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SOME RANDOM FIXED POINTS OF RANDOM MULTIVALUED OPERATORS ON PROBABILISTIC POLISH SPACES

By

V. H. Badshah

School of Studies in Mathematics, Vikram University, Ujjain-456010, M.P., India

and

Amir Khan

Department of Mathematics, MAIPS College, Ujjain-456001, M.P., India

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ABSTRACT

The main purpose is to obtain a random fixed point theorem for random multivalued operators generalized contraction in probabilistic polish spaces.

Key Words. Random fixed point; random multivalued operators; measurable mapping; probabilistic Polish space.

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1. Introduction. Dedeic and Sarapa [4] proved some theorems on common fixed points for two commuting mappings on complete Menger space. They extended the Ciric's results, which proved the fixed point theorem for generalized contractions. As another consequence of his result, they proved several fixed point theorems for two commuting mappings on complete metric spaces. Dedeic and Sarapa [5] proved some theorems on common fixed point for a sequence of mappings on complete menger space. They proved several fixed point theorems for a sequence of continuous mappings. Dedeic and Sarapa [6] also proved and generalized several known results of Sehgal and Bharucha Reid [8] and their own result and established a theorem on common fixed point for three mappings on complete Menger space. Before establishing the theorem, the assumed that (X,F,t) is a complete Menger space with continuous T-norm t, where $t(a,b)=min(a,b)$ for $a,b \in [0,1]$.

In 1992, Beg and Azam [1] gave a stochastic version as: Let (X,d) be a polish space that is a separable complete metric space and $(\Omega,\mathfrak{C})$ be a measurable space. Let 2^X be the family of all subsets of X, $K(X)$ be the family of all non-empty closed subsets of X. Let $T:X \to CB(X)$ be a mapping and $\{x_n\}$ be a sequence in X. The sequence $\{x_n\}$ is said to be asymptotically T-regular with respect of F, if $d(Fx_n,Tx_n)\to 0$.

Beg and Azam [2] proved that a random coincidence points of compatible random coincidence points of compatible random multivalued operators. Mappings $T:X\to CB(X)$ and $F:X\to X$ are compatible if whenever there is a sequence in X

satisfying $lim\ Fx_n \in lim\ Tx_n$ (provided $lim\ Fx_n$ exist and $lim\ Tx_n$ exist in $CB(X)$), then $lim\ H(FTx_n, TFx_n)=0$ where H is Hausdorff metric space on $CB(X)$ induced by the metric space on $CB(X)$ induced by the metric d. Beg and Azam [3] generalized Jungck [9] gives random operators $f: \Omega\times X \to X$ and $T:\Omega\times X \to CB(X)$ are compatible if $f(\omega,.)$ and $T(\omega,.)$ are compatible for each $\omega \in \Omega$.

2. Random Fixed Point of Random Multivalued Generalized Contractions. Random fixed point theorems are stochastic generalization of classical fixed point theorem required for the theory of random equations. This section is in continuation of generalization of Beg and Shahzad [3], the existence of a common random fixed point of two random multivalued genralized contraction by using functional inequality.

3. Preliminaries. Let (X,d) be a polish space that is a separable complete metric space and $(\Omega,\mathfrak{C})$ be measurable space. The polish space (X,F) is said to be "Probabilistic Polish Space" if the following conditions hold : (i) $F_{(x,y)}(0)=0$; (ii) $F_{(x,y)}(t)=1$ iff $x=y$; (iii) $F_{(x,y)}(t)=F_{(y,x)}(t)$; (iv) $F_{(x,y)}(t_1)=1$ and $F_{(y,z)}(t_2)=1$, then $F_{(x,z)}(t_1+t_2)=1$. Condition (ii) is equivalent to the statement $x=y$ iff $F_{x,y}=H$. We remark that any metric space may be regarded as PM-space indeed, if (X,d) is a metric space, then we define for each x and y in X, the distribution function $F_{x,y}$ as follows : $F_{x,y}(t)=H(t-d(x,y))$. Let 2^X be a family of all subsets of X and $CB(X)$ denote the family of all non-empty bounded closed subests of X. A mapping $T:\Omega\to 2^X$, is called measurable if for any open subset C of X, $T^{-1}(C)= \{\omega\in\Omega: T(\omega)\cap C\neq\phi\}\in \mathfrak{C}$. A mapping $\xi:\Omega\to X$ is said to be measurable selector of a measurable mapping $T:\Omega\to 2^X$, if ξ is measurable and for any $\omega\in\Omega$, $\xi(\omega)\in T(\omega)$. A mapping $f:\Omega\times X\to X$ is called random operator if for any $x\in X$, $f(.,X)$ is measurable. A mapping $T:\Omega\times X\to CB(X)$ is a random multivalued operator if for every $x\in X$, $T(.,X)$ is measurable. A measurable mapping $\xi:\Omega\to X$ is called random fixed point of a random multivalued operator $T:\Omega\times X\to CB(X)$ $(f:\Omega\times X\to X)$ if for every $\omega\in\Omega$, $\xi(\omega)\in T(\omega,\xi(\omega)(f(\omega,\xi(\omega)))= \xi(\omega)$. Let $T:\Omega\times X\to CB(X)$ be a random operator and $\{\xi_n\}$ a sequence of meaurable mapping $\xi_n:\Omega\to X$. The sequence $\{\xi_n\}$ is said to be asymtotically T-regular if $d(\xi_n(\omega),T(\omega,\xi_n(\omega))\to 0$.

