tijms (1992) provided an optimal routing of customers to parallel service groups. Walter (1997) developed the combinatorial analysis using non - interesting paths along with application of queueing theory. Srinivasan and Saavitri (2002) facilited a cost analysis on univariate policies of recruitment in manpower models. Srinivasan and Saavitri (2003) also proposed a cost analysis using bivariate policies of recruitment in manpower planning. Jain and Dhakad (2003) proposed a queueing network model for computerized railway reservation system. Murty et al. (2003) analyzed a three-stage recruitment process in a manpower-planning model.

However very little work has been reported for career advancement policies via queueing network model. In this paper assuming that in the recruitmtent process, the promotion processes form different stages are all poisson, the difference equations are developed and the same have been solved by product type method. The average queue lengths in each stage have been obtained. The rest of the paper is organized as follows. Section 2 presents the basic queueing network model and describes the underlying assumptions. Section 3 gives some notations used in the model. In section 4, the steady state equations are developed. Section 5 provides mathematical solution of the model. Section 6 discusses system characteristics. The formula for mean queue lengths and different stages expected number of employees are derived in section 7 . Finally, in section 8, we outline the novel features and future scope of our study.

2.Model. Service commences ingeniously when a faculty member arrives at the service channel. The faculty members are sent from one stage to the next as soon as it completes the desired requirements. The inter arrival time and service time are exponentially distributed. The waiting space has been taken infinite.

In the queueing network model, eight service channels S_i (i =1 ,2, ...,8)are taken into consideration based on University Grant Commission's (India) norm and are listed in teble 1

S_1 (L)	Lecturer
S_2 (SS)	Senior scale lecturer
S_3 (SG)	Selection grade lecturer
S_4 (RO)	Reder open category
S_5 (RP)	Reder under personal promotion scheme
S_6 (PO)	Professor open category
S_7 (PP)	Professor under personal promotion scheme
S_8 (PS)	Professor for special grade

Table: 1 **Notations are for eight sercice channels**

For construction of the governing equations of the model, the following notations are used :

Q_1	Queue length at the service stage $S_1(L)$
Q_i (i =1 , 2 ,...,8)	Queue length at the service stage $S_1(L)$ $S_2(SS)$ $S_3(SG)$, $S_4(RO)$, $S_5(RP)$, $S_6(PO)$, $S_7(PP)$, $S_8(PS)$, respectively
λ_1	Mean arrival rate for the queue formed at first stage
μ_1	Mean service time of the service stage S_1
p_i (i= 1 , 2 ,...,5)	The probability of joining at the service stages S_i (i= 2, 3,...,6) after service at S_1 (L)
q_i (i= 1 , 2 ,3,4)	The probability of joining at the service stages S_i (i= 2, 3,4,5) after service at $S_2(SS)$
r_i (i= 1 , 2 ,3,4)	The probability of joining at the service stages S_i (i= 4, 5,6,7) after service at $S_3(SG)$
s_i (i= 1, 2)	The probability of joining at the service stages S_i (i= 6, 7) after service at $S_4(RO)$
t_i (i= 1, 2, 3)	The probability of joining at the service stages S_i (i= 4, 6, 7) after service at $S_5(RP)$
β_i (i= 1, 2,...,8)	Traffic intensities for the service stages S_i(i=1,2,...,8)
P (n_1,n_2,....,n_8)	The steady state probability for the service stages S_i(i=1,2,...,8)
M_{q_i} (i=1,2,...,8)	Mean queue length for the service stages S_i(i=1,2,...,8)
L_q	Mean number of the faculty member in the system.

