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HALL EFFECTS ON COUPLE-STRESS FLUID HEATED AND SOLUTED FROM BELOW IN POROUS MEDIUM

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ABSTRACT

A layer of couple-stress fluid heated and soluted from below in porous medium is considered in the presence of uniform horizontal magnetic field to include the effect of Hall currents. For the stationary convection case, the stable solute gradient and magnetic field postpones the onset of convection while the Hall current hastens the onset of convection. The medium permeability and couple-stress both postpone and hasten the onset of convection depending on the Hall parameter M. Graphs have been plotted by giving numerical values to the parameters, to depict the stability characteristics. The stable solute gradient and the magnetic field (and corresponding Hall currents) are found to introduce oscillatory modes in the sytem, which were non-existent in their absence. The fufficient conditions for the non-existence of overstability are also obtained.

Key Words Couple-stress fluid, heated and soluted from below, Porous medium, Magnetic field, Hall currents.

1. Introduction. The formation and derivation of the basic equations of a layer of fluid heated from below in porous medium, using Boussinesq approximation, has been given in a treatise by Joseph [4]. When a fluid permeates an isotropic and homogeneous porous medium, the gross effect is represented by the Darcy's law. The study of a layer of fluid heated from below in porous media is motivated both theoretically and by its practical applications in engineering. Among the applications in engineering disciplines one can find the food process industry, chemical process indurtry, solidification and centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in porous medium. An extensive and updated account of convection in porous media has been given by Nield and Bejan [7]. The forced convection in fluid saturated porous medium channel has been studied by Nield et al. [8]. The effect of a magnetic field on the stability of such a flow is of interest in geophysics, particularly in the study of the earth's core, where the earth's mantle, which consists of conducting fluid, behaves like a porous medium that can become convectively unstable as a result of differential diffusion. Another application of the results of flow through a porous medium in the presence of a magnetic field is in the study of the stability of convective geothermal flow. A good account of the effect of magnetic field on the layer of the fluid heated from below has been given in a treatise by Chandrasekhar [2]. *MHD* finds vital applications in *MHD* generators, *MHD* flow-meters and pumps for pumping liquid metallurgy, geophysics *MHD* couplers and bearing and physiological processes such as magnetic therapy.

Double-diffusive convection concens flow that can arise when a layer of fluid with a dissolved solute (such as salt) is heated from below. Veronis [19] has investigated the problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of single component fluid and rigid boundaries and therefore it is desirable to consider a fluid acted on by solute gradient and free boundaries. The problem of the onset of thermal instability in the presence of a solute gradient is of great importance because of its applications to atmospheric physics and astrophysics, especially in the case of the ionosphere and the outer layer of the atmosphere. The thermosolutal convection problems also arise in oceanography, limnology and engineering. Thermosolutal convection problem arise in oceanography, limnology, and engineering. Examples of particular interest are provided by ponds built to trap solar heat (Tabor and Matz [17] and some Antarctic lakes (Shirtcliffe [16]). Sherman and Sutton [9] have considered the effect of current on the efficiency of a magncto-fluid-dynamic generator.

With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. Stokes [10] formulated the theory of couple-stress fluid. One of the applications of couple-stress fluid is its use to the study of the mechanisms of lubrication of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as the lubricant. When a fluid film is generated, squeeze-film action is capable of providing considerable protection to the cartilage surface.

A number of theories of the micro-continuum have been postulated and applied (Stokes [10], Lai et al. [5], Walicka [20]). The theory due to Stokes [10] allows for polar effects such as the presence of couple stresses and body couples. Stokes [10] theory has been applied to the study of some simple lubrication problems (see e.g. Sinha et al. [11], Bujurke and Jayaraman [1], Lin [6]). According to the theory of Stokes [10], couple-stresses are found to appear in noticeable magnitudes in fluids with very large molecules. Since the long chain hyaluronic acid molecules are found as additives in synovial fluid, Walicki and Walicka [21] modeled synovial fluid as couple-stress fluid in human joints. Sharma and Thakur [12] and Sharma et al. [13] have studied the problems of couple-stress fluid heated from below in porous medium in hydromagnetics and rotation, separately. The Hall effect is likely to be important in many geophysical situations as well as in flow of laboratory plasma. There is growing importance of non-Newtonian fluids in chemical technology, industry and geophysical fluid dynamics. The Hall currents have relevance and importance in geophysics, *MHD* generator and industry.

Keeping in mind the important of non-Newtonian fluids, convection in fluid layer heated and soluted from below, porous medium, magnetic field and Hall currents; the present paper attempts to study the couple-stress fluid heated and soluted from below in porous medium in the presence of uniform horizontal magnetic field to include the effect of Hall currents. The study is motivated by a model of synovial fluid. The synovial fluid is the natural lubricant of points of the vertebrates. The detailed description of the joint lubrication has very important practical implications-practically all diseases of joints are caused by or connected with a malfunction of the lubrication. The extremal efficiency of the physiological joint lubrication is caused by more mechanisms. The synovial fluid is due to the content of the hyaluronic acid a fluid of high viscosity, near to a gel. A layer of such fluid heated and soluted from below in porous medium under the action of magnetic field may find applications in physiological processes e.g. *MHD* finds applications in physiological processes such as magnetic therapy; heating may find application in physio-therapy.

2. Formulation of the Problem and Perturbation Equations. Here we consider an infinite, horizontal, incompressible, electrically conducting couple-stress fluid layer of thickness d, heated from below so that, the temperatures and densities at the bottom surface $z=0$ are T_0 and ρ_0 and at the upper surface $z=d$ and T_d and ρ_d respectively, and that a uniform temperature gradient $\beta(=|dT/dz|)$ and a uniform solute gradient $\beta'(=|dC/dz|)$ are maintained. The gravity field $\vec{g}=(0,0,-g)$ and a uniform horizontal magnetic field $\vec{H}=(H,0,0)$ pervade the system. This fluid layer is flowing through an isotropic and homogeneous porous medium of porostiy ε and medium permeability k_1.

