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(Dedicated to Professor H. M. Srivastava on his -62nd Birthday)

ON PREEMPTIVE RESUME VERSUS NON-PREEMPTIVE DISCIPLINES RELEVANT TO MONOPOLY SERVICE FACILITY

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ABSTRACT

In the present study the expected sojourn times of the n-th customer in the preemptive-resume discipline and non-preemptive discipline have been compared with the help of computer programs and graphs and interesting conclusions have been drawn.

1. Introduction. The Queue discipline is the order or manner in which customers from the queue are selected for service. There are number of ways in which customers in the queue are served. If the customers are taken into service in order of their arrival then this is known as the first come, first-served (*FCFS*) service discipline. The *FCFS* discipline is not always applicable in many service systems and customers are classified according to different priorities. Under this rule customers are grouped in priority classes on the basis of some attributes such as, service time or urgency, and *FCFS* rule used within each class to provide service. Priority service may be classfied as preemptive priority and nonpreemptive priority. Under preemptive priority rule. the highest priority customer is allowed to enter into the service immediately after entering into the system even if a customer with lower priority is already in service. That is, lower priority cutomer's service is interrupted (preempted) to start service for a special customer. This interrupted service is resumed again after the highest priority customer is served whereas in non-preemptive priority case highest priority customer goes ahead in the queue, but service is started immediately on completion of the current service.

Suppose that all customer arrivals can be classfied according to different priority classes indexed in $(1,2,...,k)$ so that $i < j$ implies that type-i customers have higher priority over type-j customers. Class-i is said to have nonpreemptive (or head-of-the line) priority over class-j, if an arriving type-i customer joins the queue at the head of the line among type-j customers. Thus the servicing of a type-j customer is not interrupted by the arrival of a type-i customer.

Class-i is said to have preemptive priority over class-j, if an arriving type-i customer not only joins the queue at the head of the line among type-j customers. but also displaces a type-j customer, who may be in service. The serivicing of a customer. who is preempted from service may continue from the point of the interruption. This rule is called preemtpive-resume, or the server may have to restart the service from the beginning, in which case we have the preemptive repeat rule. If the duration of the repeated service is identical to the initial one, this is called the preemptive-repeat identical rule, otherwise, if the repeated s service duration is different, the priority rule is called preemptive repeat different. Priority queues for a along time have been a subject of investigation in the area of queueing theory. Now-a-days, a trend of quantification of priority values in priority queues is predominantly prevailing in order to determine job scheduling in the service system. We may refer to some relevant researches, which have been done by previous authors, vide for example, [1], [2], [3], [4], [5], [6].

In a recent investigation, Pandey [7] has proved an interesting theorem on the control limit of the low priority class.

In the present study the expected Sojourn times of the n-th customers $S(n)$ in the preemptive resume discipline and non-preemptive discipline have been compared with the help of computer programs and interesting conclusions have been drawn.

2. Main Results. Let $S(n)$ denote the expected sojourn time of the n-th (the last) customer in the low priority class. Aldiri and Yechiali [2] presented this representation in a recursive form. Hanna [1] simplified their representation in the following form :

$$S(1)=\frac{1}{\mu}+\frac{\rho}{\mu(1-\rho)}$$

$$S(2)=\frac{2}{\mu}+\frac{\rho}{\mu(1-\rho)}$$

$$S(n)=\frac{n}{\mu}+\frac{\rho}{\mu(1-\rho)}\left[1+\sum_{q=0}^{n-2}\frac{\rho^q}{(1+\rho)^{2q}}\sum_{m=0}^{n-q-1}\frac{m\rho^m}{(m+q)(1+\rho)^m}\binom{m+2q-1}{q}\right], n\geq 2.$$