In 1992, Beg and Azam [1] proved a fixed point on asymptotically regular multivalued mappings. Recentrly Beg and Shahzad [3] further extended random fixed points of multivalued mapping satisfying more general contractive type conditions.

In the present paper we give generalized version of a result of Beg and Shahzad [3] in probabilistic polish space in the form of the following theorem :

4. Theorem. *Let (X,Ω) be a Probabilistic Polish Space. Let $S:\Omega\times X\to CB(X)$ be a continuous random multivalued operator, if there exists measurable mapping* $\alpha:\Omega\to(0,1)$:

$$F_{S(\omega,x),S(\omega,y)}(t) \geq F_{x,S(\omega,x)}(t/\alpha(\omega)) + F_{y,S(\omega,y)}(t/\alpha(\omega))$$

for all $x,y \in X$, $\omega \in \Omega$ and $\alpha(\omega)$ is non-negative $\alpha(\omega)<2$ with then there exists a random fixed point of S.

Proof. Let $\xi_0:\Omega \to X$ be an arbitrary measuable mapping and choose a measurable mapping $\xi:\Omega \to X$ such that $\xi_1(\omega) \in S(\omega, \xi_0(\omega))$ for each $\omega \in \Omega$ then for $\omega \in \Omega$

$$F_{S(\omega,\xi_0(\omega)),S(\omega,\xi(\omega))}(t) \geq F_{\xi_0(\omega),S(\omega,\xi_0(w))}(t/\alpha(\omega)) + F_{\xi_1(\omega),S(\omega,\xi_1(\omega))}(t/\alpha(\omega)).$$

It further implies that there exists a measurable mapping $\xi_2:\Omega \to X$ for any $\omega \in \Omega$, $\xi_2(\omega) \in S(\omega, \xi(\omega))$;

$$F_{\xi_1(\omega),\xi_2(\omega)}(t) = F_{S(\omega,\xi_0(\omega)),S(\omega,\xi(\omega))}(t),$$
$$F_{\xi_1(\omega),\xi_2(\omega)}(t) \geq F_{\xi_0(\omega),\xi_1(\omega)}(t/\alpha(\omega)) + F_{\xi_1(\omega),\xi_2(\omega)}(t/\alpha(\omega)),$$
$$F_{\xi_1(\omega),\xi_2(\omega)}(1 - t/\alpha(\omega))t \geq F_{\xi_0(\omega),\xi_1(\omega)}(t/\alpha(\omega))$$
$$\Rightarrow F_{\xi_1(\omega),\xi_2(\omega)}(t) \geq F_{\xi_0(\omega),\xi_1(\omega)}(t/(\alpha(\omega)-1))$$
$$\Rightarrow F_{\xi_1(\omega),\xi_2(\omega)}(t) \geq F_{\xi_0(\omega),\xi_1(\omega)}(t/k), \qquad \text{(I)}$$

where $k = \alpha(\omega) - 1 < 1$.

There exists a measurable mapping $\xi_3:\Omega \to X$ such that for any $\omega \in \Omega$, $\xi_3(\omega) \in S(\omega, \xi_2(\omega))$ and

$$\begin{aligned} F_{\xi_2(\omega),\xi_3(\omega)}(t) &= F_{S(\omega,\xi_1(\omega)),S(\omega,\xi_2(\omega))}(t) \\ &\geq F_{\xi_1(\omega),S(\omega,\xi_1(\omega))}(t/\alpha(\omega)) + F_{\xi_2(\omega),S(\omega,\xi_2(\omega))}(t/\alpha(\omega)) \\ &\geq F_{\xi_1(\omega),\xi_2(\omega)}(t/\alpha(\omega)) + F_{\xi_2(\omega),\xi_3(\omega)}(t/\alpha(\omega)) \end{aligned}$$
$$\Rightarrow F_{\xi_2(\omega),\xi_3(\omega)}(1 - 1/\alpha(\omega)) \geq F_{\xi_1(\omega),\xi_2(\omega)}(t/\alpha(\omega))$$
$$\Rightarrow F_{\xi_2(\omega),\xi_3(\omega)}(t) \geq F_{\xi_1(\omega),\xi_2(\omega)}(t/(\alpha(\omega)-1)).$$

Then by (I)

$$\begin{aligned} F_{\xi_1(\omega),\xi_2(\omega)}(t/(\alpha(\omega)-1)) &\geq F_{\xi_0(\omega),\xi_1(\omega)}\left(t/(\alpha(\omega)-1)^2\right) \\ &\geq F_{\xi_0(\omega),\xi_1(\omega)}\left(t/k^2\right). \end{aligned}$$

