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(Dedicated to Professor H. M. Srivastava on his 62nd Birthday)

EFFECTS OF MAGNETIC FIELD ON FREE CONVECTION MASS TRANSFER FLOW THROUGH POROUS MEDIUM WITH RADIATION AND VARIABLE PERMEABILITY IN SLIP FLOW REGIME*

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ABSTRACT

An analysis for unsteady free convection flow with radiation and mass transfer through a porous medium of variable permeability and transverse magnetic field, bounded by an infinite porous vertical plate in slip flow regime is presented. The permeability of the porous medium decreases exponentially with time about a constant mean. Approximate solutions have been obtained for the velocity, temperature, skin friction and rate of heat transfer. The effects of permeability parameter (K_0), magnetic parameter (M), rerefaction parameter (h_1) Grashof number (G_r), modified Grashof number (G_m), real constant (n), time (t), Prandtl number (P_r), Schmidt number (S_c), radiation parameter (R) and ε, on the velocity field, temperature field, skin friction and the rate of heat transfer are shown graphically and are discussed numerically.

1. Introduction. The study of the flow of a porous medium is of great importance to geophysicists and fluid dynamicists. Yamamoto and Yoshida [13] considered suction and injection flow with convective acceleration through a plane porous wall specifically for the flow outside a vortex layer. The generalisation of the above study was presented by Yamamoto and Iwamura [12]. Chawla and Singh[3] studied oscillatory flow past a porous bed. The effect of variable permeability on combined free and forced convection in porous media have studied by Chandrasekhara and Namboodiri [2], Vedhanayagam et at. [11]. Singh et al. [9], Singh and Kumar [7], Acharya et al. [1] have studied the effects of permeability variation on free convective flow through a porous medium. Recently, the effect of radiation and magnetic field on the free convective flow along infinite vertical plate have studied by Takhar et al. [10], Maharshi and Tak [5].

In geothermal region situation may arise when the flow becomes unsteady and slip at the boundary take place as well. In such situation of slip flow ordinary continuum approach fails to yield satisfactory results. Many authors have solved

Key words. *Free convection , Mass transfer, Porous medium, Radiation, Unsteady, Slip flow region.*

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problems taking slip conditions at the boundary (Singh [8]).

The object of this paper is to discuss the unsteady free convection flow through a porous medium of variable permeability with radiation, mass transfer and transverse magnetic field in slip flow regime. The permeability of porous medium decreases exponentially with time about a constant mean. The effects of different parameters entering into the problem *viz.*, K_0 (Permeability parameter), M (magnetic parameter), h_l (rarefaction parameter), G_r (Grashof number), G_m (modified Grashof number), n (real constant), t (time), P_r (Prandtl number), S_c (Schmidt number), R (radiation parameter) and ε, on the velocity field, temperature field, skin friction and the rate of transfer are shown graphically and are discussed numerically.

2. Formulation and Solution of the Problem. We consider two dimensional, unsteady, hydromagnetic, free convection with radiation and mass transfer flow, of a viscous incompressible and electrically conducting fluid, though a porous medium of variable permeability, occupying a semi-infinite region of the space bounded by an infinite vertical porous plate with constant suction in slip flow regime. A magnetic field of uniform strength is applied transversely to the direction of the flow. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected. We take the x-axis along the plate and y-axis normal to it and the flow in the medium is entirely due to buoyancy force caused by temperature difference between the wall and the fluid. The viscous dissipation and Darcy's dissipation terms are neglected for small velocities [6].

Under these conditions the flow equations can be written as:

$$\frac{\partial v}{\partial y} = 0 \qquad \text{...(1)}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + g\beta^*(C - C_\infty) + \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\upsilon}{K(t)}u - \frac{\sigma B_0^2}{\rho}u \qquad \text{...(2)}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \qquad \text{...(3)}$$

and

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}, \qquad \text{...(4)}$$

where u and v are components of velocity along x and y directions, g the acceleration due to gravity, ρ the density, β the coefficient of volume expansion, β^* the volumetric coefficient of expansion with concentration, T the temperature, C the concentration, υ the kinematic viscosity, k the thermal conductivity, C_p the specific heat of the fluid at constant pressure, σ the electrical conductivity, B_0 the megnetic field, K the permeability of the porous medium, q_r the radiative heat flux and D the coefficient of chemical molecular diffusivity.

The permeability of the porous medium is assumed to be of the form

$$K(t) = K_0 (1+\varepsilon e^{-nt}) \qquad ...(5)$$

where K_0 is the mean permeability of the medium, n the real constant, t the time and $\varepsilon(<<1)$ is a constant quantity.

The radiative heat flux q_r is given by [4]:

$$\frac{\partial q_r}{\partial y} = 4(T - T_\infty)I \qquad ...(6)$$

where $I = \int_0^\infty k_{\lambda n} \cdot \frac{\partial e_{b\lambda}}{\partial T} d\lambda, k_{\lambda n}$ is the absorption coefficient at the wall and $e_{b\lambda}$ is Plank function.

