Jñānābha, Vol. 34, 2004

FRACTIONAL DERIVATIVES OF OUR HYPERGEOMETRIC FUNCTIONS OF FOUR VARIABLES

By

R.C. Singh Chnadel* and S.Sharma

Department of Mathematics

Dayanand Vedic Postgraduate College Orai 285001, Uttar Pradesh, India

***E-Mail: rc_chandel@yahoo.com**

(Received : July 7, 2003)

ABSTRACT

In the present paper, we derive fractional derivatives of hypergeometric functions of four variables recently introduced by Chandel and Sharma [8,9]

Key Words. Fractional derivatives, Multiple Hypergeometic Functions, Malutivariable H-function

2000 Mathematics Subject Classification . Primary 33C99, Secondary 26A33

1. Introduction. Motivated by the earlier work of Srivastava and Goyal [25], Srivastava, Chandel and Vishwakarma [26] derived a number of key formulas for fractional derivatives of the multivariable H-function of Srivastava and Panda [20,21,22] for multivariable H-function also see Srivastava, Gupta and Goyal [23]). Each of these formulas can be shown to yield interesting new results for various classes of generalized hypergeometric functions of several variables of Srivastava and Daoust [19] (Also see Srivastava and Manocha [24]pp 64-65]), Lauricella [16], Exton [11,14], Chandel [1], Chandel and Gupta [2] and Karlsson [15].

Also Chandel and Vishwakarma [5] derived fractional derivatives involving confluent hypergeometric forms of Karlsson's multiple hypergeometric function ${}^{(k)}F_{CD}^{(n)}$ [15], introduced by Chandel and Vishwakarma [3,4]. Further Chandel and Vishwakarma [6] derived the multidimensional fractional derivatives involving the multiple hypergeometric functions of Lauricella [16] with their confluent forms, Exton [11,14], Chandel [1], Chandel and Gupta [2] and their confluent forms and of Karlsson [15].

Further for special interest Chandel and Vishwakarma [7] applied same techniques in order to derive fractional derivatives involving hypergeometric functions of four variables of Exton [12, 13, 14] and those functions of Sharma and Parihar [18], which are not included in the Exton [12,13,14].

In the present paper for special interest, we shall derive fractional derivatives involving hypergeometric functions of four variables (recently

introduced and studied by the authors Chandel and Sharma [8,9].

2. Use of One Fractional Derivatives Operator. Making an appeal to the formula [17, p. 67]

$$D_x^{\mu}\{x^{\lambda}\} = \frac{\Gamma(\lambda+1)}{\Gamma(\lambda-\mu+1)}x^{\lambda-\mu}, \text{ Re } (\lambda) > 1,$$

we derive the following fractional derivatives involving hypergeometric functions of four variables due to authors Chandel and Sharma [8,9]:

$$(2.1)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}H_{A_1}^{(4)}(\mu,b,c,d;e,e',e'';z_1x,z_2x,z_3x,z_4)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}H_{A_1}^{(4)}(\lambda,b,c,d;e,e',e'';z_1x,z_2x,z_3x,z_4),\ Re(\lambda) > 0;$$

$$(2.2)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}H_{A_1}^{(4)}(a,\mu,c,d;e,e',e'';z_1x,z_2,z_3,z_4x)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}H_{A_1}^{(4)}(a,\lambda,c,d;e,e',e'';z_1x,z_2,z_3,z_4x),\ Re(\lambda) > 0;$$

$$(2.3)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}H_{A_1}^{(4)}(a,b,\mu,d;e,e',e'';z_1,z_2,z_3x,z_4x)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}H_{A_1}^{(4)}(a,b,\lambda,d;e,e',e'';z_1,z_2,z_3x,z_4x),\ Re(\lambda) > 0;$$

$$(2.4)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}H_{A_1}^{(4)}(a,b,c,\mu;e,e',e'';z_1,z_2x,z_3,z_4)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}H_{A_1}^{(4)}(a,b,c,\lambda;e,e',e'';z_1,z_2x,z_3,z_4),\ Re(\lambda) > 0;$$

$$(2.5)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}H_{A_1}^{(4)}(a,b,c,d;\lambda,e',e'';z_1x,z_2,z_3x,z_4)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}H_{A_1}^{(4)}(a,b,c,d;\mu,e',e'';z_1x,z_2,z_3x,z_4),\ Re(\lambda) > 0;$$

$$(2.6)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}H_{A_1}^{(4)}(a,b,c,d;e,\lambda,e'';z_1,z_2x,z_3,z_4)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}H_{A_1}^{(4)}(a,b,c,d;e,\mu,e'';z_1,z_2x,z_3,z_4),\ Re(\lambda) > 0;$$

$$(2.7)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}H_{A_1}^{(4)}(a,b,c,d;e,e',\lambda;z_1,z_2,z_3,z_4x)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}H_{A_1}^{(4)}(a,b,c,d;e,e',\mu;z_1,z_2,z_3,z_4x),\ Re(\lambda) > 0;$$

$$(2.8)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}H_{B_1}^{(4)}(\mu,b,c,d;e_1,e_2,e_3,e_4;z_1x,z_2x,z_3,z_4x)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}H_{B_1}^{(4)}(\lambda,b,c,d;e_1,e_2,e_3,e_4;z_1x,z_2x,z_3,z_4x),\ Re(\lambda)>0;$$

$$(2.9)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}H_{B_1}^{(4)}(a,\mu,c,d;e_1,e_2,e_3,e_4;z_1x,z_2,z_3x,z_4)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}H_{B_1}^{(4)}(a,\lambda,c,d;e_1,e_2,e_3,e_4;z_1x,z_2,z_3x,z_4),\ Re(\lambda)>0;$$

$$(2.10)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}H_{B_1}^{(4)}(a,b,\mu,d;e_1,e_2,e_3,e_4;z_1,z_2,z_3x,z_4x)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}H_{B_1}^{(4)}(a,b,\lambda,d;e_1,e_2,e_3,e_4;z_1,z_2,z_3x,z_4x),\ Re(\lambda)>0;$$

$$(2.11)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}H_{B_1}^{(4)}(a,b,c,\mu;e_1,e_2,e_3,e_4;z_1,z_2x,z_3,z_4)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}H_{B_1}^{(4)}(a,b,c,\lambda;e_1,e_2,e_3,e_4;z_1,z_2x,z_3,z_4),\ Re(\lambda)>0;$$

$$(2.12)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}H_{B_1}^{(4)}(a,b,c,d;\lambda,e_2,e_3,e_4;z_1x,z_2,z_3,z_4)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}H_{B_1}^{(4)}(a,b,c,d;\mu,e_2,e_3,e_4;z_1x,z_2,z_3,z_4),\ Re(\lambda)>0;$$

$$(2.13)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}H_{B_1}^{(4)}(a,b,c,d;e_1,\lambda,e_3,e_4;z_1,z_2x,z_3,z_4)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}H_{B_1}^{(4)}(a,b,c,d;e_1,\mu,e_3,e_4;z_1,z_2x,z_3,z_4),\ Re(\lambda)>0;$$

$$(2.14)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}H_{B_1}^{(4)}(a,b,c,d;e_1,e_2,\lambda,e_4;z_1,z_2,z_3x,z_4)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}H_{B_1}^{(4)}(a,b,c,d;e_1,e_2,\mu,e_4;z_1,z_2,z_3x,z_4),\ Re(\lambda)>0;$$

