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(Dedicated to Professor H. M. Srivastava on his 62nd Birthday)

THERMOSOLUTAL CONVECTION IN WALTERS' (MODEL B') ROTATING FLUID IN POROUS MEDIUM IN HYDROMAGNETICS

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ABSTRACT

The thermosolutal convection in Walter's (model B') fluid in porous medium is considered in the presence of uniform vertical magnetic field and uniform vertical rotation. For the case of stationary convection, the stable solute gradient and rotation have stablizing effects on the system. In the presence of rotation, the medium permeability has a destablizing (or stablizing) effect and magnetic field has stablizing (or destablizing) effect on the system, whereas, in the absence of rotation, medium permeability and magnetic field have destablizing and stablizing effects respectively, on the system. The kinematic viscoelasticity has no effect for stationary convection. The kinematic viscoelasticity, rotation, stable solute gradient and magnetic field introduce oscillatory modes in the system, which were non-existent in their absence. The sufficient conditions for the non-existence of overstability are also obtained.

1. Introduction. A comprehensive account of thermal instability (Bénard convection) in a fluid layer under varying assumptions of hydromagnetics, has been summarized in the celebrated monograph by Chandrasekhar [2]. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by Veronis [11]. The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of single component fluid and rigid boundaries and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries. The problem is

Key words Thermosolutal convection, Walters' (model B') fluid, porous medium, magnetic field, rotation.

of great importance because of its application to atmospheric physics and astrophysics, especially in the case of the ionosphere and the outer layer of the atmosphere. The thermosolutal convection problems also arise in oceanography, limnology and engineering.

There is growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and petroleum industry. Bhatia and Steiner [1] have studied the problem of thermal instability of Maxwellian viscoelastic fluid in the presence of rotation and have found that the rotation has a destabilizing influence in contrast to the stabilizing effect on an ordinary viscous (Newtonian) fluid. The thermal instability of an Oldroydian viscoelastic fluid acted on by a uniform rotation has been studied by Sharma [6]. There are many elastico-viscous fluids that cannot be characterized by constitutive relations. Two such classes of fluids are Rivlin-Ericksen and Walters' (model B') fluid. Walters' has proposed the constitutive equations for such elastico-viscous fluids. Rivlin and Ericksen have proposed a theoretical model for such another elastico-viscous fluid. Such and other polymers are used in the manufacture of parts of spacecrafts, aeroplane parts, tyres, belt conveyers, ropes, cushions, seats, foams, plastics, engineering equipments, adhesives, contact lens etc. Recently, polymers are also used in agriculture, communication appliances and in bio-medical applications.

In recent years, the investigation of flow of fluids through porous media has become an important topic due to the recovery of crude oil from the pores of reservoir rocks. A great number of applications in geophysics may be found in a book by Phillips [5]. When the fluid permeates a porous material, the gross effect is represented by the Darcy's law. As a result of this macroscopic law, the usual viscous and viscoelastic terms in the equation of Walters' (model B') elastico-viscous fluid motion are replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\vec{q}\right]$, where μ and μ' are the viscosity and viscoelasticity of the Walters' (model B') elastico-viscous fluid, k_1 is the medium permeability and $\vec{q}$ is the Darcian (filter) velocity of the fluid. The problem of thermosolutal convection in fluids in a porous medium is of great importance in geophysics, soil sciences, ground water hydrology and astrophysics. Generally, it is accepted that comets consist of a dusty 'snoball' of a mixture of frozen gases, which, in the process of their journey, changes from solid to gas and vice-versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in astrophysical context (McDonnell [3]). In recent study, Sharma et al. [7] studied the instability of streaming Walters' viscoelastic fluid (model B') in porous medium. In another study, Sharma et al. ([8]-[9]) studied the thermal convection in Walters' viscoelastic fluid (model B') permeated with suspended particles through porous medium and thermosolutal instability of Walters' rotating fluid (model B') in porous medium. There is growing importance of non-Newtonian fluids in chemical technology, industry and geophysical fluid dynamics. Thermal convection in porous medium is also of interest in geophysical systems, electrochemistry and metallurgy. A comprehensive review of the literature concerning thermal convection in a fluid-saturated porous medium may be found in the book by Nield and Bejan [4].

The Walters' (model B') elastico-viscous fluid has relevance and importance in geophysical fluid dynamics, chemical technology and industry (e.g. manufacture of various items mentioned above). The present paper, therefore, deals with the combined effects of uniform vertical magnetic field and uniform vertical rotation on the thermosolutal instability of a Walters' (model B') elastico-viscous fluid in porous medium.