Then we have

$$F_{\xi_2(\omega),\xi_3(\omega)}(t) \geq F_{\xi_1(\omega),\xi_2(\omega)}(t/(\alpha(\omega)-1)) \geq F_{\xi_0(\omega),\xi_1(\omega)}\left(t/(\alpha(\omega)-1)^2\right)$$
$$\Rightarrow F_{\xi_2(\omega),\xi_3(\omega)}(t) \geq F_{\xi_1(\omega),\xi_2(\omega)}(t/k) \geq F_{\xi_0(\omega),\xi_1(\omega)}\left(t/k^2\right).$$

Similarly proceeding in the same way induction we produce a sequence of measurable mapping $\xi_n:\Omega \to X$ and

$$F_{\xi_n(\omega),\xi_{n+1}(\omega)}(t) \geq F_{\xi_0(\omega),\xi_1(\omega)}\left(t/k^n\right)$$

Further more for $m>n$

$$F_{\xi_n(\omega),\xi_m(\omega)}(t) \geq F_{\xi_n(\omega),\xi_{n+1}(\omega)}(t) + F_{\xi_{n+1}(\omega),\xi_{n+2}(\omega)}(t) + \ldots + F_{\xi_{m-1}(\omega),\xi_m(\omega)}(t)$$
$$\geq F_{\xi_0(\omega),\xi_1(\omega)}\left(1/k^n + 1/k^{n+1} + \ldots + 1/k^{m-1}\right)(t)$$
$$\geq F_{\xi_0(\omega),\xi_1(\omega)}\left\{\left(1 + 1/k + 1/k^2 + \ldots + 1/k^{m-n-1}\right)/k^n\right\}(t)$$
$$\geq F_{\xi_0(\omega),\xi_1(\omega)}\left((1-k)/k^n\right)(t) \to 1 - \lambda$$

i.e. $F_{\xi_n(\omega),\xi_m(\omega)}(t) \to 1 - \lambda$.

It follows that $\{\xi_n(\omega)\}$ is a Cauchy sequence for $\gamma > 0$ and any $\omega \in \Omega$, $\xi_{2\gamma+1}(\omega) \in S(\omega, \xi_{2\gamma}(\omega))$ then there exists a measurable mapping $\xi:\Omega \to X$ such that $\xi_n(\omega) \to \xi(\omega)$ for each $\omega \in \Omega$. It further implies that $\xi_{2\gamma+1}(\omega) \to \xi(\omega)$

$$F_{\xi(w),S(\omega,\xi(\omega))}(t) \geq \left[F_{\xi(\omega),\xi_{2\gamma+1}(\omega)}(t), F_{\xi_{2\gamma+1}(\omega),(\omega,\xi(\omega))}(t)\right]$$
$$\geq \left[F_{\xi(\omega),\xi(\omega)}(t), F_{\xi(\omega),(S(\omega),\xi(\omega))}\right](t) \to 0$$

i.e. $\{\xi_{2\gamma+1}\}$ is an asymptotically S-regular.

Consider

$$F_{\xi(\omega),S(\omega,\xi(\omega))}(t) \geq F_{\xi(\omega),\xi(\omega)}(t/\alpha(\omega)) + F_{\xi(\omega),S(\omega,\xi(\omega))}(t/\alpha(\omega))$$
$$\Rightarrow F_{\xi(\omega),S(\omega,\xi(\omega))}(1 - 1/\alpha(\omega))t \geq 1$$
$$\Rightarrow F_{\xi(\omega),S(\omega,\xi(\omega))}(t) \geq \alpha(\omega)/(\alpha(\omega) - 1)$$
$$\Rightarrow \xi(\omega) \in S(\omega, \xi(\omega))$$

i.e. $\xi(\omega)$ is a fixed point of S.

REFERENCES

[1] I. Beg and A. Azam, Fixed point of asymptotically regular multivalued mappings, *J. Austral. Math. Soc. (Ser A) Vol.* **53** (1992), 313-326.

[2] I. Beg and A. Azam, Common fixed points for commuting and compatible maps, *Discussion Mathematical Differential Inclusion,* **16** (1996), 121-135.

[3] I. Beg and N. Shahzad, Random fixed point theorems on product spaces, *J. Appl. Math. and Stochastic Analysis,* **6** (1993), 95-106.

[4] R. Dedeic and N. Sarapa, On common fixed point theorems for commuting mappings on Menger spaces, *Radovi Mathematicki,* **4** (1998), 269-278.

[5] R. Dedeic and N. Sarapa, Fixed point theorems for a sequence of mappings on Menger spaces, *Math. Japonica,* **34** (1989), 5?5-539.

[6] R. Dedeic and N. Sarapa, A common fixed point theorem for three mappings on Menger space, *Math. Japonica,* **34** (1989), 919-923.

[7] G. Jungck, Common fixed points for commuting and compatible maps on compacta, *Proc. Amer. Math. Soc.,* **103** (3) (1988), 977-983.

[8] V.M. Sehgal and A.T. Bharucha-Reid, Fixed points of contraction mappings on Probabilistic metric spaces, *Mathematicae, System Theory,* **6** (1972), 97-102.