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AVAILABILITY AND MAXIMIZING SURVIVABILITY OF CABLE T.V. TRANSMISSION SYSTEM

By

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ABSTRACT

In this short communication, we have computed availability and Maximizing Survivability for cable T.V. transmission system which is one of the example of multi-state acyclic network (MAN). The system is designed to transmit from the cable T.V. operator's satellite link to users' televisions through amplifier units. The user's televisions are connected by cables, which are the system components. The system is operational if user's televisions are connected by cables, which are the system components. The system is operational if user's televisions are connected with amplifier units and the cable T.V. operator's satellite link.

Key words. Multi-state acyclic network, Multi-state, Universal generating function; Survivability, Availability.

1. Introduction. The MAN consists of a number of positions in which multi-state elements are allocated. Each network has root position where the signal source is located. The number of leaf node (position, like television) that can only receive a signals and a number of non-leaf nodes (intermediate nodes, amplifier unit) that can retransmit the received signal to the leaf node. The system is designed to transmit from teh cable T.V. operator's satellite link to users' televisions. The users' televisions are connected by cables and amplifier units, which are the system components. The system is operational if all user televisions are connected amplifier units to cable T.V. operator's satellite link otherwise system fails. Cable T.V. network is the generalization of tree structured multi-state system. Cable T.V. network has operator's satellite link, which is the root position where the signal source is allocated and amplifier units, which are the non-leaf positions. The sink position (leaf node) is situated at user's televisions. The aim of the cable T.V. network is the transmission of signal from source to the sink (last) position. Malinowski and Preuss [1] discovered that the acyclic multi-state transmission network is a generalization of the tree-structured multi-state systems. The universal generating function technique is used for solving availability and

Maximizing Survivability of cable T.V. transmission system that is suggested by Levitin and Lisnianski [1,4].

We have taken one example of cable T.V. network for solving the availability and Survivability using the universal generating function technique with $n=4$ and $n^*=1$, where n is the total number of nodes (positions) and n^* is the number of leaf nodes. the resulting polynomial contains 2^{n^*}-1 terms. The paper organized as follows: section 2 is devoted to example of cable T.V. transmission system for estimating the availability and survivability of MAN. Section 3 present the conclusion.

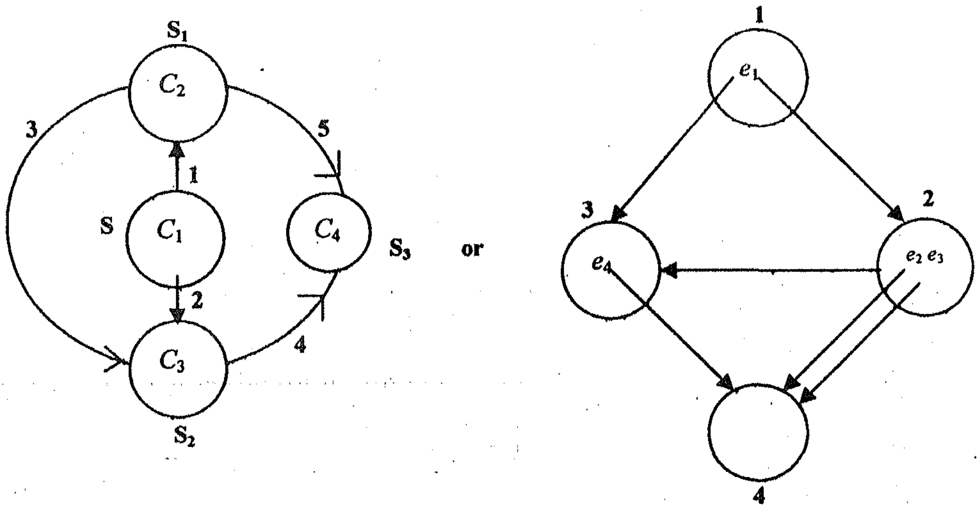

Fig. Cable T.V. Transimission System. (Simplest MAN)

2. Example.

The system is designed to transmit from the cable T.V. operator's satellite link S to users' televisions, S_3 through amplifier units S_1,S_2. the users' televisions are connected by cables numbered 4,5 and amplifier units numbered 1,2,3 which are the system components. The system is operational if all user televisions are connected from amplifier untis 1,2,3 to the cable T.V. operator's satellite link otherwise system fails.