$$(2.15)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}H_{B_1}^{(4)}(a,b,c,d;e_1,e_2,e_3,\lambda,;z_1,z_2,z_3,z_4x)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}H_{B_1}^{(4)}(a,b,c,d;e_1,e_2,e_3,\mu;z_1,z_2,z_3,z_4x),\ Re(\lambda)>0;$$

$$(2.17)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{A_1}^{(4)}(a,b,\mu,d;e,e';z_1,z_2x,z_3,z_4)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{A_1}^{(4)}(a,b,\lambda,d;e,e';z_1,z_2x,z_3,z_4),\ Re(\lambda)>0;$$

$$(2.18)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{A_1}^{(4)}(a,b,c,\mu;e,e';z_1,z_2,z_3,z_4x)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{A_1}^{(4)}(a,b,c,\lambda;e,e';z_1,z_2,z_3,z_4x), Re(\lambda)>0;$$

$$(2.19)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{A_1}^{(4)}(a,b,c,d;e,\lambda;z_1,z_2,z_3,z_4x)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{A_1}^{(4)}(a,b,c,d;e,\mu;z_1,z_2,z_3,z_4x), Re(\lambda)>0;$$

$$(2.20)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{A_2}^{(4)}(a,\mu,c,d;e,e';z_1x,z_2,z_3x,z_4x)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{A_2}^{(4)}(a,\lambda,c,d;e,e';z_1x,z_2,z_3x,z_4x), Re(\lambda)>0;$$

$$(2.21)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{A_2}^{(4)}(a,b,\mu,d;e,e';z_1,z_2x,z_3,z_4)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{A_2}^{(4)}(a,b,\lambda,d;e,e';z_1,z_2x,z_3,z_4), Re(\lambda)>0;$$

$$(2.22)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{A_2}^{(4)}(a,b,c,\mu;e,e';z_1,z_2,z_3,z_4x)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{A_2}^{(4)}(a,b,c,\lambda;e,e';z_1,z_2,z_3,z_4x), Re(\lambda)>0;$$

$$(2.23)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{A_2}^{(4)}(a,b,c,d;e,\lambda;z_1,z_2,z_3,z_4x)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{A_2}^{(4)}(a,b,c,d;e,\mu;z_1,z_2,z_3,z_4x), Re(\lambda)>0;$$

$$(2.24)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{A_3}^{(4)}(a,\mu,c,d;e,e';z_1x,z_2,z_3x,z_4x)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{A_3}^{(4)}(a,\lambda,c,d;e,e';z_1x,z_2,z_3x,z_4), Re(\lambda)>0;$$

$$(2.25)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{A_3}^{(4)}(a,b,\mu,d;e,e';z_1,z_2x,z_3,z_4x)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{A_3}^{(4)}(a,b,\lambda,d;e,e';z_1,z_2x,z_3,z_4x), Re(\lambda)>0;$$

$$(2.26)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{A_3}^{(4)}(a,b,c,\mu;e,e';z_1,z_2,z_3,z_4x)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{A_3}^{(4)}(a,b,c,\lambda;e,e';z_1,z_2,z_3,z_4x), Re(\lambda)>0;$$

$$(2.27)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{A_3}^{(4)}(a,b,c,d;e,\lambda;z_1,z_2,z_3,z_4x)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{A_3}^{(4)}(a,b,c,d;e,\mu;z_1,z_2,z_3,z_4x),\ Re(\lambda)>0;$$

$$(2.28)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{B_1}^{(4)}(a,\mu,b_2,b_3,b_4;e,e';z_1x,z_2,z_3,z_4)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{B_1}^{(4)}(a,\lambda,b_2,b_3,b_4;e,e';z_1x,z_2,z_3,z_4),\ Re(\lambda)>0;$$

$$(2.29)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{B_1}^{(4)}(a,b_1,\mu,b_3,b_4;e,e';z_1,z_2x,z_3,z_4)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{B_1}^{(4)}(a,b_1,\lambda,b_3,b_4;e,e';z_1,z_2x,z_3,z_4),\ Re(\lambda)>0;$$

$$(2.30)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{B_1}^{(4)}(a,b_1,b_2,\mu,b_4;e,e';z_1,z_2,z_3x,z_4)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{B_1}^{(4)}(a,b_1,b_2,\lambda,b_4;e,e';z_1,z_2,z_3x,z_4),\ Re(\lambda)>0;$$

$$(2.31)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{B_1}^{(4)}(a,b_1,b_2,b_3,\mu;e,e';z_1,z_2,z_3,z_4x)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{B_1}^{(4)}(a,b_1,b_2,b_3,\lambda;e,e';z_1,z_2,z_3,z_4x),\ Re(\lambda)>0;$$

$$(2.32)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{B_1}^{(4)}(a,b_1,b_2,b_3,b_4;e,\lambda;z_1,z_2,z_3,z_4x)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{B_1}^{(4)}(a,b_1,b_2,b_3,b_4;e,\mu;z_1,z_2,z_3,z_4x),\ Re(\lambda)>0;$$

$$(2.33)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{B_2}^{(4)}(a,\mu,b_2,b_3,b_4;e,e';z_1x,z_2,z_3,z_4x)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{B_2}^{(4)}(a,\lambda,b_2,b_3,b_4;e,e';z_1x,z_2,z_3,z_4x),\ Re(\lambda)>0;$$

$$(2.34)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{B_2}^{(4)}(a,b_1,\mu,b_3,b_4;e,e';z_1,z_2x,z_3,z_4)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{B_2}^{(4)}(a,b_1,\lambda,b_3,b_4;e,e';z_1,z_2x,z_3,z_4),\ Re(\lambda)>0;$$

$$(2.35)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{B_2}^{(4)}(a,b_1,b_2,\mu,b_4;e,e';z_1,z_2,z_3x,z_4)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{B_2}^{(4)}(a,b_1,b_2,\lambda,b_4;e,e';z_1,z_2,z_3x,z_4),\ Re(\lambda)>0;$$

$$(2.36)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{B_2}^{(4)}(a,b_1,b_2,b_3,\mu;e,e';z_1,z_2,z_3,z_4x)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{B_2}^{(4)}(a,b_1,b_2,b_3,\lambda;e,e';z_1,z_2,z_3,z_4x),\ Re(\lambda)>0;$$

$$(2.37)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{B_2}^{(4)}(a,b_1,b_2,b_3,b_4;e,\lambda;z_1,z_2,z_3,z_4x)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{B_2}^{(4)}(a,b_1,b_2,b_3,b_4;e,\mu;z_1,z_2,z_3,z_4x),\ Re(\lambda)>0;$$

$$(2.38)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{B_3}^{(4)}(a,\mu,b_2,b_3,b_4;e,e';z_1,z_2x,z_3,z_4x)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{B_3}^{(4)}(a,\lambda,b_2,b_3,b_4;e,e';z_1,z_2x,z_3,z_4x),\ Re(\lambda)>0;$$

$$(2.39)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{B_3}^{(4)}(a,b_1,\mu,b_3,b_4;e,e';z_1x,z_2,z_3,z_4)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{B_3}^{(4)}(a,b_1,\lambda,b_3,b_4;e,e';z_1x,z_2,z_3,z_4),\ Re(\lambda)>0;$$