2. Formulation of the Problem and Perturbation Equations. Here we consider an infinite, horizontal, incompressible Walters' (model B') elastico-viscous fluid layer of thickness d, heated and soluted from below so that, the temperatures, densities and solute concentrations at the bottom surface $z=0$ are T_0, ρ_0 and C_0 and at the upper surface $z=d$ are T_d, ρ_d and C_d respectively, and that

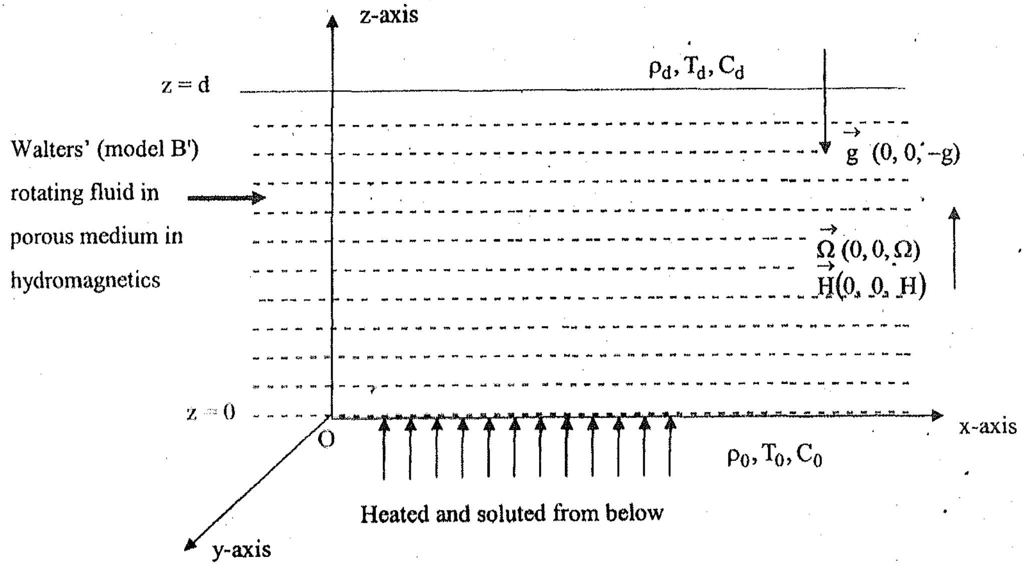

Figure: Geometry of the Problem

a uniform temperature gradient $\beta(=|dT/dz|)$ and a uniform solute gradient $\beta'(=|dC/dz|)$ are maintained. The gravity field $\vec{g}$ $(0,0,-g)$, a uniform vertical magnetic field $\vec{H}$ $(0,0,H)$ and a uniform vertical rotation $\vec{\Omega}(0,0,\Omega)$ prevade the system. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε and medium permeability k_1.

Let $p, \rho, T, C, \alpha, \alpha', g, \eta, \mu_e$ and $\vec{q}$ (u, v, w) denote, respectively, the fluid pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, gravitational acceleration, resistivity, magnetic permeability and fluid velocity. The equations expressing the conservation of momentum, mass, temperature, solute concentration and equation of state of Walters' (model B') elastico-viscous fluid are

$$\frac{1}{\varepsilon}\left[\frac{\partial \vec{q}}{\partial t}+\frac{1}{\varepsilon}(\vec{q}.\nabla)\vec{q}\right]=-\left(\frac{1}{\rho_0}\right)\nabla p+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\nu-\nu'\frac{\partial}{\partial t}\right)\vec{q}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{H}\right)\times\vec{H}+\frac{2}{\varepsilon}\left(\vec{q}\times\vec{\Omega}\right), \quad (1)$$

$$\nabla.\vec{q}=0, \quad (2)$$

$$E\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T=\kappa\nabla^2 T, \quad (3)$$

$$E'\frac{\partial C}{\partial t}+(\vec{q}.\nabla)C=\kappa'\nabla^2 C, \quad (4)$$

$$\rho=\rho_0\left[1-\alpha(T-T_0)+\alpha'(C-C_0)\right], \quad (5)$$

where the suffix zero refers to values at the reference level $z=0$ and in writing Eq. (1), use has been made of the Boussinesq approximation. The magnetic permeability, μ_e the kinematic viscosity ν, the kinematic viscoelasticity ν', the thermal diffusivity κ and the solute diffusivity κ' are all assumed to be constants.

The Maxwell's equations yield

$$\varepsilon\frac{d\vec{H}}{dt}=\left(\vec{H}.\nabla\right)\vec{q}+\varepsilon\eta\nabla^2\vec{H}, \quad (6)$$

$$\nabla.\vec{H}=0, \quad (7)$$

where $\frac{d}{dt}=\frac{\partial}{\partial t}+\varepsilon^{-1}\vec{q}.\nabla$ stands for the convective derivative.

Here $E=\varepsilon+(1-\varepsilon)\left(\frac{\rho_s c_s}{\rho_0 c_i}\right)$ is a constant and E' is a constant analogous to E but corresponding to solute rather than heat. ρ_s, c_s and ρ_0, c_i stand for density and heat capacity of solid (porous matrix) material and fluid, respectively. The steady state solution is

$$\vec{q}=(0,0,0), \qquad T=-\beta z+T_0,$$

$$C=-\beta' z+C_0, \qquad \rho=\rho_0(1+\alpha\beta z-\alpha'\beta' z) \quad (8)$$