In the queueing network approach to analyse the existing caeeer advancement policy we consider eight stages. The length of queues at be assumed to be distributed geometrically with different parameters. We assume that the arrival choose the stage $S_1(L)$ in poisson fashion with parameter λ_1, after spending a random time from the queue Q_1 an employee joins the stages $S_2(SS)$, $S_3(SG)$, S_4 (RO), $S_5(RP)$, $S_6(PO)$, respectively with parameters p_i (i= 1,2,...,5). The employees in stage $S_2(SS)$ will be promoted after spending a random time with parameters q_i (i =1 , 2, 3 , 4,) to the stages $S_3(CG)$, $S_4(RO)$, $S_5(RP)$, $S_6(PO)$. The employees in stage $S_3(CG)$ be promoted to the stages $S_4(RO)$, $S_5(RP)$, $S_6(PO)$ and $S_7(PP)$ with parameters r_i (i=1,2,3,4). Also the employees in stage $S_4(RO)$ will be promoted after spending a random time with parameter s_i(i=1,2) to the stages $S_6(RP)$ and $S_7(PP)$ and the employees in stage $S_5(RP)$ will be promoted to the stages $S_4(RO)$, $S_6(PO)$ and $S_7(PP)$ with parameters t_i (i=1,2, 3). Similarly the employees in stage $S_6(PO)$ are promoted to the stages S_8(PS) and from the stage $S_7(PP)$ are promoted $S_6(PO)$ from the last stage $S_8(PS)$ the employees leave the system with probability one. The service is said to be completed when an employee goes through all relevant service channels in series from the eight parallel service channels. The routing amery eight- service channels are shown in figure 1. For illustration the possible transitions originating from $S_1(L)$ are also enlisted as follow :

$S_1(L)\rightarrow S_2(SS)\rightarrow S_3(SG) \rightarrow S_4(RO) \rightarrow S_6(PO) \rightarrow S_8(PS) \rightarrow$

$S_1(L)\rightarrow S_2(SS)\rightarrow S_3(SG) \rightarrow S_4(RO) \rightarrow S_7(PP) \rightarrow S_6(PO)\rightarrow S_8(PS) \rightarrow$

$S_1(L)\rightarrow S_2(SS)\rightarrow S_3(SG) \rightarrow S_5(RP) \rightarrow S_6(PO)\rightarrow S_8(PS) \rightarrow$

$S_1(L)\rightarrow S_2(SS)\rightarrow S_3(SG) \rightarrow S_5(RP) \rightarrow S_4(RO) \rightarrow S_6(PO)\rightarrow S_8(PS) \rightarrow$

$S_1(L)\rightarrow S_2(SS)\rightarrow S_3(SG) \rightarrow S_5(RP) \rightarrow S_4(RO) \rightarrow S_7(PP) \rightarrow S_6(PO)\rightarrow S_8(PS) \rightarrow$

$S_1(L)\rightarrow S_2(SS)\rightarrow S_3(SG) \rightarrow S_5(RP) \rightarrow S_7(PP) \rightarrow S_6(PO)\rightarrow S_8(PS) \rightarrow$

$S_1(L)\rightarrow S_2(SS)\rightarrow S_3(SG) \rightarrow S_6(PO)\rightarrow S_8(PS) \rightarrow$

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$S_1(L)\rightarrow S_2(SS)\rightarrow S_4(RO) \rightarrow S_6(PO)\rightarrow S_8(PS) \rightarrow$

$S_1(L)\rightarrow S_2(SS)\rightarrow S_4(RO) \rightarrow S_7(PP) \rightarrow S_6(PO)\rightarrow S_8(PS) \rightarrow$

$S_1(L)\rightarrow S_2(SS)\rightarrow S_5(RP) \rightarrow S_4(RO) \rightarrow S_6(PO)\rightarrow S_8(PS) \rightarrow$

$S_1(L)\rightarrow S_2(SS)\rightarrow S_5(RP) \rightarrow S_4(RO) \rightarrow S_7(PP) \rightarrow S_6(PO)\rightarrow S_8(PS) \rightarrow$

$S_1(L)\rightarrow S_2(SS)\rightarrow S_6(PO)\rightarrow S_8(PS) \rightarrow$

$S_1(L)\rightarrow S_3(SG) \rightarrow S_4(RO) \rightarrow S_6(PO)\rightarrow S_8(PS) \rightarrow$

$S_1(L)\rightarrow S_3(SG) \rightarrow S_4(RO) \rightarrow S_7(PP) \rightarrow S_6(PO)\rightarrow S_8(PS) \rightarrow$

