Jñānābha, Vol. 34, 2004

SOME COMMON FIXED POINT THEOREMS IN FUZZY 3-METRIC SPACES

By

R.S. Chandel
Department of Mathematics
Gitanjali College, Bhopal-462003, Madhya Pradesh, India
email : rsschandel1@yahoo.com
Sushil Sharma
Department of Mathematics, Madhav Scinece College, Vikram University,
Ujjain-456010, Madhya Pradesh, India
email : sksharma2005@yahoo.com
and
Rashmi Tiwari
Department of Mathematics
Government Postgraduate College
Pipariya (Hoshangabad), Madhya Pradesh, India

(Received : December 26, 2004)

ABSTRACT

The aim of this paper is to prove some common fixed point theorems in fuzzy 3-metric spaces by removing the assuption of continuity, relaxing the compatibility or compatibility of type (α) or compatibility of type (β), to weak compatibility and replacing the completeness of the space with a set of alternative condition.

Keywords . Fuzzy metric spaces, coincidence point, common fixed point, compatible maps, weakly compatible maps.
2000 AMS Subject Code. 47H10, 54H25.

1. Introduction and Preliminaries . The concept of fuzzy sets was introduced initially by Zadeh [39] in 1965. Since then, to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and applications. Especially, Deng [5], Erceg [6], Kaleva and Seikkala [20], Kramosil and Michalek [22] have introduced the concept of fuzzy metric spaces in different ways. Recently many authors have also studied the fixed point theory in these fuzzy metric spaces ([1], [2], [7], [9], [10], [18],[19],[22], [32],[33],[35]). Mishra et al. [23] proved common fixed point theorems on complete fuzzy metric spaces, which generalized, extended and fuzzified several known fixed point theorems for contractive type maps on metric and other spaces. They assumed continuity of one map in each of the two pairs of compatible maps and also the commutativity of continuous maps. Cho [4] and Jung et al. [18] extended and generalized several fixed point theorems on metric spaces, Menger probabilistic metric spaces, uniform spaces and proved common fixed point theorems on complete fuzzy metric spaces. The result of Cho [4] was extended by Sharma [34] and Sharma and Deshpande [35].

Jung et al. [18], Hadzic [10], Jungck et al. [16] and Singh et al. [31] have proved common fixed point theorems for mappings under the condition of continuity and compatibility of type (α) in complete fuzzy metric, probabilistic metric and metric spaces.

In this paper we remove the assumption of continuity, relaxing compatibility or compatibility of type (α) or compatibility of type (β) to weak compatibility and replacing the completenss of space with set of alternative conditions. We also remove the assumption of commutativity of continuous maps in case of two pairs of maps. We extend the results of Sharma and Deshpande [36].

Now we begin with some definitions.

Definition 1. (Sharma [33]) A binary operation$*:[0,1]^4\to[0,1]$ is called a continuous t-norm if $([0,1],*)$ is an abelian topological monoid with unit 1 such that $a_1*b_1*c_1*d_1\le a_2*b_2*c_2*d_2$ whenever $a_1\le a_2$, $b_1\le b_2, c_1\le c_2$ and $d_1\le d_2$ for all a_1,a_2,b_1,b_2, c_1, c_2, and d_1, d_2 are in [0,1].

Definition 2. The 3-tuple $(X,M,*)$ is called a fuzzy 3-metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set in $X^4\times[0,\infty)$ satisfying the following conditions for all $x,y,z,u,w\in X$ and $t_1,t_2,t_3,t_4>0$:

(FM-1) $M(x,y,z,w,0)=0$,

(FM-2) $M(x,y,z,w,t)=1$, for all $t>0$,

(Only when the three simplex $\langle x,y,z,w\rangle$ degenerate)

(FM-3) $M(x,y,z,w,t)=M(x,w,z,y,t)=M(y,z,w,x,t)=M(z,w,x,y,t)=\dots$

(FM-4) $M(x,y,z,w,t_1+t_2+t_3+t_4)\ge M(x,y,z,u,t_1)*M(x,y,u,w,t_2)$
$*M(x,u,z,w,t_3)*M(u,y,z,w,t_4)$

(FM-5) $M(x,y,z,w,.):[0,1)\to[0,1]$ is left continuous.

Definition 3. Let $(X,M,*)$ be a fuzzy 3-metric space:

(1) A sequence $\{x_n\}$ in fuzzy 3-metric space X is said to be convergent to a point $x\in X$, if

$$\lim_{n\to\infty} M(x_n,x,a,b,t)=1$$

for all $a,b\in X$ and $t>0$.

(2) A sequence $\{x_n\}$ in fuzzy 3-metric space X is called a Cauchy sequence, if

$$\lim_{n\to\infty} M(x_{n+p},x_n,a,b,t)=1$$

for all $a,b \in X$ and $t>0, p>0$.

(3) A fuzzy 3-metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 4. A pair of mappings A and S is said to be weakly compatible in fuzzy 3-metric space if they commute at coincidence points.

Example 1. Let (X,d) be 3-metric space define $a*b=ab$ or $a*b=min\{a,b\}$ and for all $x,y,a,b \in X$ and $t>0$

$$M(x,y,b,t)=t/(t+d(x,y,a,b)). \tag{1.b}$$

Then $(X,M,*)$ is fuzzy 3-metric space. We call the fuzzy metric M induced by the metric d the standard fuzzy metric.

Remark (Sharma [33]) Since * is continuous, it follows from (*FM-4*) that the limit of the sequence is fuzzy 3-metric space is uniquely determined. *Let*$(X,M,*)$ is a fuzzy 3-metric space with the following condition :

(*FM-6*) $\lim_{t\to\infty} M(x,y,z,w,t)=1$ for all $x,y,z,w \in X$.

Example 2. Define $A,S : [0,3]\to[0,3]$ by

$$A(x)\begin{bmatrix} x \text{ if } x\in[0,1) \\ x \text{ if } x\in[1,3] \end{bmatrix} \text{ and } S(x)=\begin{bmatrix} 3-x & if & x\in[0,1) \\ 3 & if & x\in[1,3) \end{bmatrix}$$

Then for any $x\in[1,3]$, $ASx=SAx$, showing that A,S are weakly compatible maps on $[0,3]$.