Let $p, \rho, T, C, \alpha, \alpha', g, \eta, \mu_e, N, e$ and $\vec{q} = (u,v,w)$ denote, respectively, the fluid pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, gravitational acceleration, resistivity, magnetic permeability, electron number density, charge of an electron and fluid velocity. The equations expressing the conservation of momentum, mass, temperature solute concentration and equation of state of couple-stress fluid through porous medium (Stokes [10], Joseph [4]) are

$$\frac{1}{\varepsilon}\left[\frac{\partial \vec{q}}{\partial t} + \frac{1}{\varepsilon}(\vec{q}.\nabla)\vec{q}\right] = -\frac{1}{\rho_0}\nabla p + \vec{g}\left(1 + \frac{\delta\rho}{\rho_0}\right) - \frac{1}{k_1}\left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\vec{q} + \frac{\mu_e}{4\pi\rho_0}(\nabla \times \vec{H}) \times \vec{H}, \tag{1}$$

$$\nabla.\vec{q} = 0, \tag{2}$$

$$E\,\partial T/\partial t + (\vec{q}.\nabla)T = \kappa\nabla^2 T, \tag{3}$$

$$E'\,\partial C/\partial t + (\vec{q}.\nabla)C = \kappa'\nabla^2 C, \tag{4}$$

$$\rho = \rho_0[1 - \alpha(T - T_0) + \alpha'(C - C_0)], \tag{5}$$

where the suffix zero refers to values at the reference level $z=0$ and in writing Eq. (1), use has been made of the Boussinesq approximation which states that the density variations are ignored in all terms in the equation of motion except the external force term. The magnetic permeability μ_e, the kinematic viscosity, ν, the kinematic viscoelasticity ν', the thermal diffusivity κ and the solute diffusivity κ' are all assumed to be constants.

The Maxwell's equations yield

$$\varepsilon\frac{d\vec{H}}{dt} = (\vec{H}.\nabla)\vec{q} + \varepsilon\eta\nabla^2\vec{H} - \frac{c\varepsilon}{4\pi Ne}\nabla \times \left[(\nabla \times \vec{H}) \times \vec{H}\right], \tag{6}$$

$$\nabla.\vec{H} = 0, \tag{7}$$

where $d/dt = \partial/\partial t + \varepsilon^{-1}\vec{q}.\nabla$ stands for the convection derivative.

Here $E = \varepsilon + (1-\varepsilon)((\rho_s c_s)/(\rho_0 c_i))$ is a constant and E' is a constant analogous to E but corresponding to solute rather than heat. ρ_s, c_s and ρ_0, c_i stand for density and heat capacity of solid (porous matrix) material and fluid, respectively. The steady state solution is

$$\vec{q} = (0,0,0), \quad T = -\beta z + T_0,$$
$$C = -\beta' z + C_0, \quad \rho = \rho_0(1 + \alpha\beta z - \alpha'\beta' z). \tag{8}$$

Here we use linearized stability theory and normal mode analysis method. Assume small perturbations around the basic solution, and let $\delta\rho$, δp, θ, γ, $\vec{q}$ (u,v,w) and $\vec{h} = (h_x, h_y, h_z)$ denote respectively the perturbations in fluid density ρ, pressure p, temprature T, solute concentration C, velocity $(0,0,0)$ and magnetic field $\vec{H} = (H,0,0)$. The change in density $\delta\rho$, caused mainly by the perturbations θ and γ

in temperature and concentration, is given by

$$\delta\rho = -\rho_0(\alpha\theta - \alpha'\gamma). \tag{9}$$

Then the linearized perturbation equations of the couple-stress fluid become

$$\frac{1}{\varepsilon}\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_0}(\nabla\delta p) - \vec{g}(\alpha\theta - \alpha'\gamma) - \frac{1}{k_1}\left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\vec{q} + \frac{\mu_e}{4\pi\rho_0}(\nabla\times\vec{H})\times\vec{H}, \tag{10}$$

$$\nabla.\vec{q} = 0, \tag{11}$$

$$E\frac{\partial\theta}{\partial t} = \beta w + \kappa\nabla^2\theta, \tag{12}$$

$$E'\frac{\partial\gamma}{\partial t} = \beta' w + \kappa'\nabla^2\gamma, \tag{13}$$

$$\varepsilon\frac{d\vec{h}}{dt} = (\vec{H}.\nabla)\vec{q} + \varepsilon\eta\nabla^2\vec{h} - \frac{c\varepsilon}{4\pi Ne}\nabla\times\left[(\nabla\times\vec{H})\times\vec{H}\right], \tag{14}$$

$$\nabla.\vec{h} = 0. \tag{15}$$

3. The Dispersion Relation. Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[w, h_z, \theta, \gamma, \zeta, \xi] = [W(z), K(z), \Theta(z), \Gamma(z), Z(z), Z(z)]\exp(ik_x x + ik_y y + nt), \tag{16}$$

where k_x, k_y are the wave number along the x-and y-directions respectively, $k = \sqrt{(k_x^2 + k_y^2)}$ is the resultant wave number and n is the growth rate, which is, in general, a complex constant. Here $\zeta = \partial v/\partial x - \partial u/\partial y$ and $\xi = \partial h_y/\partial x - \partial h_x/\partial y$ stand for the z-components of vorticity and current density, respectively.