For the non preemptive resume case

$$S(1)=1/\mu$$

$$S(2)=2/\mu+\rho/\mu(1-\rho)$$

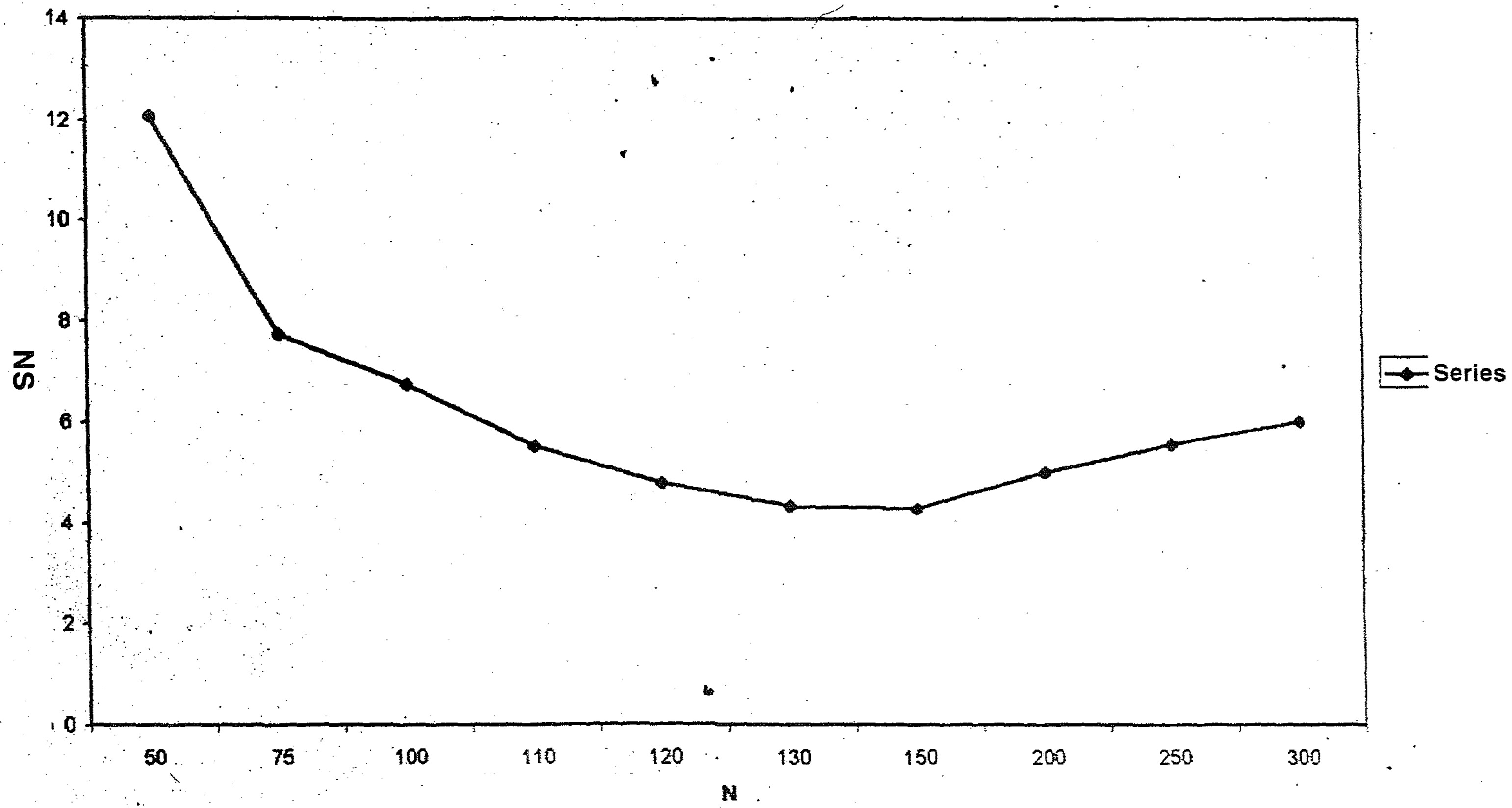
$$S(n)=\frac{n}{\mu}+\frac{\rho}{\mu(1-\rho)}\left[1+\sum_{q=0}^{n-3}\frac{\rho^q}{(1+\rho)^{2q}}\sum_{m=1}^{n-q-2}\frac{m\rho^m}{(m+q)(1+\rho)^m}\binom{m+2q-1}{q}\right], n\geq 3$$