Here we use linearized stability theory and normal mode analysis method. Consider a small perturbation on the steady state solution, and let δp, $\delta\rho$, θ, γ, $\vec{h}(h_x,h_y,h_z)$ and $\vec{q}\,(u,v,w)$ denote, respectively, the perturbations in pressure p, density ρ, temperature T, solute concentration C, magnetic field $\vec{H}\,(0,0,H)$ and velocity $\vec{q}\,(0,0,0)$. The change in density $\delta\rho$, caused mainly by the perturbations θ and γ in temperature and concentration, is given by

$$\delta\rho = -\rho_0(\alpha\theta - \alpha'\gamma). \tag{9}$$

Then the linearized perturbation equations become

$$\frac{1}{\varepsilon}\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_0}(\nabla\delta\rho) - \vec{g}(\alpha\theta - \alpha'\gamma) - \frac{1}{k_1}\left(\nu - \nu'\frac{\partial}{\partial t}\right)\vec{q} + \frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{h}\right)\times\vec{H} + \frac{2}{\varepsilon}\left(\vec{q}\times\vec{\Omega}\right), \tag{10}$$

$$\nabla.\vec{q} = 0, \tag{11}$$

$$E\frac{\partial\theta}{\partial t} + \beta w = \kappa\nabla^2\theta, \tag{12}$$

$$E'\frac{\partial\gamma}{\partial t} = \beta' w + \kappa'\nabla^2\gamma, \tag{13}$$

$$\varepsilon\frac{\partial\vec{h}}{\partial t} = \left(\vec{H}.\nabla\right)\vec{q} + \varepsilon\eta\nabla^2\vec{h}, \tag{14}$$

$$\nabla.\vec{h} = 0. \tag{15}$$

3. The Dispersion Relation. Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[w, h_z, \theta, \gamma, \zeta, \xi] = [W(z), K(z), \Theta(z), \Gamma(z), Z(z), X(z)]exp(ik_x x + ik_y y + nt), \tag{16}$$

where k_x, k_y are the wave numbers along the x-and y-directions respectively, $k = \sqrt{\left(k_x^2 + k_y^2\right)}$ is the resultant wave number and n is the growth rate which is, in general, a complex constant $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ stand for the z-components of vorticity and current density, respectively.

Expressing the coordinates x, y, z in the new unit of length d and letting $a = kd$, $\sigma = \frac{nd^2}{\upsilon}$, $p_1 = \frac{\upsilon}{\kappa}$, $p_2 = \frac{\upsilon}{\eta}$, $q = \frac{\upsilon}{\kappa'}$, $F = \frac{\upsilon'}{d^2}$, $P_l = \frac{k_1}{d^2}$ and $D = \frac{d}{dz}$, Eqs. (10)-(15), using (16), yield

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{p_l}(1 - \sigma F)\right]\left(D^2 - a^2\right)W + \frac{ga^2d^2}{\upsilon}(\alpha\Theta - \alpha'\Gamma) - \frac{\mu_e Hd}{4\pi\rho_0\upsilon}\left(D^2 - a^2\right)DK + \frac{2\Omega d^3}{\varepsilon\upsilon}DZ = 0, \tag{17}$$

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{p_l}(1 - \sigma F)\right]Z = \left(\frac{\mu_e Hd}{4\pi\rho_0\upsilon}\right)DX + \left(\frac{2\Omega d}{\varepsilon\upsilon}\right)DW, \tag{18}$$

$$\left[D^2 - a^2 - p_2\sigma\right]K = \left(\frac{Hd}{\varepsilon\eta}\right)DW, \tag{19}$$

$$\left[D^2 - a^2 - p_2\sigma\right]X = \left(\frac{Hd}{\varepsilon\eta}\right)DZ, \tag{20}$$

$$\left[D^2 - a^2 - Ep_l\sigma\right]\Theta = \left(\frac{\beta d^2}{\kappa}\right)W, \tag{21}$$

$$\left[D^2 - a^2 - E'q\sigma\right]\Gamma = \left(\frac{\beta' d^2}{\kappa'}\right)W. \tag{22}$$

Consider the case where both boundaries are free as well as perfect conductors of both heat and solute concentration, while the adjoining medium is perfectly conducting. The case of two free boundaries is a little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The appropriate boundary conditions, with respect to which Eqs. (17)-(22)must be solved are

$W = D^2W = X = DZ = 0$, $\Theta = 0$, $\Gamma = 0$, at $z = 0$ and 1,

$DX = 0$, $K = 0$ on a perfectly conducting boundary

and $X = 0$, h_x, h_y, h_z are continuous with an external vacuum field on a non-conducting boundary. (23)

The case of two free boundaries, though little artificial, is the most appropriate for stellar atmospheres as studied by Spiegel [10]. Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for $z=0$ and 1 and hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \, Sin\ \pi z, \tag{24}$$

where W_0 is a constant.