Expressing the coordinates x,y,z in the new unit of length d and letting $a = kd$, $\sigma = nd^2/\nu$, $p_1 = \nu/\kappa$, $p_2 = \nu/\eta$, $q = \nu/\kappa'$, $P_l = k_1/d^2$, $F = \mu'/(\rho_0 d^2\nu)$ and $D = d/dz$, Eqs. (10)-(15), using (16), yield

$$(D^2 - a^2)\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_\ell} - \frac{F}{P_\ell}(D^2 - a^2)\right]W - \frac{ik_x\mu_e Hd^2}{4\pi\rho_0\nu}(D^2 - a^2)K + \frac{ga^2d^2}{\nu}(\alpha\Theta - \alpha'\Gamma) = 0, \tag{17}$$

$$\left[\sigma/\varepsilon + 1/P_\ell - F/P_\ell(D^2 - a^2)\right]Z = ik_x\mu_e Hd^2X/(4\pi\rho_0\nu), \tag{18}$$

$$(D^2 - a^2 - p_2\sigma)K = -(ik_xHd^2/(\eta\varepsilon))W + cik_xHd^2X/(4\pi Ne\eta), \tag{19}$$

$$(D^2 - a^2 - p_2\sigma)X = -(ik_xHd^2/(\eta\varepsilon))Z - cik_xH/(4\pi Ne\eta)(D^2 - a^2)K, \tag{20}$$

$$(D^2 - a^2 - Ep_1\sigma)\Theta = -(\beta d^2/\kappa)W, \tag{21}$$

$$(D^2 - a^2 - E'q\sigma)\Gamma = -(\beta' d^2/\kappa')W. \tag{22}$$

Consider the case where both boundaries are free as well as perfect conductors of both heat and solute concentration, while the adjoining medium is perfectly conducting. The case of two free boundaries is a little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The appropriate boundary conditions, with respect to which Eqs. (17)-(22) must be solved are (Chandrasekhar [2])

$$W = D^2W = X = DZ = 0, \Theta = 0, \Gamma = 0, \text{ at } z = 0 \text{ and } 1,$$

$DX=0$, $K=0$ on a perfectly conducting boundary

and $X=0$, h_x, h_y, h_z are continuous with an external vacuum field on a non-conducting boundary. (23)

The case of two free boundaries, though little artificial, is the most appropriate for steller atmospheres (Spiegel [14]). Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for $z=0$ and 1 and hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z. \tag{24}$$

where W_0 is a constant.

Eliminating Θ, Γ, K, Z and X between Eqs. (17)-(22) and substituting the proper solution $W=W_0 \sin\pi z$, in the resultant equation, we obtain the dispersion relation

$$R_1 = \left(\frac{1+x}{x}\right)\left[\frac{i\sigma_1}{\varepsilon} + \frac{1}{P} + \frac{\pi^2 F(1+x)}{P}\right]\left[1+x+iEp_1\sigma_1\right] + S_1\frac{(1+x+iEp_1\sigma_1)}{(1+x+iE'q\sigma_1)}$$

$$+ Q_1\cos^2\theta \frac{(1+x)\left[1+x+iEp_1\sigma_1\right]\left\{\left(\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}+\frac{\pi^2F(1+x)}{P}\right)\left[1+x+ip_2\sigma_1\right]+Q_1x\cos^2\theta\right\}}{\left\{\begin{array}{l}\left[\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}+\frac{\pi^2F(1+x)}{P}\right]\left[1+x+ip_2\sigma_1\right]^2+Q_1x\cos^2\theta\left[1+x+ip_2\sigma_1\right]+\\ Mx\cos^2\theta(1+x)\left[\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}+\frac{\pi^2F(1+x)}{P}\right]\end{array}\right\}}, \tag{25}$$

where

$$R_1 = \frac{g\alpha\beta d^4}{\nu\kappa\pi^4}, S_1 = \frac{g\alpha'\beta' d^4}{\nu\kappa'\pi^4}, Q_1 = \frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta\varepsilon\pi^2}, M = \left(\frac{cH}{4\pi Ne\eta}\right)^2, x = \frac{a^2}{\pi^2}, i\sigma_1 = \frac{\sigma}{\pi^2},$$

$k_x = k \cos\theta$ and $P = \pi P_\ell$.

Equation (25) is the required dispersion relation including the effects of magnetic field, Hall current, stable solute gradient and medium permeability on a layer of couple-stress fluid heated and soluted from below in porous medium in the presence of a uniform horizontal magnetic field and Hall currents.

4. The Stationary Convection. When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma=0$. Putting $\sigma=0$, the dispersion relation (25) reduces to

$$R_1=\left(\frac{1+x}{x}\right)\frac{\left(\frac{(1+x)\{1+\pi^2F(1+x)\}}{P}+Q_1x\cos^2\theta\right)^2+\frac{Mx\cos^2\theta(1+x)\{1+\pi^2F(1+x)\}^2}{P^2}}{\left\{\frac{(1+x+Mx\cos^2\theta)\{1+\pi^2F(1+x)\}}{P}+Q_1x\cos^2\theta\right\}}+S_1, \tag{26}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters S_1,Q_1,M,F and P.

To study the effects of stable solute gradient, horizontal magnetic field, Hall currents, medium permeability and couple-stress parameter, we examine the natures of $dR_1/dS_1, dR_1/dQ_1, dR_1/dM, dR_1/dP$ and dR_1/dF analytically. Equation (26) yields

$$dR_1/dS_1=+1, \tag{27}$$

$$\frac{dR_1}{dQ_1}=\left(\frac{1+x}{x}\right)x\cos^2\theta\frac{\left(\frac{(1+x)\{1+\pi^2F(1+x)\}}{P}+Q_1x\cos^2\theta+\frac{Mx\cos^2\theta\{1+\pi^2F(1+x)\}\,Q_1x\cos^2\theta}{P\left(\frac{1+x+Mx\cos^2\theta\{1+\pi^2F(1+x)\}}{P}+Q_1x\cos^2\theta\right)}\right)}{\left\{\frac{(1+x+Mx\cos^2\theta)\{1+\pi^2F(1+x)\}}{P}+Q_1x\cos^2\theta\right\}}, \tag{28}$$

and

$$\frac{dR_1}{dM}=-Q_1x\cos^4\theta(1+x)\frac{\left(\frac{(1+x)\{1+\pi^2F(1+x)\}}{P}+Q_1x\cos^2\theta\right)\left(\frac{\{1+\pi^2F(1+x)\}}{P}\right)}{\left\{\frac{(1+x+Mx\cos^2\theta)\{1+\pi^2F(1+x)\}}{P}+Q_1x\cos^2\theta\right\}^2}. \tag{29}$$