Example 3. *Let* $X=[0,2]$ with the metric d defined by $d(x,y) = |x-y|$. For each $t\in(0,\infty)$ define

$$M(x,y,t)=t/(t+d(x,y)), M(x,y,0)=0, \; x,y\in X.$$

Clearly $M(X,M,*)$ is a fuzzy metric space on X where * is defined by $a*b=ab$ or $a*b=min\,\{a,b\}$. Define $A,B : X\to X$ by $Ax=x$ if $x\in[0,1/3)$, $Ax=1/3$ if $x\geq 1/3$ and $Bx=x/(1+x), x\in[0,2]$.

Consider the sequence $\{x_n=1/2+1/n;\ n\geq 1\}$ in X. Then

$Lim_{n\to\infty} Ax_n=1/3$ and $Lim_{n\to\infty} Bx_n=1/3$. But

$Lim_{n\to\infty} M(ABx_n,BAx_n,t)=t/(t+|1/3\text{-}1/4|)\neq 1$.

Thus A and B are non compatible, but A and B are commuting at their coincidence point $x=0$, that is, weakly compatible at $x=0$. Also

$Lim_{n\to\infty} M(ABx_n,BBx_n,t)=t/(t+|1/3\text{-}1/4|)\neq 1$.

and

$Lim_{n\to\infty} M(BAx_n,AAx_n,t)=t/(t+|1/4\text{-}1/3|)\neq 1$.

Further

$Lim_{n\to\infty} M(AAx_n,BBx_n,t)=t/(t+|1/3\text{-}1/4|)\neq 1$.

Thus A and B are not compatible of type (β).

In view of this example, we observe that

(i) Weakly compatible maps need not to compatible.

(ii) Weakly compatible maps need not be compatible of type (α).

(iii) Weakly compatible maps need not be compatible of type (β).

Lemma 1 : (Sharma [33]). For all $x,y,z,w \in X$, $M(x,y,z,w,.)$ is non decreasing.

Lemma 2. (Sharma [33]). Let $\{y_n\}$ be sequence in a fuzzy 3-metric space $(X,M,*)$ with the condition (*FM-6*). If there exists a number $k \in (0,1)$ such that

$$M(y_{n+2},y_{n+1},a,b,kt) \geq M(y_{n+1},y_n,a,b,t)$$

for all $t>0$ and $a,b \in X$ and $n=1,2,...$ then $\{y_n\}$ is a Cauchy sequence in X.

Lemma 3. (Sharma [33]). If for all $x,y,a,b \in X$, $t>0$ and for a number $k \in (0,1)$,

$$M(x,y,a,b,kt) \geq M(x,y,a,b,t), \text{ then } x=y.$$

2. Main results. Sharma and Deshpande [36] proved the following

Theorem A. Let (X,M^*) be a fuzzy metric space with $t*t \geq t$ for all $t \in [0,1]$ and the condition (*FM-6*). Let A, B, S, T be mappings from X into itself such that

(1) $A(X) \subset T(X)$, $B(X) \subset S(X)$,

(2) there exists a constant $k \in (0,1)$ such that

$$M(Ax,By,kt) \geq M(Ty,By,t)*M(Sx,Ax,t)*M(Sx,By,\alpha t)*M(Ty,Ax,(2-\alpha)t)*M(Ty,Sx,t)$$

for all $x,y \in X$, $\alpha \in (0,2)$ and $t>0$.

(3) One of $A(X)$, $B(X)$, $S(X)$ or $T(X)$ is a complete subspace of X, then

(i) A and S have a coincidence point

(ii) B and T have a coincidence point

Further if

(4) the pairs $\{A,S\}$ and $\{B,T\}$ are weakly compatible, then

(iii) A,B,S and T have a unique fixed point in X.

For fuzzy 2-metric space, Sharma and Tiwari [37] proved

Theorem B. Let $(X,M,*)$ be a fuzzy 2-metric space with $t*t \geq t$ for all $t \in [0,1]$ and the condition (*FM-6*). Let A, B, S and T be mappings from X into itself such that

(1) $A(X) \subset T(X)$, $B(X) \subset S(X)$,

(2) there exists a constant $k \in (0,1)$ such that

$$M(Ax,By,a,kt) \geq M(Ty,By,a,t)*M(Sx,Ax,a,t)*M(Sx,By,a,\alpha t)$$
$$*M(Ty,Ax,a,(2-\alpha)t)*M(Ty,Sx,a,t)$$

for all $x,y,a \in X$, $\alpha \in (0,2)$ and $t>0$.

(3) One of $A(X)$, $B(X)$, $S(X)$ or $T(X)$ is a complete subspace of X, then

(i) A and S have a coincidence point

(ii) B and T have a coincidence point

Further if

(4) the pairs $\{A,S\}$ and $\{B,T\}$ are weakly compatible, then

(iii) A,B,S and T have a unique fixed point in X.

Theorem 1. Let $(X,M,*)$ be a fuzzy 3-metric space with $t*t \geq t$ for all $t \in [0,1]$ and the condition (*FM-6*). Let *A, B, S* and *T* be mappings from *X* into itself satisying the conditions (1),(3) and

(2.1) there exists a constant $k \in (0,1)$ such that

$$M(Ax,By,a,b,kt) \geq M(Ty,By,a,b,t)*M(Sx,Ax,a,b,t)*M(Sx,By,a,b,\alpha t)$$
$$*M(Ty,Ax,a,b,(2-\alpha)t)*M(Ty,Sx,a,b,t)$$

for all $x,y,a,b \in X$, $\alpha \in (0,2)$ and $t>0$.

(i) *A* and *S* have a coincidence point

(ii) *B* and *T* have a coincidence point

Further if

(2.2) the pairs $\{A,S\}$ and $\{B,T\}$ are weakly compatible, then

(iii) *A,B,S* and *T* have a unique fixed point in *X*.

Proof. By (1), since $A(X) \subset T(X)$, so for any arbitrary $x_0 \in X$, there exists a point $x_1 \in X$ such that $Ax_0 = Tx_1$. Since $B(X) \subset S(X)$, for the point x_1 we can choose a point $x_2 \in X$ such that $Bx_1 = Sx_2$ and so on. Inductively we can define a sequence $\{y_n\}$ in *X* such that

$$y_{2n} = Tx_{2n+1} = Ax_{2n} \text{ and } y_{2n+1} = Sx_{2n+2} = Bx_{2n+1} \quad \text{for } n=0,1,2,...$$