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$S_1(L)\rightarrow S_3(SG) \rightarrow S_5(RP) \rightarrow S_4(RO) \rightarrow S_7(PP) \rightarrow S_6(PO)\rightarrow S_8(PS) \rightarrow$

$S_1(L)\rightarrow S_3(SG) \rightarrow S_5(RP) \rightarrow S_7(PP) \rightarrow S_6(PO)\rightarrow S_8(PS) \rightarrow$

$S_1(L)\rightarrow S_3(SG) \rightarrow S_6(PO)\rightarrow S_8(PS) \rightarrow$

$S_1(L)\rightarrow S_3(SG) \rightarrow S_7(PP) \rightarrow S_6(PO)\rightarrow S_8(PS) \rightarrow$

In the model proposed here the teaching faculty joins the college/university from outside the system in poisson fashion. After having entered the system the first service facility has eight channels. Thereafter at each node of promotion, the channels are indicated in the following diagram:

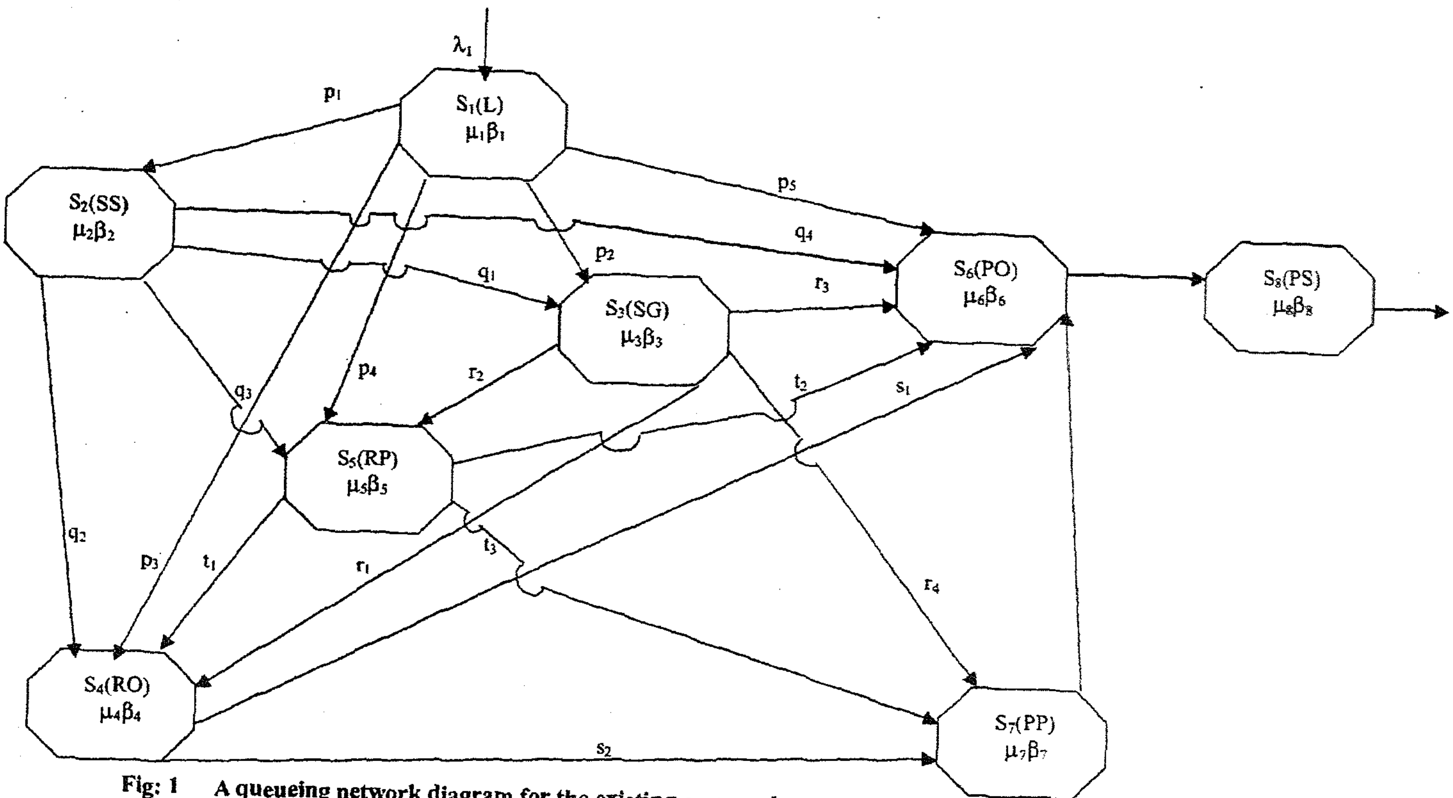

Fig: 1 A queueing network diagram for the existing career advancement policy

3. The Steady State Equations and Analysis. The steady state equations and the normalization condition for probabilities are given by

$$\mu_1\beta_1 = \lambda_1 \qquad \text{... (1.a.1)}$$

$$\mu_2\beta_2 = \mu_1\beta_1 p_1 \qquad \text{...(1.a.2)}$$

$$\mu_3\beta_3 = \mu_1\beta_1 p_2 + \mu_2\beta_2 q_1 \qquad \text{...(1.a.3)}$$

$$\mu_4\beta_4 = \mu_1\beta_1 p_3 + \mu_2\beta_2 q_2 + \mu_3\beta_3 r_1 + \mu_5\beta_5 t_1 \qquad \text{...(1.a.4)}$$