It is clear from (27), (28) and (29) that, for stationary convection, the magnetic field and stable solute gradient postpones the onset of convection whereas, the Hall currents hastens the onset of convection on the thermosolutal instability of couple-stress fluid in porous medium in hydromagnetics in the presence of Hall currents. Equation (26) also yields

$$\frac{dR_1}{dP}=-\frac{(1+x)\{1+\pi^2F(1+x)\}}{xP^2}\frac{\left(\begin{array}{l}\frac{(1+x)}{P^2}(1+x+Mx\cos^2\theta)^2\{1+\pi^2F(1+x)\}^2\\+\frac{2Q_1x\cos^4\theta(1+x)(1+x+Mx\cos^2\theta)\{1+\pi^2F(1+x)\}}{P}\\+Q_1^2x^2\cos^4\theta(1+x-Mx\cos^2\theta)\end{array}\right)}{\left[\frac{(1+x+Mx\cos^2\theta)\{1+\pi^2F(1+x)\}}{P}+Q_1x\cos^2\theta\right]^2}. \tag{30}$$

$$\frac{dR_1}{dF}=\left(\frac{(1+x)^2\pi^2}{xP}\right)\cdot\frac{\begin{pmatrix}\left(\frac{(1+x)}{P^2}\right)\left(1+x+Mx\cos^2\theta\right)^2\left\{1+\pi^2F(1+x)\right\}^2\\ +\frac{2Q_1x\cos^2\theta\,(1+x)\left(1+x+Mx\cos^2\theta\right)\left\{1+\pi^2F(1+x)\right\}}{P}\\ +Q_1^2x^2\cos^4\theta\left(1+x-Mx\cos^2\theta\right)\end{pmatrix}}{\left[\frac{\left(1+x+Mx\cos^2\theta\right)\left\{1+\pi^2F(1+x)\right\}}{P}+Q_1x\cos^2\theta\right]^2}. \tag{31}$$

Hence, it is clear from (30) and (31) that, for stationary convection, the meidum permeability hastens the onset of convection whereas, the couple-stress postpones the onset of convection on the thermal instability of couple-stress fluid in porous medium in hydromagnetics in the presence of Hall currents for all wave numbers

$$(1+x)>Mx\cos^2\theta,$$

which is normally satisfied as the Hall currents parameter M is very small compared to unity.

The dispersion relation (26) is analysed numerically. In Figure 1, R_1 is plotted against x for $S_1=10, 20, 30$; $P=50$, $\theta=45^0$, $F=2$, $Q_1=10$ and $M=10$. It is clear that the stable solute gradient postpones the onset of convection in a couple-stress fluid heated and soluted from below in a porous medium in the presence of Hall currents as the Rayleigh number increases with the increase in stable solute gradient parameter. In Figure 2, R_1 is plotted against x for $Q_1=10,20,30$; $P=50$, $\theta=45^0$, $F=2$, $S_1=10$ and $M=10$. It is clear that the magnetic field postpones the onset of convection in a couple-stress fluid heated from below in a porous medium in the presence of Hall currents as the Rayleigh number increases with the increase in magnetic field parameter. In Figure 3, R_1 is plotted against x for $M=10,20, 30$; $P=50$, $\theta=45^0$, $F=2$, $S_1=10$ and $Q_1=10$. Here we find that the Hall currents hastens the onset of convection for all wave numbers as the Rayleigh number decreases with an increase in the Hall currents parameter.

In Figure 4, R_1 is plotted against x for $P=10, 20, 30$; $M=0.1$, $\theta=45^0$, $F=2$, $S_1=10$ and $Q_1=100$. Here we find that when $M<1$, the medium permeability always hastens the onset of convection for all wave numbers as the Rayleigh number decreases with an increase in medium permeability parameter. In Figure 5, R_1 is plotted against x for $P=10, 20,30$; $M=100$, $\theta=45^0$, $F=2$, $S_1=10$ and $Q_1=100$. Here we find that when $M>1$, the medium permeability postpones the onset of convection for small wave numbers only as the Rayleigh number increases with an increase in medium permeability parameter and hastens the onset of convection for higher wave numbers as the Rayleigh numbers decreases with an increase in medium permeability parameter.

In Figure 6, R_1 is plotted against x for $F=1,2,3,4;\ Q_1=100,\ \theta=45^0,\ P=10,\ S_1=10$ and $M=0.1$. Here we find that when $M<1$, the couple-stress postpones the onset of convection for all wave numbers as the Rayleigh number increases with the increase in couple-stress parameter. In Figure 7, R_1 is plotted against x for $F=1,2,3;\ Q_1=100,\ \theta=45^0,\ P=10,\ S_1=10$ and $M=100$. Here we find that when $M>1$, the couple-stress hastens the onset of convection for small wave numbers as the Rayleigh number decreases with the increase in couple-stress parameter and postpones the onset of convection in the for higher wave number as the Rayleigh number increases with the increase in couple-stress parameter.

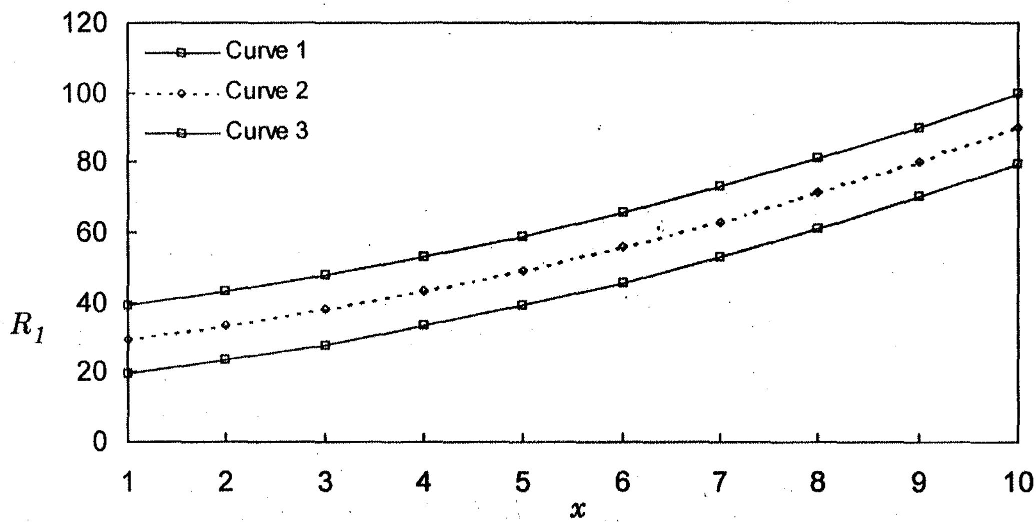

Fig.1: The variation of Rayleigh number (R_1) with wave number (x) for $P=50$, $F=2$, $M=10$, $Q_1=10$; $S_1=10$ for curve 1, $S_1=20$ for curve 2 and $S_1=30$ for curve 3.