The boundary conditions are :

$$\left.\begin{aligned} & u = L_1\left[\frac{\partial u}{\partial y}\right], T = T_w, C = C_w \; at \; y = 0 \\ & u \to 0, \; T \to T_\infty, C \to C_\infty \; as \; y \to \infty \end{aligned}\right\} \qquad ...(7)$$

where $L_1 = \left[\frac{2-m_1}{m_1}\right] L, L$ being the mean free path and m_1 the Maxwell's reflection coefficient.

The continuity equation (1) gives

$$v = -v_0 \qquad ...(8)$$

where $v_0>0$ is the constant suction velocity at the plate.

We introduce the following non-dimensional quantities:

$$y^* = \frac{yv_0}{v}, \; t^* = \frac{v_0^2 t}{4v}, \; n^* = \frac{4vn}{v_0^2}, \; u^* = \frac{u}{v_0}, \; \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$$C^* = \frac{C - C_\infty}{C_w - C_\infty}, \; G_r = \frac{vg\beta(T_w - T_\infty)}{v_0^3}, \; G_m = \frac{vg\beta^*(C_w - C_\infty)}{v_0^3},$$

$$K_0^* = \frac{K_0 v_0^2}{v^2}, \; P_r = \frac{\mu C_p}{k}. \; R = \frac{4vl}{\rho C_p v_0^2}, \; S_c = \frac{v}{D}, \; M^2 = \frac{\sigma B_0^2 v}{\rho v_0^2}, \; h_1 = \frac{L_1 v_0}{v}.$$

The equations (2) to (4) in view of (5) and (6) in non-dimensional form after dropping the asterisks over them reduce to :

$$\frac{1}{4}\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=G_r\theta+G_mC+\frac{\partial^2 u}{\partial y^2}-\left[M^2+1/\left\{K_0\left(1+\varepsilon e^{-nt}\right)\right\}\right]u \quad ...(9)$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t}-\frac{\partial \theta}{\partial y}=\frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2}-R\theta \quad ...(10)$$

$$\frac{1}{4}\frac{\partial C}{\partial t}-\frac{\partial C}{\partial y}=\frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} \quad ..(11)$$

with corresponding boundary conditions

$$\left.\begin{array}{l} u=h_1\left[\dfrac{\partial u}{\partial y}\right],\theta=1,C=1\ at\ y=0 \\ u\to 0,\ \theta\to 0,C\to 0\ as\ y\to\infty \end{array}\right\} \quad ...(12)$$

The partial differential equations (9) to (11) are reducedd to ordinary one by assuming the following expressions for velocity, temperature and concentration

$$u\ (y,t)\ =\ u_0(y)+\varepsilon\, e^{-nt}u_1(y) \quad ...\ (13)$$

$$\theta\ (y,t)\ =\ \theta_0(y)+\varepsilon\, e^{-nt}\theta_1(y) \quad ...\ (14)$$

$$C\ (y,t)\ =\ C_0(y)+\varepsilon\, e^{-nt}C_1(y) \quad ...\ (15)$$

Substituting equations (13) to (15) in equations (9) to (11) and equating the coefficients of like powers of ε , the following set of ordinary differential equations are obtained:

$$u_0''+u_0'-\left[M^2+1/K_0\right]u_0=-G_r\theta_0-G_mC_0 \quad ....(16)$$

$$u_1''+u_1'-\left[M^2+1/K_0-n/4\right]u_1=-G_r\theta_1-G_mC_1-u_0/K_0 \quad ...(17)$$

$$\frac{1}{P_r}\theta_0''+\theta_0'-R\theta_0=0 \quad ...(18)$$

$$\frac{1}{P_r}\theta_1''+\theta_1'-[R-n/4]\theta_1=0 \quad ...(19)$$

$$C_0''+S_cC_0'=0 \quad ...(20)$$

$$C_1''+S_cC_1'+n/4\,S_cC_1=0 \quad ...(21)$$

with corresponding boundary conditions

$$\left.\begin{array}{l} u_0=h_1u'_0,\ u_1=h_1u'_1,\ \theta_0=1,\ \theta_1=0,\ C_0=1,\ C_1=0,\ at\ y=0 \\ u_0\to 0,\ u_1\to 0,\ \theta_0\to 0,\ \theta_1\to 0,\ C_0\to 0,\ C_1\to 0,\ as\ y\to\infty \end{array}\right\} \quad ...(22)$$

where primes denote differentiation with respect to y.

