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UNSTEADY STATE RADIAL HEAT FLOW IN THE OUTER LAYERS OF SPHERICAL REGIONS OF HUMAN OR ANIMAL BODY

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ABSTRACT

Exact solution has been obtained using Laplace transform for one dimensional unsteady state heat migration problem in annular layers in spherical regions of a human or animal body under different atmospheric and physiological conditions assuming that skin surface is exposed to the environmental and loss of heat from the surface is due to radiation, convection and evaporation. The mathematical model incorporates the variations of blood mass flow rate and metabolic heat generating in different layers taking metabolic heat generation tissue temperature dependent.

Key Words. Stratum Corneum, Stratum Germinativum, Blood Mass Flow Rate, Metabolic Heat Generation, Laplace Transform.

1. Introduction. The body core temperature remains almost constant under normal conditions. The skin and underlying tissue layers undergoes temperature variation due to change in physiological parameters and environmental conditions in order to maintain a uniform body core temperature. The skin mainly consists of two layers, epidermis and dermis. The epidermis is composed of a living stratum germinativum which rests upon the dermis, and a dead, horny, superficial stratum corneum. The dermis is composed of matted masses of connective tissues and elastic fibres through which pass numerous bleed vessels, lymphatics and nerves. There are no blood vessels in the epidermis (Fig. 1). The population density of blood vessels in the dermis is very thin near the interface of epidermis and dermis, but increases gradually becoming almost uniform in the sub-dermal part. This help to understand the variation of quantities like rate of metabolic heat generation, rate of blood mass flow and thermal conductivity of tissue in this region with respect to its position.

In the present study, it is assumed that outer skin surface is exposed to atmosphere and the loss of heat is due to convection, radiation and evaporation of sweat. We consider only those part of human or animal body which are almost

spherical in s. .irection only. We
divide the reg

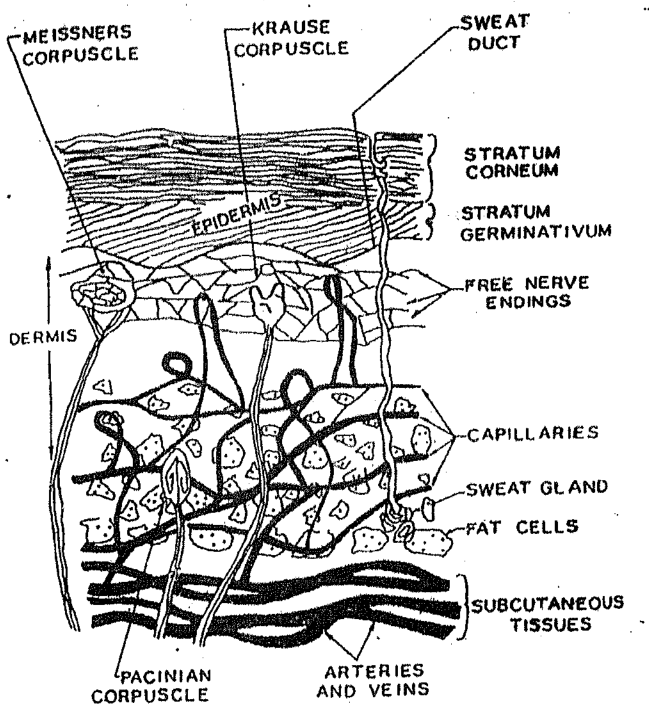

Fig 1. *A section through hairless skin showing epidermis, dermis and part of subcutaneous tissues*

its structure. It is assumed that rate of blood mass flow and rate of metabolic heat generation are position dependent which are negligible (zero) in epidermis vary linearly in dermis and constant in the sbdermal part. Further the rate of metabolic heat generation depends on the tissue temperature and its variation is in a manner that makes it self controlled. The thermal conductivity is taken as constant but different in three layers.

Mathematical Formualtion. The general mathematical model for the heat flow in skin and underlying tissues is given by

$$div[K\,gard\,T] + M(T_b - T) + S = \rho c(\partial T/\partial t) \qquad ...(1)$$

and $$-K(\partial T/\partial n) = h(T - T_a) + LE \qquad ...(2)$$

at the skin surface, here $M = m_b c_b$.

The inner boundary is taken at the interface of subdermal tissues and body core, where

$$T = T_b = 37.2^0\text{C}. \qquad ...(3)$$

Here K is the thermal conductivity of the tissue, m_b the rate of blood mass flow, c_b the specific heat of the blood. T_b the body core temperature, S the rate of metabolic heat generation ρ the tissue density, c the specific heat of the tissue, h is the heat transfer coefficient, L the latent heat of evaporation, E the rate of evaporation, T_a the atmospheric temperature and $\partial T/\partial n$ the partical derivative of T along normal to the boundary. We apply equation (1) and (2) to one dimensional unsteady state case taking skin surface insulated at time $t=0$.