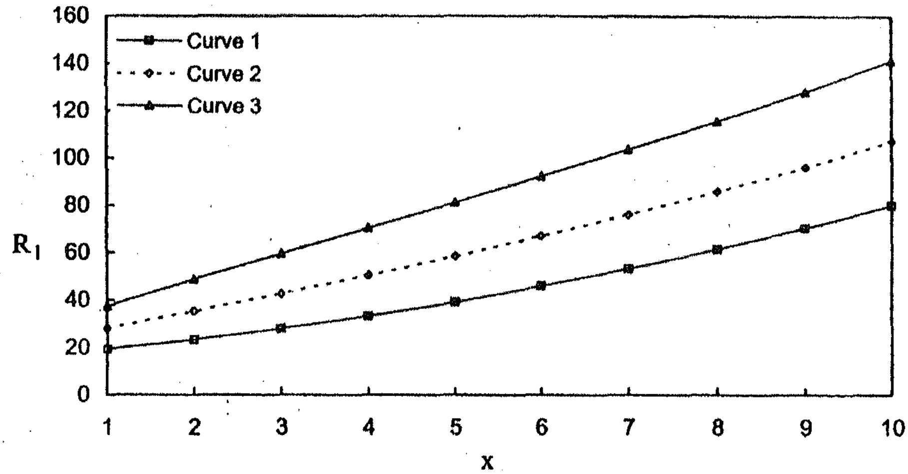

Fig.2: The variation of Rayleigh number (R_1) with wave number (x) for P = 50, F = 2 $\theta = 45^0$, M = 10, S_1 =10; Q_1= 10 for curve 1, Q_1= 20 for curve 2 and Q_1= 30 for curve 3.

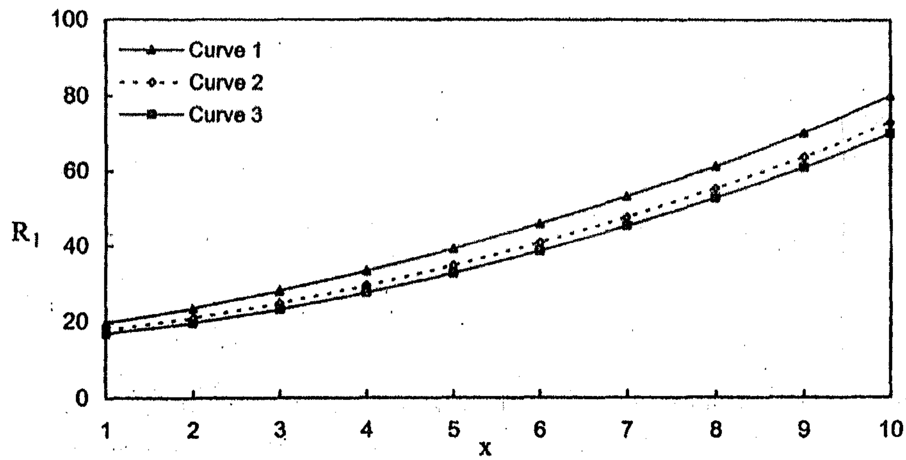

Fig.3: The variation of Rayleigh number (R_1) with wave number (x) for P = 50, F = 2 $\theta = 45^0$, Q_1 = 10, S_1 =10; M= 10 for curve 1, M= 20 for curve 2 and M= 30 for curve 3.

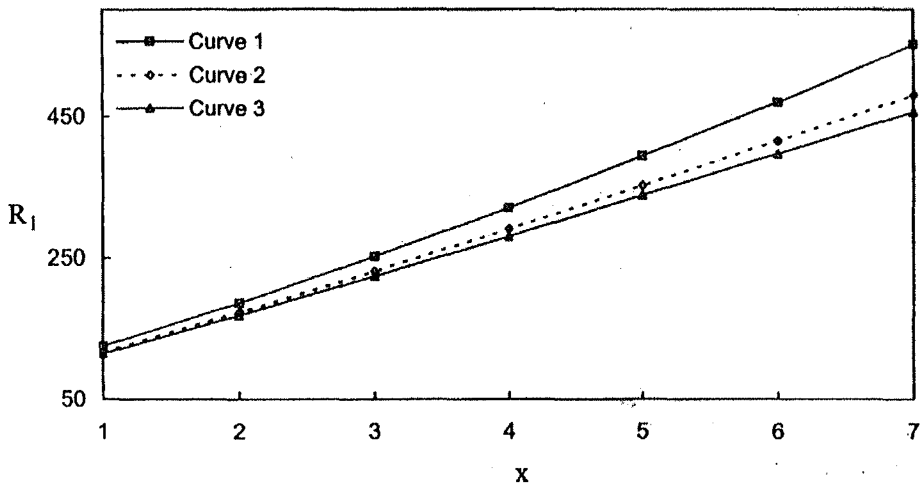

Fig.4: The variation of Rayleigh number (R_1) with wave number (x) for F = 2, Q_1 = 100 $\theta = 45^0$, M = 0.1, S_1 =10; P = 10 for curve 1, P = 20 for curve 2 and P = 30 for curve 3