Equations (16) to (21) are second order linear differential equations with constant coefficients, the solutions of which are straight forward. The solutions of (16) to (21), satisfying the boundary conditions (22), are substitued in equations (13) to (15), the give

$$u(y,t) = A\,e^{-R_1 y} - \frac{G_r}{L_1} e^{-ay} - \frac{G_m}{L_2} e^{-S_c y} + \frac{\varepsilon e^{-nt}}{K_0}$$

$$\left[BK_0 e^{-R_2 y} - \frac{A}{L_3} e^{-R_1 y} + \frac{G_r e^{-ay}}{L_1(L_1 + n/4)} + \frac{G_m e^{-S_c y}}{L_2(L_2 + n/4)} \right] \qquad ...(23)$$

$$\theta(y,t) = e^{-ay} \qquad ...(24)$$

where

$$L_1 = a(a-1) - \left[M^2 + 1/K_0\right]$$

$$L_2 = S_c(S_c - 1) - \left[M^2 + 1/K_0\right]$$

$$L_3 = R_1(R_1 - 1) - \left[M^2 + 1/K_0 - n/4\right]$$

$$a = \frac{P_r + \sqrt{P_r^2 + 4P_r R}}{2}$$

$$R_1 = \frac{1 + \sqrt{1 + 4\left[M^2 + 1/K_0\right]}}{2}$$

$$R_2 = \frac{1 + \sqrt{1 + 4\left[M^2 + 1/K_0 - n/4\right]}}{2}$$

$$A = \frac{1}{(1 + h_1 R_1)} \left[\frac{G_r}{L_1}(1 + a h_1) + \frac{G_m}{L_2}(1 + S_c h_1) \right]$$

$$B = \frac{1}{K_0(1 + h_1 R_2)} \left[\frac{A}{L_3}(1 + h_1 R_1) - \frac{G_r(1 + a h_1)}{L_1(L_1 + n/4)} - \frac{G_m(1 + S_c h_1)}{L_2(L_2 + n/4)} \right].$$

From equation (23) we calculate the skin friction

$$\tau = -AR_1 + \frac{G_r a}{L_1} + \frac{G_m S_c}{L_2} + \frac{\varepsilon e^{-nt}}{K_0}$$

$$\left[-BR_2K_0 + \frac{AR_1}{L_3} - \frac{G_r a}{L_1(L_1+n/4)} - \frac{G_m S_c}{L_2(L_2+n/4)}\right] \quad ...(25)$$

From equation (24) we calculate the rate of heat transfer in terms of Nusselt number. Thus

$$Nu = \frac{P_r + \sqrt{P_r^2 + 4P_r R}}{2} \quad ...(26)$$

3. Discussion and Conclusion. In order to understand the physical solution, we have calculated the numerical values of the velocity distribution [*Fig. 1 and Fig. 2*], skin friction at the plate [*Fig. 3 and Fig. 4*], temperature distribution [*Fig.* 5] and rate of heat transfer at the plate [*Fig. 6*] for different values of K_0 (permeability parameter), M (magnetic parameter), h_1 (rarefaction parameter), G_r (Grashof number), G_m (modified Grashof number), n (real constant), t (time), P_r (Prandtl number), S_c (Schmidt number), R (radiation parameter) and ε.

In *figure* 1 the velocity distribution is plotted against y for fixed values of $\varepsilon=0.01$, $n=1.0$, $t=0.1$, $P_r=0.71$, $S_c=0.6$, $R=1.0$ and different values of K_0, h_1, M, G_r and G_m. It is being observed that when M and G_m are increased velocity is decreased but the phenomena reverses for the case of K_0, h_1, and G_r.

In *figure 2* the velocity distribution is plotted against y for fixed values of $\varepsilon=0.01$, $K_0=1.0$, $h_1=0.4$, $M=0.2$, $G_r=5.0$, $G_m=2.0$ and different values of n, t, P_r, S_c and R. It is being observed that when n, t, P_r, S_c and R are increased velocity is decreased.

In *figure 3* the skin friction is plotted against M for fixed values of $\varepsilon=0.01$, $n=1.0$, $t=0.1$, $P_r=0.71$, $S_c=0.6$, $R=1.0$ and different values of K_0, h_1, G_r and G_m. It is being observed that when h_1 is increased skin friction is decreased but the phenomena reverses for the case of K_0, G_r and G_m.

In *figure 4* the skin friction is plotted against M for fixed values of $\varepsilon=0.01$, $K_0=1.0$, $h_1=0.4$, $G_r=5.0$, $G_m=2.0$, $R=1.0$ and different values of n, t, P_r, S_c and R. It is being observed that when n, t, P_r, S_c and R are increased skin friction is decreased.

In *figure 5* the temperature distribution is plotted against y for different values of P_r and R. It is being observed that when P_r and R are increased temperature is decreased.

In *figure 6* the rate of heat transfer is plotted against R for different values of P_r. It is being observed that when P_r is increased rate of heat transfer is increased.