$$\mu_5\beta_5 = \mu_1\beta_1 p_4 + \mu_2\beta_2 q_3 + \mu_3\beta_3 r_2 \qquad \text{...(1.a.5)}$$

$$\mu_6\beta_6 = \mu_1\beta_1 p_5 + \mu_2\beta_2 q_4 + \mu_3\beta_3 r_3 + \mu_4\beta_4 s_1 + \mu_5\beta_5 t_2 + \mu_7\beta_7 \qquad \text{...(1.a.6)}$$

$$\mu_7\beta_7 = \mu_3\beta_3 r_4 + \mu_4\beta_2 s_2 + \mu_5\beta_5 t_3 \qquad \text{...(1.a.7)}$$

$$\mu_8\beta_8 = \mu_6\beta_6 \qquad \text{...(1.a.8)}$$

$$\sum_{i=1}^{6} p_i = 1,\ \sum_{i=1}^{4} q_i = 1,\ \sum_{i=1}^{4} r_i = 1,\ \sum_{i=1}^{2} s_i = 1,\ \sum_{i=1}^{2} t_i = 1.$$

Solving equations (1.a.1)-(1.a.8), we obtain intensities as follows

$$\beta_1 = \lambda_1/\mu_1 \qquad \text{...(2.a.1)}$$

$$\beta_2 = \lambda_1 p_1/\mu_2 \qquad \text{...(2.a.2)}$$

$$\beta_3 = \lambda_1\delta_1/\mu_3 \qquad \text{...(2.a.3)}$$

$$\beta_4 = \lambda_1[\delta_2 + r_1\delta_1 + t_1(\delta_3 + r_2\delta_1)]/\mu_4 \qquad \text{...(2.a.4)}$$

$$\beta_5 = \lambda_1[\delta_3 + r_2\delta_1]/\mu_3 \qquad \text{...(2.a.5)}$$

$$\beta_6 = \frac{\lambda_1[\delta_4 + \delta_1(r_3 + r_4) + (s_1 + s_2)\{\delta_2 + r_1\delta_1 + t_1(\delta_3 + r_2\delta_1)\} + (s_1 t_2 + t_3)(\delta_3 + r_2\delta_1)]}{\mu_6} \qquad \text{...(2.a.6)}$$

$$\beta_7 = \frac{\lambda_1[r_4 + \delta_1 + s_2\{\delta_2 + r_1\delta_1 + t_1(\delta_3 + r_2\delta_1)\} + t_3(\delta_3 + r_2\delta_1)]}{\mu_7} \qquad \text{...(2.a.7)}$$