Consider multi-state acyclic network (MAN) with n=4 and n^*=1 presented in above figure. Assume that *ME* e_1 is located at C_1 and provides a connection between C_1 and any subset of $\{C_2,C_3\}$. *ME* e_2 is located at C_2 and provides a conncection between C_2 and any subst of $\{C_3,C_4\}$. *ME* e_3 is also located at C_2 and provides a connection between C_2 and C_4. *ME* e_4 is located at C_3 and provides a connection between C_3 and C_4 . Using the universal generating function technique, the u-functions of individual *MEs* located at nodes C_1,C_2 and C_3 are :

$u_{11}(z) = p_{1\phi}\, z^{0000} + p_{1\{2\}} z^{0100} + p_{1\{3\}} z^{0010} + p_{1\{2,3\}} z^{0110}$,

$u_{22}(z) = p_{2\phi}\, z^{0000} + p_{2\{3\}} z^{0010} + p_{2\{4\}} z^{0001} + p_{2\{3,4\}} z^{0011}$,

$u_{32}(z) = p_{3\phi}\, z^{0000} + p_{3\{4\}} z^{0001}$,

$u_{43}(z) = p_{4\phi}\, z^{0000} + p_{4\{4\}} z^{0001}$.

The u-functions of groups of *MEs* located at the same nodes are,

$U_1(z) = u_{11}(z)$,

$U_3(z) = u_{43}(z)$,

$$U_2(z) = \Omega_{max}\,(u_{11}(z), u_{32}(z))$$

$$= p_{2\phi}\, p_{3\phi} z^{0000} + p_{2\{3\}} p_{3\phi} z^{0010} + p_{2\{4\}} p_{3\phi} z^{0001} + p_{2\{3,4\}} p_{3\phi} z^{0011} + p_{2\phi}\, p_{3\{4\}} z^{0001} + p_{2\{3\}} p_{3\{4\}} z^{0011} + p_{2\{4\}} p_{3\{4\}} z^{0001} + p_{2\{3,4\}} p_{3\{4\}}{}^{0011}$$

$$= q_{2\phi} z^{0000} + q_{2\{3\}} z^{0010} + q_{2\{4\}} z^{0001} + q_{2\{3,4\}} z^{0011},$$

where $q_{2\phi} = p_{2\phi}\, p_{3\phi}, q_{2\{3\}} = p_{2\{3\}}\, p_{3\phi}, q_{2\{4\}} = p_{2\{4\}}\, p_{3\phi} + p_{2\phi}\, p_{3\{4\}} + p_{2\{4\}} p_{3\{4\}}$

$q_{2\{3,4\}} = p_{2\{3,4\}}\, p_{3\phi} + p_{2\{3\}}\, p_{3\{4\}} + p_{2\{3,4\}} p_{3\{4\}}$

Since $p_{3\phi} + p_{3\{4\}} = 1$, the equations for $q_{2\{4\}} = p_{2\{4\}} + p_{2\phi}\, p_{3\{4\}}$

$q_{2\{3,4\}} = p_{2\{3,4\}} + p_{2\{3\}}\, p_{3\{4\}}$.

2.1. Availability of MAN. Following the consecutive procedure, we obtain:

$$\widetilde{U}_1(z) = U_1(z).$$

$$\varphi(\widetilde{U}_1(z)) = p_{1\{2\}} z^{0100} + p_{1\{3\}} z^{0010} + p_{1\{2,3\}} z^{0110},$$

$$\widetilde{U}_2(z) = \Omega_\phi(\varphi(\widetilde{U}_1(z)), U_2(z)),$$

$$= p_{1\{2\}} q_{2\phi} z^{0100} + p_{1\{2\}} q_{2\{3\}} z^{0110} + p_{1\{2\}} q_{2\{4\}} z^{0101} + p_{1\{2\}} q_{2\{3,4\}} z^{0111} + p_{1\{3\}} z^{0010}$$

$$+ p_{1\{2,3\}} q_{2\phi} z^{0110} + p_{1\{2,3\}} q_{2\{3\}} z^{0110} + p_{1\{2,3\}} q_{2\{4\}} z^{0111} + p_{1\{2,3\}} q_{2\{3,4\}} z^{0111},$$

and after simplification :

$$\varphi(\widetilde{U}_2(z)) = (p_{1\{2\}} q_{2\{3\}} + p_{1\{3\}} + p_{1\{2,3\}} q_{2\phi} + p_{1\{2,3\}} q_{2\{3\}}) z^{0010}$$

$$+ (p_{1\{2\}} q_{2\{3,4\}} + p_{1\{2,3\}} q_{2\{4\}} + p_{1\{2,3\}} q_{2\{3,4\}}) z^{0011} + p_{1\{2\}} q_{2\{4\}} z^{0001}.$$