Eliminating Θ, Γ, K, Z and X between Eqs. (17)-(22) and substituting the proper solution $W = W_0 \sin \pi z$, in the resultant equation, we obtain the dispersion relation

$$R_l = \left(\frac{1+x}{x}\right)\left[\frac{i\sigma_l}{\varepsilon} + \frac{1}{P}\left(1 - i\sigma_l\pi^2 F\right)\right]\left[1 + x + iEp_l\sigma_l\right] + Q_l\,\frac{(1+x)(1+x+iEp_l\sigma_l)}{x(1+x+ip_2\sigma_l)} +$$

$$S_1\frac{(1+x+iEp_1\sigma_1)}{(1+x+iE'q\sigma_1)} + T_{A_1}\left[\frac{(1+x+iEp_1\sigma_1)(1+x+ip_2\sigma_1)}{x\left\{\left(\frac{i\sigma_1}{\varepsilon} + \frac{1}{P}\left(1 - i\sigma_1\pi^2 F\right)\right)(1+x+ip_2\sigma_1) + Q_1\right\}}\right], \tag{25}$$

where

$$R_1 = \frac{g\alpha\beta d^4}{\nu\kappa\pi^4},\ S_1 = \frac{g\alpha'\beta' d^2}{\nu\kappa'\pi^4},\ Q_1 = \frac{\mu_e H^2 d^4}{4\pi\rho_0\nu\eta\varepsilon\pi^2},\ T_{A_1} = \left(\frac{2\Omega d^2}{\varepsilon\nu\pi^2}\right)^2,\ x = \frac{a^2}{\pi^2},\ i\sigma_l = \frac{\sigma}{\pi^2}$$

and $P = \pi^2 P_l$.

Equation (25) is the required dispersion relation including the effects of magnetic field, rotation, medium permeability, kinematic viscoelasticity and stable solute gradient on the thermosolutal instability of Walters' (model B') rotating fluid in a porous medium in hydromagnetics.

4. The Stationary Convection. When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma=0$. Putting $\sigma=0$, the dispersion relation (25) reduces to

$$R_1=\left(\frac{1+x}{x}\right)\left[(1+x)/P+Q_1\right]+T_{A_1}\frac{(1+x)^2}{x\{(1+x)/P+Q_1\}}+S_1, \tag{26}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters S_1, T_{A_1}, Q_1 and P. The parameter F accounting for the kinematic viscoelasticity effect vanishes for the stationary convection.

To investigate the effects of stable solute gradient, rotation, magnetic field and medium permeability, we examine the behavior of $\frac{dR_1}{dS_1}, \frac{dR_1}{dT_{A_1}}, \frac{dR_1}{dQ_1}$ and $\frac{dR_1}{dP}$ analytically. Equation (26) yields

$$\frac{dR_1}{dS_1}=+1, \tag{27}$$

$$\frac{dR_1}{dT_{A_1}}=\left[(1+x)/x\right]\frac{(1+x)}{((1+x)/P+Q_1)}. \tag{28}$$

This shows that, for a stationary convection, the stable solute gradient and rotation are found to have stabilizing effects on the system.
It can easily be derived from (26) that

$$\frac{dR_1}{dP}=-\frac{(1+x)^2}{x}\left[\frac{1}{P^2}-T_{A_1}\frac{(1+x)}{(1+x+PQ_1)^2}\right], \tag{29}$$

$$\frac{dR_1}{dQ_1}=\frac{(1+x)}{x}\left[1-T_{A_1}\frac{(1+x)P^2}{(1+x+PQ_1)^2}\right]. \tag{30}$$

In the absence of rotation $(T_{A_1}=0)$, $\frac{dR_1}{dP}$, is always negative and $\frac{dR_1}{dQ_1}$ is always positive, which means that medium permeability has a destabilizing effect, whereas, magnetic field has a stabilizing effect on the system in the absence of rotation. For a rotation system, the medium permeability has a destabilizing (or stabilizing) effect and magnetic field has a stabilizing (or destabilizing) effect on thermosolutal instability of Walters' (model B') rotating fluid in porous medium in hydromagnetics if

$$T_{A_1}<(or>)\frac{(1+x+PQ_1)^2}{(1+x)P^2} \tag{31}$$

The dispersion relation (26) is also analysed numerically. In Figure 1, R_1 is

plotted against x for Q_1=20, T_{A_1} =25, P=10 and S_1=10, 15 and 20 . The stabilizing role of the stable solute gradient is clear from the increase of the Rayleigh number with increasing stable solute gradient parameter value. Figure 2 gives R_1 plotted against x for Q_1=20, P=10, S_1=10 and T_{A_1} =25, 35 and 45 . Here we also find the stablizing role of the rotation as the Rayleigh number increases with the increase in rotation parameter T_{A_1}. Figure 3 gives R_1 plotted against x for Q_1=20, S_1=20, T_{A_1} =0 and P=1, 3 and 5. Here we find the destabilizing role of the medium permeability as the Rayleigh number decreases with the increase in medium permeability parameter P in the absence of rotation. Figure 4 gives R_1 plotted against x for Q_1=*10*, S_1=*20*, T_{A_1} =20, and P=*1*,*3* and *5*. Here, there is a competition between the destabilizing role of medium permeability and the stabilizing role of the rotation. Figure 5 gives R_1 plotted against x for P=*10*, S_1=*20*, T_{A_1} =0, and Q_1=*10*, *30* and *50*. Here we also find the stabilizing role of the magnetic field as the Rayleigh number increases with the increase in magnetic field parameter Q_1 in the absence of rotation. Figure 6 gives R_1 plotted against x for P=*10*, S_1=*20*, T_{A_1} =*20*, and Q_1=*10*, *12* and *14*. Here we also find the stabilizing role of the magnetic field for small wave numbers in the presence of rotation as the Rayleigh number increases with the increase in magnetic field parameter Q_1, but for higher wave numbers the
magnetic field have destabilizing effect in the presence of rotation.

