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(Dedicated to Professor H. M. Srivastava on his 62nd Birthday)

ON TWO BOUNDARY VALUE PROBLEMS

By
R.C. Singh Chandel and Shailja Sengar
Department of Mathematics, D.V. Postgraduate College, Orai-285001, U.P., India

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ABSTRACT

In the present paper, first we evaluate an integral involving product of the general class of polynomials of Srivastava [6] and multivariable H-function of Srivastava and Panda ([10], [11] and also see [12]) and then make its applications to solve two boundary value problems on

I) heat conduction in a rod

II) diflection of vibrating string under certain conditions and to establish an expansion formula involving product of above polynomials and H-function of several complex variables. In the last some interesting special cases will also be discussed. Our results are generalization of the results due to Chandel-Tiwari [1], Chaurasia-Patni [2] and Srivastava-Srivastava [16]. In special case V of problem I, it is also shown that all the results due to Chaurasia-Patni ([2],(8),(11),(12), (13), (14), (15), (16), (17), (18), (19)) are wrongly expressed. This remark also suggests that all the results due to Chaurasia and Gupta ([3], (2.1), (2.2), (3.1), (3.3), (4.1) to (4.12)) are also wrongly expressed.

1. Introduction. Chandel and Tiwari [1] have employed multiple hypergeometric function of several variables of Srivastava and Daoust ([7], [8], [9]; also see modified form Srivastava and Karlsson [14, p.37, eqns (2.1) to (2.3)]) in two boundary value problems.

In the present paper, first we evaluate an integral involving the product of the H-function of several variables of Srivastava and Panda ([10],[11], also see [12]) and several general classes of polynomials of Srivastava [6, p.1, eqn. (1.1)] defined by

$$(1.1)\qquad S_n^{[m]}[x] = \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} F_{n,s} x^s, n = 0,1,2,\ldots$$

where m is arbitrary positive integer and the coefficients $F_{n,s}$ $(n, s \geq 0)$ are arbitrary constants, real or complex.

In the last some interesting special cases will also be discussed.

Our results will be generalizations of the results due to Chandel-Tiwari [1], Chaurasia-Patni [2], and Srivastava-Srivastava [16].

2. Main integral. In this section, making an appeal to modified form of [4,p.372, (1)]

$$\int_0^L (\sin \pi x/L)^{W-1} \sin \pi x\lambda_m/L \, dx = \frac{\Gamma(W) \sin \pi\lambda_m/2}{2^{W-1}\Gamma\left(\frac{W+\lambda_m+1}{2}\right)\Gamma\left(\frac{W-\lambda_m+1}{2}\right)}, Re(W) > 0,$$

we evaluate the following integral very useful in our investigations:

$$(2.1)\quad \int_0^L (\sin \pi x/L)^{W-1} \sin(\pi x\lambda_m/L) S_{n_1}^{m_1}\left[y_1(\sin \pi x/L)^{2\rho_1}\right]\ldots S_{n_r}^{m_r}\left[y_r(\sin \pi x/L)^{2\rho_r}\right]$$

$$H^{0,\lambda:(\mu',\nu');\ldots;(\mu^{(n)},\nu^{(n)})}_{A,C:[B',D'];\ldots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]:[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}];\\ [(c):\psi',\ldots,\psi^{(n)}]:[(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\, z_1(\sin \pi x/L)^{2\xi_1},\ldots,z_n(\sin \pi x/L)^{2\xi_n}\right)dx$$

$$= \frac{L\sin(\pi\lambda_m/2)}{2^{W-1}} \sum_{s_1=0}^{[n_1/m_1]}\cdots\sum_{s_r=0}^{[n_r/m_r]} \frac{(-n_1)_{m_1 s_1} A_{n_1,s_1}\ldots(-n_r)_{m_r s_r} A_{n_r,s_r}}{2^{2(\rho_1 s_1+\ldots+\rho_r s_r)}}$$

$$\frac{y_1^{s_1}}{s_1!}\ldots\frac{y_r^{s_r}}{s_r!} H(s_1,\ldots,s_r),$$

where

$$H(s_1,\ldots,s_r) = H^{0,\lambda+1:(\mu',\nu');\ldots;(\mu^{(n)},\nu^{(n)})}_{A+1,C+2:[B',D'];\ldots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}],[1-W-2(\rho_1 s_1+\ldots+\rho_r s_r):2\xi_1,\ldots,2\xi_n]:\\ [(c):\psi',\ldots,\psi^{(n)}],\left[\frac{1-W-2(\rho_1 s_1+\ldots+\rho_r s_r)\pm\lambda_m}{2}:\xi_1,\ldots,\xi_n\right]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}];\\ [(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\, z_1/4^{\xi_1}\ldots z_n/4^{\xi_n}\right)$$