$$\beta_8 = \frac{\lambda_1[\delta_4 + \delta_1(r_3 + r_4) + (s_1 + s_2)\{\delta_2 + r_1\delta_1 + t_1(\delta_3 + r_2\delta_1)\} + (s_1 t_2 + t_3)(\delta_3 + r_2\delta_1)]}{\mu_8} \qquad \text{...(2.a.8)}$$

where $\delta_i = (p_{i+1} + p_1 q_i), i = 1,2,3,4.$

Let p_{n_i} be the probability that there n employee being in $i^{th}(i=1,2,\ldots,8)$ stages in the system, the n steady state solution is obtained as

$$p(n_1, n_2, \ldots, n_8) = p(0,0,\ldots,0)\prod_{i=1}^{8}\beta_i \qquad \text{... (3)}$$

We can determine $P(0,0,\ldots,0)$ from the normalizing condition:

$$\sum_{i=1}^{8}\sum_{n=0}^{\infty} p_{n_i} = 1,$$

which gives,

$$p(0,0,\ldots,0) = \prod_{i=1}^{8}(1-\beta_i) \qquad \ldots (4)$$

Using equations (2.a.1)-(2.a.8) and (4) in (3), the steady state solution can be expressed as

$$p_{n_i} = \frac{(\mu_1-\lambda_1)(\mu_2-\lambda_1 p_1)(\mu_3-\lambda_1\delta_1)(\mu_4-\Lambda_1)(\mu_5-\Lambda_2)(\mu_6-\Lambda_3)(\mu_7-\Lambda_4)(\mu_8-\Lambda_5)\,\lambda_1^{n_1}(\lambda_1 p_1)^{n_2}(\lambda_1\delta_1)^{n_3}(\lambda_1\Lambda_1)^{n_4}(\lambda_1\Lambda_2)^{n_5}(\lambda_1\Lambda_3)^{n_6}(\lambda_1\Lambda_4)^{n_7}(\lambda_1\Lambda_5)^{n_8}}{\mu_1^{n_1+1}\left(\prod_{i=2}^{7}\mu_i^{n_i+1}\right)} \qquad \ldots (5)$$

where

$\Lambda_1 = [\delta_2 + r_1\delta_1 + t_1(\delta_3 + r_2\delta_1)]$

$\Lambda_2 = [\delta_3 + r_2\delta_1]$

$\Lambda_3 = [\delta_4 + \delta_1(r_3 + r_4) + (s_1 + s_2)\{\delta_2 + r_1\delta_1 + t_1(\delta_3 + r_2\delta_1)\} + (s_1 t_2 + t_3)(\delta_3 + r_2\delta_1)]$

$\Lambda_4 = [r_4 + \delta_1 + s_2\{\delta_2 + r_1\delta_1 + t_1(\delta_3 + r_2\delta_1)\} + t_3(\delta_3 + r_2\delta_1)]$

$\Lambda_5 = [\delta_4 + \delta_1(r_3 + r_4) + (s_1 + s_2)\{\delta_2 + r_1\delta_1 + t_1(\delta_3 + r_2\delta_1)\} + (s_1 t_2 + t_3)(\delta_3 + r_2\delta_1)]$.

4. System Characteristics. The steady state solution given in (5) exists subject to the conditions

$$\lambda_1 < \mu_1, \lambda_1 p_1 < \mu_2, \lambda_1\delta_1 < \mu_3, \Lambda_i < \mu_{i+3} (i = 1,2,\ldots,5) \qquad \ldots (6)$$