By (2.1) for all $t>0$ and $\alpha = 1-q$ with $q \in (0,1)$, we have

$$M(Ax_{2n+2},Bx_{2n+1},a,b,kt)$$
$$\geq M(Tx_{2n+1},Bx_{2n+1},a,b,t) *M(Sx_{2n+2},Ax_{2n+2},a,b,t)$$
$$*M(Sx_{2n+2},Bx_{2n+1},a,b,\alpha t)*M(Tx_{2n+1},Ax_{2n+2},a,b,(2-\alpha)t)$$
$$*M(Tx_{2n+1},Sx_{2n+2},a,b,t)$$

$$M(y_{2n+2},y_{2n+1},a,b,kt) \geq M(y_{2n},y_{2n+1},a,b,t)*M(y_{2n+1},y_{2n+2},a,b,t)$$
$$*M(y_{2n+1},y_{2n+1},a,b,\alpha t)*M(y_{2n},y_{2n+2},a,b,(2-\alpha)t)$$
$$*M(y_{2n},y_{2n+1},a,b,t)$$
$$\geq M(y_{2n},y_{2n+1},a,b,t)*M(y_{2n+1},y_{2n+2},a,b,t)*1$$
$$*M(y_{2n},y_{2n+2},a,b(1+q)t)*M(y_{2n},y_{2n+1},a,b,t)$$
$$\geq M(y_{2n},y_{2n+1},a,b,t)*M(y_{2n+1},y_{2n+2},a,b,t)*1$$
$$*M(y_{2n},y_{2n+2},a,b(1+q)t)*M(y_{2n},y_{2n+1},a,b,t)$$
$$\geq M(y_{2n},y_{2n+1},a,b,t)*M(y_{2n+1},y_{2n+2},a,b,t)*M(y_{2n},y_{2n+2},a,b(1+q)t)$$
$$\geq M(y_{2n},y_{2n+1},a,b,t)*M(y_{2n+1},y_{2n+2},a,b,t)*M(y_{2n},y_{2n+2},a,b,tq+t/3+t/3+t/3)$$

(Following Sharma [33] and using (*FM-4*))

$$\geq M(y_{2n},y_{2n+1},a,b,t)*M(y_{2n+1},y_{2n+2},a,b,t)*M(y_{2n},y_{2n+2},a,y_{2n+1},qt)$$
$$*M(y_{2n},y_{2n+2},y_{2n+1},b,t/3)*M(y_{2n},y_{2n+1},a,b,t/3)*M(y_{2n+1},y_{2n+2},a,b,t/3)$$
$$\geq M(y_{2n},y_{2n+1},a,b,t)*M(y_{2n+1},y_{2n+2},a,b,t)*M(y_0,y_2,a,y_1,t/4q^{2n})$$
$$*M(y_0,y_2,y_1,y_1,t/4q^{2n})*M(y_0,y_1,a,y_1,t/4q^{2n})*M(y_1,y_2,a,y_1,t/4q^{2n})$$
$$*M(y_0,y_2,y_1,b,t/3q^{2n})*M(y_{2n},y_{2n+1},a,b,t/3)*M(y_{2n+1},y_{2n+2},a,b,t/3).$$

Thus since $M(y_0,y_2,y_1,b,t/3q^{2n}) \to 1$ and $M(y_0,y_2,a,y_1,t/4q^{2n}) \to 1$ as $n \to \infty$.

Thus we have

(2.3) $M(y_{2n+1},y_{2n+2},a,b,kt) \geq M(y_{2n},y_{2n+1},a,b,t)*M(y_{2n+1},y_{2n+2},a,b,t)$.

Similarly we have

(2.4) $M(y_{2n+2},y_{2n+3},a,b,kt)\geq M(y_{2n+1},y_{2n+2},a,b,t)*M(y_{2n+2},y_{2n+3},a,b,t)$.

From (2.3) and (2.4) it follows that

$M(y_{n+1},y_{n+2},a,b,kt)\geq M(y_n,y_{n+1},a,b,t)*M(y_{n+1},y_{n+2},a,b,t)$

for $n=1,2,....$ and also for positive integer n,p

$M(y_{n+1},y_{n+2},a,b,kt)\geq M(y_n,y_{n+1},a,b,t)*M(y_{n+1},y_{n+2},a,b,t/k^P)$.

Thus since $M(y_{n+1},y_{n+2},a,b,t/k^P)\to 1$ as $p\to\infty$ we have

$$M(y_{n+1},y_{n+2},a,b,kt)\geq M(y_n,y_{n+1},a,b,t).$$

So by Lemma 2, $\{y_n\}$ is a Cauchy sequence in X.

Now suppose $S(X)$ is complete. Note that the subsequence $\{y_{2n+1}\}$ is contained in $S(X)$ and has a limit in $S(X)$, call it z. Let $u=S^{-1}z$, thus $Su=z$. We shall use the fact that the subsequence $\{y_{2n}\}$ also converges to z.

By (2.2) with $\alpha=1$, we have

$$\begin{aligned}M(Au,y_{2n+1},a,b,kt)&=M(Au,Bx_{2n+1},a,b,kt)\\&\geq M(Tx_{2n+1},Bx_{2n+1},a,b,t)*M(Su,Au,a,b,t)*M(Su,Bx_{2n+1},a,b,t)\\&\quad *M(Tx_{2n+1},Au,a,b,t)*M(Tx_{2n+1},Su,a,b,t)\\&=M(y_{2n},y_{2n+1},a,b,t)*M(Su,Au,a,b,t)*M(Su,y_{2n+1},a,b,t)*M(y_{2n},Au,a,b,t)\\&\quad *M(y_{2n},Su,a,b,t),\end{aligned}$$

which implies that as $n\to\infty$,

$$\begin{aligned}M(Au,z,a,b,kt)&\geq 1*M(z,Au,a,b,t)*1*M(z,Au,a,b,t)*1\\&\geq M(Au,z,a,b,t).\end{aligned}$$

Therefore by Lemma 3, we have $Au=z$. Thus $Au=z=Su$ i.e. u is a coincidence point of A and S. This proves (i).

Since $A(X)\subset T(X)$, $Au=z$ implies that $z\in T(X)$. Let $v=T^{-1}z$, then $Tv=z$. It can be easily verified by using similar arguments of the previous part of the proof that $Bv=z$. Thus $Bv=z=Tv$, i.e. v is a coincidence point of B and T. This proves (ii).

If we assume that $T(X)$ is complete, then argument analogous to the previous completeness argument estabilishes (i) and (ii). The remaining two cases pertain essentially to the previous cases. Indeed if $B(X)$ is complete, then by (1), $z\in B(X)\subset S(X)$. Similarly if $A(X)$ is complete, then $z\in A(X)\subset T(X)$. Thus (i) and (ii) are completely established.

Since the par $\{A,S\}$ is weakly compatible therefore A and S commute at their coincidence point i.e. $ASu=SAu$ or $Az=Sz$. Similarly

$BTv=TBv$ or $Bz=Tz$.