$Re\ (W) > 0$, $Re\left(W+\sum_{i=1}^{n}\xi_i d_j^{(i)}/\delta_j^{(i)}\right) > 0$, all ξ_i are real positive numbers, λ, A, C, $\mu^{(i)}$, $\nu^{(i)}$, $B^{(i)}$, $D^{(i)}$ are such that $A \geq \lambda \geq 0$, $C \geq 0$, $D^{(i)} \geq \mu^{(i)} \geq 0$, $B^{(i)} \geq \nu^{(i)} \geq 0$ and $\theta_j^{(i)}, j= 1,\ldots, A$; $\phi_j^{(i)}, j= 1,\ldots, B^{(i)}$; $\psi_j^{(i)}, j= 1,\ldots,C$; $\delta_j^{(i)}, j= 1,\ldots,D^{(i)}$ are positive real numbers,

$$\left|arg\left[z_i(\sin \pi x/L)^{2\xi_i}\right]\right| \leq \Delta_i\, \pi/2;$$

$$\Delta_i = -\sum_{j=\lambda+1}^{A}\theta_j^{(i)} + \sum_{j=1}^{\nu^{(i)}}\phi_j^{(i)} - \sum_{j=\nu^{(i)}+1}^{B^{(i)}}\phi_j^{(i)} - \sum_{j=1}^{C}\psi_j^{(i)} + \sum_{j=1}^{\mu^{(i)}}\delta_j^{(i)} - \sum_{j=\mu^{(i)}+1}^{D^{(i)}}\delta_j^{(i)} > 0;\quad i=1,\ldots,n.$$

Also ρ_k are real positive numbers, n_k, m_k are arbitrary positive integers, A_{n_k,s_k} are arbitrary functions of n_k and s_k real or complex independent of x, y_k, ρ_k ; $k=1,\ldots,r$.

Problem I

3. Application to heat conduction in a rod. In this section, we consider a problem on outer heat conduction in a rod under certain boundary conditions. If the thermal coefficients are constants and there is no source of thermal energy, then the temperature $u(x,t)$ in one dimensional rod $0 \leq x \leq L$ satisfies the following heat equation

$$(3.1)\quad \frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, t \geq 0.$$

If we consider the following boundary conditions

(3.2) $u(0, t) = 0,$

(3.3) $\frac{\partial u(L,t)}{\partial x} + hu(L,t) = 0, h > 0,$

(3.4) $u(x, t)$ is finite as $t \to \infty$,

and initial condition

(3.5) $u(x,0) = f(x),$

then the solution of partial differential equation (3.1) is given by Sommerfield [5]

$$(3.6)\quad u(x,t) = \sum_{m=1}^{\infty} A_m \sin(\lambda_m \pi x/L) \exp\{-(\pi\lambda_m/L)^2 kt\},$$

where $\lambda_1,\dots,\lambda_m$ are the roots of the transcendental equation

(3.7) $\tan \pi\lambda_m = \pi\lambda_m/kL.$

Here we consider the problem of determining $u(x,t)$, where

$$(3.8)\quad u(x,0) = f(x) = (\sin \pi x/L)^{W-1} S_{n_1}^{m_1}\left[y_1(\sin \pi x/L)^{2\rho_1}\right]\dots S_{n_r}^{m_r}\left[y_r(\sin \pi x/L)^{2\rho_r}\right]$$

$$H^{0,\lambda:(\mu',v');\dots;(\mu^{(n)},v^{(n)})}_{A,C:[B',D'];\dots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]:[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}];\\ [(c):\psi',\dots,\psi^{(n)}]:[(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\, z_1(\sin \pi x/L)^{2\xi_1},\dots,z_n(\sin \pi x/L)^{2\xi_n}\right)$$

