

**RELAXATION LABELING BASED GRAPH MATCHING METHODOLOGY  
TO COMPARE VHR SATELLITE IMAGES WITH GIS DATA**

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**ABSTRACT**

In this paper, we propose a graph matching methodology based on relaxation labeling to compare road junction on *VHR* satellite images with *GIS* road data. Use is made of the spatial layout between points based on the relative angle. The technique finds correspondences between sets of points taking into account error on the spatial location and spurious or missing points. An analysis is given to determine the weights of the algorithm based on the expected graph error for remotely sensed *GIS* data.

**Key Words.** registration, change detection, graph matching

**1. Introduction.** A major challenge in the production and use of geographic information is assessment and control of the quality of the database. The rapid growing number of sources of geospatial data, ranging from high-resolution satellite and airborne sensors, *GPS*, and derivative geospatial products, pose severe problems for integrating data. One of the major challenges for content providers to face is the problem of upgrading their current databases to a higher accuracy any ensuring the quality of the information. Current techniques can't support this in a cost-effective way due to the necessary manpower. Automated detection of change and anomalies in the existing databases using very high resolution (*VHR*)

satellite images can form an essential tool to support quality control and maintenance of spatial information. We examine to what extent remote sensing and computer vision can be used to detect deviation of a *GIS* database vs "real world".

We introduce a system that is based on object recognition and searches for the correspondences between the road information in images and existing road vector data. The information that is used consists out of road junctions. Junctions have the advantage that the representation extracted from the image and the vector representations are semantically close: in the simple case, junctions can be represented by a single point in the image and vector data. In contrast, roads are much more difficult in the sense that road detection in the image often leads to fragmented pixel chains with no easy correspondence with the idealized polylines in the vector data. In [1], a simple technique based on ridge detection is introduced for the automatic detection of road junctions.

In this paper, we examine the correspondence problem between two datasets containing junctions, modeled as a graph matching problem, and how this can be used for registration and change detection. Using the spatial layout of the junctions (e.g. the angle formed by two junctions), we define local constraints that determine which correspondences are allowed. This defines a constraint satisfaction problem comparing two graphs that is solved using continuous relaxation labeling [2]. The matching is error robust, meaning that corresponding road junctions need not be at the exact same spatial position but deviations are allowed. This is important for change detection, allowing the corresponding junction to be found although it has changed position from one dataset to the other. This sensitivity for local changes or deformations make graph matching an interesting alternative for registration techniques based on a global spatial transformation model [3]. Of course, since the technique is error robust one needs to be able to define which errors are allowable and which are not. The constraint satisfaction problem holds this information implicitly, but we have derived an analytical expression, which allows the user to determine the error thresholds within the system. The expression is general and useful for optimisation techniques other than relaxation labeling like e.g. Markov random fields.

**2. Continuous Relaxation Labeling.** Error tolerant graph matching techniques look for graph morphism between two graphs, which allows for distortions. A general distortion model contains the deletion and addition of nodes/arcs and the replacement of attribute values. Based on this model, a distance function is defined between graphs that penalize the occurring distortions when comparing graphs. The matching problem can be defined as a graph labelling problem. The following are defined:

1. a set of object  $i$ , corresponding to image features;
2. a set of labels  $x_i$ , corresponding to *GIS* features;
3. a neighbour relationship over the objects;
4. restrictions on possible labels between pairs of neighbouring objects.

Relaxation labeling techniques use an iterative process to determine the probabilities of each object. Different update rules have been proposed. In [2] the relation between different update rules is analytically shown. The problem of finding consistent solutions is shown to be equivalent to solving a variational inequality which is based on the mathematical concept of "consistency". This concept is interesting because it lays bare the foundations of the labeling process and offers guidance in determining good compatibility coefficients.

To each object  $i$  a probability distribution is associated that  $i$  has label  $X_k(p_i(x_1), \dots, p_i(x_m))$ ,

$$0 \leq (p_i(x_k)) \leq 1, \quad \sum_{k=1}^m p_i(x_k) = 1. \quad (2.1)$$

For each pair of neighbouring objects  $i$  and  $j$  and for each pair of labels  $X_k$  and  $X_l$  a compatibility coefficient  $r_{ij}(X_k, X_l)$  exists. These coefficients express the compatibility of assigning label  $X_k$  to object  $i$  in combination with assigning label  $x_l$  to object  $j$ . Negative values express incompatibility, positive values compatibility. Given these quantities, the support of a label  $X_k$  for the object  $i$  given by the correspondence  $\hat{a}$  is defined by

$$S_i(X_k) = s_i(X_k, \hat{a}) \sum_{j=1}^n \sum_{l=1}^m r_{ij}(x_k, x_l) p_j(x_l). \quad (2.2)$$