Now we prove $Az=z$ by (2.1) with $\alpha=1$, we have

$$\begin{aligned}&M(Az,y_{2n+1},a,b,kt)=M(Az,Bx_{2n+1},a,b,kt)\\&\geq M(Tx_{2n+1},Bx_{2n+1},a,b,t)*M(Sz,Az,a,b,t)*M(Sz,Bx_{2n+1},a,b,t)*M(Tx_{2n+1},Az,a,b,t)\\&\quad *M(Tx_{2n+1},Sz,a,b,t)\\&=M(y_{2n},y_{2n+1},a,b,t)*M(Az,Az,a,b,t)*M(Az,y_{2n+1},a,b,t)*M(y_{2n},Az,a,b,t)*M(y_{2n},Az,a,b,t)\end{aligned}$$

Taking limit as $n \to \infty$, we have

$M(Az,z,a,b,kt) \geq 1*1*M(Az,z,a,b,t)*M(z,Az,a,b,t)*M(z,Az,a,b,t)$

$\geq M(Az,z,a,b,t).$

Therefore by Lemma 3, we have $Az=z$. Thus $Az=z=Sz$.

Similarly we have $Bz=z=Tz$. This means that z is a common fixed point of A,B,S and T.

For uniqueness of common fixed point let $w \neq z$ be another common fixed point of A,B,S and T. Then by (2.1) with $\alpha=1$, we have

$M(z,w,a,b,kt) = M(Az,Bw,a,b,kt)$

$\geq M(Tw,Bw,a,b,t)*M(Sz,Az,a,b,t)*M(Sz,Bw,a,b,t)*M(Tw,Az,a,b,t)*M(Tw,Sz,a,b,t)$

$\geq 1*1*M(z,w,a,b,t)*M(w,z,a,b,t)*M(w,z,a,b,t)$

$\geq M(z,w,a,b,t).$

Therefore by Lemma 1.3, we have $z=w$. This completes the proof.

Theorem 2. Let $(X,M,*)$ be a fuzzy 3-metric space with $t*t \geq t$ for all $t \in [0,1]$ and the condition (*FM-6*). Let A,B,S,T and P be mappings from X into itself such that

(2.5) $P(X) \subset AB(X)$, $P(X) \subset ST(X)$,

(2.6) there exists a constant $k \in (0,1)$ such that

$$M(Px,Py,a,b,kt) \geq M(ABy,Py,a,b,t)*M(STx,Px,a,b,t)*M(STx,Py,a,b,\alpha t)$$
$$*M(ABy,Px,a,b,(2\text{-}\alpha)t)*M(ABy,STx,a,b,t)$$

for all $x,y,a,b \in X$, $\alpha \in (0,2)$ and $t>0$.

(2.7) If one of $P(X)$, $AB(X)$ or $ST(X)$ is a complete subspace of X, then

(i) P and AB have a coincidence point

(ii) P and ST have a coincidence point

Further if

(2.8) $PB=BP$; $AB=BA$; $PT=TP$ and $ST=TS$,

(2.9) the pairs $\{P,AB\}$ and $\{P,ST\}$ are weakly compatible then

(iii) A,B,S,T and P have a unique common fixed point in X.

Proof. By (2.5) since $P(X) \subset AB(X)$, for any point $x_0 \in X$, there exists a point $x_1 \in X$ such that $Px_0=ABx_1$. Since $P(X) \subset ST(X)$, for the point x_1 we can choose a point $x_2 \in X$ such that $Px_1=Sx_2$ and so on. Inductively we can define a sequence $\{y_n\}$ in X such that

$y_{2n}=Px_{2n}=ABx_{2n+1}$ and $y_{2n+1}=Px_{2n+1}=STx_{2n+2}$ for $n=0,1,2,...$

By (2.6) for all $t>0$ and $\alpha=1-q$ with $q \in (0,1)$, we have

$$M(Px_{2n+2},Px_{2n+1},a,b,kt)$$
$$\geq M(ABx_{2n+1},Px_{2n+1},a,b,t)*M(STx_{2n+2},Px_{2n+2},a,b,t)$$
$$*M(STx_{2n+2},Px_{2n+1},a,b,\alpha t)*M(ABx_{2n+1},Px_{2n+2},a,b,(2-\alpha)t)$$
$$*M(ABx_{2n+1},STx_{2n+2},a,b,t),$$

$$M(y_{2n+2},y_{2n+1},a,b,kt)$$
$$\geq M(y_{2n},y_{2n+1},a,b,t)*M(y_{2n+1},y_{2n+2},a,b,t)*M(y_{2n+1},y_{2n+1},a,b,\alpha t)$$

$$*M(y_{2n},y_{2n+2},a,b,(1+q)t)*M(y_{2n},y_{2n+1},a,b,t)$$
$$\geq M(y_{2n},y_{2n+1},a,b,t)*M(y_{2n+1},y_{2n+2},a,b,t)*1*M(y_{2n},y_{2n+2},a,b,(1+q)t)$$
$$*M(y_{2n},y_{2n+1},a,b,t)$$
$$\geq M(y_{2n},y_{2n+1},a,b,t)*M(y_{2n+1},y_{2n+2},a,b,t)*M(y_{2n},y_{2n+2},a,b(1+q)t) .$$
$$\geq M(y_{2n},y_{2n+1},a,b,t)*M(y_{2n+1},y_{2n+2},a,b,t)*M(y_{2n},y_{2n+2},a,b,tq+t/3+t/3+t/3)$$

(Following Sharma [33] and using (FM-4))

$$\geq M(y_{2n},y_{2n+1},a,b,t)*M(y_{2n+1},y_{2n+2},a,b,t)*M(y_{2n},y_{2n+2},a,y_{2n+2},qt)$$
$$*M(y_{2n},y_{2n+2},y_{2n+1},b,t/3)*M(y_{2n},y_{2n+1},a,b,t/3)*M(y_{2n+1},y_{2n+2},a,b,t/3)$$
$$\geq M(y_{2n},y_{2n+1},a,b,t)*M(y_{2n+1},y_{2n+2},a,b,t)*M(y_0,y_2,a,y_1,t/4q^{2n})$$
$$*M(y_0,y_2,y_1,y_1,t/4q^{2n})*M(y_0,y_1,a,y_1,t/4q^{2n})*M(y_1,y_2,a,y_1,t/4q^{2n})$$
$$*M(y_0, y_2, y_1, b, t/3, q^{2n})*M(y_{2n},y_{2n+1},a,b,t/3)*M(y_{2n+1},y_{2n+2},a,b,t/3).$$

Thus since $M(y_0,y_2,y_1,b,t/3q^{2n})\to 1$ and $M(y_0,y_2,a,y_1,t/4q^{2n})\to 1$ as $n\to\infty$.