4. Solution of the problem. Making an appeal to (3.6), (3.8) and (2.1) we derive

$$(4.1)\quad A_m = \frac{\pi\lambda_m \sin(\pi\lambda_m/2)}{2^{W-3}[2\pi\lambda_m - \sin 2\pi\lambda_m]} \sum_{s_1=0}^{[n_1/m_1]}\dots\sum_{s_r=0}^{[n_r/m_r]} \frac{(-n_1)_{m_1 s_1} A_{n_1,s_1}\dots(-n_r)_{m_r s_r} A_{n_r,s_r}}{4^{(\rho_1 s_1+\dots+\rho_r s_r)}}$$

$$H(s_1,\dots,s_r)\frac{y_1^{s_1}}{s_1!}\dots\frac{y_r^{s_r}}{s_r!},$$

where all conditions of (2.1) are satisfied and λ_m are the roots of transcendental equation (3.7) and

$$H(s_1,\dots,s_r) = H^{0,\lambda+1:(\mu',v');\dots;(\mu^{(n)},v^{(n)})}_{A+1,C+2:[B',D'];\dots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}],[1-W-2(\rho_1 s_1+\dots+\rho_r s_r):2\xi_1,\dots,2\xi_n]:\\ [(c):\psi',\dots,\psi^{(n)}],\left[\frac{1-W-2(\rho_1 s_1+\dots+\rho_r s_r)\pm\lambda_m}{2}:\xi_1,\dots,\xi_n\right]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}];\\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\, z_1/4^{\xi_1}\dots z_n/4^{\xi_n}\right).$$

Now substituting the value of A_m from (4.1) in (3.6), we derive the following solution of the problem:

$$(4.2)\quad u(x,t) = \frac{\pi}{2^{W-3}} \sum_{m=1}^{\infty} \sin(\lambda_m \pi x/L) \exp\{-(\pi\lambda_m/L)^2 kt\}$$

$$\frac{\lambda_m \sin(\pi\lambda_m/2)}{[2\pi\lambda_m - \sin 2\pi\lambda_m]} \sum_{s_1=0}^{[n_1/m_1]} \cdots \sum_{s_r=0}^{[n_r/m_r]} \frac{(-n_1)_{m_1 s_1} A_{n_1,s_1} \cdots (-n_r)_{m_r s_r} A_{n_r,s_r}}{4^{(\rho_1 s_1 + \ldots + \rho_r s_r)}}$$

$$H(s_1,\ldots,s_r)\frac{y_1^{s_1}}{s_1!}\cdots\frac{y_r^{s_r}}{s_r!},$$

where all conditions of (4.1) are satisfied.

5. Expansion Formula. Making an appeal to (3.8) and (4.2), we derive the expansion formula :

$$(5.1) \qquad (\sin \pi x/L)^{W-1} S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left(y_1(\sin \pi x/L)^{2\rho_1},\ldots,y_r(\sin \pi x/L)^{2\rho_r}\right)$$

$$H_{A,C:[B',D'];\ldots;[B^{(n)},D^{(n)}]}^{0,\lambda:(\mu',\nu');\ldots;(\mu^{(n)},\nu^{(n)})}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]: \\ [(c):\psi',\ldots,\psi^{(n)}]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix} z_1(\sin \pi x/L)^{2\xi_1},\ldots,z_n(\sin \pi x/L)^{2\xi_n}\right)$$

$$= \frac{\pi}{2^{W-3}} \sum_{m=1}^{\infty} \sin(\pi\lambda_m x/L) \frac{\lambda_m \sin(\pi\lambda_m/2)}{[2\pi\lambda_m - \sin 2\pi\lambda_m]} \sum_{s_1=0}^{[n_1/m_1]} \cdots \sum_{s_r=0}^{[n_r/m_r]} \frac{(-n_1)_{m_1 s_1} A_{n_1,s_1} \cdots (-n_r)_{m_r s_r} A_{n_r,s_r}}{4^{(\rho_1 s_1 + \ldots + \rho_r s_r)}}$$

$$H[s_1,\ldots,s_r]\frac{y_1^{s_1}}{s_1!}\cdots\frac{y_r^{s_r}}{s_r!},$$

provided that all the conditions of (4.1) are satisfied.