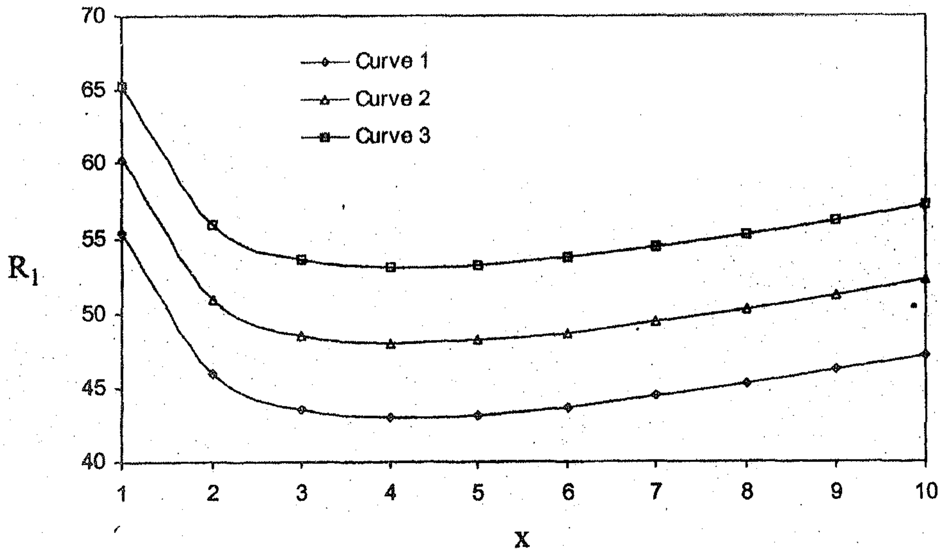

Fig.1 : The variation of Rayleigh number (R_1) with wave number (x) for Q_1=*20*, T_{A_1} =*25*, P=*10*, S_1=*10* for curve 1, S_1=*15* for courve 2 and S_1=*20* for curve 3.

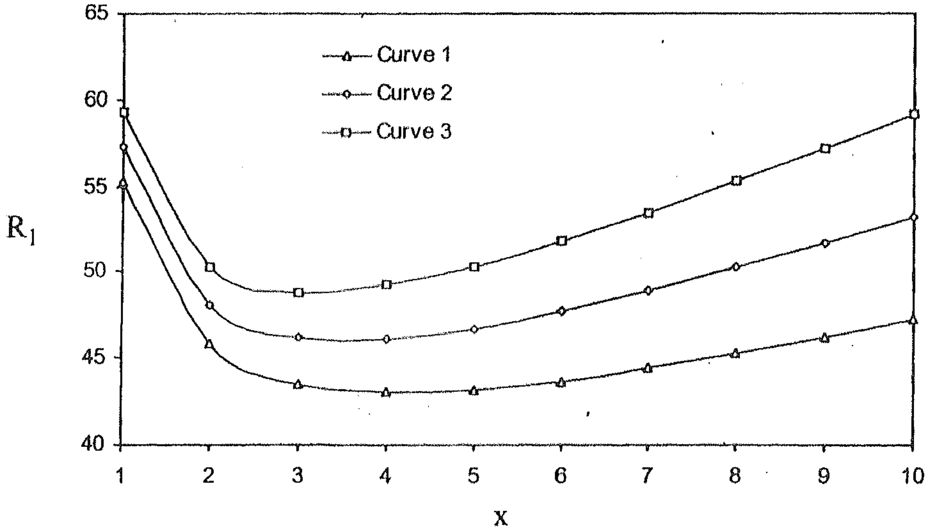

Fig.2 : The variation of Rayleigh number (R_1) with wave number (x) for Q_1=*20*, S_1=*10*, P=*10*, T_{A_1}=*25* for curve 1, T_{A_1}=35 for curve 2 and T_{A_1}=45 for curve 3.