To find a consistent labeling, we optimize the average local consistency, given by

$$A(\hat{a}) = \sum_{j=1}^n \sum_{k=1}^m p_i(x_k) s_i(X_k, \hat{a}), \quad (2.3)$$

which is a quadratic function which we optimize using a constrained gradient descent method, taking into account the restrictions of Eq. (2.1)

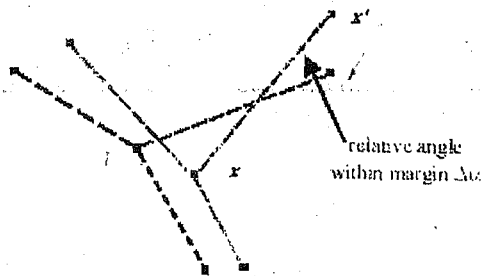


Figure 1. Illustration of relative angle constraint.

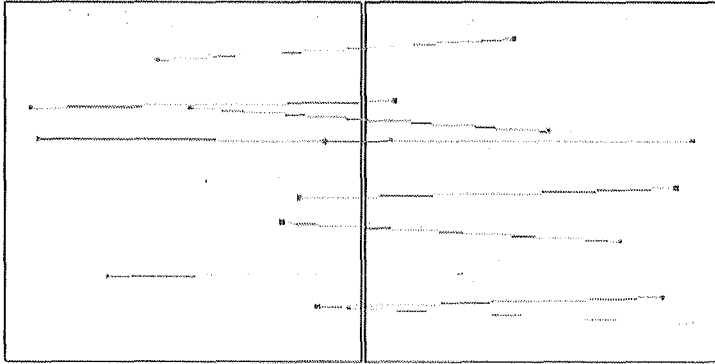


Figure 2. Matching of point sets using relative angle constraint.

**3. Matching Sets of Points.** In [1] road junctions were proposed as stable registration objects. The problem of comparing images with a *GIS* road database can then be represented as finding the correspondence between two sets of points: one originating from the image and one which can be extracted from the database. This can of course be applied to situations other than image-to-*GIS* registration (e.g. image-to-image or *GIS* -to-*GIS*).

The technique makes use of the spatial relations between points to find correspondences. This makes the correspondence technique less vulnerable for road junctions which are detected in the right location but whose detected shape does not fully correspond to its counterpart in the database (defects can occur due to image noise). So instead of using features based on the shape of single crossroad to guide matching, we use geometric relations (e.g. angle, distance) between a crossroad and its neighbours to find correspondences matching tree [4]. These are much stabler features given the detection quality which we can realistically expect from line detection. In these experiments we rely only on the relative angle between pairs of points. Fig.1 shows an illustration of the constraint. The black points show object points  $i$  and the grey points show the lable points  $x$ . In mapping a pair of points  $i$  and  $j$  on  $x$  and  $x'$  the relative angle between the lines  $ij$  and  $xx'$  should lie within a margin  $\Delta\alpha$  (e.g.  $\pm\pi/4$ ). Although this is a weak constraint, it has been shown that the constraint is adequate to find correspondences between sets of points, disturbed by location noise [1].

**4. Null Lable Assignment.** To guarantee a good performance of the matching algorithm, the compatibility matrix  $r_{ij}(X, X')$  needs to be determined correctly. In most applications, the value of these coefficients are determined using heuristics which basically impose a relative ordering on the restrictions. Stronger restrictions receive a higher absolute value than weaker restrictions. The specific proportion of the value is usually determined through trial-and-error. For the null-label assignment it is however difficult to determine a correct value for the compatibility coefficient (noted as  $r_{ij}^0$ ). Since each object is a priori a possible null

object, every assignment is consistent with the null assignment, i.e.  $r_{ij}^0(X_0, X') > 0$  for  $\forall j, X'$ . The problem is to assess the relative importance of the null-label assignment with respect to the other restrictions. It should be avoided that the null solution is the most consistent solution of the system. On the other hand, false matchings of spurious points should be less consistent than the null assignment.