Pearl [1] and Cooper and Trezek [2] have studied solution of some simple problems of infinite tissue media assuming all the parameters as constant throughout the region. Saxena [3] and Saxena and Arya [4] applied analytical and numerical methods to find solutions of certain unsteady state and steady state problems of temperature distribution in skin and subcutaneous tissues. Saxena [5] has studied the heat transport in peripheral layers of human body under normal and abnormal conditions. Some related work has been done by Pardasani and Saxena [6], Saxena, Juneja and Yadav [8] and Saxena, Pardasani and Saxena [7].

Solution of the Problem. The equations (1) and (2) are transformed into spherical polar coordinates for a one-dimensional unsteady state case to obtain

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(Kr^2(\partial T/\partial r)\right)+M(T_b-T)+S=\rho c(\partial T/\partial t) \quad \text{...(4)}$$

and $$-K(\partial T/\partial r)=h(T-T_a)+LE \quad \text{...(5)}$$

at the skin surface

In equation (4) the parameter $M(=m_b c_b)$ and S are assumed to be position dependent. The following assumptions have been made for each layer when the total thickness of the region under consideration is $r_0 \leq r \leq r_3$:

(i) For Epidermis

$K=K_1,\ M_1=0,\ S_1=0$

The thickness of epidermis is $r_2 \leq r \leq r_3$

(ii) For Dermis

$K=K_2,\ M_2=m[(r_2-r)/(r_2-r_1)]$

$S_2=S\ [(r_2-r)/(r_2-r_1)][1+q_d(T_b-T_2)]$,

where $q_d=2/(T_a+T_b)$. The thickness of dermis is $r_1 \leq r \leq r_2$.

At interface of epidermis and dermis the temperature is same i.e. $T_1=T_2$ where T_1 is temperature at r_2.

(iii) For Subdermal Tissues

$K=K_3,\ M_3=m,\ S_3=S[1+q_s(T_b-T_3)]$,

where $q_s=1/T_b$. The thickness of subdermal layer is $r_0 \leq r \leq r_1$ and the temperature at interface of subdermal dermal layers is same i.e. $T_2=T_3$ if T_2 is temperature at r_1.

Now taking Laplace transformation with respect to t on both sides of equation (4), we get

$$d/(r^2dr)(Kr^2(d\overline{T}/dr))+M(T_b/p-\overline{T})+S/p=\rho cp\overline{T}-\rho cpT(0,r). \quad ...(9)$$

The initial condition is given by

$T(0,r)=T_b$.

Here $\overline{T}(r,p)$ is Laplace Transform of $T(r,t)$.

Now applying the initial condition, equation (9) becomes

$$d^2\overline{T}/dr^2+(2/r)(d\overline{T}/dr)-(M+\rho cp)\overline{T}/K=-(MT_b+\rho cpT_b+S)/(pK). \quad ...(10)$$

The solution of equation (10) is

$$\overline{T}(r,p)=\frac{1}{r}\left(Ae^{\alpha r}+Be^{-\alpha r}\right)+\frac{MT_b+\rho cpT_b+S}{p(M+\rho cp)}, \quad ...(11)$$

where $\alpha=\sqrt{(M+\rho cp)/K}$.

Now taking Laplace transformation with respect to t on both sides of equation (3) and equation (5) we get

$$\overline{T}(r_3,p)=T_a/p-LE/(hp) \quad ...(12)$$

at skin surface,

and

$$\overline{T}(r_0,p)=T_a/p \quad ...(13)$$

at inner boundary.

Applying boundary conditions (12) and (13) to equation (11) and substituting the value of α we get

$$\overline{T}(r,p)=\frac{r_3}{r}\left[\left(T_a-T_b-\frac{LE}{h}-\frac{S}{M}\right)\frac{1}{p}\left\{\frac{sinh(p+M/\rho c)^{1/2}(\rho c/K)^{1/2}(r-r_0)}{sinh(p+M/\rho c)^{1/2}(\rho c/K)^{1/2}(r_3-r_0)}\right\}+\frac{S}{M(p+M/\rho c)}\right.$$

$$\left.\left\{\frac{sinh(p+M/\rho c)^{1/2}(\rho c/K)^{1/2}(r-r_0)}{sinh(p+M/\rho c)^{1/2}(\rho c/K)^{1/2}(r_3-r_0)}\right\}\right]+\frac{Sr_0}{Mr}\left(\left\{\frac{sinh(p+M/\rho c)^{1/2}(\rho c/K)^{1/2}(r_3-r)}{p\,sinh(p+M/\rho c)^{1/2}(\rho c/K)^{1/2}(r_3-r_0)}\right\}\right)$$

$$-\frac{1}{(p+M/\rho c)}\left\{\frac{sinh(p+M/\rho c)^{1/2}(\rho c/K)^{1/2}(r_3-r)}{sinh(p+M/\rho c)^{1/2}(\rho c/K)^{1/2}(r_3-r_0)}\right\}\Bigg]+\frac{(M+\rho cp)T_b+S}{p(M+\rho cp)} \quad ...(14)$$