$$(2.40)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{B_3}^{(4)}(a,b_1,b_2,\mu,b_4;e,e';z_1,z_2,z_3x,z_4)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{B_3}^{(4)}(a,b_1,b_2,\lambda,b_4;e,e';z_1,z_2,z_3x,z_4),\ Re(\lambda)>0;$$

$$(2.41)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{B_3}^{(4)}(a,b_1,b_2,b_3,\mu;e,e';z_1,z_2,z_3,z_4x)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{B_3}^{(4)}(a,b_1,b_2,b_3,\lambda;e,e';z_1,z_2,z_3,z_4x),\ Re(\lambda)>0;$$

$$(2.42)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{B_3}^{(4)}(a,b_1,b_2,b_3,b_4;e,\lambda;z_1,z_2,z_3,z_4x)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{B_3}^{(4)}(a,b_1,b_2,b_3,b_4;e,\mu;z_1,z_2,z_3,z_4x),\ Re(\lambda)>0;$$

$$(2.43)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{C_1}^{(4)}(\mu,b,c,d;e,e';z_1x,z_2,z_3x,z_4x)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{C_1}^{(4)}(\lambda,b,c,d;e,e';z_1x,z_2,z_3x,z_4x),\ Re(\lambda)>0;$$

$$(2.44)\ D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{C_1}^{(4)}(a,\mu,c,d;e,e';z_1x,z_2x,z_3,z_4)\right\}$$

$$=\frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{C_1}^{(4)}(a,\lambda,c,d;e,e';z_1x,z_2x,z_3,z_4),\ Re(\lambda)>0;$$

(2.45) $$D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{C_1}^{(4)}(a,b,c,\mu;e,e';z_1,z_2,z_3,z_4x)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{C_1}^{(4)}(a,b,c,\lambda;e,e';z_1,z_2,z_3,z_4x),\ Re(\lambda)>0;$$

(2.46) $$D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{C_1}^{(4)}(a,b,c,d;e,\lambda;z_1,z_2,z_3,z_4x)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{C_1}^{(4)}(a,b,c,d;e,\mu;z_1,z_2,z_3,z_4x),\ Re(\lambda)>0;$$

(2.47) $$D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{C_2}^{(4)}(\mu,b,c,d;e,e';z_1x,z_2x,z_3,z_4x)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{C_2}^{(4)}(\lambda,b,c,d;e,e';z_1x,z_2x,z_3,z_4x),\ Re(\lambda)>0;$$

(2.48) $$D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{C_2}^{(4)}(a,\mu,c,d;e,e';z_1x,z_2,z_3x,z_4)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{C_2}^{(4)}(a,\lambda,c,d;e,e';z_1x,z_2,z_3x,z_4),\ Re(\lambda)>0;$$

(2.49) $$D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{C_2}^{(4)}(a,b,c,\mu;e,e';z_1,z_2,z_3,z_4x)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{C_2}^{(4)}(a,b,c,\lambda;e,e';z_1,z_2,z_3,z_4x),\ Re(\lambda)>0;$$

(2.50) $$D_x^{\lambda-\mu}\left\{x^{\lambda-1}G_{C_2}^{(4)}(a,b,c,d;e,\lambda;z_1,z_2,z_3,z_4x)\right\}$$

$$= \frac{\Gamma(\lambda)}{\Gamma(\mu)}x^{\mu-1}G_{C_2}^{(4)}(a,b,c,d;e,\mu;z_1,z_2,z_3,z_4x),\ Re(\lambda)>0;$$

3. Use of Two Fractional Derivative Operators.

(3.1) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}H_{A_1}^{(4)}(\mu,\mu',c,d;e,e',e'';z_1xy,z_2x,z_3x,z_4y)\right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}H_{A_1}^{(4)}(\lambda,\lambda',c,d;e,e',e'';z_1xy,z_2x,z_3x,z_4y),$$

$$Re(\lambda)>0,\ Re(\lambda')>0;$$

(3.2) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}H_{A_1}^{(4)}(\mu,b,\mu',d;e,e',e'';z_1x,z_2x,z_3xy,z_4y)\right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}H_{A_1}^{(4)}(\lambda,b,\lambda',d;e,e',e'';z_1x,z_2x,z_3xy,z_4y),$$

$$Re(\lambda)>0,\ Re(\lambda')>0;$$

(3.3) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}H_{A_1}^{(4)}(a,\mu,\mu',d;e,e',e'';z_1x,z_2,z_3y,z_4xy)\right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} H_{A_1}^{(4)}(a,\lambda,\lambda',d;e,e',e'';z_1x,z_2,z_3y,z_4xy),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.4) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} H_{A_1}^{(4)}(\mu,b,c,\mu';e,e',e'';z_1x,z_2xy,z_3x,z_4) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} H_{A_1}^{(4)}(\lambda,b,c,\lambda';e,e',e'';z_1x,z_2xy,z_3x,z_4),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.5) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} H_{A_1}^{(4)}(a,\mu,c,\mu';e,e',e'';z_1x,z_2y,z_3,z_4x) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} H_{A_1}^{(4)}(a,\lambda,c,\lambda';e,e',e'';z_1x,z_2y,z_3,z_4x),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.6) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} H_{A_1}^{(4)}(a,b,\mu,\mu';e,e',e'';z_1,z_2y,z_3x,z_4x) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} H_{A_1}^{(4)}(a,b,\lambda,\lambda';e,e',e'';z_1,z_2y,z_3x,z_4x),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.7) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} H_{A_1}^{(4)}(a,b,c,d;\lambda,\lambda',e'';z_1x,z_2y,z_3x,z_4) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} H_{A_1}^{(4)}(a,b,c,d;\mu,\mu',e'';z_1x,z_2y,z_3x,z_4),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.8) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} H_{A_1}^{(4)}(a,b,c,d;\lambda,e',\lambda';z_1x,z_2,z_3x,z_4y) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} H_{A_1}^{(4)}(a,b,c,d;\mu,e',\mu';z_1x,z_2,z_3x,z_4y),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.9) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} H_{B_1}^{(4)}(\mu,\mu',c,d;e_1,e_2,e_3,e_4;z_1xy,z_2x,z_3y,z_4x) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} H_{B_1}^{(4)}(\lambda,\lambda',c,d;e_1,e_2,e_3,e_4;z_1xy,z_2x,z_3y,z_4x),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.10) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} H_{B_1}^{(4)}(\mu,b,\mu',d;e_1,e_2,e_3,e_4;z_1x,z_2x,z_3y,z_4xy) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} H_{B_1}^{(4)}(\lambda,b,\lambda',d;e_1,e_2,e_3,e_4;z_1x,z_2x,z_3y,z_4xy),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

$$(3.11)\quad D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}H_{B_1}^{(4)}(a,\mu,\mu',d;e_1,e_2,e_3,e_4;z_1x,z_2,z_3xy,z_4y)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}H_{B_1}^{(4)}(a,\lambda,\lambda',d;e_1,e_2,e_3,e_4;z_1x,z_2,z_3xy,z_4y),$$

$$Re(\lambda)>0, Re(\lambda')>0;$$

$$(3.12)\quad D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}H_{B_1}^{(4)}(a,b,c.d;\lambda,\lambda',e_3,e_4;z_1x,z_2y,z_3,z_4)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}H_{B_1}^{(4)}(a,b,c,d,;\mu,\mu',e_3,e_4;z_1x,z_2y,z_3,z_4),$$

$$Re(\lambda)>0, Re(\lambda')>0.$$

Five more results similar to (3.12) can also be obtained but we have left them due to lack of space.