$\varepsilon=0.01,\ n=1.0,\ t=0.1,\ P_r=0.71,\ S_c=0.6,\ R=1.0$

NO	K_0	h_1	M	G_r	G_m
1	1.0	0.4	0.2	5.0	2.0
2	1.2	0.4	0.2	5.0	2.0
3	1.0	0.6	0.2	5.0	2.0
4	1.0	0.6	0.5	5.0	2.0
5	1.0	0.6	0.5	7.0	2.0
6	1.0	0.6	0.5	7.0	3.0

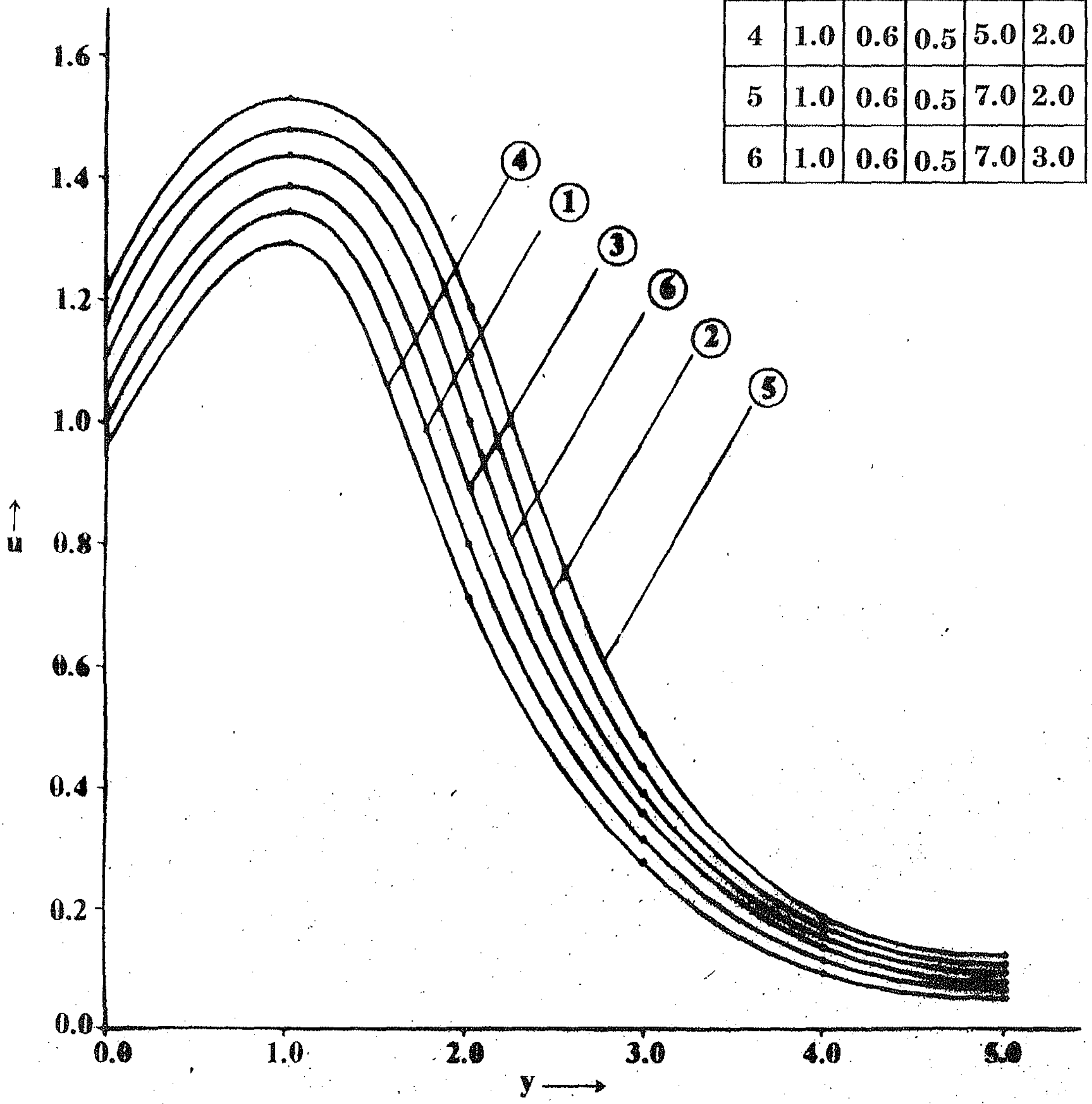


Fig. 1 Velocity distribution u plotted against y for different values of K_0, h_1, M, G_r and G_m

$\varepsilon=0.01, K_0=1.0, h_1=0.4, M=0.2, G_c = 5.0, G_m = 2.0$

NO	n	t	P_r	S_c	R
1	1.0	0.1	0.71	0.6	1.0
2	2.0	0.1	0.71	0.6	1.0
3	1.0	0.2	0.71	0.6	1.0
4	1.0	0.2	2.0	0.6	1.0
5	1.0	0.2	2.0	0.8	1.0
6	1.0	0.2	2.0	0.8	2.0