Problem II

6. Application to Homogeneous Wave Problem. In this section, we shall determine the shape (deflection) $u(x,t)$ of vibrating string. If the deflection due to weight of string is negligible (usually the case), then $u(x,t)$ satisfies the partial differential equation:

$$(6.1) \qquad \frac{\partial^2 u}{\partial x^2} = \frac{1}{c}\frac{\partial^2 u}{\partial t^2}, \qquad 0 < x < L,\ t > 0.$$

If we assume the boundary conditions

$$(6.2) \qquad u(0,t) = 0, \qquad u(L,t) = 0, \qquad t > 0$$

and initial conditions

$$(6.3) \qquad \frac{\partial u(x,0)}{\partial t} = g(x) \quad \text{(initial velocity)}$$

and

$$(6.4) \qquad u(x,0) = f(x).$$

Then the solution of partial differential equation (6.1) is given by

$$(6.5) \qquad u(x,t) = \sum_{m=1}^{\infty}\left[A_m \cos(\pi\lambda_m ct/L) + B_m \sin(\pi\lambda_m ct/L)\right]\sin(\pi\lambda_m x/L).$$

Now we consider the problem of determining $u(x,t)$, where $u(x,0)=f(x)$ is given by (3.8) and

$$(6.6)\qquad g(x)=\left(\sin \pi x/L\right)^{W'-1} S_{p_1}^{q_1}\left[x_1\left(\sin \pi x/L\right)^{2\sigma_1}\right]\ldots S_{p_l}^{q_l}\left[x_l\left(\sin \pi x/L\right)^{2\sigma_l}\right]$$

$$H_{V,W:[P',Q'];\ldots;[P^{(n)},Q^{(n)}]}^{0,u:(M',N');\ldots;(M^{(n)},N^{(n)})}\left(\begin{matrix}[(e):E',\ldots,E^{(n)}]:[(f'):F'];\ldots;[(f^{(n)}):F^{(n)}];\\ [(g):G',\ldots,G^{(n)}]:[(h'):H'];\ldots;[(h^{(n)}):H^{(n)}];\end{matrix}\right.$$

$$\left. u_1 \sin^{2\eta_1} \pi x/L,\ldots,u_n \sin^{2\eta_n} \pi x/L \right).$$

Now making an appeal to (6.4) and (6.5), we obtain

$$(6.7)\qquad u(x,0)=f(x)=\sum_{m=1}^{\infty} A_m \sin\left(\pi x\lambda_m/L\right),$$

while an appeal to (6.3) and (6.5) gives

$$(6.8)\qquad \frac{\partial(u(x,0))}{\partial t}=g(x)=\frac{\pi c}{L}\sum_{m=1}^{\infty} B_m\lambda_m \sin(\pi\lambda_m x/L).$$

Now making an appeal to (2.1), (3.8) and (6.4), A_m is given by (4.1).

Again by (6.6), (6.8) and (2.1), we also derive

$$(6.9)\qquad B_m=\frac{L\sin(\pi\lambda_m/2)}{2^{W'-3}c[2\pi\lambda_m-\sin 2\pi\lambda_m]}\sum_{s_1=0}^{[p_1/q_1]}\cdots\sum_{s_l=0}^{[p_l/q_l]}\frac{(-p_1)_{q_1s_1}A_{p_1,s_1}\cdots(-p_l)_{q_ls_l}A_{p_l,s_l}}{2^{2(\sigma_1s_1+\ldots+\sigma_ls_l)}}$$

$$\bar{H}[s_1,\ldots,s_l]\frac{x_1^{s_1}}{s_1!}\cdots\frac{x_l^{s_l}}{s_l!},$$

where

$$\bar{H}[s_1,\ldots,s_l]=H_{v+1,w+2:[P',Q'];\ldots;[P^{(n)},Q^{(n)}]}^{0,u+1:(M',N');\ldots;(M^{(n)},N^{(n)})}\left(\begin{matrix}[(e):E',\ldots,E^{(n)}],\\ [(g):G',\ldots,G^{(n)}],\end{matrix}\right.$$

$$\left(\begin{matrix}[1-W'-2(\sigma_1s_1+\ldots+\sigma_ls_l):2\eta_1,\ldots,2\eta_n]: & [(f'):F'];\ldots;[(f^{(n)}):F^{(n)}]; & \\ \left[\dfrac{1-W'-2(\sigma_1s_1+\ldots+\sigma_ls_l)\pm\lambda_m}{2}:\eta_1,\ldots,\eta_n\right]: & [(h'):H'];\ldots;[(h^{(n)}):H^{(n)}]; & \dfrac{u_1}{4^{\eta_1}};\ldots;\dfrac{u_n}{4^{\eta_n}}\end{matrix}\right).$$