$$(3.13)\quad D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}x^{\lambda-1}y^{\lambda'-1}G_{A_1}^{(4)}(a,\mu,\mu',d;e,e';z_1x,z_2y,z_3x,z_4)$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{A_1}^{(4)}(a,\lambda,\lambda',d;e,e';z_1x,z_2y,z_3x,z_4),$$

$$Re(\lambda)>0, Re(\lambda')>0;$$

$$(3.14)\quad D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{A_1}^{(4)}(a,\mu,c,\mu';e,e';z_1x,z_2,z_3x,z_4y)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{A_1}^{(4)}(a,\lambda,c,\lambda';e,e';z_1x,z_2,z_3x,z_4y),$$

$$Re(\lambda)>0, Re(\lambda')>0;$$

$$(3.15)\quad D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{A_1}^{(4)}(a,\mu,c,d;e,\lambda';z_1x,z_2,z_3x,z_4y)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{A_1}^{(4)}(a,\lambda,c,d;e,\mu';z_1x,z_2,z_3x,z_4y),$$

$$Re(\lambda)>0, Re(\lambda')>0;$$

$$(3.16)\quad D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{A_2}^{(4)}(a,\mu,\mu',d;e,e';z_1x,z_2y,z_3x,z_4x)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{A_2}^{(4)}(a,\lambda,\lambda',d;e,e';z_1x,z_2y,z_3x,z_4x),$$

$$Re(\lambda)>0, Re(\lambda')>0;$$

$$(3.17)\quad D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{A_2}^{(4)}(a,\mu,c,\mu';e,e';z_1x,z_2,z_3x,z_4xy)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{A_2}^{(4)}(a,\lambda,c,\lambda';e,e';z_1x,z_2,z_3x,z_4xy),$$

$$Re(\lambda)>0, Re(\lambda')>0;$$

$$(3.18)\quad D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{A_2}^{(4)}(a,\mu,c,d;e,\lambda';z_1x,z_2,z_3x,z_4xy)\right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} G^{(4)}_{A_2}(a,\lambda,c,d;e,\mu';z_1x,z_2,z_3x,z_4xy),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.19) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} G^{(4)}_{A_2}(a,b,\mu,d;e,\lambda';z_1,z_2x,z_3,z_4y) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} G^{(4)}_{A_2}(a,b,\lambda,d;e,\mu';z_1,z_2x,z_3,z_4y),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.20) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} G^{(4)}_{A_2}(a,b,c,\mu;e,\lambda';z_1,z_2,z_3,z_4xy) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} G^{(4)}_{A_2}(a,b,c,\lambda;e,\mu';z_1,z_2,z_3,z_4xy),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.21) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} G^{(4)}_{A_3}(a,\mu,\mu',d;e,e';z_1x,z_2y,z_3x,z_4y) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} G^{(4)}_{A_3}(a,\lambda,\lambda',d;e,e';z_1x,z_2y,z_3x,z_4y),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.22) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} G^{(4)}_{A_3}(a,\mu,c,\mu';e,e';z_1x,z_2,z_3x,z_4y) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} G^{(4)}_{A3}(a,\lambda,c,\lambda';e,e';z_1x,z_2,z_3x,z_4y),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.23) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} G^{(4)}_{A_3}(a,b,\mu,\mu';e,e';z_1,z_2x,z_3,z_4xy) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} G^{(4)}_{A_3}(a,b,\lambda,\lambda';e,e';z_1,z_2x,z_3,z_4xy),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.24) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} G^{(4)}_{A_3}(a,,\mu,c,d;e,\lambda';z_1x,z_2,z_3x,z_4y) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} G^{(4)}_{A_3}(a,\lambda,c,d;e,\mu';z_1x,z_2,z_3x,z_4y),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.25) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} G^{(4)}_{A_3}(a,b,\mu,d;e,\lambda';z_1,z_2x,z_3,z_4yx) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} G^{(4)}_{A_3}(a,b,\lambda,d;e,\mu';z_1,z_2x,z_3,z_4xy),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.26) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{A_3}^{(4)}(a,b,c,\mu;e,\lambda';z_1,z_2,z_3,z_4xy)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{A_3}^{(4)}(a,b,c,\lambda;e,\mu';z_1,z_2,z_3,z_4xy),$$

$$Re(\lambda)>0,\ Re(\lambda')>0;$$

(3.27) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}x^{\lambda-1}y^{\lambda'-1}G_{B_1}^{(4)}(a,\mu,\mu',b_3,b_4;e,e';z_1x,z_2y,z_3,z_4)$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{B_1}^{(4)}(a,\lambda,\lambda',b_3,b_4;e,e';z_1x,z_2y,z_3,z_4),$$

$$Re(\lambda)>0,\ Re(\lambda')>0;$$

We can also obtain five more relations similar to (3.27).

(3.28) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{B_1}^{(4)}(a,\mu,b_2,b_3,b_4';e,\lambda;z_1x,z_2,z_3,z_4y)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{B_1}^{(4)}(a,\lambda,b_2,b_3,b_4;e,\mu';z_1x,z_2,z_3,z_4y)$$

$$Re(\lambda)>0,\ Re(\lambda')>0.$$

We can also obtain three more relations similar to (3.28).

(3.29) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{B_2}^{(4)}(a,\mu,\mu',b_3,b_4;e,e';z_1x,z_2y,z_3,z_4x)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{B_2}^{(4)}(a,\lambda,\lambda',b_3,b_4;e,e';z_1x,z_2y,z_3,z_4x),$$

$$Re(\lambda)>0,\ Re(\lambda')>0;$$

(3.30) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1'}G_{B_2}^{(4)}(a,\mu,b_2,\mu',b_4;e,e';z_1x,z_2,z_3y,z_4x)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{B_2}^{(4)}(a,\lambda,b_2,\lambda',b_4;e,e';z_1x,z_2,z_3y,z_4x),$$

$$Re(\lambda)>0,\ Re(\lambda')>0;$$

(3.31) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{B_2}^{(4)}(a,\mu,b_2,b_3,\mu';e,e';z_1x,z_2,z_3,z_4xy)\right\}$$

$$\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{B_2}^{(4)}(a,\lambda,b_2,b_3,\lambda';e,e';z_1x,z_2,z_3y,z_4xy),$$

$$Re(\lambda)>0,\ Re(\lambda')>0;$$

(3.32) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{B_2}^{(4)}(a,\mu,b_2,b_3,b_4;e,\lambda';z_1x,z_2,z_3,z_4xy)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{B_2}^{(4)}(a,\lambda,b_2,b_3,b_4;e,\mu';z_1x,z_2,z_3,z_4xy),$$

$$Re(\lambda)>0,\ Re(\lambda')>0;$$

(3.33) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{B_2}^{(4)}(a,b_1,\lambda,b_3,b_4;e,\mu';z_1,z_2x,z_3,z_4y)\right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} G_{B_2}^{(4)}(a, b_1, \mu, b_3, b_4; e, \lambda'; z_1, z_2 x, z_3, z_4 y),$$