PREEMPTIVE

λ	μ	n	S_n	S_1	S_2
1	5	50	12.0634110046	.25000	.45000
1	10	75	7.72741529	.111111	.211111
1	15	100	6.7326101956	.07142857	.1380952
1	20	110	5.52786027	.05263151	.102631
1	25	120	4.814413465	.0416666	.0816666
1	30	130	4.341796100	.034482758	.06781609
1	35	150	4.29113429	.0294117647	.0579831932
1	40	200	5.0036992577	.0256410256	.05064102564
1	45	250	5.558203172	.02272727	.0449494949
1	50	300	6.001967032	.0204081632	.0404081632

```
C     PROGRAM TO CALCULATE THE VALUE FOR SN
      SUBROUTINE FACT (N, IFN)
      J= 1
      DO 10 I=1, N
      J=J*I
10    CONTINUE
      IFN =J
      RETURN
      END
      INTEGER Q, M, N, FR, FN, FNR
      DOUBLE PRECISION LEM, MU, NCR, MQ, S1, S2, SQ, SM, R, RO
      READ (*,*) LEM, MU
      RO=LEM/MU
      S1=(1.0/MU)
      S2=(2.0/MU)+(RO/(MU*(1.0-RO)))
      READ (*,*)N
      MQ=M+(2*Q)-1
      SQ =0
      SM =0
      DO  1Q = 0, N-3
      SQ=SQ+((RO**Q)/((1+RO)**(2*Q)))
      DO 2 M = 1, N–Q–2
      CALL FACT (M+2*Q)-1, FN)
      CALL FACT (Q, FR)
      CALL FACT (M+(2*Q)–1–Q, FNR)
      NCR=FN/(FR*FNR)
      SM=SM+(M*RO**M)/((M+Q)*(1+RO)**M)*NCR
2     CONTINUE
1     CONTINUE
      SN=(N/MU)+((RO/(MU*(1–RO)))*(1+SQ*SM))
      WRITE (*,*) SN
      WRITE (*,*) S1,S2
      STOP
      END
```

