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(Dedicated to the memory of Professor J.N. Kapur)

$2\omega_p$ EMISSIONS BY NONLINEAR MIXING OF PLASMA AND ION-ACOUSTIC WAVES

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ABSTRACT

A large amplitude plasma wave $(\omega_p, \vec{k}_0)$ propagating through a plasma produces oscillatary electron velocity at the harmonics of ω_p. The electron velocity at second harmonic couples with the density oscillations of an ion acoustic wave $(\omega_s, \vec{k}\ )$ to produce a nonlinear current at ($2\omega_p - \omega_s w \approx 2\omega_p$), that produces $2\omega_p$, emissions. The process is sensitive to power of the plasma wave.

1. Introduction. Wave mixing and harmonic generation are the areas of intense research in plasma physics during past few decades [3, 7, 8, 11]. Wave mixing has applications in image processing, dispersion correction, tracking and pointing, optical computing and charged particle acceleration. Harmonic generation has applications in wavelength conversion, generation of coherent *XUV* and soft *X*-rays etc. Both of these processes also serves as diagnostics and much of the knowledge about plasma can be acquired [4]. These techniques are very attractive since they render possible the study of different processes also in cases when a local measurement in situ in the plasma is not possible for various reasons. Both of these processes involves excitation of natural plasma modes. Two such modes are plasma or Langmuir wave and ion-acoustic wave. Plasma wave is the high frequency mode obeying the dispersion relation $\omega^2=\omega_p^2 + k^2 v_{th}^2$ and v_{th} are the plasma frequency and electron thermal velocity respectively. Ion acoustic wave is the low frequency mode with dispersion relation $\omega = kc_s$, c_s being the ion sound speed [2]. Second harmonic generation by an electromagnetic wave is often interpreted as a two-step process[1]. In the first step some electron density oscillations at a frequency equal to or very close to the fundamental frequency are produced. In the second step the incident fundamental is scattered from the electron density fluctuations or two electron plasma waves near fundamental frequency interact to produce second harmonic. Alternatively electron plasma waves can also be excited by parametric decay instability in which the incident electromagnetic wave decay into electron plasma wave and ion acoustic wave. The nonlinearity of ion acoustic wave may play an important role in the development of parametric instabilities leading to the excitation of ion acoustic waves. In particular, harmonic generation by a strong ion acoustic wave results in an increase of its effective dissipation and thus instability saturation. In various space and astrophysical plasmas, signatures if harmonic emissions have been reported. One of the possible scenario may be due to plasma wave coalescence [5,9]. In this paper we study generation of electromagnetic waves at $2\omega_p$ via mixing of plasma and ion acoustic waves. The electron velocity at $2\omega_p$, due to plasma wave couples with the density oscillations due to ion acoustics wave to produce nonlinear current at $2\omega_p$, which results in $2\omega_p$, emissions. In section we obtain electric field at $2\omega_p$, and in section 3 we discuss our results.

2. Second Harmonic Field. Consider the propagation of a large amplitude Langmuir wave,

$$\phi_p = A_0 e^{-i(\omega_0 t - \vec{k}_0 \cdot \vec{r})} \tag{1}$$

and an ion-acoustic wave

$$\phi = A e^{-i(\omega t - \vec{k} \cdot \vec{r})} \tag{2}$$

through a plasma of electron density n_0^0 and temperature Te. Here

$k_0 = \sqrt{\omega_0^2 - \omega_p^2} / \upsilon_{th}$, $k = \omega / c_s$, $\upsilon_{th} = k_B T_e / m_e$, $c_s = k_B T_e / m_i$, k_B is Boltzman constant,

$$\omega_p = \sqrt{4\pi n_0^0 - e^2 / m_e},$$

e and m_e being electronic charge and mass respectively and m_i is ion mass.

On solving equation of motion $m(d\vec{\upsilon}/dt) = -e\vec{E}$, electron velocity due to Langmuir wave, can be written as

$$\vec{\upsilon}_0 = \frac{e\vec{E}_0}{mi\omega_0} \quad , \vec{E}_0 = i\,\vec{k}_0\,\phi_0 . \tag{3}$$

On integrating Eq. (3) we obtain,

$$\vec{r} = \vec{r}_0 - e\vec{E}_0 / m\omega_0^2 , \tag{4}$$

where r_0 is the electron's position at $t = 0$. Using (4) in (1) and applying the identity

$e^{if \sin\theta} = \sum_n J_n (f) e^{in\theta}$, we get the plasma wave potential as

$$\phi_0 = \sum_n A_0 J_n(\alpha) e^{-i[(n+1)\omega_0 t - (n+1)k_0' r']} \tag{5}$$

where $\alpha = k_0 v_{osc} / \omega_0$.

For $n=1$, we obtain the potential of the plasma wave at second harmonic as,

$$\phi_2 \approx A_0 J_1 (\alpha) e^{-i(2\omega_0 t - k_0 r)} \tag{6}$$

Electron velocity at second harmonic due to ϕ_{02} and self consistent potential $\phi_2 = \phi_2 e^{-i(2\omega_0 t - k_0 r)}$ after solving the equation of motion is

$$\vec{v}_{02} = \frac{e\vec{k}_0}{m\omega_0} (\phi_{02} + \phi_2). \tag{7}$$

Using (7) in the equation of continuity $\partial n / \partial t + \nabla . (n\vec{v}) = 0$, we obtain the electron density oscillations at $(2\omega_0, 2\vec{k}_2)$ as

$$n_{02} = \frac{n_0^0 \vec{k}\, \vec{v}_{02}}{\omega_0} \tag{8}$$

Using (8) in the Poisson's equation $\nabla^2 \phi = 4\pi n e$, we get

$$\varepsilon_2 \phi_2 = \chi_{e2} \phi_{02} , \tag{9}$$

where $\chi_{e2} = -\omega_p^2 / 4\omega_0^2$, $\varepsilon_2 = 1 + \chi_{e2}$.