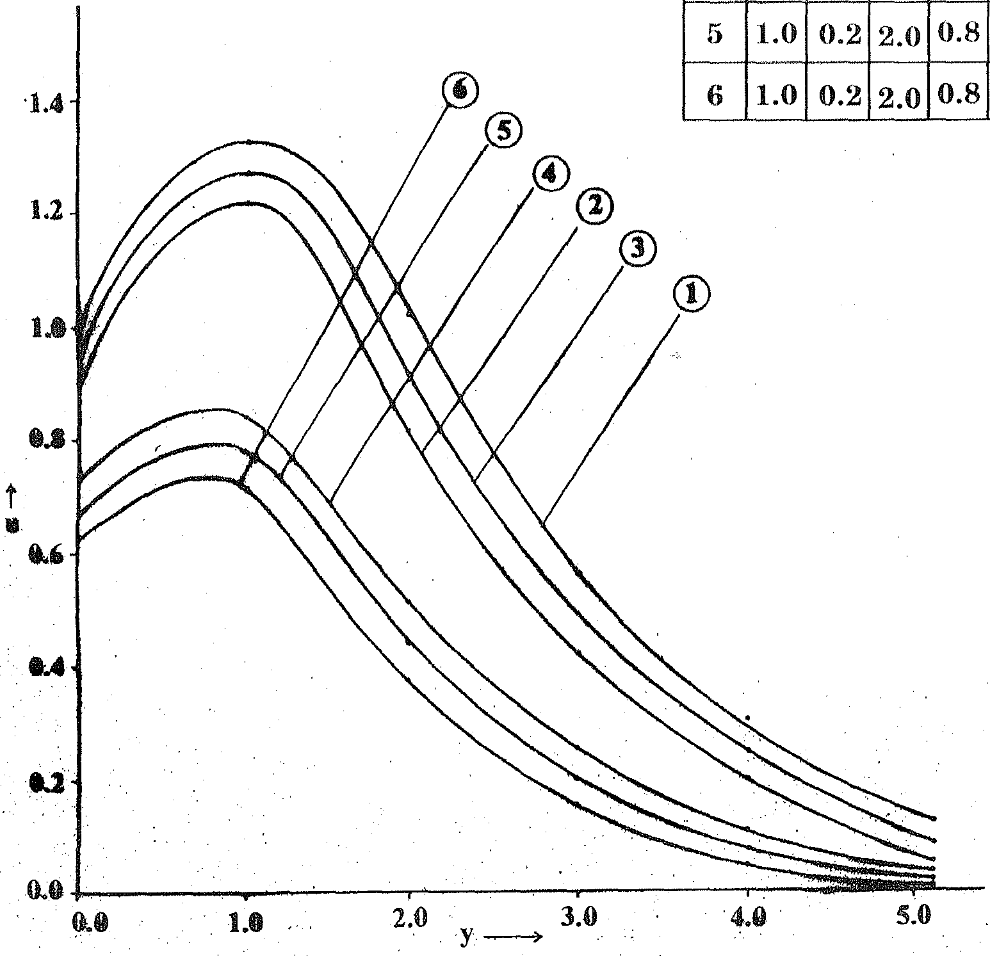


Fig. 2 Velocity distribution u plotted against y for different values of n, t, P_r, S_c and R.

$\varepsilon=0.01,\ n=1.0,\ t=0.1,\ P_r=0.71,\ S_c=0.6,\ R=1.0$

NO	K_0	h_1	G_r	G_m
1	1.0	0.4	5.0	2.0
2	1.2	0.4	5.0	2.0
3	1.0	0.6	5.0	2.0
4	1.0	0.6	7.0	2.0
5	1.0	0.6	7.0	3.0