Now taking inverse Laplace transformation on both side of equation (14) we obtain

$$T(r,t)=\frac{4(K\rho c)^{1/2}}{r}\left[r_3\left\{\left(T_a-T_b-\frac{LE}{h}\right)\sum_{n=1}^{\infty}(-1)^n\left(\frac{1}{M/\rho c+2\pi^2Kn^2}\right)\times\right.\right.$$

$$\left(e^{-(M/\rho c+2\pi^2Kn^2)t}-1\right)n\,sin(2n\rho c(r-r_0)(r_3-r_0))\Big\}+(S/M)r_3\times\left\{\sum_{n=1}^{\infty}(-1)^n\right.$$

$$\left(\frac{Me^{-(M/\rho c+2\pi^2Kn^2)t}}{2\pi^2Kn^2\rho c(M/\rho c+2\pi^2Kn^2)}-\frac{e^{-(M/\rho c)t}}{2\pi^2Kn^2}+\frac{1}{M/\rho c+2\pi^2Kn^2}\right)\times n\,sin(2n\rho c(r-r_0)(r_3-r_0))\Bigg\}$$

$$+\frac{S}{M}\left\{\sum_{n=1}^{\infty}(-1)^n\left(\frac{Me^{-(M/\rho c+2\pi^2Kn^2)t}}{2\pi^2Kn^2\rho c(M/\rho c+2\pi^2Kn^2)}-\frac{e^{-(M/\rho c)t}}{2\pi^2Kn^2}+\frac{1}{M/\rho c+2\pi^2Kn^2}\right)\right.$$

$$n\,sin(2n\rho c(r_3-r_0)(r_3-r_0))\}\Big]+T_b+S/M-(S/M)e^{-(M/\rho c)t}. \quad ...(15)$$

The equation (15) gives the value of temperature distribution in different layers of skin and subcutaneous tissue (*SST*) region.

Numerical Results and Discussion. The numerical results have been obtained using the following values (Saxena and Bindra [9])
$K_1=0.030$ *cal* cm^{-1} min^{-1} $^0C^{-1}$, $K_2=0.045$ *cal* $cm^{-1}min^{-1}C^{-1}$, $K_3=0.060$ *cal* cm^{-1} min^{-1} $^0C^{-1}$, $L=579$ *cal/g*, $h=0.009$ *cal* cm^{-2} min^{-1} $^0C^{-1}$, $\rho=1.05$ g/cm^2, $c=0.83$ *cal/g*.
The numerical calculations have been made for three cases of atmospheric temperatures and the values of *m*, *S* and *E* have been taken accordingly as follows:

(i) $T_a=15^0C$, $m=m_bc_b=0.003$ *cal* cm^{-3} min^{-1} $^0C^{-1}$,
$S=0.0357$ *cal* cm^{-3} min^{-1} and $E=0$ *g* cm^{-2} min^{-1}

(ii) $T_a=23^0C$, $m=m_bc_b=0.018$ *cal* cm^{-3} min^{-1} $^0C^{-1}$,
$S=0.018$ *cal* cm^{-3} min^{-1} and $E=0,\ 0.24\times10^{-3},0.48\times10^{-3}$ *g* cm^{-2} min^{-1}

(iii) $T_a=33^0C$, $m=m_bc_b=0.0315$ *cal* cm^{-3} min^{-1} $^0C^{-1}$,
$S=0.018$ *cal* cm^{-3} min^{-1} and
$E=0.24\times10^{-3},0.48\times10^{-3}$, 0.72×10^{-3} *g* cm^{-2} min^{-1}.

Depending upon the structure of skin and underlying tissues the constant $r_i(i=0,1,2,3)$ are assigned following values

$r_0=8cm$, $r_1=8.5cm$, $r_2=8.9cm$, $r_3=9.1cm$.

We have taken total thickness of skin 1.1*cm*.