Thus we have

(2.10) $M(y_{2n+1},y_{2n+2},a,b,kt)\geq M(y_{2n},y_{2n+1},a,b,t)*M(y_{2n+1},y_{2n+2},a,b,t)$.

Similarly we have

(2.11) $M(y_{2n+2},y_{2n+3},a,b,kt)\geq M(y_{2n+1},y_{2n+2},a,b,t)*M(y_{2n+2},y_{2n+3},a,b,t)$.

From (2.10) and (2.11) it follows that

$M(y_{n+1},y_{n+2},a,b,kt)\geq M(y_n,y_{n+1},a,b,t)*M(y_{n+1},y_{n+2},a,b,t)$

for $n=1,2,....$ and also for positive integer n,p

$M(y_{n+1},y_{n+2},a,b,kt)\geq M(y_n,y_{n+1},a,b,t)*M(y_{n+1},y_{n+2},a,b,t/k^P)$.

Thus since $M(y_{n+1},y_{n+2},a,b,t/k^P)\to 1$ as $p\to\infty$ we have

$$M(y_{n+1},y_{n+2},a,b,kt)\geq M(y_n,y_{n+1},a,b,t) .$$

So by Lemma 2, $\{y_n\}$ is a Cauchy sequence in X.

Now suppose $ST(X)$ is complete. Note that the subsequence $\{y_{2n+1}\}$ is contained in $ST(X)$ and has a limit in $ST(X)$, call it z.

Let $u=(ST)^{-1}z$, thus $STu=z$. We shall use the fact that the subsequence $\{y_{2n}\}$ also converges to z.

By (2.6) with $\alpha=1$, we have

$$M(Pu,y_{2n+1},a,b,kt)=M(Pu,Px_{2n+1},a,b,kt)$$
$$\geq M(ABx_{2n+1},Px_{2n+1},a,b,t)*M(STu,Pu,a,b,t)*M(STu,Px_{2n+1},a,b,t)$$
$$*M(ABx_{2n+1},Pu,a,b,t) *M(ABx_{2n+1},STu, a,b,t)$$
$$=M(y_{2n},y_{2n+1},a,b,t)*M(z,Pu,a,b,t)*M(z,y_{2n+1},a,b,t)*M(y_{2n},Pu,a,b,t)$$
$$*M(y_{2n},z,a,b,t),$$

which implies that as $n\to\infty$

$$M(Pu,z,a,b,kt) \geq 1*M(z,Pu,a,b,t)*1*M(z,Pu,a,b,t)*1$$
$$\geq M(Pu,z,a,b,t) .$$

Therefore by Lemma 3, we have $Pu=z$. Since $STu=z$, thus $Pu=z=STu$ i.e. u is coincidence point of P and ST. This proves (i).

Since $P(X)\subset AB(X)$, $Pu=z$ implies that $z\in AB(X)$.

Let $v=(AB)^{-1}z$, then $ABv=z$. It can be easily be verified by using similar argument of the previouis part of the proof that $Pv=z$. If we assume that $AB(X)$ is complete then argument analogous to the previous completeness argument estabilishes (i) and (ii).

The remaining one case pertain essentially to the previous cases. Indeed if $P(X)$ is complete, then by (2.5), $z\in P(X)\subset ST(X)$ or $z\in P(X)\subset AB(X)$. Thus (i) and (ii) are completely established. Since the pair $\{P,ST\}$ is weakly compatible therefore P and ST commute at their coincidence point i.e. $P(STu)=(ST)Pu$ or $Pz=STz$. Similarly $P(ABv)=(AB)Pv$ or $Pz=ABz$.

Now we prove that $Pz=z$ by (2.6) with $\alpha=1$, we have

$$\begin{aligned}M(Pz,y_{2n+1},a,b,kt) &= M(Pz,Px_{2n+1},a,b,kt)\\ &\geq M(ABx_{2n+1},Px_{2n+1},a,b,t)*M(STz,Pz,a,b,t)*M(STz,\ Px_{2n+1},a,b,t)\\ &\quad *M(ABx_{2n+1},Pz,a,b,t)*M(ABx_{2n+1},STz,a,b,t)\\ &= M(y_{2n},y_{2n+1},a,b,t)*M(Pz,Pz,a,b,t)*M(Pz,y_{2n+1},a,b,t)*M(y_{2n},Pz,a,b,t)\\ &\quad *M(y_{2n},Pz,a,b,t).\end{aligned}$$

Taking limit as $n\to\infty$, we have

$$\begin{aligned}M(Pz,z,a,b,kt) &\geq 1*1*M(Pz,z,a,b,t)*M(z,Pz,a,b,t)*M(z,Pz,a,b,t)\\ &\geq M(Pz,z,a,b,t)\ .\end{aligned}$$

Therefore by Lemma 3, we have $Pz=z$. Thus $ABz=z=Pz=STz$.

Now we shall show that $Bz=z$. in fact by (2.6) with $\alpha=1$ and (2.8), we have

$$\begin{aligned}M(z,Bz,a,b,kt) &= M(Pz,BPz,a,b,kt)=M(Pz,PBz,a,b,kt)\\ &\geq M(AB(Bz),PBz,a,b,t)*M(STz,Pz,a,b,t)*M(STz,PBz,a,b,t)*M(AB(Bz),Pz,a,b,t)\\ &\quad *M(AB(Bz),STz,a,b,t)\\ &= 1*1*M(z,Bz,a,b,t)*M(Bz,z,a,b,t)*M(Bz,z,a,b,t)\\ &\geq M(z,Bz,a,b,t),\end{aligned}$$

which implies by Lemma 3, that $Bz=z$. Since $ABz=z$, therefore $Az=z$. Finally we know that $Tz=z$. Indeed by (2.6) with $\alpha=1$ and (2.8)

$$\begin{aligned}M(Tz,z,a,b,kt) &= M(TPz,Pz,a,b,kt)=M(PTz,Pz,a,b,kt)\\ &\geq M(ABz,Pz,a,b,t)*M(ST(Tz),PTz,a,b,t)*M(ST(Tz),Pz,a,b,t)*M(ABz,P(Tz),a,b,t)\\ &\quad *M(ABz,ST(Tz),a,b,t)\\ &= 1*1*M(Tz,z,a,b,t)*M(z,Tz,a,b,t)*M(z,Tz,a,b,t)\\ &\geq M(tz,z,a,b,t),\end{aligned}$$

which implies by Lemma 3, that $Tz=z$. Since $STz=z$, we have $z=Stz=Sz$. Therefore by combining the above results, we have $Az=Bz=Sz=Tz=Pz=z$, that is z is a common fixed point of A,B,S,T and P.