(Note that operator φ reduces the number of different terms in this u-function from nine to three.)

$$\widetilde{U}_3(z) = \Omega_\phi(\varphi(\widetilde{U}_2(z)), U_3(z))$$

$$= (p_{1\{2\}} q_{2\{3\}} + p_{1\{3\}} + p_{1\{2,3\}} q_{2\phi} + p_{1\{2,3\}} q_{2\{3\}}) p_{4\phi} z^{0010}$$

$$+ (p_{1\{2\}} q_{2\{3,4\}} + p_{1\{2,3\}} q_{2\{4\}} + p_{1\{2,3\}} q_{2\{3,4\}}) p_{4\phi} z^{0011} + p_{1\{2\}} q_{2\{4\}} p_{4\phi} z^{0001}$$

$$+ (p_{1\{2\}} q_{2\{3\}} + p_{1\{3\}} + p_{1\{2,3\}} q_{2\phi} + p_{1\{2,3\}} q_{2\{3\}}) p_{4\{4\}} z^{0011}$$

$$+ (p_{1\{2\}} q_{2\{3,4\}} + p_{1\{2,3\}} q_{2\{4\}} + p_{1\{2,3\}} q_{2\{3,4\}}) p_{4\{4\}} z^{0011} + p_{1\{2\}} q_{2\{4\}} p_{4\{4\}} z^{0001}$$

and after simplification:

$$\varphi(\widetilde{U}_3(z)) = [(p_{1\{2\}}q_{2\{3,4\}} + p_{1\{2,3\}}q_{2\{4\}} + p_{1\{2,3\}}q_{2\{3,4\}})p_{4\phi} + p_{1\{2\}}q_{2\{4\}}p_{4\phi}$$
$$+ (p_{1\{2\}}q_{2\{3\}} + p_{1\{3\}} + p_{1\{2,3\}}q_{2\phi} + p_{1\{2,3\}}q_{2\{3\}})p_{4\{4\}}$$
$$+ (p_{1\{2\}}q_{2\{3,4\}} + p_{1\{2,3\}}q_{2\{4\}} + p_{1\{2,3\}}q_{2\{3,4\}})p_{4\{4\}} + p_{1\{2\}}q_{2\{4\}} + p_{4\{4\}}]z^{0001}.$$

The coefficient in the term with the vector $\widetilde{G}^{(3)} = 0001$ in $\varphi(\widetilde{U}_3(z))$ is the probability that the signal reaches C_4, which is equal to MAN availability:

$$A_4 = (p_{1\{2\}}q_{2\{3,4\}} + p_{1\{2,3\}}q_{2\{4\}} + p_{1\{2,3\}}q_{2\{3,4\}})p_{4\phi} + p_{1\{2\}}q_{2\{4\}}p_{4\phi}$$
$$+ (p_{1\{2\}}q_{2\{3\}} + p_{1\{3\}} + p_{1\{2,3\}}q_{2\phi} + p_{1\{2,3\}}q_{2\{3\}})p_{4\{4\}}$$
$$+ (p_{1\{2\}}q_{2\{3,4\}} + p_{1\{2,3\}}q_{2\{4\}} + p_{1\{2,3\}}q_{2\{3,4\}})p_{4\{4\}} + p_{1\{2\}}q_{2\{4\}}p_{4\{4\}}$$

2.2 Survivability of MAN. In this section, the first *ME* is located at position *1* second and third *ME* are located at posion 2 and fourth *ME* is located at position 3.

Following the consecutive procedure, we obtain:

$$\hat{U}_1(z) = \theta\{U_1(z)\} = (\beta + \alpha p_{1\phi})z^{0000} + \alpha p_{1\{2\}}z^{0100} + \alpha p_{1(3)}z^{0010} + \alpha p_{1(2,3)}z^{0110},$$
$$\hat{U}_2(z) = \theta\{U_2(z)\} = (\beta + \alpha p_{2\phi})z^{0000} + \alpha q_{2\{3\}}z^{0010} + \alpha q_{2(4)}z^{0001} + \alpha q_{2(3,4)}z^{0011},$$
$$\hat{U}_3(z) = \theta\{U_3(z)\} = (\beta + \alpha p_{4\phi})z^{0000} + \alpha p_{4\{4\}}z^{0001}.$$
$$\overline{U}_1(z) = \hat{U}_1(z),$$
$$\varphi(\overline{U}_1(z)) = \alpha p_{1\{2\}}z^{0100} + \alpha p_{1\{3\}}z^{0010}\alpha p_{1\{2,3\}}z^{0110},$$
$$\overline{U}_2(z) = \psi(\varphi(\overline{U}_1(z)),(\hat{U}_2(z)))$$
$$= \Psi(\alpha p_{1\{2\}}z^{0100} + \alpha p_{1\{3\}}z^{0010} + \alpha p_{1\{2,3\}}z^{0110}, (\beta + \alpha q_{2\phi})z^{0000} + \alpha q_{2\{3\}}z^{0010} + \alpha q_{2\{4\}}z^{0001} + \alpha q_{2\{3,4\}}z^{0011})$$
$$= a p_{1\{2\}}(\beta + \alpha q_{2\phi})z^{0100} + \{\alpha p_{1\{3\}}(\beta + \alpha q_{2\phi}) + \alpha^2 p_{1\{3\}}q_{2\{3\}} + \alpha^2 p_{1\{2\}}q_{2\{3\}}\}z^{0010}$$
$$+ \{\alpha p_{1\{2,3\}}(\beta + \alpha q_{2\phi}) + \alpha^2 p_{1\{2,3\}}q_{2\{3\}}\}z^{0110} + \alpha^2 p_{1\{2\}}q_{2\{4\}}z^{0101} + \{\alpha^2 p_{1\{3\}}q_{2\{4\}} + \alpha^2 p_{1\{3\}}q_{2\{3,4\}}\}z^{0011}$$
$$+ \{\alpha^2 p_{1\{2,3\}}q_{2\{4\}} + \alpha^2 p_{1\{2\}}q_{2\{3,4\}} + \alpha^2 p_{1\{2,3\}}q_{2\{3,4\}}\}z^{0111}$$
$$= \alpha p_{1\{2\}}(\beta + \alpha q_{2\phi})z^{0100} + \{\alpha p_{1\{3\}} + \alpha^2 p_{1\{3\}}(q_{2\phi} + q_{2\{3\}}) + \alpha^2 p_{1\{2\}}q_{2\{3\}}\}z^{0010}$$
$$+ \{\alpha\beta p_{1\{2,3\}} + \alpha^2 p_{1\{2,3\}}(q_{2\phi} + q_{2\{3\}})\}z^{0110} + \alpha^2 p_{1\{2\}}q_{2\{4\}}z^{0101} + \{\alpha^2 p_{1\{2\}}(q_{2\{4\}} + q_{2\{3,4\}})\}z^{0011}$$
$$+ \{\alpha^2 p_{1\{2,3\}}(q_{2\{4\}} + q_{2\{3,4\}}) + \alpha^2 p_{1\{2\}}q_{2\{3,4\}}\}z^{0110},$$

$$= \alpha p_{1\{2\}}(\beta + \alpha q_{2\phi})z^{0100} + \{\alpha\beta p_{1\{3\}} + \alpha^2 p_{1\{3\}}(q_{2\phi} + q_{2\{3\}}) + \alpha^2 p_{1\{2\}} p_{2\{3\}} + \alpha\beta p_{1\{2,3\}} + \alpha^2 p_{1\{2,3\}}(q_{2\phi} + q_{2\{3\}})\} z^{0010} + \alpha^2 p_{1\{2\}} q_{2\{4\}} z^{0101} + \{\alpha^2 p_{1\{2,3\}}(q_{2\{4\}} + q_{2\{3,4\}}) + \alpha^2 p_{1\{2\}} q_{2\{3,4\}}\} z^{0011},$$

$$\varphi(\overline{U}_2(z)) = \{\alpha\beta p_{1\{3\}} + \alpha^2 p_{1\{3\}}(q_{2\phi} + q_{2\{3\}}) + \alpha^2 p_{1\{2\}} p_{2\{3\}} + \alpha\beta p_{1\{2,3\}} + \alpha^2 p_{1\{2,3\}}(q_{2\phi} + q_{2\{3\}})\} z^{0010} + \alpha^2 p_{1\{2\}} q_{2\{4\}} z^{0101} + \{\alpha^2 p_{1\{2,3\}}(q_{2\{4\}} + q_{2\{3,4\}}) + \alpha^2 p_{1\{2\}} q_{2\{3,4\}}\} z^{0011}.$$