The steady state queue size distribution p_{n_1} can be obtained by using the equations (2.a.1)-(2.a.8)

$$p_{n_1} = (\mu_1 - \lambda_1)\lambda_1 / \mu_1^{n_1+1}. \qquad \ldots (7)$$

Similarly, p_{n_1} (i=2,...,7) the probabilities of these being n_i faculty members waiting for service in the Q_i before stages S_i(i=1,2,..,8) are given by

$$p_{n_2} = (\mu_2 - \lambda_1 p_1)/\mu_2^{n_2+1} \qquad \ldots (8)$$

$$p_{n_3} = (\mu_3 - \lambda_1\delta_1)/\mu_3^{n_3+1} \qquad \ldots (9)$$

$$p_{n_i} = (\mu_i - \Lambda_i)\Lambda_i^{n_i}/\mu_i^{n_i+1}, \quad i = 4,5,\ldots,8. \qquad \ldots (10)$$

Mean queue length. The mean queue length $M_{q_i}(i = 1,2,\ldots,8)$ before the service stage $S_i (i=1,2,...,8)$ and the expected number L_q of the employees in the queue can be obtained in a straightforward manners as

$$M_{q_i} = \sum_{i=1}^{\infty} n_i p_{n_i}, \quad L_q = \sum_{i=1}^{8} M_{q_i} \ .$$

Substituting the value of P_{n_i} from (7)-(10), we have

$$\left.\begin{aligned} M_{q_1} &= \frac{\lambda_1}{(\mu_1 - \lambda_1)}, & M_{q_2} &= \frac{\lambda_1 p_1}{(\mu_2 - \lambda_1 p_1)} \\ M_{q_3} &= \frac{\lambda_1 \delta_1}{(\mu_3 - \lambda_1 \delta_1)}, & M_{q_i} &= \frac{\Lambda_i}{(\mu_{i+3} - \Lambda_i)}, \ i = 1,2,3,4,5 \end{aligned}\right\} \quad ...(11)$$

We obtain the expected number of employees in the queue as

$$L_q = \sum_{i=1}^{8} M_{q_i} = \frac{\lambda_1}{(\mu_1 - \lambda_1)} + \frac{\lambda_1 p_1}{(\mu_2 - \lambda_1 p_1)} + \frac{\lambda_1 \delta_1}{(\mu_3 - \lambda_1 \delta_1)} + \sum_{i=1}^{5} \frac{\Lambda_i}{(\mu_{i+3} - \Lambda_i)} \quad ...(12)$$

5. Conclusion. In this paper, we have discussed a mathematical model for the career advancement policy via queueing theoretic approach. The model developed may be helpful to determine the optimal recruitment process for the teachers in the universities and collages by using the different eight stages in service scheme. The various performance indices have been computed in explicit form, which can be easily computed.

REFERENCES

[1] R.C. Grinold, Manpowr planning with uncertain recruitment, *Oper. Res.* **24** (3) (1976), 387-399.

[2] N. Hadidi, Busy period of queues with state dependent arrival and service rate., *Jour. App. Prob.,* **11** (1974), 842-848.

[3] M. Jain and M.R. Dhakad, Queueing network model for computerized railway reservation system, *Int. J. Infor. Computing Sci.,* **6** (1) (2003), 30-39.

[4] T. Nakagawa, Optimal number of units for a parallel system, *Jour. App. Prob.,* **21** (1984) 431-436.

[5] R.D. Noble and H.C. Tijms, Optimal routing of customers to parallel service groups in: W. Biihler, (ed.), *19th Oper. Res. Proceedings,* Springer-Verlay, Berlin (1992), 173-180.

[6] S.D. Sharma and S.S. Ahuja, Serial and parallel queueing network in managing promotion/career advancement policy, *Oper. Res. Information Technology and Industry* (Eds. M. Jain and G.C. Sharma) S.R. Sciencetific Publication Agra (2004), 156-188.

[7] A. Srinivasan and V. Saavitri, Cost analysis using bivariate policies of recruitment in manpower planning-a shock model approach, *Jour. De. Math Sci.* **8** (1-3)(2003) 71-78.

[8] W. Walter, Non-interesting paths and applications in queueing theory, *Advances in Combinatorial Methods and Applied Probability Statistics,* N. Bal Krishnan (eds) (1997).

[9] R. Weber, On the optimal assignment of customers to parallel servers, *Jour. App. Prob.,* **15** (1978), 406-413.