$$Re(\lambda) > 0, Re(\lambda') > 0;$$

(3.34) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} G_{B_2}^{(4)}(a, b_1, b_2, \lambda, b_4; e, \mu'; z_1, z_2, z_3 x, z_4 y) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} G_{B_2}^{(4)}(a, b_1, b_2, \mu, b_4; e, \lambda'; z_1, z_2, z_3 x, z_4 y),$$

$$Re(\lambda) > 0, Re(\lambda') > 0;$$

(3.35) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} G_{B_2}^{(4)}(a, b_1, b_2, b_3, \lambda; e, \mu'; z_1, z_2, z_3, z_4 xy) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} G_{B_2}^{(4)}(a, b_1, b_2, b_3, \mu; e, \lambda'; z_1, z_2, z_3, z_4 xy),$$

$$Re(\lambda) > 0, Re(\lambda') > 0;$$

(3.36) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} G_{B_3}^{(4)}(a, \mu, \mu', b_3, b_4; e, e'; z_1 y, z_2 x, z_3, z_4 x) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} G_{B_3}^{(4)}(a, \lambda, \lambda', b_3, b_4; e, e'; z_1 y, z_2 x, z_3, z_4 x),$$

$$Re(\lambda) > 0, Re(\lambda') > 0;$$

(3.37) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} G_{B_3}^{(4)}(a, \mu, b_2, \mu', b_4; e, e'; z_1, z_2 x, z_3 y, z_4 x) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} G_{B_3}^{(4)}(a, \lambda, b_2, \lambda', b_4; e, e'; z_1, z_2 x, z_3 y, z_4 x),$$

$$Re(\lambda) > 0, Re(\lambda') > 0;$$

(3.38) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} G_{B_3}^{(4)}(a, \mu, b_2, b_3, \mu'; e, e'; z_1, z_2 x, z_3, z_4 xy) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} G_{B_3}^{(4)}(a, \lambda, b_2, b_3, \lambda'; e, e'; z_1, z_2 x, z_3, z_4 xy),$$

$$Re(\lambda) > 0, Re(\lambda') > 0;$$

(3.39) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} G_{B_3}^{(4)}(a, \mu, b_2, b_3, b_4; e, \lambda'; z_1, z_2 x, z_3, z_4 xy) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} G_{B_3}^{(4)}(a, \lambda, b_2, b_3, b_4; e, \mu'; z_1, z_2 x, z_3, z_4 xy),$$

$$Re(\lambda) > 0, Re(\lambda') > 0;$$

(3.40) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} \left\{ x^{\lambda-1} y^{\lambda'-1} G_{B_3}^{(4)}(a, b_1, \mu, b_3, b_4; e, \lambda'; z_1 x, z_2, z_3, z_4 y) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')} x^{\mu-1} y^{\mu'-1} G_{B_3}^{(4)}(a, b_1, \lambda, b_3, b_4; e, \mu'; z_1 x, z_2, z_3, z_4 y),$$

$$Re(\lambda) > 0, Re(\lambda') > 0;$$

(3.41) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{B_3}^{(4)}(a,b_1,b_2,\mu,b_4;e,\lambda';z_1,z_2,z_3x,z_4y)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{B_3}^{(4)}(a,b_1,b_2,\lambda,b_4;e,\mu';z_1,z_2,z_3x,z_4y),$$

$$Re(\lambda)>0, Re(\lambda')>0;$$

(3.42) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{B_3}^{(4)}(a,b_1,b_2,b_3,\mu;e,\lambda';z_1,z_2,z_3,z_4xy)\right\}$$

$$\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{B_3}^{(4)}(a,b_1,b_2,b_3,\lambda;e,\mu';z_1,z_2,z_3,z_4xy),$$

$$Re(\lambda)>0, Re(\lambda')>0;$$

(3.43) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{C_1}^{(4)}(\ \mu,\mu',,c,d;e,e';z_1xy,z_2y,z_3x,z_4x)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{C_1}^{(4)}(\lambda,\lambda',c,d;e,e';z_1xy,z_2y,z_3x,z_4x),$$

$$Re(\lambda)>0, Re(\lambda')>0;$$

(3.44) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{C_1}^{(4)}(\ \mu,b,c,\mu';e,e';z_1x,z_2,z_3x,z_4xy)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{C_1}^{(4)}(\lambda,b,c,\lambda';e,e';z_1x,z_2,z_3x,z_4xy),$$

$$Re(\lambda)>0, Re(\lambda')>0;$$

(3.45) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{C_1}^{(4)}(\ \mathrm{a},\mu,c,\mu';e,e';z_1x,z_2x,z_3,z_4y)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{C_1}^{(4)}(a,\lambda,c,\lambda';e,e';z_1x,z_2x,z_3,z_4y),$$

$$Re(\lambda)>0, Re(\lambda')>0;$$

(3.46) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{C_1}^{(4)}(\ \mu,b,c,d;e,\lambda';z_1x,z_2,z_3x,z_4xy)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{C_1}^{(4)}(\lambda,b,c,d;e,\mu';z_1x,z_2,z_3x,z_4xy),$$

$$Re(\lambda)>0, Re(\lambda')>0;$$

(3.47) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{C_1}^{(4)}(\ \mathrm{a},\mu,c,d;e,\lambda';z_1x,z_2x,z_3,z_4y)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{C_1}^{(4)}(a,\lambda,c,d;e,\mu';z_1x,z_2x,z_3,z_4y),$$

$$Re(\lambda)>0, Re(\lambda')>0;$$

(3.48) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{C_1}^{(4)}(\ \mathrm{a},\mathrm{b},c,\mu;e,\lambda';z_1,z_2,z_3,z_4xy)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{C_1}^{(4)}(a,b,c,\lambda;e,\mu';z_1,z_2,z_3,z_4xy),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.49) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{C_1}^{(4)}(\mu,\mu',c,d;e,e';z_1xy,z_2x,z_3y,z_4x)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{C_1}^{(4)}(\lambda,\lambda',c,d;e,e';z_1xy,z_2x,z_3y,z_4x),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.50) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{C_1}^{(4)}(\mu,b,c,\mu';e,e';z_1x,z_2x,z_3,z_4xy)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{C_1}^{(4)}(\lambda,b,c,\lambda';e,e';z_1x,z_2x,z_3,z_4xy),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.51) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{C_1}^{(4)}(a,\mu,c,\mu';e,e';z_1x,z_2,z_3x,z_4y)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{C_1}^{(4)}(a,\lambda,c,\lambda';e,e';z_1x,z_2,z_3x,z_4y),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.52) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{C_1}^{(4)}(\mu,b,c,d;e,\lambda';z_1x,z_2x,z_3,z_4xy)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{C_1}^{(4)}(\lambda,b,c,d;e,\mu';z_1x,z_2x,z_3,z_4xy),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.53) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{C_1}^{(4)}(a,\mu,c,d;e,\lambda';z_1x,z_2,z_3x,z_4y)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{C_1}^{(4)}(a,\lambda,c,d;e,\mu';z_1x,z_2,z_3x,z_4y),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

(3.54) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'}\left\{x^{\lambda-1}y^{\lambda'-1}G_{C_1}^{(4)}(a,b,c,\mu;e,\lambda';z_1,z_2,z_3,z_4xy)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')}{\Gamma(\mu)\Gamma(\mu')}x^{\mu-1}y^{\mu'-1}G_{C_1}^{(4)}(a,b,c,\lambda;e,\mu';z_1,z_2,z_3,z_4xy)$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0;$$