Now substituting the values of A_m and B_m in (6.5), the solution of the problem in given by

$$(6.10)\qquad u(x,t)=\frac{\pi}{2^{W-3}}\sum_{m=1}^{\infty}\frac{\lambda_m \sin(\pi\lambda_m x/L)\cos(\pi\lambda_m ct/L)\sin(\pi\lambda_m/2)}{[2\pi\lambda_m-\sin 2\pi\lambda_m]}$$

$$\sum_{s_1=0}^{[n_1/m_1]}\cdots\sum_{s_r=0}^{[n_r/m_r]}\frac{(-n_1)_{m_1s_1}A_{n_1s_1}\cdots(-n_r)_{m_rs_r}A_{n_rs_r}}{2^{2(\rho_1s_1+\ldots+\rho_rs_r)}}H(s_1,\ldots,s_r)\frac{y_1^{s_1}}{s_1!}\cdots\frac{y_r^{s_r}}{s_r!}+\frac{\pi L}{c.2^{W'\cdot 3}}$$

$$\sum_{m=1}^{\infty}\frac{sin(\pi\lambda_m x/L)sin(\pi\lambda_m ct/L)sin(\pi\lambda_m/2)}{[2\pi\lambda_m-sin\,2\pi\lambda_m]}$$

$$\sum_{s_1=0}^{[p_1/q_1]}\cdots\sum_{s_k=0}^{[p_k/q_k]}\frac{(-p_1)_{q_1s_1}A_{p_1,s_1}\cdots(-p_l)_{q_ls_l}A_{p_l,s_l}}{2^{2(\sigma_1s_1+\ldots+\sigma_ls_l)}}\overline{H}(s_1,\ldots s_l)\frac{x_1^{s_1}}{s_1!}\cdots\frac{x_l^{s_l}}{s_l!},$$

where λ_m are the roots of equation (3.7), $Re(w)>0$, $Re(W')>0$, $Re\left(W'+\sum_{i=1}^{n}\eta_i h_j^{(i)}/H_j^{(i)}\right)>0$, all x_i, h_i are positive real numbers and u,v,w, $N^{(i)},M^{(i)},P^{(i)}$, $Q^{(i)}$ are such that $v\geq u\geq 0$, $w\geq 0$, $Q^{(i)}\geq M^{(i)}\geq 0$, $P^{(i)}\geq N^{(i)}\geq 0$ and $E_j^{(i)}$, $j=1,\ldots,v$; $F_j^{(i)}$, $j=1,\ldots,P^{(i)}$; $G_j^{(i)}$, $j=1,\ldots,w$; $H_j^{(i)}$ $j=1,\ldots,Q^{(i)}$ are positive real numbers;

$$\left|arg\left(y_i\,sin^{2\eta_i}\,\pi x/L\right)\right|<T_i\,\pi/2,$$

where

$$T_i=-\sum_{j=u+1}^{v}E_j^{(i)}+\sum_{j=1}^{N^{(i)}}F_j^{(i)}-\sum_{j=1+N^{(i)}}^{P^{(i)}}F_j^{(i)}-\sum_{j=1}^{w}G_j^{(i)}+\sum_{j=1}^{M^{(i)}}H_j^{(i)}-\sum_{j=1+M^{(i)}}^{Q^{(i)}}H_j^{(i)}>0;\,i=1,\ldots,n.$$

p_r, q_r are arbitrary positive integers, A_{p_r,s_r} are arbitrary functions of p_r, s_r real or complex independent of x, x_r, s_r; $r=1,\ldots,k$ and also all conditions of (2.1) are satisfied.