For uniqueness of common fixed point let $w\neq z$ be another common fixed point of A,B,S,T, and P.

Then by (2.6) with $\alpha=1$, we have

$$M(z,w,a,b,kt)\geq M(ABw,Pw,a,b,t)\ *M(STz,Pz,a,b,t)\ *M(STz,Pw,a,b,\alpha t)$$

$*M(ABw,Pz,a,b,t)\ *M(ABw,STz,a,b,t)$
$\geq 1*1*M(z,w,a,b,t)*M(w,z,a,b,t)*M(w,z,a,b,t)$
$\geq M(z,w,a,b,t).$

Therefore by Lemma 3, we have $z=w$. This completes the proof.

Theorem 3. Let $(X,M,*)$ be a fuzzy 3-metric space with $t*t\geq t$ for all $t\in[0,1]$ and the condition (*FM-6*). Let A,B,S,T,P and Q be mappings from X into itself such that

(2.12) $P(X)\subset AB(X)$, $Q(X)\subset ST(X)$,

(2.13) there exists a constant $k\in(0,1)$ such that

$$M(Px,Qy,a,b,kt) \geq M(ABy,Qy,a,b,t)*M(STx,Px,a,b,t)*M(STx,Qy,a,b,\alpha t)$$
$$*M(ABy,Px,a,b,(2-\alpha)t)*M(ABy,STx,a,b,t)$$

for all $x,y,a,b\in X$, $\alpha\in(0,2)$ and $t>0$.

(2.14) If one of $P(X)$, $Q(X)$ $AB(X)$ or $ST(X)$ is a complete subspace of X, then

(i) P and ST have a coincidence point

(ii) Q and AB have a coincidence point

Further if

(2.15) $AB=BA$, $QB=BQ$, $PT=TP$ and $ST=TS$,

(2.16) the pairs $\{Q,AB\}$ and $\{P,ST\}$ are weakly compatible then

(iii) A,B,S,T,P and Q have a unique common fixed point in X.

Proof. By (2.12) since $P(X)\subset AB(X)$, for any point $x_0\in X$, there exists a point $x_1\in X$ such that $Px_0=ABx_1$. Since $Q(X)\subset ST(X)$, for the point x_1 we can choose a point $x_2\in X$ such that $Qx_1=STx_2$ and so on. Inductively, we can define a sequence $\{y_n\}$ in X such that

$y_{2n}=Px_{2n}=ABx_{2n+1}$ and $y_{2n+1}=Qx_{2n+1}=STx_{2n+2}$ for $n=0,1,2,...$

for all $t>0$ and $\alpha=1-q$ with $q\in(0,1)$. As proved in Theorem 1 and Theorem 2, we can prove that $\{y_n\}$ is a Cauchy sequence in X. Now suppose $ST(X)$ is complete. Note that the subsequence $\{y_{2n+1}\}$ is contained in $ST(X)$ and has a limit in $ST(X)$, call it z. Let $u=(ST)^{-1}z$. then $STu=z$ we shall use the fact that the subsequence $\{y_{2n}\}$ also converges to z.

By (2.13) with $\alpha=1$, we have

$$M(Pu,Qx_{2n+1},a,b,kt)\geq M(ABx_{2n+1},Qx_{2n+1},a,b,t)*M(STu,Pu,a,b,t)$$
$$*M(STu,Qx_{2n+1},a,b,t)*M(ABx_{2n+1},Pu,a,b,t)\ *M(ABx_{2n+1},STu,\ a,b,t)$$
$$=M(y_{2n},y_{2n+1},a,b,t)*M(STu,Pu,a,b,t)*M(STu,y_{2n+1},a,b,t)*M(y_{2n},Pu,a,b,t)*M(y_{2n},STu,a,b,t),$$

which implies that as $n\to\infty$

$M(Pu,z,a,b,kt)\geq M(Pu,z,a,b,t).$

Therefore by Lemma 3, we have $Pu=z$. Since $STu=z$, thus $Pu=z=STu$ i.e. u is a coincidence point of P and ST. This proves (i). Since $P(X)\subset AB(X)$, $Pu=z$ implies that $z\in AB(X)$. Let $v=(AB)^{-1}z$, then $ABv=z$.

By (2.13) with $\alpha=1$, we have

$M(Pz,y_{2n+1},a,b,kt) = M(Pz,Qx_{2n+1},a,b,kt)$
$\geq M(ABx_{2n+1},Qx_{2n+1},a,b,t)*M(STz,Pz,a,b,t)*M(STz,Qx_{2n+1},a,b,t)$
$\quad *M(ABx_{2n+1},Pz,a,b,t)*M(ABx_{2n+1},STz,a,b,t)$
$\geq M(y_{2n},y_{2n+1},a,b,t)*M(STz,Pz,a,b,t)*M(STz,y_{2n+1},a,b,t)*M(y_{2n},Pz,a,b,t)$
$\quad *M(y_{2n},STz,a,b,t)$.

Taking limit as $n\to\infty$, we have
$M(Pz,z,a,b,kt) \geq M(Pz,z,a,b,t)$.
Therefore by Lemma 3, we have $Pz=z=STz$.
Now we show that $Qz=z$. in fact by (2.13) with $\alpha=1$ and (2.15) we have
$M(y_{2n},Qz,a,b,kt) = M(Px_{2n},Qz,a,b,kt)$
$\geq M(ABz,Qz,a,b,t)*M(STx_{2n},Px_{2n},a,b,t)*M(STx_{2n},Qz,a,b,t)*M(ABz,Px_{2n},a,b,t)$
$\quad *M(ABz,STx_{2n},a,b,t)$.
Taking the limit $n\to\infty$, we have
$M(z,Qz,a,b,t) \geq M(Qz,z,a,b,t)$.
Therefore by Lemma 3, we have $Qz=z=ABz$. Thus $Pz=Qz=ABz=STz=z$.
By putting $x=z$ and $y=Bz$ with $\alpha=1$ in (2.13), using (2.15) and Lemma 3, it is easy to see that $Bz=z$. Since $ABz=z$, therefore $Az=z$.
Similarly by putting $x=Tz$ and $y=z$ and $\alpha=1$ in (2.13), using (2.15) and Lemma 3, it is easy to prove that $Tz=z$. Since $STz=z$, we have $Sz=z$. Therefore, by combining the above results, we have

$$Az=Bz=Sz=Tz=Qz=z$$

that is z is the common fixed point of A, B, S, T, P and Q.
It is easy to prove uniqueness.