Using (9) in (7) we get,

$$\vec{v}_{02} = \frac{e\vec{k}_0 \phi_{02}}{m\omega_0 \varepsilon_2} . \tag{10}$$

Electron density perturbation due to ion acoustic wave is given by

$$n = n_0^0 \; e \, \phi / T_e . \tag{11}$$

From (10) and (11) we obtain the nonlinear current density at $2\omega_0 - \omega \approx 2\omega_p$ as

$$\vec{J}^{NL} = - n^* e\vec{v}_{02} . \tag{12}$$

Using (12) in the wave equation [10], we get

$$\nabla^2 \vec{E}_2 - \nabla (\nabla . \vec{E}_2) = \frac{4\pi i\omega_p}{c^2} n^* e\vec{v}_{02} - \frac{4\omega_p^2}{c^2} \vec{E}_2 - \frac{8\pi i\omega_p}{c^2} \vec{J}_2^L , \tag{13}$$

where, $\vec{J}_2^L$ is the linear component of current density. Taking ∇. of (13) we get

$$\nabla . \vec{E}_2 = \frac{(2\vec{k}_0 - \vec{k})}{\omega_p} n^* e\vec{v}_{02} , \tag{14}$$

Using (14) back in (13), we get

$$\nabla^2 \vec{E}_2 + \frac{3\omega_p^2}{c^2} \vec{E}_2 - \frac{4\pi i \omega_p}{c^2} \overset{*}{n} e \vec{v}_{02} - \frac{(2\vec{k}_0 - \vec{k})}{\omega_p} n^* e \vec{k}_0 \cdot \vec{v}_{02}. \tag{15}$$

Considering the variation of $\vec{E} = \vec{A}_2 (z) e^{-i[(2\omega_0 - \omega)t - (2\vec{k}_0 - \vec{k}).\vec{r}]}$ and applying the condition $(2\vec{k}_0 - \vec{k})^2 c^2 + \omega_p^2 = (2\omega_0 - \omega)^2$ in (15), we obtain

$$c^2 \frac{\partial A_2}{\partial z} = \frac{2\pi\omega_p n^* e v_{02}}{(2k_{0z} - k_z)} + \frac{n^* e \vec{k}_0 \cdot \vec{v}_{02} c^2}{2\omega_p}, \tag{16}$$

$$A_2 = J_1(\alpha) \frac{2\omega_p^3 v_{02} k_z}{c^2 k_z^2 v_{th}^2} \phi^*. \tag{17}$$

In Fig.(l) we have plotted the efficiency of the power coupling with the parameter α.

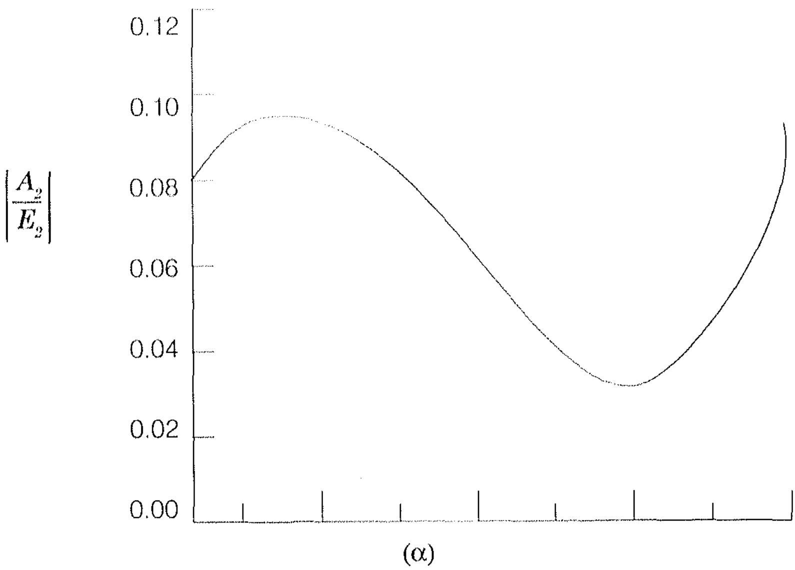
3. Results and discussions. The plasma wave on coupling with the ion acoustic wave can produce signatures of electromagnetic radiations at $2\omega_p$ frequencies. The efficiency of the process shows a peak around $\alpha =$ 2 This may be one of the mechanisms observed in solar radio bursts. This mechanism could also be employed as a diagnostic tool in laboratory produced plasmas and also in the generation of high frequency waves. The scheme can be extended to study generation of higher harmonics also.

We can express Eq.(17) in terms of I-function of one variable[10], an extension of Mijer's G-function, Fox,s H-function of one variable.

$$A_2 = \frac{2\omega_p^3}{c^2 k_z^2} \frac{v_{02} k_z}{v_{th}^2} \phi^* I^{1,0}_{0,2:1} \left[\frac{\alpha^2}{4} \middle| \left(\frac{1}{2}, 1\right), \left(-\frac{1}{2}, 1\right) \right]. \tag{18}$$

Figure Captions

Figure 1 : Effciency of the process variation with α.



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