$$\overline{U}_3(z) = \psi(\varphi(\overline{U}_2(z)), \hat{U}_3(z))$$

$$= \Psi\Big[\{\alpha\beta p_{1\{3\}} + \alpha^2 p_{1\{3\}}(q_{2\phi} + q_{2\{3\}}) + \alpha^2 p_{1\{2\}} p_{2\{3\}} + \alpha\beta p_{1\{2,3\}} + \alpha^2 p_{1\{2,3\}}(q_{2\phi} + q_{2\{3\}})\} z^{0010} + \alpha^2 p_{1\{2\}} q_{2\{4\}} z^{0101} + \{\alpha^2 p_{1\{2,3\}}(q_{2\{4\}} + q_{2\{3,4\}}) + \alpha^2 p_{1\{2\}} q_{2\{3,4\}}\} z^{0011}, (\beta + \alpha p_{4\phi}) z^{0000} + \alpha p_{4\{4\}} z^{0001}\Big]$$

$$= \Big[\{\alpha\beta p_{1\{3\}} + \alpha^2 p_{1\{3\}}(q_{2\phi} + q_{2\{3\}}) + \alpha^2 p_{1\{2\}} p_{2\{3\}} + \alpha\beta p_{1\{2,3\}} + \alpha^2 p_{1\{2,3\}}(q_{2\phi} + q_{2\{3\}})\} (\beta + \alpha p_{4\phi}) z^{0010} + \alpha^2 p_{1\{2\}} q_{2\{4\}}(\beta + \alpha p_{4\phi}) z^{0001} + \{\alpha^2 p_{1\{2,3\}}(q_{2\{4\}} + q_{2\{3,4\}}) + \alpha^2 p_{1\{2\}} q_{2\{3,4\}} + \alpha^3 p_{1\{2\}} q_{2\{4\}} p_{4\{4\}}\} z^{0001} + \{\alpha\beta p_{1\{3\}} + \alpha^2 p_{1\{3\}}(q_{2\phi} + q_{2\{3\}}) + \alpha^2 p_{1\{2\}} p_{2\{3\}} + \alpha\beta p_{1\{2,3\}} + \alpha^2 p_{1\{2,3\}}(q_{2\{4\}} + q_{2\{3,4\}}) + \alpha^2 p_{1\{2\}} q_{2\{3,4\}}\} z^{0011}\Big]$$

$$= \Big[\{\alpha\beta p_{1\{3\}} + \alpha^2 p_{1\{3\}}(q_{2\phi} + q_{2\{3\}}) + \alpha^2 p_{1\{2\}} p_{2\{3\}} + \alpha\beta p_{1\{2,3\}} + \alpha^2 p_{1\{2,3\}}(q_{2\phi} + q_{2\{3\}})\}(\beta + \alpha p_{4\phi})\Big] z^{0010} + \Big[\alpha^2 p_{1\{2\}} q_{2\{4\}}(\beta + \alpha p_{4\phi}) + \alpha^2 p_{1\{2,3\}}(q_{2\{4\}} + q_{2\{3,4\}}) + \alpha^2 p_{1\{2\}} q_{2\{3,4\}} + \alpha^3 p_{1\{2\}} q_{2\{4\}} p_{4\{4\}} + \alpha\beta p_{1\{3\}} + \alpha^2 p_{1\{3\}}(q_{2\phi} + q_{2\{3\}}) + \alpha^2 p_{1\{2\}} p_{2\{3\}} + \alpha\beta p_{1\{2,3\}} \alpha^2 p_{1\{2,3\}}(q_{2\phi} + q_{2\{3\}}) + \alpha^2 p_{1\{2,3\}}(q_{2\{4\}} + q_{2\{3,4\}}) + \alpha^2 p_{1\{2\}} q_{2\{3,4\}}\Big] z^{0001},$$

$$\varphi(\overline{U}_3(z)) = \alpha^2 p_{1(2)} q_{2\{4\}}(\beta + \alpha p_{4\phi}) + \alpha^2 p_{1\{2,3\}}(q_{2\{4\}} + q_{2\{3,4\}}) + \alpha^2 p_{1\{2\}} q_{2\{3,4\}} + \alpha^3 p_{1\{2\}} q_{2\{4\}} p_{4\{4\}} + \alpha\beta p_{1\{3\}} + \alpha^2 p_{1\{3\}}(q_{2\phi} + q_{2\{3\}}) + \alpha^2 p_{1\{2\}} p_{2\{3\}} + \alpha\beta p_{1\{2,3\}} + \alpha^2 p_{1\{2,3\}}(q_{2\phi} + q_{2\{3\}}) + \alpha^2 p_{1\{2,3\}}(q_{2\{4\}} + q_{2\{3,4\}}) + \alpha^2 p_{1\{2\}} q_{2\{3,4\}}.$$