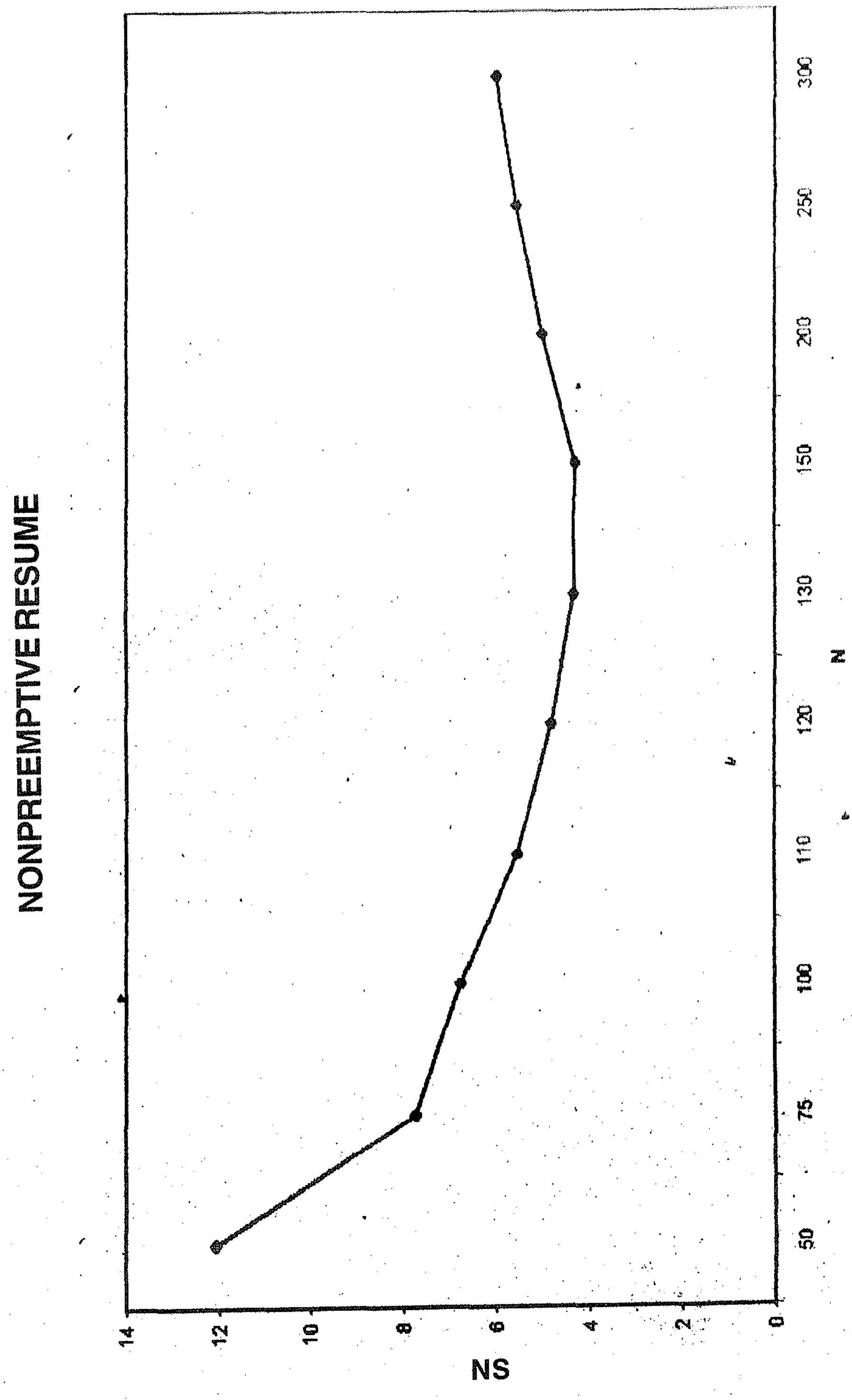
PREEMPTIVE RESUME



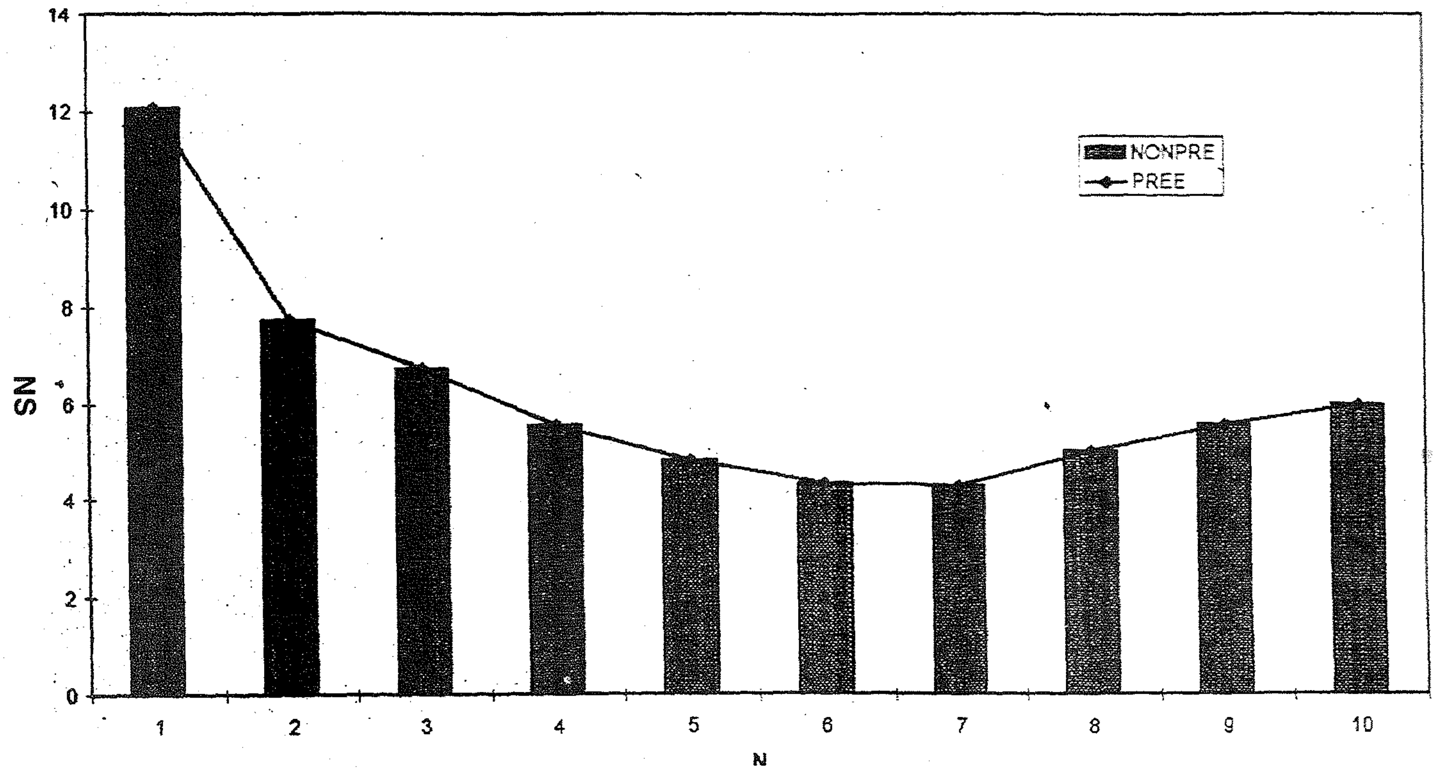
NON - PREEMPTIVE

λ	μ	n	S_n	S_1	S_2
1	5	50	12.06312660	.20000	.450000
1	10	75	7.72739728	.10000	.211111
1	15	100	6.73260656	.066666	.1380952
1	20	110	5.527858943	.050000	.10263157
1	25	120	4.8144128607	.040000	.0816666
1	30	130	4.341795784125	.03333	.06781609
1	35	150	4.29113341202	.02857142	.0579831
1	40	200	5.0036991731	.0250000	.0506410256
1	45	250	5.5582031258	.02222	.04494949
1	50	300	6.001967004	.020000	.0404081632

```
C       PROGRAM TO CALCULATE THE VALUE FOR SN
        SUBROUTINE FACT (N, IFN)
        J= 1
        DO 10 I=1, N
        J=J*I
10      CONTINUE
        IFN =J
        RETURN
        END
        INTEGER Q, K, N, FR, FN, FNR
        DOUBLE PRECISION LEM, MU, NCR, MQ, S1, S2, SQ, SN, SM, R, RO
        READ (*,*) LEM, MU
        RO=LEM,MU
        S1=(1.0/MU)+(RO/(MU*(1.0-RO)))
        S2=(2.0/MU)+(RO/(MU*(1.0-RO)))
        READ (*,*)N
        MQ=M+(2*Q)-1
        SQ =0
        SM =0
        DO  1Q = 0, N-2
        SQ=SQ+((RO**Q)/((1+RO)**(2*Q)))
        DO 2 M = 1, N–Q–1
        CALL FACT (M+(2*Q)-1, FN)
        CALL FACT (Q, FR)
        CALL FACT (M+(2*Q)–1–Q, FNR)
        NCR=FN/(FR*FNR)
        SM=SM+(M*RO**M)/((M+Q)*(1+RO)**M)*NCR
2       CONTINUE
1       CONTINUE
        SN=(N/MU)+((RO/(MU*(1–RO)))*(1+SQ*SM))
        WRITE (*,*) SN
        WRITE (*,*) S1,S2
        STOP
        END
```


NONPREEMPTIVE RESUME
NS
14
12
10
8
6
4
2
0
N
50
75
100
110
120
130
150
200
250
300

COMPARISION BETWEEN PREEMPTIVE & NONPREEMPTIVE RESUME



3. Discussion and Conclusion. Assigning various values to n, A and μ the recursive representations (the expected sojourn times of the n^{th} customer denoted by $S(n)$ in the low priority class) for the preemptive resume discipline and non-preemptive resume case have been computed.

Figure-1 illustrates the possible shapes of the prevalence of sojourn times in preemptive resume, while figure-2 depicts the possible shapes of the prevalence of sojourn times in non-preemptive resume. With the increase in the number of customers, sojourn times in both disciplines decrease. A comparison of two disciplines (Figure-3) shows that sojourn times in non-preemptive discipline are less than sojourn times in preemptive discipline. In case of fewer customers main characteristics of graphs rapidly change and sojourn times in both disciplines be almost but not exactly the same.

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