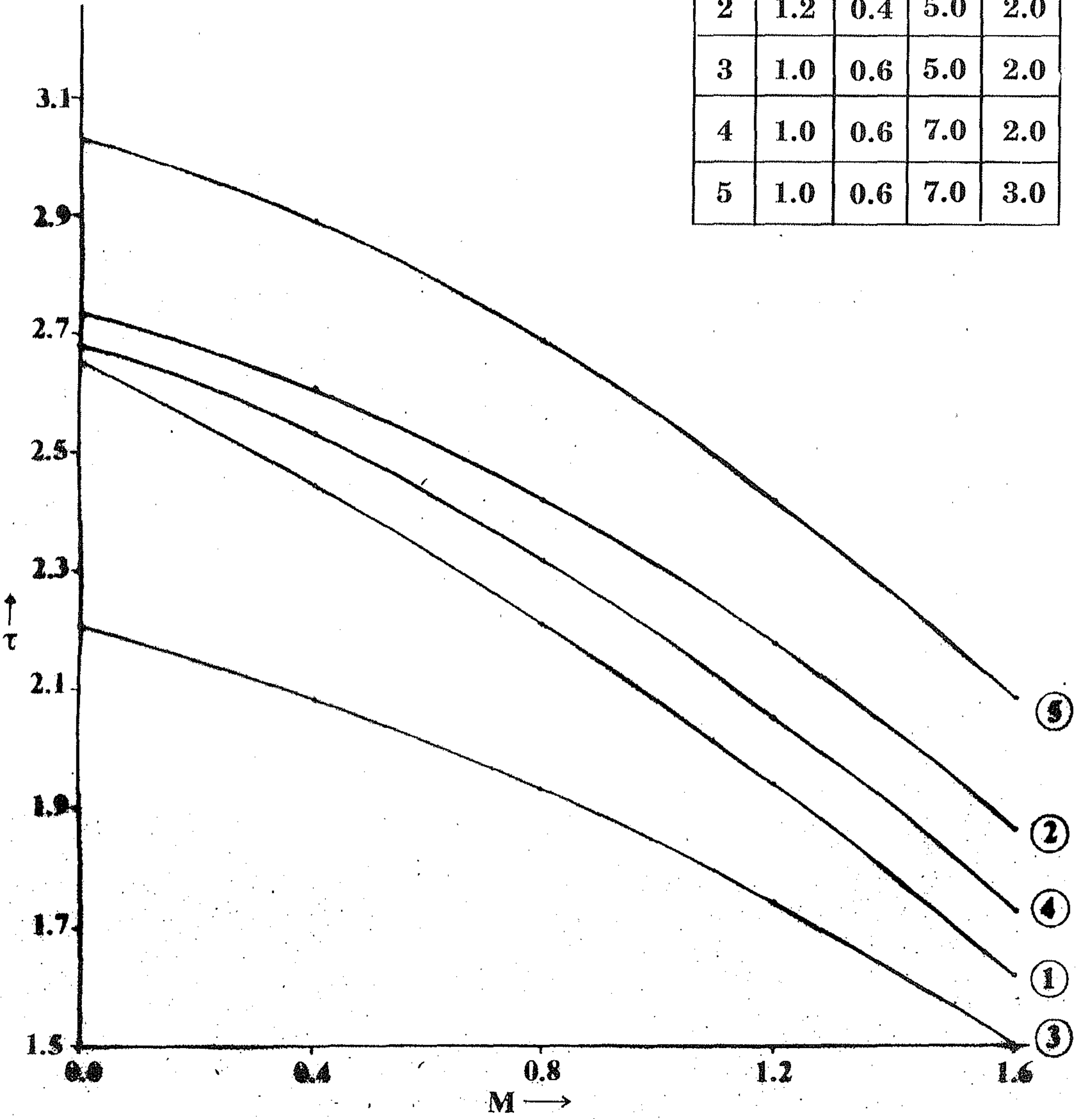
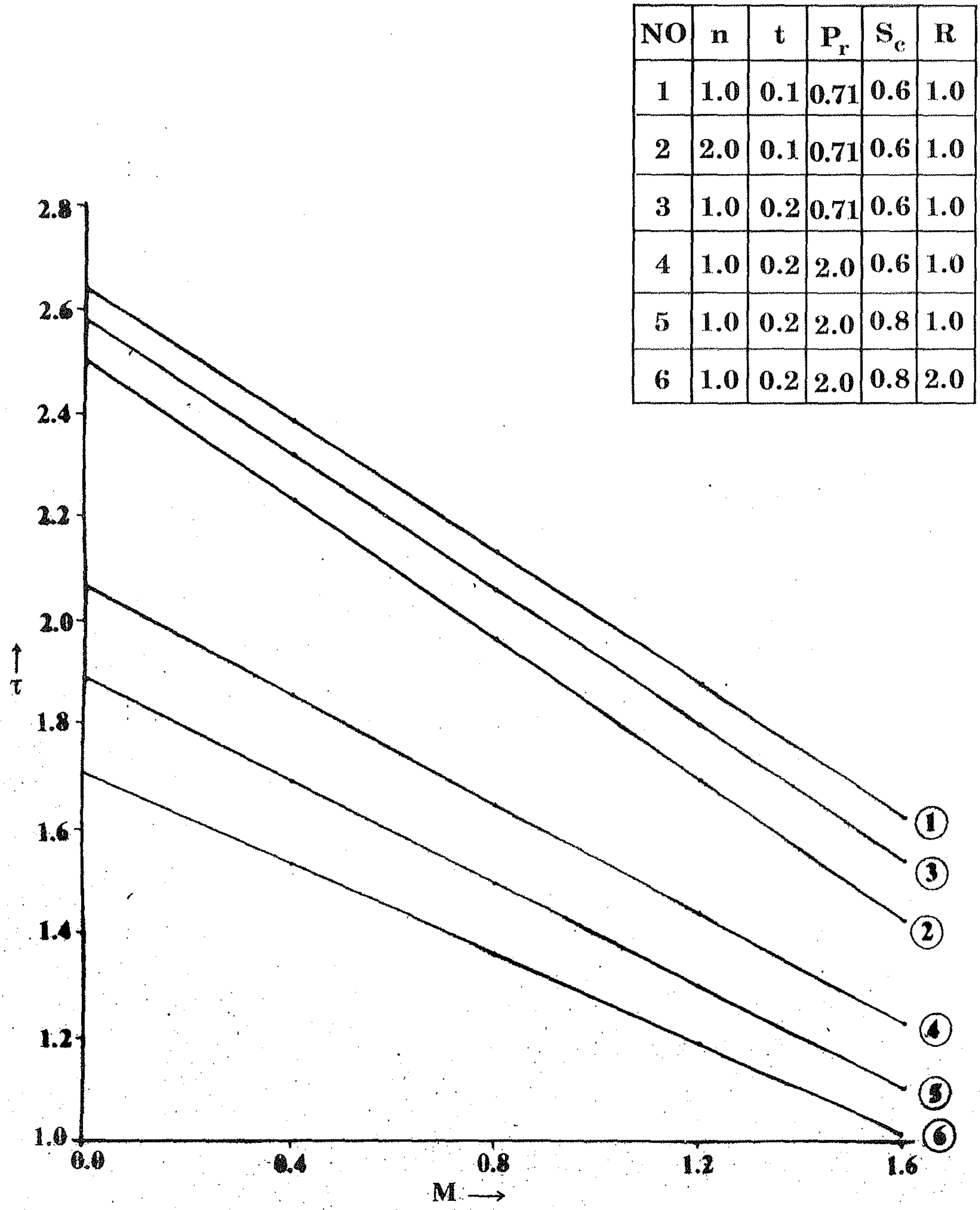