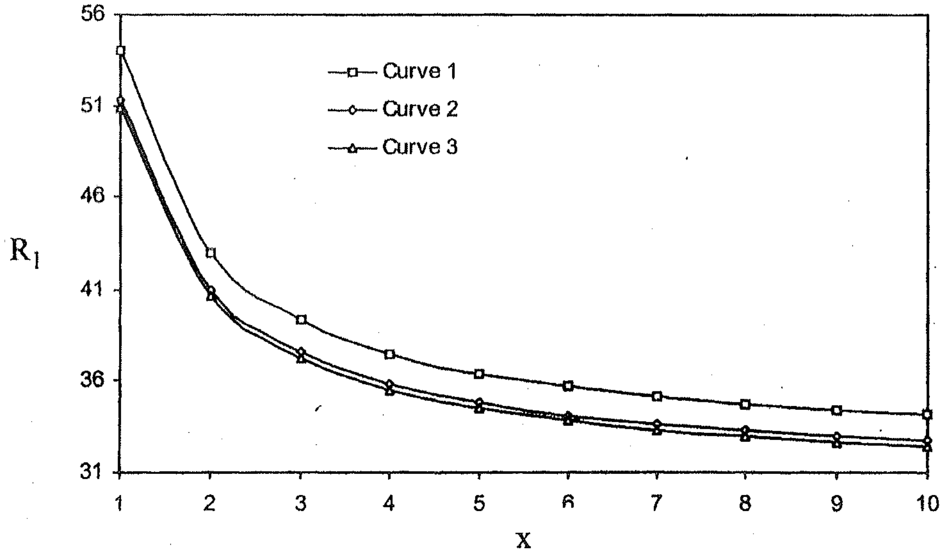

Fig.3 : The variation of Rayleigh number (R_1) with wave number (x) for Q_1=*20*, S_1=*10*, T_{A_1}=*0*, P=*1* for curve 1, P=3 for curve 2 and P=5 for curve 3.

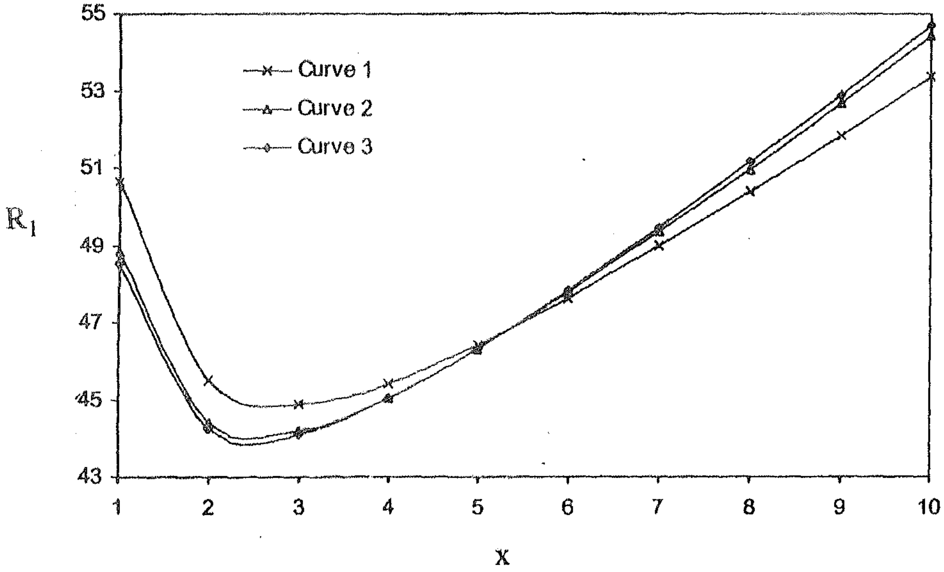

Fig.4 : The variation of Rayleigh number (R_1) with wave number (x) for $Q_1=10$, $S_1=20$, $T_{A_1}=20$, $P=1$ for curve *1*, $P=3$ for curve *2* and $P=5$ for curve *3*.

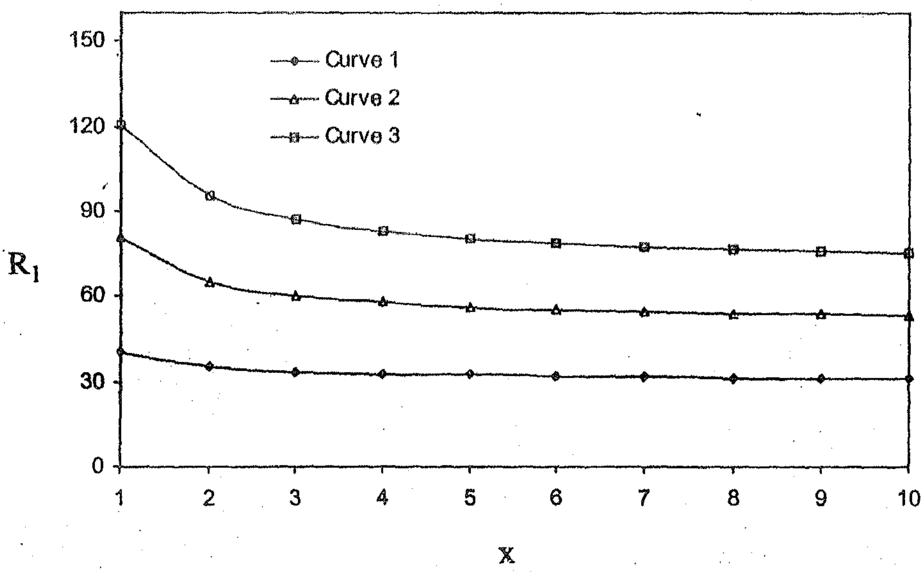

Fig.5 : The variation of Rayleigh number (R_1) with wave number (x) for $P=10$, $S_1=20$, $T_{A_1}=0$, $Q_1=10$ for curve *1*, $Q_1=20$ for curve *2* and $Q_1=30$ for curve *3*.