8. Special Cases of Problem 1.

Case I. For each $m_i=2, A_{n_i,s_i}=(-1)^{s_i}, S_{n_i}^{2}(x)\to x^{n_i/2}H_{n_i}\left(\frac{1}{2\sqrt{x}}\right)$, we derive the result from (2.1) for Hermite polynomials ([15] p.106, eq. (5.5.4) and [13],p.158)

$$(8.1)\quad \int_0^L(sin\,\pi x/L)^{W-1}sin(\pi x\lambda_m/L)\left[y_1(sin\,\pi x/L)^{2\rho_1}\right]^{n_1/2}H_{n_1}\left(\frac{1}{2\sqrt{y_1(sin\,\pi x/L)^{2\rho_1}}}\right)$$

$$\ldots\left[y_r(sin\,\pi x/L)^{2\rho_r}\right]^{n_r/2}H_{n_r}\left(\frac{1}{2\sqrt{y_r(sin\,\pi x/L)^{2\rho_r}}}\right)H^{0,\lambda:(\mu',\nu');\ldots;(\mu^{(n)},\nu^{(n)})}_{A,C:[B',D'];\ldots;[B^{(n)},D^{(n)}]}$$

$$\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]:[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}];\\ [(c):\psi',\ldots,\psi^{(n)}]:[(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\,z_1(sin\,\pi x/L)^{2\xi_1},\ldots,z_n(sin\,\pi x/L)^{2\xi_n}\right)dx$$

$$=\frac{L\,sin(\pi\lambda_m/2)}{2^{W-1}}\sum_{s_1=0}^{[n_1/2]}\cdots\sum_{s_r=0}^{[n_r/2]}\frac{(-n_1)_{2s_1}}{s_1!}\cdots\frac{(-n_r)_{2s_r}}{s_r!}\frac{(-1)^{s_1+\dots+s_r}\,y_1^{s_1}\dots y_r^{s_r}}{2^{2(\rho_1 s_1+\dots+\rho_r s_r)}}H(s_1,\dots,s_r)$$

valid if all conditions of (2.1) hold.
Then by (4.2), the solution of problem I, is given by

$$(8.2)\qquad u(x,t)=\frac{\pi}{2^{W-3}}\sum_{m=1}^{\infty}sin(\lambda_m\pi x/L)\,exp\left\{-(\pi\lambda_m/L)^2kt\right\}$$

$$\frac{\lambda_m\,sin(\pi\lambda_m/2)}{[2\pi\lambda_m-sin\,2\pi\lambda_m]}\sum_{s_1=0}^{[n_1/2]}\cdots\sum_{s_r=0}^{[n_r/2]}\frac{(-n_1)_{2s_1}\cdots(-n_r)_{2s_r}(-1)^{s_1+\dots+s_r}}{2^{2(\rho_1 s_1+\dots+\rho_r s_r)}}\frac{y_1^{s_1}\dots y_r^{s_r}}{s_1!\dots s_r!}H(s_1,\dots,s_r)$$

where all conditions of (4.2) are satisfied.
From (5.1), we derive expansion formula

$$(8.3)\qquad (sin\,\pi x/L)^{W-1}y_1^{n_1/2}\dots y_r^{n_r/2}(sin\,\pi x/L)^{n_1\rho_1+\dots+n_r\rho_r}H_{n_1}\left(\frac{1}{2\sqrt{y_1}(sin\,\pi x/L)^{\rho_1}}\right)$$

$$\dots H_{n_r}\left(\frac{1}{2\sqrt{y_r}(sin\,\pi x/L)^{\rho_r}}\right)H^{0,\lambda:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})}_{A,C:[B',D'];\dots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]:\\ [(c):\psi',\dots,\psi^{(n)}]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}];\\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\;z_1(sin\,\pi x/L)^{2\xi_1},\dots,z_n(sin\,\pi x/L)^{2\xi_n}\right)$$

$$=\frac{\pi}{2^{W-3}}\sum_{m=1}^{\infty}\frac{sin(\pi\lambda_m x/L)\lambda_m\,sin(\pi\lambda_m/2)}{[2\pi\lambda_m-sin\,2\pi\lambda_m]}\sum_{s_1=0}^{[n_1/2]}\cdots\sum_{s_r=0}^{[n_r/2]}\frac{(-n_1)_{2s_1}\cdots(-n_r)_{2s_r}(-1)^{s_1+\dots+s_r}}{2^{2(\rho_1 s_1+\dots+\rho_r s_r)}}$$

$$H[s_1,\dots,s_r]\frac{y_1^{s_1}}{s_1!}\cdots\frac{y_r^{s_r}}{s_r!}.$$

Case II. For each $m_i=1, A_{n_i s_i}=\dfrac{(1+\alpha_i)_{n_i}}{n_i!}\dfrac{1}{(1+\alpha_i)_{s_i}}, i=1,\dots,r;$ we derive the result for laguerre polynomials $S^1_{n_i}(x)\to L^{(\alpha_i)}_{n_i}$ ([15], p. 101, Eq. (5.1.6) and [13], p.159)