The physical parameter rate of metabolic heat generation has been taken as self controlled, described by factor $q_d(T_b-T)$ where q_d depends on the atmospheric temperature and body core temperature. As the temperature changes, the thermoreceptors present in the skin send information to the hypothalamus centres which in turn controls the rate of blood mass flow and rate of metabolic heat generation. At low temperature for e.g. at 15^0C, the blood vessels constrict, causing low blood mass flow rate and q_d and (T_b-T) increases, causing increase in the rate of metabolic heat generation. Therefore the structure of skin and underlying tissues decreases the heat carried by blood to the surface and increases the rate of metabolic heat generation to regulate the body core temperature at low atmospheric temperature.

At higher atmospheric tgemperature for e.g. at 33^0C these process are reversed that is more heat is carried by the blood to the surface due to increased blood flow rate. The metabolic heat generation also decreases in order to regulate the body core temperature. In spite of these processes, the rate of sweat evaporation at the skin surface also play an important role in heat regulation in *SST* region.

The time factor also play an imporatnat role in order to regulate the body temperature. Initially when blood enters in the body tissue (i.e. at time zero) the temperature of the *SST* region is same as the blood temperature (i.e. 37.2^0C) because no heat loss takes place at time zero but after some time (say t) the skin temperature varies in different layers i.e. in epidermis, dermis and subcutaneous part of skin. The body core temperature remains constant i.e. 37.2^0C. The temperature of region changes up to 10 min and very small change is observed upto 20 and 30 minute but after that it will remain nearly constant. The values of temperatures in *SST* region at time 0.5 minute for different values of T_a, E are r_i $(i=0,1,2,3)$ are given in table-I

Table -I

Temperature of SST region at time 0.5 min. at different values of atmospheric temperature and for different values of E

Atmospheric temperature →		$T_a=15^0C$	$T_a=23^0C$ E in $gcm^{-2}min^{-1}$			$T_a=33^0C$, E in $gcm^{-2}min^{-1}$		
Temperature of SST region for r_0, r_1, r_2 and r_3 →	Radial Distance	$E=0$	0	0.24×10^{-3}	0.48×10^{-3}	0.24×10^{-3}	0.48×10^{-3}	0.72×10^{-3}
	r_3=9.1cm	27.289750	30.861011	23.968477	17.075942	28.432554	21.540019	14.647485
	r_2=8.9cm	33.050795	34.546008	31.660251	28.774499	33.539262	30.787608	27.757756
	r_1=8.5cm.	35.839102	36.337438	35.375649	34.413860	35.998570	35.036781	34.074992
	r_0=8.0cm.	37.20	37.20	37.20	37.20	37.20	37.20	37.20

Fig 2 is drawn for different values of temperature and radial distance given in Table -I

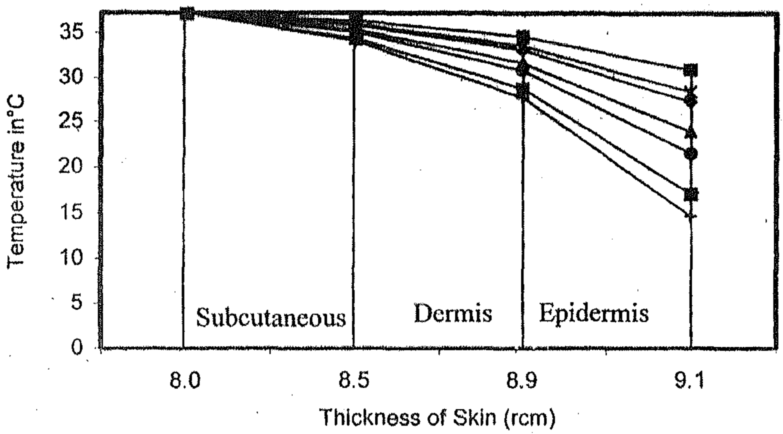

Fig 2.

In the above figure (Fig. 2) the first, fourth and sixth curves from top to bottom are for T_a=23^0C and E=0, 0.24×10^{-3} and 0.4×10^{-3} gcm-$^2m^{-1}$, second, fifth and seventh curves from top to bottom are for T_a=33^0C and E=0.24×10^{-3}, 0.48×10^{-3} and 0.72×10^{-3} gcm^{-2} min^{-1} and third curve from top to bottom is for T_a=15^0C and E=0.

From Fig. 2 we make the following observation :

We see in Fig.2 that the gaps between the curves for different values of E at each T_a shows that the rate of evaporation has significant effect on temperature distribution and this effect decreases as we move towards the body core. The skin surface temperature for T_a=15^0C and E=0 is less than that when T_a=23^0C and E=0 because the rate of blood mass flow is much less than at T_a=15^0C then that at T_a=23^0C. Lastly the skin surface temperature for some evaporation rate is lower at T_a=23^0C than that at T_a=33^0C. This may be because of the effect of lower atmospheric temperature and lower rate of blood mass flow at T_a=23^0C.

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