4. Use of Three Fractional Derivative Operators.

(4.1) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}y^{\lambda''-1}H_{A_1}^{(4)}(\mu,\mu',\mu'',d;e,e',e'';xyu_1,xu_2,xzu_3,yzu_4)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}H_{A_1}^{(4)}(\lambda,\lambda',\lambda'',d;e,e',e'';xyu_1,xu_2,xzu_3,yzu_4),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

(4.2) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}y^{\lambda''-1}H_{A_1}^{(4)}(\mu,\mu',c,d;\lambda'',e',e'';zxyu_1,xu_2,xzu_3,yu_4)\right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')} x^{\mu-1} y^{\mu'-1} z^{\mu''-1} H_{A_1}^{(4)}(\lambda, \lambda', c, d; \mu'', e', e''; xyzu_1, xu_2, zxu_3, yu_4),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

(4.3) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} D_z^{\lambda''-\mu''} \left\{ x^{\lambda-1} y^{\lambda'-1} y^{\lambda''-1} H_{A_1}^{(4)}(\mu, b, \mu', d; \lambda'', e', e''; u_1 xz, xu_2, u_3 xyz, u_4 y) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')} x^{\mu-1} y^{\mu'-1} z^{\mu''-1} H_{A_1}^{(4)}(\lambda, b, \lambda', d; \mu'', e', e''; u_1 xz, u_2 x, u_3 xyz, u_4 y),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

(4.4) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} D_z^{\lambda''-\mu''} \left\{ x^{\lambda-1} y^{\lambda'-1} y^{\lambda''-1} H_{A_1}^{(4)}(a, \mu, \mu', d; \lambda'', e', e''; u_1 xz, u_2, u_3 yz, u_4 yx) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')} x^{\mu-1} y^{\mu'-1} z^{\mu''-1} H_{A_1}^{(4)}(a, \lambda, \lambda', d; \mu'', e', e''; u_1 xz, u_2, u_3 yz, u_4 xy),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

(4.5) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} D_z^{\lambda''-\mu''} \left\{ x^{\lambda-1} y^{\lambda'-1} y^{\lambda''-1} H_{B_1}^{(4)}(\mu, \mu', \mu'', d; e_1, e_2, e_3, e_4; u_1 xy, u_2 x, u_3 yz, u_4 xz) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')} x^{\mu-1} y^{\mu'-1} z^{\mu''-1} H_{B_1}^{(4)}(\lambda, \lambda', \lambda'', d; e_1, e_2, e_3, e_4; u_1 xy, u_2 x, u_3 yz, u_4 xz),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

(4.6) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} D_z^{\lambda''-\mu''} \left\{ x^{\lambda-1} y^{\lambda'-1} y^{\lambda''-1} H_{B_1}^{(4)}(\mu, \mu', c, d; \lambda'', e_2, e_3, e_4; u_1 xyz, u_2 x, u_3 y, u_4 x) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')} x^{\mu-1} y^{\mu'-1} z^{\mu''-1} H_{B_1}^{(4)}(\lambda, \lambda', c, d; \mu'', e_2, e_3, e_4; u_1 xyz, u_2 x, u_3 y, u_4 x),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

(4.7) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} D_z^{\lambda''-\mu''} \left\{ x^{\lambda-1} y^{\lambda'-1} y^{\lambda''-1} H_{B_1}^{(4)}(\mu, b, \mu', d; \lambda'', e_2, e_3, e_4; u_1 xz, u_2 x, u_3 y, u_4 xy) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')} x^{\mu-1} y^{\mu'-1} z^{\mu''-1} H_{B_1}^{(4)}(\lambda, \lambda', c, d; \mu'', e_2, e_3, e_4; u_1 xyz, u_2 x, u_3 y, u_4 x)$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

(4.8) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} D_z^{\lambda''-\mu''} \left\{ x^{\lambda-1} y^{\lambda'-1} y^{\lambda''-1} H_{B_1}^{(4)}(a, \mu, \mu', d; \lambda'', e_2, e_3, e_4; u_1 xz, u_2, u_3 xy, u_4 y) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')} x^{\mu-1} y^{\mu'-1} z^{\mu''-1} H_{B_1}^{(4)}(a, \lambda, \lambda', d; \mu'', e_2, e_3, e_4; u_1 xz, u_2, u_3 xy, u_4 y),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

(4.9) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} D_z^{\lambda''-\mu''} \left\{ x^{\lambda-1} y^{\lambda'-1} y^{\lambda''-1} G_{A_1}^{(4)}(a, \mu, \mu', \mu''; e, e'; u_1 x, u_2 y, u_3 x, u_4 z) \right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')} x^{\mu-1} y^{\mu'-1} z^{\mu''-1} G_{A_1}^{(4)}(a, \lambda, \lambda', \lambda''; e, e'; u_1 x, u_2 y, u_3 x, u_4 z),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

(4.10) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}y^{\lambda''-1}G_{A_1}^{(4)}(a,\mu,\mu',d;e,\lambda'';u_1x,u_2y,u_3x,u_4z)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}G_{A_1}^{(4)}(a,\lambda,\lambda',d;e,\mu'';u_1x,u_2y,u_3x,u_4z),$$

$$Re(\lambda)>0,\ Re(\lambda')>0, Re(\lambda'')>0;$$

(4.11) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}z^{\lambda''-1}G_{A_2}^{(4)}(a,\mu,\mu',\mu'';e,e';u_1x,u_2y,u_3x,u_4xz)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}G_{A_2}^{(4)}(a,\lambda,\lambda',\lambda'';e,e';u_1x,u_2y,u_3x,u_4xz),$$

$$Re(\lambda)>0,\ Re(\lambda')>0, Re(\lambda'')>0;$$

(4.12) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}z^{\lambda''-1}G_{A_2}^{(4)}(a,\mu,\mu',d;e,\lambda'';u_1x,u_2y,u_3x,u_4xz)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}G_{A_2}^{(4)}(a,\lambda,\lambda',d;e,\mu'';u_1x,u_2y,u_3x,u_4xz),$$

$$Re(\lambda)>0,\ Re(\lambda')>0, Re(\lambda'')>0;$$

(4.13) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}z^{\lambda''-1}G_{A_3}^{(4)}(a,\mu,\mu',\mu'';e,e';u_1x,u_2y,u_3x,u_4yz)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}G_{A_3}^{(4)}(a,\lambda,\lambda',\lambda'';e,e';u_1x,u_2y,u_3x,u_4yz),$$

$$Re(\lambda)>0,\ Re(\lambda')>0, Re(\lambda'')>0;$$

(4.14) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}z^{\lambda''-1}G_{A_3}^{(4)}(a,\mu,\mu',d;e,\lambda'';u_1x,u_2y,u_3x,u_4yz)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}G_{A_3}^{(4)}(a,\lambda,\lambda',d;e,\mu'';u_1x,u_2y,u_3x,u_4yz),$$

$$Re(\lambda)>0,\ Re(\lambda')>0, Re(\lambda'')>0;$$

(4.15) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}z^{\lambda''-1}G_{B_1}^{(4)}(a,\mu,\mu',\mu'',b_4;e,e';u_1x,u_2y,u_3z,u_4)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}G_{B_1}^{(4)}(a,\lambda,\lambda',\lambda'',b_4;e,e';u_1x,u_2y,u_3z,u_4),$$

$$Re(\lambda)>0,\ Re(\lambda')>0, Re(\lambda'')>0;$$

(4.15) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}z^{\lambda''-1}G_{B_1}^{(4)}(a,\mu,\mu',\mu'',b_4;e,e';u_1x,u_2y,u_3z,u_4)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}G_{B_1}^{(4)}(a,\lambda,\lambda',\lambda'',b_4;e,e';u_1x,u_2y,u_3z,u_4),$$

$$Re(\lambda)>0,\ Re(\lambda')>0, Re(\lambda'')>0;$$

(4.16) $$D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}z^{\lambda''-1}G_{B_1}^{(4)}(a,b_1,\mu,\mu',\mu'';e,e';u_1,u_2x,u_3y,u_4z)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}G_{B_1}^{(4)}(a,b_1,\lambda,\lambda',\lambda'';e,e';u_1,u_2x,u_3y,u_4z),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