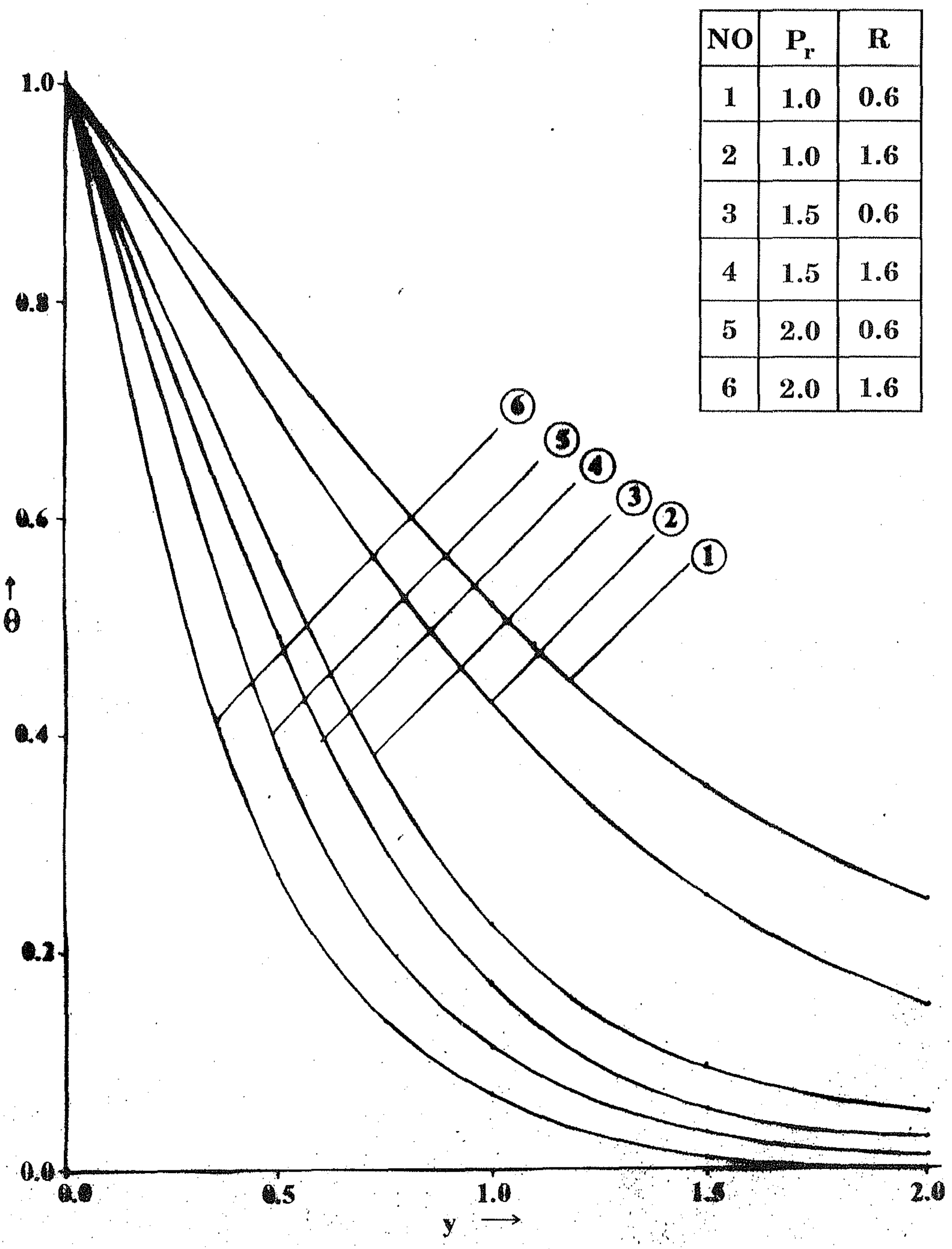
Fig. 3 Skin friction τ plotted against M for different values of K_0, h_1, G_r and G_m.