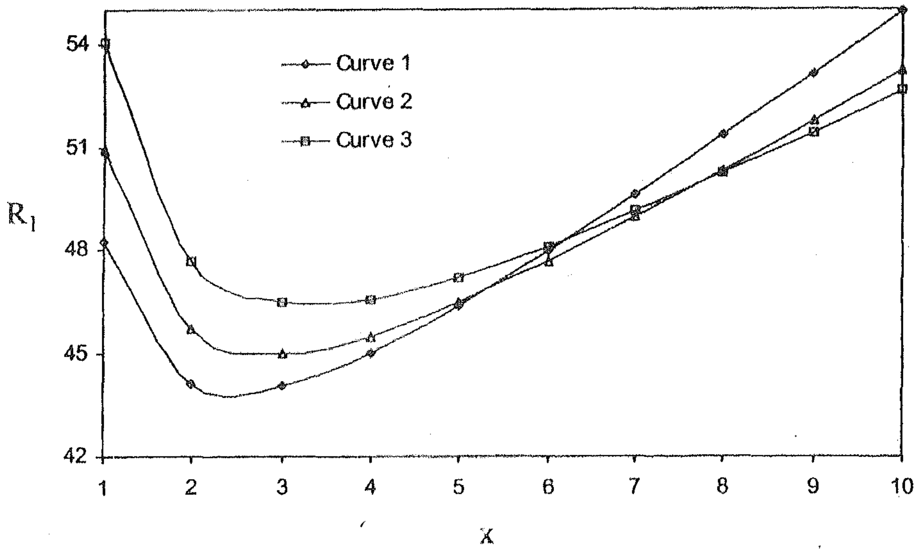

Fig.6 : The variation of Rayleigh number (R_1) with wave number (x) for $P=10$, $S_1=20$, $T_{A_1}=20$, $Q_1=10$ for curve *1*, $Q_1=12$ for curve *2* and $Q_1=14$ for curve *3*.

5. Stability of the System and Oscillatory Modes. Here we examine the possibility of oscillatory modes, if any, on stability problem due to the presence of kinematic viscoelasticity, stable solute gradient and magnetic field. Multiplying (17) by W^*, the complex conjugate of W, and using (18)-(22) together with the boundary conditions (23), we obtain

The integrals $I_1,\ldots,I_{10}$ are all positive definite. Putting and equating the real and imaginary parts of equation (33), we obtain

$$\left[\frac{\sigma}{\varepsilon}+\frac{1}{P_l}(1-\sigma F')\right]I_1+\left(\frac{g\alpha'\kappa' a^2}{\upsilon\beta'}\right)\left[I_4+E'q\sigma^* I_5\right]+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}\left[I_6+p_2\sigma^* I_7\right]$$

$$+\frac{\mu_e\varepsilon\eta d^2}{4\pi\rho_0\nu}\left[I_8+p_2\sigma I_9\right]+d^2\left[\frac{\sigma^*}{\varepsilon}+\frac{1}{P_l}(1-\sigma^* F)\right]I_{10}-\left(\frac{g\alpha\kappa a^2}{\upsilon\beta}\right)\left[I_2+Ep_1\sigma^* I_3\right]=0, \quad (32)$$

where $I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz,\quad I_2=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz,\quad I_3=\int_0^1\left(|\Theta|^2\right)dz,$

$$I_4=\int_0^1\left(|D\Gamma|^2+a^2|\Gamma|^2\right)dz,\quad I_5=\int_0^1\left(|\Gamma|^2\right)dz,$$

$$I_6=\int_0^1\left(\left|D^2K\right|^2+2a^2|DK|^2+a^4|K|^2\right)dz,\quad I_7=\int_0^1\left(|DK|^2+a^2|K|^2\right)dz,$$

$$I_8=\int_0^1\left(|DX|^2+a^2|X|^2\right)dz,\quad I_9=\int_0^1\left(|X|^2\right)dz,\quad I_{10}=\int_0^1\left(|Z|^2\right)dz. \quad (33)$$

The integrals $I_1,...,I_{10}$ are all positive definite. Putting $\sigma=\sigma_r+i\sigma_i$ and equating the real and imaginary parts of equation (33), we obtain

$$\left[\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)I_1+\frac{g\alpha'\kappa'a^2}{\upsilon\beta'}E'qI_5+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}p_2\left(I_7+d^2I_9\right)+d^2\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)I_{10}-\frac{g\alpha\kappa a^2}{\upsilon\beta}Ep_1I_3\right]\sigma_r$$

$$=-\left[\frac{I_1}{P_l}+\frac{g\alpha'\kappa'a^2}{\upsilon\beta'}I_4+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}\left(I_6+d^2I_8\right)+d^2\frac{1}{P_l}I_{10}-\frac{g\alpha\kappa a^2}{\upsilon\beta}I_2\right], \qquad (34)$$

$$\left[\left(\frac{1}{\varepsilon}-\frac{F}{P_l}\right)I_1-\frac{g\alpha'\kappa'a^2}{\upsilon\beta'}E'qI_5+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}p_2\left[I_7-d^2I_9\right]-d^2\left(\frac{1}{\varepsilon}-\frac{F}{P_l}\right)I_{10}+\frac{g\alpha\kappa a^2}{\upsilon\beta}Ep_1I_3\right]\sigma_i=0.$$