Taking into account than the MAN surviability

$$S = \alpha^2 p_{1\{2\}} q_{2\{4\}}(\beta + \alpha p_{4\phi}) + \alpha^2 p_{1\{2,3\}}(q_{2\{4\}} + q_{2\{3,4\}}) + \alpha^2 p_{1\{2\}} q_{2\{3,4\}} + \alpha^3 p_{1\{2\}} q_{2\{4\}} p_{4\{4\}} + \alpha\beta p_{1\{3\}} + \alpha^2 p_{1\{3\}}(q_{2\phi} + q_{2\{3\}}) + \alpha^2 p_{1\{2\}} p_{2\{3\}} + \alpha\beta p_{1\{2,3\}} + \alpha^2 p_{1\{2,3\}}(q_{2\phi} + q_{2\{3\}}) + \alpha^2 p_{1\{2,3\}}(q_{2\{4\}} + q_{2\{3,4\}}) + \alpha^2 p_{1\{2\}} q_{2\{3,4\}},$$

Since $\beta = 1 - \alpha$.

$$S = \alpha^2 p_{1\{2\}} q_{2\{4\}}((1-\alpha) + \alpha p_{4\phi}) + \alpha^2 p_{1\{2,3\}}(q_{2\{4\}} + q_{2\{3,4\}}) + \alpha^2 p_{1\{2\}} q_{2\{3,4\}} + \alpha^3 p_{1\{2\}} q_{2\{4\}} p_{4\{4\}} + \alpha(1-\alpha) p_{1\{3\}} + \alpha^2 p_{1\{3\}}(q_{2\phi} + q_{2\{3\}}) + \alpha^2 p_{1\{2\}} p_{2\{3\}} + \alpha(1-\alpha) p_{1\{2,3\}} + \alpha^2 p_{1\{2,3\}}(q_{2\phi} + q_{2\{3\}})$$

$$+\alpha^2 p_{1\{2,3\}}\left(q_{2\{4\}}+q_{2\{3,4\}}\right)+\alpha^2 p_{1\{2\}}q_{2\{3,4\}}.$$

3. Conclusion. The short communication suggests an availability and survivability of cable T.V. transmission system which are generalization of the tree-structured multi-state networks and multistate acyclic networks. The method is based on universal generating function technique extended for representing random binary vectors. The resulting polynomial contains 2^{n^*}-1 terms. Therefore, the suggested method can be applied for MAN with moderate values of n^*. The suggested method allows one to obtain probability that the signal generated at the cable T.V. operator's satellite link of a MAN reaches users' televisions.

REFERENCES

[1] A. Lisnianski and G. Levitin, Multi-State System Reliability, New Jersey, 2003.

[2] G. Levtin, Optimal allocation of multi-state retransmitters in acyclic transmission network, *Reliab Engng Syst. Safety,* **75** (2002), 73-82.

[3] G. Levitin, Reliability evaluation for acyclic consecutively connected networks with multi-state elements, *Reliab Engng Syst. Safety,* **73** (2001), 137-143.

[4] G. Livitin, Maximizing survivability of acyclic transmission networks with multi-state retransmitters and vulnerable nodes, *Reliab Engng Syst. Safety,* **77** (2002), 189-199.

[5] J. Malinowski and W. Preuss, Reliability evaluation for tree-structured systems with multistate components, *Microelectron Reliab,* **36** (1996), 9-17.

[6] R.S. Gaur and S. Chaudhary, A study of an algorithm for reliability of ACCN, *Proceedings of National Conference on Modern Mathematics,* Agra, (2003).

[7] R.S. Gaur and S. Chaudhary, Reliability evaluation of acyclic multi-state transmission networks (AMTNs) using the universal generating function technique, *IEEE on Reliab Trans.* (under communication)

[8] R.S. Gaur and S. Chaudhary, Reliability enhancement and fixed transmission times of acyclic transmission multi-state networks (ATMNs). Proceedings of the *Ramanujan Mathematical Society,* Agra, 2004.