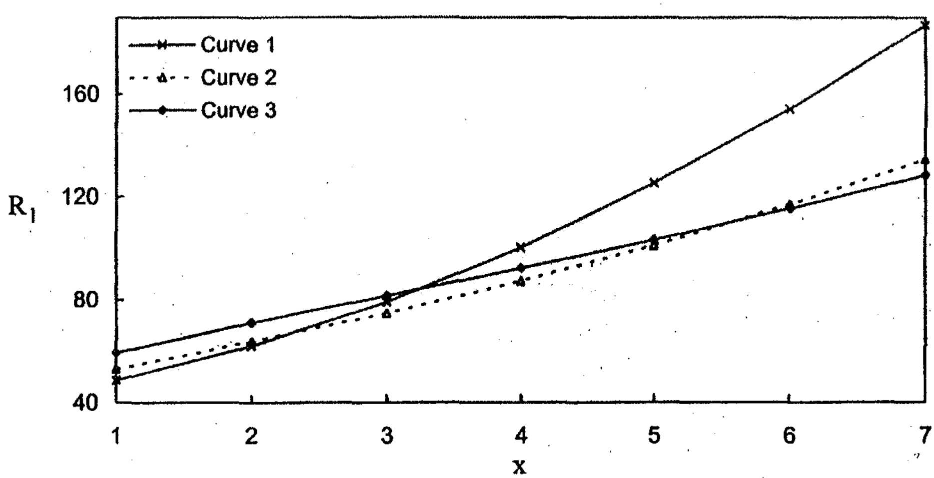

Fig.5: The variation of Rayleigh number (R_1) with wave number (x) for Q_1 = 100, F = 2 $\theta = 45^0$, M = 100, S_1 =10; P = 10 for curve 1, P = 20 for curve 2 and P = 30 for curve 3

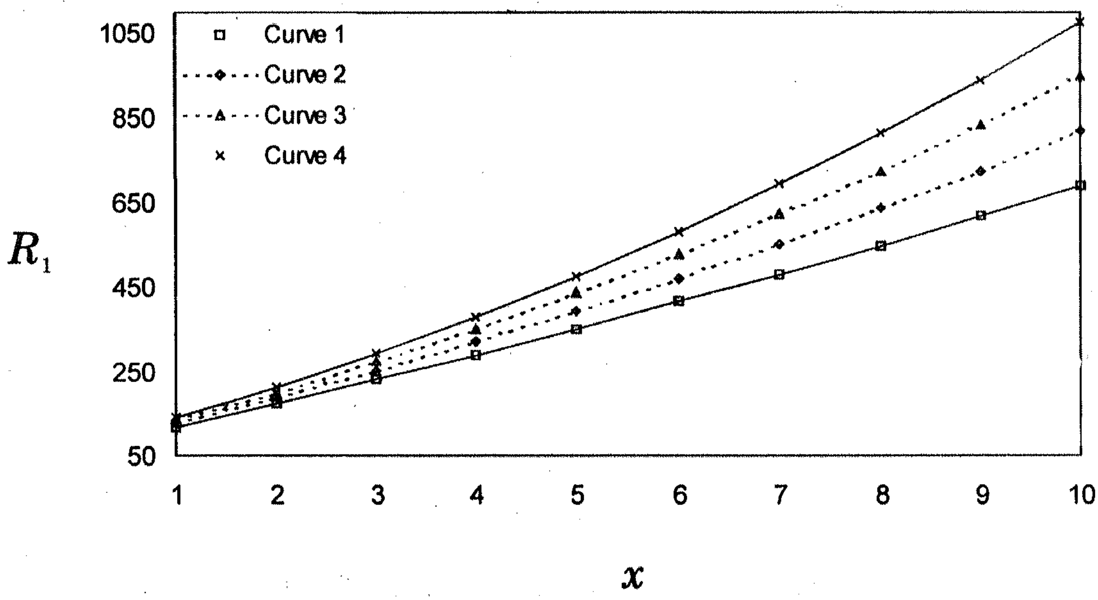

Fig.6: The variation of Rayleigh number (R_1) with wave number (x) for $P = 10, \theta = 45^0$, $M = 0.1, Q_1 = 100; S_1 = 10$; F=1 for curve 1, F=2 for curve 2, F=3 for cur 3 and F= 4 for curve 4.

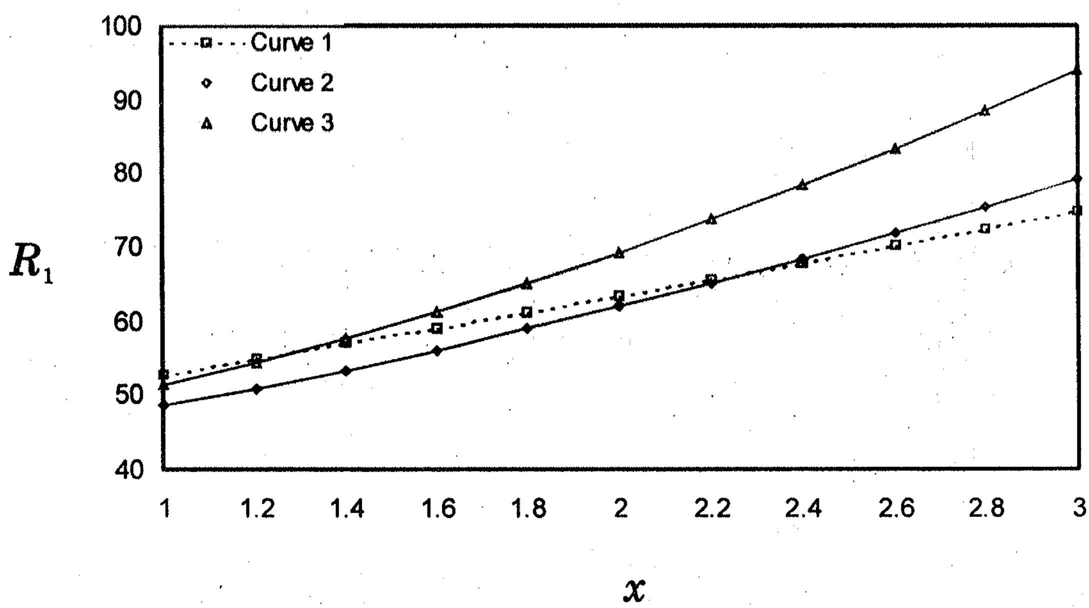

Fig.7: The variation of Rayleigh number (R_1) with wave number (x) for $P = 10, \theta = 45^0$, $M = 100, Q_1 = 100; S_1 = 10$; F=1 for curve 1, F=2 for curve 2, F=3 for curve 3.