Theorem 4. Let $(X,M,*)$ be a fuzzy 3-metric space with $t*t\geq t$ for all $t\in[0,1]$ and the condition (FM-6). Let A,B,S,T and $\{P_i\}_{i\in I}$ be mappings from X into itself
(2.17) $\cup_{i\in I}P_i(X)\subset AB(X)$, $\cup_{i\in I}P_i(X)\subset ST(X)$, where I is an index set,
(2.18) there exists a constant $k\in(0,1)$ such that

$$M(P_ix,P_iy,a,b,kt) \geq M(ABy,P_iy,a,b,t)*M(STx,P_ix,a,b,t)*M(STx,P_iy,a,b,\alpha t)$$
$$*M(ABy,P_ix,a,b,(2-\alpha)t)*M(ABy,STx,a,b,t)$$

for all $x,y,a,b\in X$, $\alpha\in(0,2)$ $i\in I$ and $t>0$,
(2.19) If one of $AB(X)$ or $ST(X)$ or $P_i(X)(i\in I)$ is a complete subspace of X, then
(i) for all $i\in I$, P_i and AB have a coincidence point ,
(ii) for all $i\in I$, P_i and ST have a coincidence point .
Further if
(2.20) For all $i\in I$, $P_iB=BP_i$; $AB=BA$; $P_iT=TP_i$ and $ST=TS$,
(2.21) For all $i\in I$, the pairs $\{P_i,AB\}$ and $\{P_i,ST\}$ are weakly compatible then
(iii) A,B,S,T,P and $\{P_i\}_{i\in I}$ have a unique common fixed point in X.
If we put $B=T=I_x$ (the identity map on X) in Theorem 2, we get the following result :

Corollary 1. Let $(X,M,*)$ be a fuzzy 3-metric space with $t*t \geq t$ for all $t \in [0,1]$ and the condition $(FM\text{-}6)$. Let A,S and P be mappings from X into itself such that

(2.22) $P(X) \subset A(X)$, $P(X) \subset S(X)$

(2.23) there exists a constant $k \in (0,1)$ such that

$$M(Px,Py,a,b,kt) \geq M(Ay,Py,a,b,t)*M(Sx,Py,a,b,\alpha t)*M(Ay,Py,a,b,(2\text{-}\alpha)t)$$
$$*M(Ay,Sx,a,b,t)$$

for all $x,y,a,b \in X$, $\alpha \in (0,2)$ and $t>0$.

(2.24) If one of $P(X)$ or $S(X)$ is a complete subspace of X, then

(i) P and A have a coincidence point,

(ii) P and S have a coincidence point.

Further if

(2.25) The pairs $\{P,A\}$ and $\{P,S\}$ are weakly compatible then

(iii) A,S and P have a unique common fixed point in X.

If we put $A=B=S=T=I_x$ (the identity map on X) in Theorem 2.2, we get the following result :

Corollary 2. Let $(X,M,*)$ be a fuzzy 3-metric space with $t*t \geq t$ for all $t \in [0,1]$ and the condition $(FM\text{-}6)$. Let P be mappings from X into itself such that

(2.26) there exists a constant $k \in (0,1)$ such that

$$M(Px,Py,a,b,kt) \geq M(y,Py,a,b,t)*M(x,Px,a,b,t)*M(x,Py,a,b,\alpha t)$$
$$*M(y,Px,a,b,(2\text{-}\alpha)t) *M(y,x,a,b,t)$$

for all $x,y,a,b \in X$, $\alpha \in (0,2)$ and $t>0$.

If $P(X)$ is complete subspace of X then P has a unique common fixed point in X.

By using theorem 1, we get the following result.

Theorem 5. Let $(X,M,*)$ be a fuzzy 3-metric space with $t*t \geq t$ for all $t \in [0,1]$ and the condition $(FM\text{-}6)$. Let A,B and T be mappings from X into itself such that

(2.27) $A(X) \cup B(X) \subset T(X)$

(2.18) there exists a constant $k \in (0,1)$ such that

$$M(Ax,By,a,b,kt) \geq M(Ty,By,a,b,t)*M(Tx,Ax,a,b,t)*M(Tx,By,a,b,\alpha t)$$
$$*M(Ty,Ax,a,b,(2\text{-}\alpha)t)*M(Ty,Tx,a,b,t)$$

for all $x,y,a,b \in X$, $\alpha \in (0,2)$ $i \in I$ and $t>0$.

(2.29) One of $A(X)$, $B(X)$ or $T(X)$ is a complete subspace of X, then A,B, T have a coincidence point.

We establish Theorem 1 for sequence of mappings in the following manner.

Theorem 6. Let $(X,M,*)$ be a fuzzy 3-metric space with $t*t \geq t$ for all $t \in [0,1]$ and the condition $(FM\text{-}6)$. Let S,T, $A_i:X \to X$, $i=0,1,2...$ such that

(2.30) $A_0(X) \subset T(X)$, $A_i(X) \subset S(X)$, $i \in N$

(2.31) there exists a constant $k \in (0,1)$ such that

$$M(A_0x,A_iy,a,b,kt) \geq M(Ty,A_iy,a,b,t)*M(Sx,A_0x,a,b,t)*M(Sx,A_iy,a,b,\alpha t)$$
$$*M(Ty,A_0x,a,b,(2\text{-}\alpha)t)*M(Ty,Sx,a,b,t)$$

for all $x,y,a,b \in X$, $\alpha \in (0,2)$ and $t>0$.

(2.32) The pairs $\{A_0,S\}$ and $\{A_i,T\}$ ($i \in N$) are weakly compatible,

(2.33) if one of $S(X)$, $T(X)$ or $A_0(X)$ is a complete subspace of X or altrnatively A_i, $i \in N$ are complete subspace of X.

Then S,T, and $A_i, i=0,1,2,...$ have a unique common fixed point.

ACKNOWLEDGEMENTS

The authors extend their thanks to Professor P. Veeramani for their kind help in the preparation of this paper.

REFERENCES

[1] R. Badard, Fixed point theorems for fuzzy numbers, *Fuzzy Sets and Systems,* **13** (1984), 291-302.