The definition of consistency can be used to determine the correct values. The definition not only determines the optimal solution of the labeling problem, it also determines what values the compatibility coefficients should take for an "ideal" solution to become the optimal solution of the system. The ideal solution is the matching we wish to find given the noise properties of the detection. For a correct null label assignment, we need to determine when the errors, which occur in the neighbour structure of a node, are that the null label should be assigned. To analyse this, we should look at the support of the different assignments. In the case of the null label assignment, the support can be written as:

$$\begin{aligned} S_i(X_0, \hat{a}) &= \sum_{j=1}^n \sum_{x'=1}^{m+1} r_{ij}^0(X_0, X') p_i(x') \\ &= r^0 \sum_{j \in \Omega} \sum_{x'=1}^{m+1} p_i(X') = r^0 d(i) \end{aligned} \quad (4.1)$$

with  $d(i)$  the degree of node  $i$  (i.e. the  $n$  number of neighbours). We have simplified  $r_{ij}^0(X_0, X') = r_i^0$  if  $j \in \Omega$  (else  $r_{ij}^0(X_0, X') = 0$ ). The constant factor  $r^0$  is reasonable in the absence of prior knowledge of assignments.

The support for a non-full label can be split up into three classes, namely positive coefficients which express compatibility, negative coefficients which express incompatibility and negative coefficients which control the null assignment. If we consider the first two coefficients constant within the neighbourhood of node  $i$ , resp., then the support for  $c_i$  can be simplified to

$$S_i(X_i) = \sum_{j=1}^{n_+} r_{ij}^+(X_i, X_j) + \sum_{j=n_++1}^l r_{ij}^-(X_i, X_j) + \sum_{j=n_++1}^{n_0} r_i^0 \approx \hat{c}_i^+ n_+ + \hat{c}_i^- n_- + r_i^0 n_0. \quad (4.2)$$

Here  $n_+$  is the number of compatible neighbours,  $n_-$  the number of incompatible neighbours and  $n_0$  the number of nullneighbours, with  $n_+ + n_- + n_0 = d(i)$ . The number of non-full neighbours is given by  $n_1 = n_+ + n_-$ . Eq. (4.1) and (4.2) give the following condition which holds in the optimal solution:

$$\hat{c}_i^+ n_+ + \hat{c}_i^- n_- + r_i^0 n_0 > r_i^0 d(i) \quad (4.3)$$

or equivalently

$$(1 - f^0) r_i^0 < f^+ r_i^+ + f^- r_i^- \quad (4.4)$$

where  $f^+$ ,  $f^-$  and  $f^0$  are the fractions of the number of compatible, incompatible

and null assignments in the neighbourhood of object  $i$ . Equation (4.4) can be used to determine the weights for the compatibility matrix given the expected graph error. Based on this expression a distinction can be made between allowable distortions, which should find a correspondent in the other dataset, and significant changes, which should be assigned the null label.

## 5. Applications

In [1], reports were given on artificially generated sets of points. We have investigated the problem of automatic registration of satellite the images with a remotely sensed *GIS* database based on feature points like road junctions. The road junctions can be manually selected or through automatic detection based on the results of [1] as shown in Fig. 3. In this example we work with manually selected junctions. 82 points were selected in the image. The equivalent region in the road database consists of 200 junctions. A rough initial tie point is given between image map [5] and *GIS* data after which a search radius of 150 meters is defined around each point in which the corresponding point is sought. We have manually registered the 82 image points to the corresponding points in the dataset. The *RMS* on the position is 7 meters and the maximum error is 50 meters. Of the 82 points, 71 were correctly matched, 11 were assigned the null label and no false matching was performed. Based on the matching results, a rubber sheeting transformation is performed which compensates the local distortions between image and *GIS* data. Fig 4. shows the result of this registration.

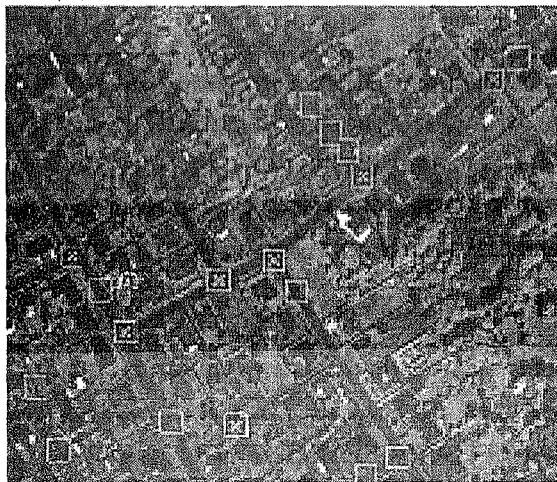


Figure 3. Example of automatic detection of road junctions in a panchromatic ikonos image  
(green crosses show detected junctions)



Figure 4. Results after rubbersheet transformation based on matching results.

## 6. Conclusions

We have proposed a methodology for comparing *VHR* images with remotely sensed *GIS* data based on the corresponding road junctions using continuous relaxation labeling. A thorough analysis is given of the null label assignment, which leads to an expression on the optimal weights based on the expected graph error. The method allows for local distortions between two remotely sensed datasets to be compensated and to distinguish these distortions from significant changes.

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