5. Stability of the System and Oscillatory Modes. Here we examine the possibility of oscillatory modes, on a stability problem due to the presence of the magnetic field and Hall currents. Multplying (17) by W^*, which is the complex conjugate of W, and using (18)-(22) together with the boundary conditions (23), we obtain

$$FI_1+(1+P_\ell\sigma/\varepsilon)I_2-(g\alpha\kappa a^2P_\ell/\nu\beta)[I_3+Ep_1\sigma^*I_4]+(g\alpha'\kappa'a^2P_\ell/\nu\beta')[I_5+E'q\sigma^*I_6]$$
$$+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}P_\ell(I_7+p_2\sigma^*I_8)+\frac{\mu_e\varepsilon\eta d^2}{4\pi\rho_0\nu}P_\ell(I_{11}+p_2\sigma I_{12})+d^2[(1+P_\ell\sigma^*/\varepsilon)I_{10}+FI_9]=0, \quad (32)$$

where

$$I_1=\int_0^1\left(|D^2W|^2+2a^2|DW|^2+a^4|W|^2\right)dz, \qquad I_2=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz,$$
$$I_3=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz, \qquad I_4=\int_0^1\left(|\Theta|^2\right)dz,$$
$$I_5=\int_0^1\left(|D\Gamma|^2+a^2|\Gamma|^2\right)dz, \qquad I_6=\int_0^1\left(|\Gamma|^2\right)dz,$$
$$I_7=\int_0^1\left(|D^2K|^2+2a^2|DK|^2+a^4|K|^2\right)dz, \qquad I_8=\int_0^1\left(|DK|^2+a^2|K|^2\right)dz,$$
$$I_9=\int_0^1\left(|DZ|^2+a^2|Z|^2\right)dz, \qquad I_{10}=\int_0^1\left(|Z|^2\right)dz,$$
$$I_{11}=\int_0^1\left(|DX|^2+a^2|X|^2\right)dz, \qquad I_{12}=\int_0^1\left(|X|^2\right)dz \quad (33)$$

The integrals $I_1,\ldots,I_{12}$ are all positive-definite. Putting $\sigma=\sigma_r+i\sigma_i$ where σ_r,σ_i are real and equating the real and imaginary parts of Eq. (32), we obtain

$$\sigma_r\left[\frac{I_2}{\varepsilon}-\frac{g\alpha\kappa a^2}{\nu\beta}Ep_1I_4+\frac{g\alpha'\kappa'a^2}{\nu\beta'}E'qI_6+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}p_2(I_8+d^2I_{12})+\frac{d^2}{\varepsilon}I_{10}\right]$$
$$=-\left[\frac{F}{P_\ell}I_1+\frac{1}{P_\ell}I_2-\frac{g\alpha\kappa a^2}{\nu\beta}I_3+\frac{g\alpha'\kappa'a^2}{\nu\beta'}I_5+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}(I_7+d^2I_{11})+\frac{d^2}{P_\ell}(I_{10}+FI_9)\right], \quad (34)$$

$$i\sigma_i\left[\frac{I_2}{\varepsilon}+\frac{g\alpha\kappa a^2}{\nu\beta}Ep_1I_4-\frac{g\alpha'\kappa'a^2}{\nu\beta'}E'qI_6-\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}p_2(I_8-d^2I_{12})-\frac{d^2}{\varepsilon}I_{10}\right]=0. \quad (35)$$

It is evident from Eq. (34) that σ_r is either positive or negative. The system is, therfore, either stable or unstable. It is clear from Eq. (35) that σ_i may be either zero or non-zero, meaning that the modes may be either non-oscillatory or oscillatory. In the absence of stable solute gradient and magnetic field, equation (35) reduces to

$$[I_2/\varepsilon+g\alpha\kappa a^2Ep_1I_4/\nu\beta]\sigma_i=0. \quad (36)$$

and the terms in brackets are positive definite. Thus $\sigma_i=0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilitties is satisfied for a porous medium, in the absence of stable solute gradient and magnetic field. This result is true for the porous as well as non-porous medium as studied in Chandrasekhar [2]. The oscillatory modes are introduced due to the presence of the stable solute gradient and the magnetic field (and corresponding Hall currents), which were non-existent in their absence.

6. The Case of Overstability. Here we discuss the possibility of whether instability may occur as overstability. Since we wish to determine the critical Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which (25) will admit of solutions with σ_1 real. Equating real and imaginary parts of Eq. (25) and eliminating R_1 between them, we obtain

$$A_4 c_1^4 + A_3 c_1^3 + A_2 c_1^2 + A_1 c_1 + A_0 = 0, \tag{37}$$

where we have put $c_1 = \sigma_1^2, b = 1 + x$ and

$$A_4 = E'^2 q^2 p_2^4/\varepsilon^2 \left[\left(1/\varepsilon + Ep_1\pi^2 F/P\right)b + Ep_1/P \right], \tag{38}$$