It is evident from (34) that σ_r is positive or negative. The system is, therefore, stable or unstable.It is clear from (35) that σ_i may be zero or non-zero, meaning that the modes may be non-oscillatory or oscillatory. In the absence of kinematic viscoelasticity, stable solute gradient, rotation and magnetic field, equation (35) reduces to

$$\left[\frac{I_1}{\varepsilon}+\frac{g\alpha\kappa a^2}{\upsilon\beta}Ep_1I_3\right]\sigma_i=0, \qquad (36)$$

and the terms in brackets are positive definite. Thus $\sigma_i=0$ which means that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied for a porous medium, in the absence of kinematic viscoelasticity, stable solute gradient, rotation and magnetic field. This result is ture for the porous as well as non-porous medium as studied in [2]. The oscillatory modes are introduced due to the pressence of kinematic viscoelasticity, stable solute gradient, rotation and magnetic field, which were non-existent in their absence.

6. The case of overstability. Here we discuss the possibility of whether instability may occur as overstability. Since we wish to determine the critical Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which (25) will admit of solutions with σ_1 real.

If we equate real and imaginary parts of (25) and eliminate R_1 between them, we obtain

$$A_4c_1^4+A_3c_1^3+A_2c_1^2+A_1c_1+A_0=0, \qquad (37)$$

where we have put $c_1=\sigma_i^1$, $b=1+x$ and

$$A_4=E'^2q^2p_2^2\left(\frac{1}{\varepsilon}-\frac{\pi^2F}{P}\right)^2\left[\left(\frac{1}{\varepsilon}-\frac{\pi^2F}{P}\right)b+\frac{Ep_1}{P}\right], \qquad (38)$$

$$A_3 = \left[\left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P}\right)\left(2E''^2 q^2 + p_2^2\right)\right]b^4 + \left[\frac{Ep_1 p_2^2}{P}\left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P}\right)^2 + 2E''^2 q^2\left\{\frac{Ep_1}{P}\left(\frac{1}{\varepsilon} + \frac{\pi^2 F}{P}\right)^2\right\}\right]b^3$$

$$+\left[E''^2 q^2 p_2\left(\frac{1}{\varepsilon} + \frac{\pi^2 F}{P}\right)\left\{\frac{p_2}{P^2} - \frac{3Q_1}{\varepsilon} + \frac{3Q_1\pi^2 F}{P}\right\} + 2Ep_1 E''^2 q^2 Q_1\left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P}\right)^2\right]b^2$$

$$+p_2\left[E''^2 q^2\left\{\frac{Ep_1}{2P}\left(\frac{p_2}{P^2} - \frac{4Q_1}{\varepsilon} + \frac{4Q_1\pi^2 F}{P}\right) + p_2\left(\frac{Ep_1}{2P^3} - \frac{T_{A_1}}{\varepsilon} + \frac{T_{A_1}\pi^2 F}{P}\right)\right\}\right.$$

$$\left.+S_1(b-1)p_2\left(\frac{1}{\varepsilon} + \frac{\pi^2 F}{P}\right)^2 (Ep_1 - E'q)\right]b + \frac{T_{A_1} Ep_1 E''^2 q^2 p_2^2}{P}. \tag{39}$$

Since σ_1 is real for overstability, the four values of $c_1\left(=\sigma_1^2\right)$ are positive. The sum of roots of (37) is $-\frac{A_3}{A_4}$, and if this is to be negative, then $A_3>0$ $A_4>0$. It is clear from (38) and (39) that A_3 and A_4 are always positive if

$$\frac{\pi^2 F}{P} < \frac{1}{\varepsilon},\ Ep_1 > E'q,\ Ep_1 > \frac{2T_{A_1}P^3}{\varepsilon} \text{ and } p_2 > \frac{2Q_1 P^2}{\varepsilon}, \tag{40}$$

which imply that

$$\nu' < \frac{k_1}{\varepsilon}, E\frac{\nu}{\kappa} > E'\frac{\nu}{\kappa'}, E\frac{\nu}{\kappa} \gtrsim k_1\left(\frac{2\Omega\pi k_1}{\varepsilon\upsilon d}\right)^2 \text{ and } \upsilon > \frac{k_1^2\pi}{\varepsilon^2 d^2}\left(\frac{3\mu_e H^2}{4\upsilon\rho_0}\right). \tag{41}$$

i.e.

$$\nu' < \frac{k_1}{\varepsilon}, E\frac{\nu}{\kappa} > \min\left\{E'\frac{\nu}{\kappa'}, k_1\left(\frac{2\Omega\pi k_1}{\varepsilon\upsilon d}\right)^2\right\} \text{ and } \upsilon > \frac{k_1^2\pi}{\varepsilon^2 d^2}\left(\frac{3\mu_e H^2}{4\upsilon\rho_0}\right). \tag{42}$$

Thus $\nu' < \frac{k_1}{\varepsilon}, E\frac{\nu}{\kappa} > \min\left\{E'\frac{\nu}{\kappa'}, k_1\left(\frac{2\Omega\pi k_1}{\varepsilon\upsilon d}\right)^2\right\}$ and $\upsilon > \frac{k_1^2\pi}{\varepsilon^2 d^2}\left(\frac{3\mu_e H^2}{4\upsilon\rho_0}\right)$. are the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

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