(4.17) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} D_z^{\lambda''-\mu''}\left\{x^{\lambda-1} y^{\lambda'-1} z^{\lambda''-1} G_{B_1}^{(4)}\left(a, b_1, \mu, \mu', b_4; e, \lambda''; u_1, u_2 x, u_3 x, u_4 z\right)\right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')} x^{\mu-1} y^{\mu'-1} z^{\mu''-1} G_{B_1}^{(4)}\left(a, b_1, \lambda, \lambda', b_4; e, \mu''; u_1, u_2 x, u_3 y, u_4 z\right),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

(4.18) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} D_z^{\lambda''-\mu''}\left\{x^{\lambda-1} y^{\lambda'-1} z^{\lambda''-1} G_{B_1}^{(4)}\left(a, b_1, b_2, \lambda, \mu'; e, \lambda''; u_1, u_2, u_3 x, u_4 yz\right)\right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')} x^{\mu-1} y^{\mu'-1} z^{\mu''-1} G_{B_1}^{(4)}\left(a, b_1, b_2, \lambda, \lambda'; e, \mu''; u_1, u_2, u_3 x, u_4 yz\right),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

(4.19) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} D_z^{\lambda''-\mu''}\left\{x^{\lambda-1} y^{\lambda'-1} z^{\lambda''-1} G_{B_2}^{(4)}\left(a, \mu, \mu', \mu'', b_4; e, e'; u_1 x, u_2 y, u_3 z, u_4 x\right)\right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')} x^{\mu-1} y^{\mu'-1} z^{\mu''-1} G_{B_2}^{2}\left(a, \lambda, \lambda', \lambda'', b_4; e, e'; u_1 x, u_2 y, u_3 z, u_4 x\right),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

(4.20) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} D_z^{\lambda''-\mu''}\left\{x^{\lambda-1} y^{\lambda'-1} z^{\lambda''-1} G_{B_2}^{(4)}\left(a, \mu, \mu', b_3, b_4; e, \lambda''; u_1 x, u_2 y, u_3, u_4 xz\right)\right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')} x^{\mu-1} y^{\mu'-1} z^{\mu''-1} G_{B_2}^{(4)}\left(a, \lambda, \lambda', b_3, b_4; e, \mu''; u_1 x, u_2 y, u_3, u_4 xz\right),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

(4.21) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} D_z^{\lambda''-\mu''}\left\{x^{\lambda-1} y^{\lambda'-1} z^{\lambda''-1} G_{B_2}^{(4)}\left(a, \mu, b_2, \mu', b_4; e, \lambda''; u_1 x, u_2, u_3 y, u_4 xz\right)\right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')} x^{\mu-1} y^{\mu'-1} z^{\mu''-1} G_{B_2}^{(4)}\left(a, \mu, b_2, \mu', b_4; e, \lambda''; u_1 x, u_2, u_3 y, u_4 xz\right),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

(4.22) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} D_z^{\lambda''-\mu''}\left\{x^{\lambda-1} y^{\lambda'-1} z^{\lambda''-1} G_{B_2}^{(4)}\left(a, \mu, b_2, b_3, \mu'; e, \lambda''; u_1 x, u_2, u_3, u_4 xyz\right)\right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')} x^{\mu-1} y^{\mu'-1} z^{\mu''-1} G_{B_2}^{(4)}\left(a, \lambda, b_2, b_3, \lambda'; e, \mu''; u_1 x, u_2, u_3, u_4 xyz\right),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

(4.23) $$D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} D_z^{\lambda''-\mu''}\left\{x^{\lambda-1} y^{\lambda'-1} z^{\lambda''-1} G_{B_3}^{(4)}\left(a, \mu, \mu', \mu'', b_4; e, e'; u_1 y, u_2 x, u_3 z, u_4 x\right)\right\}$$

$$= \frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')} x^{\mu-1} y^{\mu'-1} z^{\mu''-1} G_{B_3}^{(4)}\left(a, \lambda, \lambda', \lambda'', b_4; e, e'; u_1 y, u_2 x, u_3 z, u_4 x\right)$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

$$(4.24)\quad D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}z^{\lambda''-1}G_{B_3}^{(4)}(a,\mu,\mu',b_3,\mu'';e,e';u_1y,u_2x,u_3,u_4xz)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}G_{B_3}^{(4)}(a,\lambda,\lambda',b_3,\lambda'';e,e';u_1y,u_2x,u_3,u_4xz),$$

$$Re(\lambda)>0,\ Re(\lambda')>0, Re(\lambda'')>0;$$

$$(4.25)\quad D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}z^{\lambda''-1}G_{B_3}^{(4)}(a,\mu,\mu',b_3,b_4;e,\lambda'';u_1y,u_2x,u_3,u_4xz)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}G_{B_3}^{(4)}(a,\lambda,\lambda',b_3,b_4;e,\mu'';u_1y,u_2x,u_3,u_4xz),$$

$$Re(\lambda)>0,\ Re(\lambda')>0, Re(\lambda'')>0;$$

$$(4.26)\quad D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}z^{\lambda''-1}G_{C_1}^{(4)}(\mu,\mu',c,\mu'';e,e';u_1xy,u_2y,u_3x,u_4zx)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}G_{C_1}^{(4)}(\lambda,\lambda',c,\lambda'',;e,e';u_1xy,u_2y,u_3x,u_4xz),$$

$$Re(\lambda)>0,\ Re(\lambda')>0, Re(\lambda'')>0;$$

$$(4.27)\quad D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}z^{\lambda''-1}G_{C_1}^{(4)}(\mu,\mu',c,d;e,\lambda'';u_1xy,u_2y,u_3x,u_4zx)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}G_{C_1}^{(4)}(\lambda,\lambda',c,d;e,\mu'';u_1xy,u_2y,u_3x,u_4xz),$$

$$Re(\lambda)>0,\ Re(\lambda')>0, Re(\lambda'')>0;$$

$$(4.28)\quad D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}z^{\lambda''-1}G_{C_1}^{(4)}(\mu,b,c,\mu';e,\lambda'';u_1x,u_2,u_3x,u_4xyz)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}G_{C_1}^{(4)}(\lambda,b,c,\lambda',;e,\mu'';u_1x,u_2,u_3x,u_4xyz),$$

$$Re(\lambda)>0,\ Re(\lambda')>0, Re(\lambda'')>0;$$

$$(4.29)\quad D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}z^{\lambda''-1}G_{C_1}^{(4)}(a,\mu,c,\mu';e,\lambda'';u_1x,u_2x,u_3,u_4yz)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}G_{C_1}^{(4)}(a,\lambda,c,\lambda';e,\mu'';u_1x,u_2x,u_3,u_4yz)$$