$$\begin{aligned} A_3 = &\left[E'^2 q^2 \left(\frac{1}{\varepsilon} + \frac{Ep_1\pi^2 F}{P}\right)\left(\frac{p_2^2\pi^4 F^2}{P^2} + \frac{2}{\varepsilon^2}\right)p_2^2 + \frac{p_2^4}{\varepsilon^2}\left(\frac{1}{\varepsilon} + \frac{Ep_1\pi^2 F}{P}\right)\right]b^3 \\ &+ \left[2E'^2 q^2 \left(\frac{1}{\varepsilon} + \frac{Ep_1\pi^2 F}{P}\right)\left(\frac{p_2^2\pi^4 F^2}{P^2} - \frac{Mx\cos^2\theta}{\varepsilon^2}\right)p_2^2 + \frac{p_2^4}{\varepsilon^2}\frac{Ep_1}{P} + \frac{E'^2 q^2 Ep_1}{P}\left(\frac{p_2^2\pi^2 F^2}{P^2} + \frac{2}{\varepsilon}\right)p_2^2 \right]b^2 \\ &+ E'^2 q^2 \left[\left(\frac{1}{\varepsilon} + \frac{Ep_1\pi^2 F}{P}\right)\left(\frac{p_2}{P^2} - \frac{2Q_1 x\cos^2\theta}{\varepsilon}\right)p_2^3 + \frac{2Ep_1 p_2^2}{P}\left(\frac{p_2^2\pi^2 F^2}{P^2} - \frac{Mx\cos^2\theta}{\varepsilon^2}\right) + \left(\frac{p_2^2}{\varepsilon^2}(Ep_1 - p_2)\right)\right]b \\ &+ \left[\frac{E'^2 q^2 Ep_1}{P}\left(\frac{p_2}{P^2} - \frac{2Q_1 x\cos^2\theta}{\varepsilon}\right)p_2^3 + + \left(\frac{p_2^4}{\varepsilon^2}(Ep_1 - p_2)(b-1)\right)\right], \end{aligned} \tag{39}$$

and the coefficients A_0, A_1 and A_2, being quite lengthy and not needed in the discussion of overstability, have not been written here.

Since σ_1 is real for overstability, the four values of $c_1 \left(= \sigma_1^2\right)$ are positive. The sum of roots of (37) is $-A_3/A_4$, and if this is to be negative, then $A_3 > 0$ $A_4 > 0$. It is clear from (38) and (39) that A_3 and A_4 are always positive if

$$Ep_1 > p_2, Ep_1 > E'q, p_2^2\pi^2 F/P^2 > Mx\left(\cos^2\theta\right)/\varepsilon^2 \text{ and } p_2/P^2 > 2Q_1 x\left(\cos^2\theta\right)/\varepsilon \tag{40}$$

which imply that

$$\kappa < E\eta, \kappa < \frac{E\kappa'}{E'}, \nu > \frac{\rho_0 k_x^2}{\mu'}\left(\frac{cHk_1}{4\varepsilon Ne}\right)^2, \nu > \left(\frac{\mu_e}{2\pi\rho_0}\right)^{1/2}\left(\frac{k_1 Hk_x}{\varepsilon}\right) \tag{41}$$

i.e. $$\kappa < \min\left(E\eta, \frac{E\kappa'}{E'}\right), \nu > \max\left[\frac{\rho_0 k_x^2}{\mu'}\left(\frac{cHk_1}{4\varepsilon Ne}\right)^2, \left(\frac{\mu_e}{2\pi\rho_0}\right)^{1/2}\left(\frac{k_1 Hk_x}{\varepsilon}\right)\right] \tag{42}$$

thus $\kappa < min\left(E\eta, \frac{E\kappa'}{E'}\right), \nu > max\left[\frac{\rho_0 k_x^2}{\mu'}\left(\frac{cHk_1}{4\varepsilon Ne}\right)^2, \left(\frac{\mu_e}{2\pi\rho_0}\right)^{1/2}\left(\frac{k_1 H k_x}{\varepsilon}\right)\right]$, therefore, are sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply occurrence of overstability.

7. Discussion. The inclusion of Hall currents gives rise to a cross flow i.e., a flow at right angles to the primary flow in a channel in the presence of a transverse magnetic field, has been shown by Sato [15] whereas Tani [18]. Tani [18] has found that Hall effect produces a cross-flow of double-swirl pattern in incompressible flow through a straight channel with arbitrary cross-section. This breakdown of the primary flow and formation of a secondary flow may be attributed to the inherent instability of the primary flow in the presence of Hall current. Sato [15] has pointed out that even if the distribution of the primary flow velocity be stable to external distrubances. the whole layer may become turbulent if the distribution of the cross-flow velocity is unstable. A similar situation occurs on the three-dimensional boundary layer along a swept-back wing. Gupta [3] has found that the presence of Hall current induces a vertical component of vorticity and this may well the reason for the destabilizing influence.

Therefore, for stationary convection, the magnetic field postpones the onset of convection whereas, the Hall currents hastens the onset of convection on the thermal instability of couple-stress fluid in porous medium in hydromagnetics in the presence of Hall currents. The medium permeability hastens the onset of convection whereas, the couple-stress postpones the onset of convection on the thermal instability of couple-stress fluid in porous medium in hydromagnetics in the presence of Hall currents for all wave numbers $(1+x) > Mx\cos^2\theta$, which is normally satisfied as the Hall currents parameter M is very small compared to unity. Graphs have been plotted by giving some numerical values to the parameters, to depict the stability characteristics. The stable solute gradient and the magnetic field (and corresponding Hall currents) introduce oscillatory modes in the system, which were non-existent in their absence. The sufficient condition for the non-existence of overstability for thermal instability in couple-stress fluid in presence of magnetic field in porous medium are

$\kappa < min\left(E\eta, \frac{E\kappa'}{E'}\right), \nu > max\left[\frac{\rho_0 k_x^2}{\mu'}\left(\frac{cHk_1}{4\varepsilon Ne}\right)^2, \left(\frac{\mu_e}{2\pi\rho_0}\right)^{1/2}\left(\frac{k_1 H k_x}{\varepsilon}\right)\right]$. Moreover, in the absence of couple-stress vicosity ($\mu'=0$) and solute gradient, sufficient condition for non-existence of overstability for thermal instability in viscous, Newtonian fluid (Gupta [3]), as expected, reduces to $E\eta > \kappa$. Porosity 'ε' factor played an important role in developing the sufficient conditions for the non-existence of overstability. The result of the satisfaction of principal of exchange of stabilities

[Sec. 5] is true in porous as well as non-porous medium.

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