$\varepsilon=0.01,\ K_0=1.0,\ h_1=0.4,\ G_r = 5.0,\ G_m = 2.0$



NO	n	t	P_r	S_c	R
1	1.0	0.1	0.71	0.6	1.0
2	2.0	0.1	0.71	0.6	1.0
3	1.0	0.2	0.71	0.6	1.0
4	1.0	0.2	2.0	0.6	1.0
5	1.0	0.2	2.0	0.8	1.0
6	1.0	0.2	2.0	0.8	2.0

Fig. 3 Skin friction τ plotted against M for different values of n, t, P_r, S_c and R.



NO	P_r	R
1	1.0	0.6
2	1.0	1.6
3	1.5	0.6
4	1.5	1.6
5	2.0	0.6
6	2.0	1.6

Fig. 5 Temperature θ plotted against y for different values of P_r, and R.

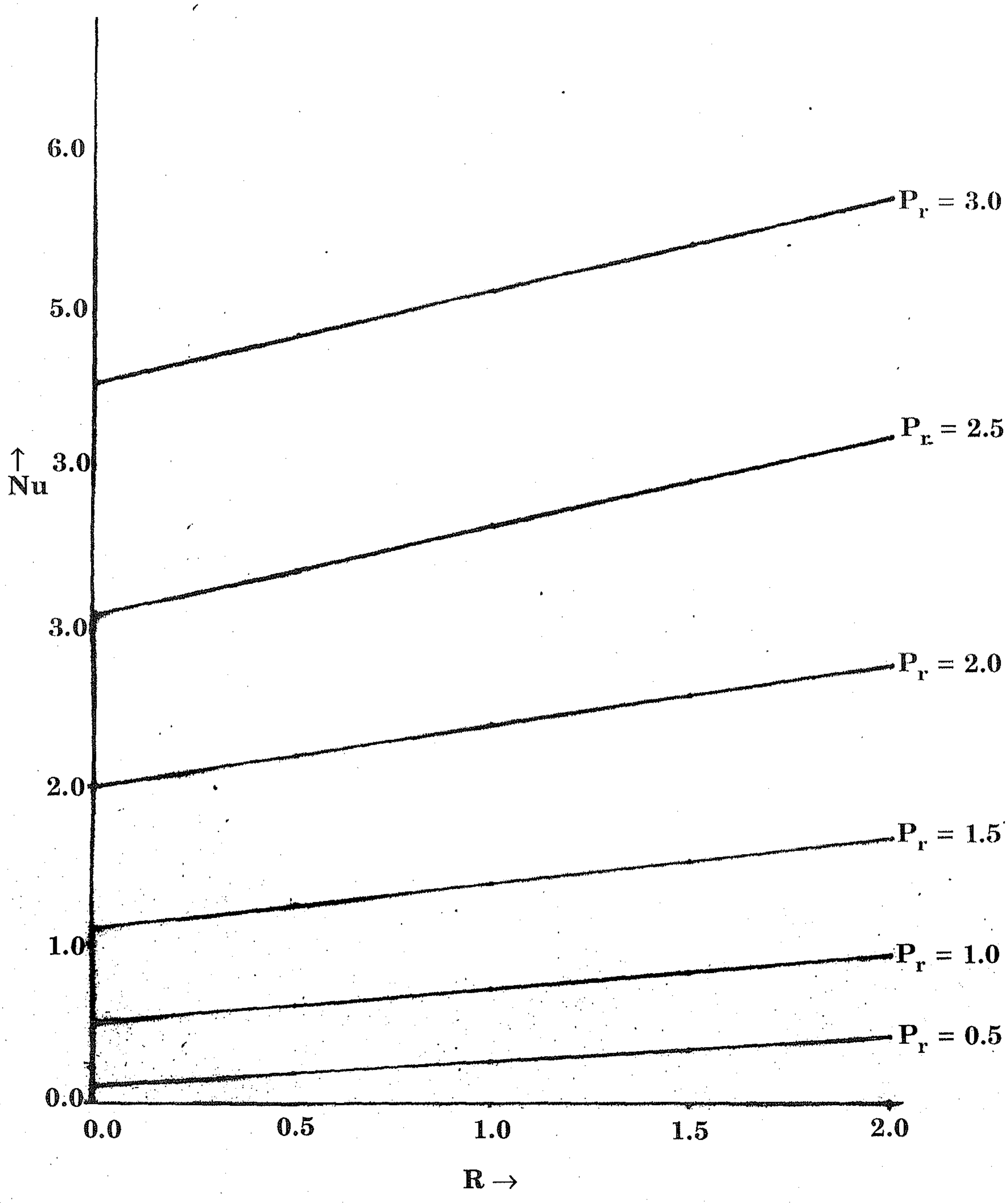


Fig. 6 Rate of heat transfer N_u plotted against R for different values of Pr.

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