$$Re(\lambda)>0,\ Re(\lambda')>0, Re(\lambda'')>0;$$

$$(4.30)\quad D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}z^{\lambda''-1}G_{C_2}^{(4)}(\mu,\mu',c,\mu'';e,e';u_1xy,u_2x,u_3y,u_4xz)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}G_{C_2}^{(4)}(\lambda,\lambda',c,\lambda'';e,e';u_1xy,u_2x,u_3y,u_4xz),$$

$$Re(\lambda)>0,\ Re(\lambda')>0, Re(\lambda'')>0;$$

$$(4.31)\quad D_x^{\lambda-\mu}D_y^{\lambda'-\mu'}D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}z^{\lambda''-1}G_{C_2}^{(4)}(\mu,\mu',c,d;e,\lambda'';u_1xy,u_2x,u_3y,u_4xz)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}G_{C_2}^{(4)}(\lambda,\lambda',c,d;e,\mu'';u_1xy,u_2x,u_3y,u_4xz),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

$$(4.32)\quad D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}z^{\lambda''-1}G_{C_2}^{(4)}(a,\mu,c,\mu';e,\lambda'';u_1x,u_2,u_3x,u_4yz)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}G_{C_2}^{(4)}(a,\lambda,c,\lambda';e,\mu'';u_1x,u_2,u_3x,u_4yz),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

$$(4.33)\quad D_x^{\lambda-\mu} D_y^{\lambda'-\mu'} D_z^{\lambda''-\mu''}\left\{x^{\lambda-1}y^{\lambda'-1}z^{\lambda''-1}G_{C_2}^{(4)}(\mu,b,c,\mu';e,\lambda'';u_1x,u_2x,u_3,u_4yz)\right\}$$

$$=\frac{\Gamma(\lambda)\Gamma(\lambda')\Gamma(\lambda'')}{\Gamma(\mu)\Gamma(\mu')\Gamma(\mu'')}x^{\mu-1}y^{\mu'-1}z^{\mu''-1}G_{C_2}^{(4)}(\lambda,b,c,\lambda';e,\mu'';u_1x,u_2x,u_3,u_4xyz),$$

$$Re(\lambda) > 0,\ Re(\lambda') > 0, Re(\lambda'') > 0;$$

Applying the same techniques, we have also derived fractional derivatives involving hypergeometric functions of four variables introduced by Chandel, Agrawal and Kumar [10] but we have not recorded them due to lac of space.

REFERENCES

[1] R.C.S. Chandel, On some multiple hypergomotric functions related to Lauricella's functions, *Jñānābha* Sect A, **3** (1973),119-136. Errata and Addenda, ibid. **5** (1975), 177-180.

[2] R.C.S. Chandel and A.K. Gupta, Multiple hypergeometic functions related to Lauricella's functions, *Jñānābha* , **16** (1986),195-209.

[3] R.C.S. Chandel and P.K. Vishwakarma, Karlsson's multiple hypergeometric function and its confluent forms, *Jñānābha* , **19** (1989),173-185.

[4] R.C. S. Chandel and P.K. Vishwakarma, Fractional integration and integral representations of Karlsson's multiple hypergeometric function and its confluent forms, *Jñānābha* **20** (1990),101-110.

[5] R.C.S. Chandel and P.K. Vishwakrma, Fractional derivatives of confluent hypergemetric forms of Karlsson's multiple hypergeometric function, ${}^{(k)}F_{CD}^{(n)}$, *Pure Appl. Math. Sci.* **35** (1992), 31-39.

[6] R.C.S. Chandel and P.K. Vishwakarma, multidimensional fractional derivatives of the multiple hypergeometric functions of several variables, *Jñānābha* , **24** (1994),19-27.

[7] R.C.S. Chandel and P.K. Vishwakarma, Fractional derivatives of the multiple hypergeometric functions of four variables, *Jñānābha* , **26** (1996),83-86.

[8] R.C.S. Chandel and S. Sharma, Some new hypergeometric functions of four variables II, *Bull. Vijñāna Parishad of India,* **2** (1994), 44 .

[9] R.C.S. Chandel and S. Sharma, Hypergeometric functions of four variables, *Pure Appl. Math. Sci.* **58** (2003), 7-18.

[10] R.C.S. Chandel, R.D. Agrawal and H. Kumar, Hypergeometric functions of four variables and their integral representations, *Math. Eduction,* **26** (1992), 76-94.

[11] H. Exton, On two multiple hypergeometric funtions related to Lauricella's $F_D^{(n)}$, *Jñānābha* Sect. A, **2** (1992),59-73.

[12] H. Exton, Certain hypergeometric functionsof four variables, *Bull. Soc. Math. Gréce (N.S.)* **14** (1973), 132-140.

[13] H. Exton, Some integral representations and transformations of hypergeometric functions of four variables, *Bull. Soc. Math. Grece* (N.S.) **14** (1973), 132-140.

[14] H. Exton, *Multiple hypergeometric Functions and Applications,* Jhon Wiley and Sons, New York, London, Sydney and Toronto, 1976.

[15] P.W. Karlsson, On intermediate Lauricella functions, *Jñānābha* , **16** (1986),211-222.

[16] G. Lauricella, Sulle funzioni ipergeometriche a più variabili, *Rend. Circ. Mat. Palermo,* **7** (1893), 111-158.

[17] K.B. Oldham and J. Spanier, *The Fractional Calculus,* Academic Press, New York, London 1974.

[18] C. Sharma and C.L. Parihar, Hypergeometric functions of four variables I, *J. Indian, Acad. Math.* **11** (1989), 121-133.

[19] H. M. Srivastava and M.C. Daoust, Certain generalized Neumann expansions associated with the Kampé de Fériet function, *Nederl. Akad. Wetensch. Proc. Ser.* A **72** *Indag. Math.* **31** (1969), 449-457.

[20] H.M. Srivastava and R. Panda, Some bilateral generating functions for a class of generalized hypergeometric polynomials, *J. Reine. Angew. Math.* **283/284** (1976), 265-274.

[21] H.M. Srivastava and R. Panda, Expansion theorems for the *H*-function of several complex variables, *J. Reine Angew. Math.,* **288** (1976), 129-145.

[22] H.M. Srivastava and R. Panda, Some Expanssion theorems and generating relations for the *H*-function of several complex variables I and II, *Commet. Math. Univ. St. Paul.* **24** fasc **2** (1975), 119-137; **25** fasc **2** (1976), 167-197.

[23] H.M. Srivastava , K.C. Gupta and S.P. Goyal, *The H-Function of One and Two Variables with Applications,* South Asian Publisher, New Delhi/Madrass, 1982.

[24] H.M. Srivastava and H.L. Manocha, *A Treatise On Generating Functions,* John Wiley and Sons, New York, Chichester, Brisbane and Toronto, 1984.

[25] H.M. Srivastava and S.P. Goyal, Fractional dervatives of the *H*-function of several variables, *J. Math. Anal. Appl.,* **112** (1985), 641-651.

[26] H.M. Srivastava, R.C.S. Chandel and P.K. Vishwakarma, Fractional derivatives of certain generalized hypergeometric functions of several variables, *J. Math. Anal. Appl.* **184**