[2] R. Chugh and S.Kumar, *Proc. Indian Acad. Sci., Math. Sci.,* **111 (2)** (2001), 241-247.

[3] S.S. Chang, Y.J. Cho, B.S. Lec, J.S. Jung and S.M. Kang, Coincidence points and minimization theorems in fuzzy metric spaces, *Fuzzy Sets and Systems,* **88**(1), 5(4) (1997), 949-962.

[4] Y.J. Cho, Fixed point in fuzzy metric space, *Journal of Fuzzy Mathematics,* **5(4)** (1997), 949-962.

[5] Z.K. Deng, Fuzzy pseudo metric spaces, *J. Math., Anal. Appl.,* **86** (1982), 75-84.

[6] M.A. Erceg, Metric spcaes in fuzzy set theory, *J. Math. Anal. Appl.* **69** (1970), 205-230.

[7] J.X. Fang, On fixed points theorems in fuzzy metric spaces, *Fuzzy Sets and Systems,* **64** (1994), 395-399.

[8] A. George and P. Veeramani, On some results in fuzzy metric space, *Fuzzy Sets and Systems,* **64** (1994), 395-399.

[9] M. Grabiec, Fixed point in fuzzy metric spaces, *Fuzzy Sets and Systems,* **27** (1988), 385-389.

[10] O. Hadzic, Fixed point theorems for multivalued mappings in classes of fuzzy metric spaces, *Fuzzy Sets and Systems,* **29** (1989), 115-125.

[11] O. Hadzic, *Fixed Point Theorems in Probabilistic Metric Spaces,* Serbian Aacademy of Sciences and Arts, Institute of Mathematics, University of Navisad, Yogoslovia, 1995.

[12] I. Istratescu, A fixed point theorem for mappings with probabilistic contractive iterated, *Rev. Roumaric. Math. Pure Appl.,* **26** (1981), 431-435.

[13] K. Iseki, Some applications of Banach type contraction principles *Math. Sem. Notes, Kobe Univ.,* **4** Arts Institute of Mathematics, Univ of Novisad, Yugoslovia, 1995.

[14] G. Jungck, Commuting mapping and fixed points, *Amer. Math. Monthly,* **83** (1976), 261-263.

[15] G. Jungck, Common fixed points for commuting and compatible mappings on compacta, *Proc. Amer. Math. Soc.,* **103** (1988), 977-983.

[16] G. Jungck, P.P. Murthy and Y.J. Cho, Compatible mappings of type (A) and common fixed points, *Math. Japonica,* **38** (1983), 381-390.

[17] G. Jungck and B.E. Rhoades, Fixed point for set valued functions without continuity, *Indian J. Pure Appl. Math.,* **29** (3) (1998), 227-238.

[18] J.S. Jung, Y.J. Cho and J.K. Kim, Minimization theorems for fixed point theorems in fuzzy metric spaces and applications, *Fuzzy Sets and Systems,* **61** (1994), 199-207.

[19] J.S. Jung, Y.J. Cho, S.S. Chang and S.M. Kang, Coincidence theorems for set-valued mappings and ekelands variational principle in fuzzy metric spaces, *Fuzzy Sets and Systems,* **79** (1996), 239-250.

[20] O. Kaleva and S. Sheikkala, On fuzzy metric space, *Fuzzy Sets and Systems*, **12** (1984), 215-229.

[21] R. Kannan, Some results on fixed points, *Bull. Cal. Math. Soc.,* **60** (1968), 71-76.

[22] I. Kramosil and J. Michalek, Fuzzy metric and statistical metric spaces, *Kybernetica* **11** (1975), 336-344.

[23] S. N. Mishra, N. Sharma, and S.L. Singh, Common fixed point of maps of fuzzy metric spaces, *Internet. J. Math. Sci.*, **17** (1994), 253-258.

[24] H.K. Pathak, Y.J. Cho, S.S. Chang and S.M. Kang, Compatible mappings of type (P) and fixed point theorems in metric spaces and probabilistic metric spaces, *Novi Sad J. Math.*, **26** No.2 (1996), 87-109.

[25] B.E. Rhoades, A comparison of various definitions of contractive mappings, *Trans. Amer. Math. Soc.*, **226** (1977), 257-290.

[26] B.E. Rhoades, Contractive definition revisited, *Contemp. Math.*, **21** (1983), 313-334.

[27] B. Schweizer and A. Sklar, Statistical metric space, *Pacific J. Math.*, **10** (1960), 189-205.

[28] S. Sessa, On a weak commutativity condition of mappings in fixed point considerations, *Publ. Inst. Math.* (Beograd) (N.S.) **32** (46) (1982), 149-155.

[29] S. Sessa, B.E. Rhoades and M.S. Khan, On common fixed point of compatible mappings in metric space and Banach space, *Internat. J. Math. and Math. Sci.*, **11** (2) (1988), 375-392.

[30] S.L. Singh, Some common fixed point theorems in *L*-space, *Math. Sem. Notes Kobe Univ.*, **7** (1979), 91-97.

[31] S.L. Singh and S. Kasahara, On some recent results on common fixed points, *Indian J. Pure Appl. Math.*, **13** (1982), 757-761, Corrigendum, **14** (1983), 1075.

[32] S. L. Singh and B. Ram, Common fixed point of commuting mappings in 2-metric space, *Math., Sem. Notes Kob Univ.*, **10** (1982), 197-208.

[33] S. Sharma, On fuzzy metric spaces, *South East Asian Bulletin of Math., Springer Verlag,* **26** (2002), 133-145.

[34] S. Sharma, Common fixed point theorem in fuzzy metric spaces *Fuzzy Sets and Systems,* **125** (2001), 345-352.

[35] S. Sharma and B. Deshpande, Commond fixed point of compatible of type (β) on fizzyk metric spaces, *Demonstratio Math.*, **35** No.1 (2002), 165-174.

[36] S. Sharma and B. Deshpande, Discontinuity and weak compatibility in fixed point consideration on non-complete fuzzy metric spaces, *J. Fuzzy Maths.*, **11** No. 2 (2003), 671-686.

[37] S. Sharma and J. K. Tiwari, Some common fixed point theorems in fuzzy 2-metric spaces, *Pak. J. C.S.I.R.* (accepted for publication)

[38] B.M.L. Tiwari and S.L. Singh, A note on recent generalization of Jungck contraction principle, *J, U.P.G.C. Soc.* **3** (1986), 13-18.

[39] L.A. Zadeh, *Fuzzy sets, Inform control* **8** (1965), 338-353.