$$(8.4)\qquad \int_0^L(sin\,\pi x/L)^{W-1}sin(\pi x\lambda_m/L)L^{(\alpha_1)}_{n_1}\left[y_1(sin\,\pi x/L)^{2\rho_1}\right]\dots L^{(\alpha_r)}_{n_r}\left[y_r(sin\,\pi x/L)^{2\rho_r}\right]$$

$$H^{0,\lambda:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})}_{A,C:[B',D'];\dots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]:[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}];\\ [(c):\psi',\dots,\psi^{(n)}]:[(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\;z_1(sin\,\pi x/L)^{2\xi_1},\dots,z_n(sin\,\pi x/L)^{2\xi_n}\right)dx$$

$$= \frac{L\,sin(\pi\lambda_m/2)}{2^{W-1}} \frac{(1+\alpha_i)_{n_1}}{n_1!} \cdots \frac{(1+\alpha_r)_{n_r}}{n_r!} \sum_{s_1=0}^{n_1} \cdots \sum_{s_r=0}^{n_r} \frac{(-n_1)_{s_1} \cdots (-n_r)_{s_r}}{(1+\alpha_1)_{s_1} \cdots (1+\alpha_r)_{s_r}}$$

$$\frac{(y_1/4^{\rho_1})^{s_1}}{s_1!} \cdots \frac{(y_r/4^{\rho_r})^{s_r}}{s_r!} H[s_1,\ldots,s_r],$$

where all condition of (2.1) are satisfied.
Now by (4.2) the solution of peoblem I, reduces to

(8.5) $$u(x,t) = \frac{\pi}{2^{W-3}} \sum_{m=1}^{\infty} sin(\lambda_m \pi x / L)\, exp\left\{-(\pi\lambda_m/L)^2 kt\right\}$$

$$\frac{\lambda_m\, sin(\pi\lambda_m/2)}{[2\pi\lambda_m - sin\, 2\pi\lambda_m]} \frac{(1+\alpha_1)_{n_1}}{n_1!} \cdots \frac{(1+\alpha_r)_{n_r}}{n_r!} \sum_{s_1=0}^{n_1} \cdots \sum_{s_r=0}^{n_r} \frac{(-n_1)_{s_1}}{(1+\alpha_1)_{s_1}} \cdots \frac{(-n_r)_{s_r}}{(1+\alpha_r)_{s_r}}$$

$$H[s_1',\ldots,s_r] \frac{y_1^{s_1}}{s_1!} \cdots \frac{y_r^{s_r}}{s_r!} \frac{1}{4^{(\rho_1 s_1 + \ldots + \rho_r s_r)}},$$

where all conditions of (4.1) are satisfied.

Also from (5.1) we derive expansion formula

(8.6) $$(sin\,\pi x/L)^{W-1} L_n^{(\alpha_1)}\left[y_1(sin\,\pi x/L)^{2\rho_1}\right] \ldots L_{n_r}^{(\alpha_r)}\left[y_r(sin\,\pi x/L)^{2\rho_r}\right]$$

$$H^{0,\lambda:(\mu',\nu');\ldots;(\mu^{(n)},\nu^{(n)})}_{A,C:[B',D'];\ldots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]: \\ [(c):\psi',\ldots,\psi^{(n)}]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix} z_1(sin\,\pi x/L)^{2\xi_1},\ldots,z_n(sin\,\pi x/L)^{2\xi_n}\right)$$

$$= \frac{\pi}{2^{W-3}} \sum_{m=1}^{\infty} sin(\pi\lambda_m x/L) \frac{\lambda_m\, sin(\pi\lambda_m/2)}{[2\pi\lambda_m - sin\, 2\pi\lambda_m]} \frac{(1+\alpha_1)_{n_1}}{n_1!} \cdots \frac{(1+\alpha_r)_{n_r}}{n_r!}$$

$$\sum_{s_1=0}^{[n_1/m_1]} \cdots \sum_{s_r=0}^{[n_r/m_r]} \frac{(-n_1)_{s_1} \cdots (-n_r)_{s_r}}{(1+\alpha_1)_{s_1} \cdots (1+\alpha_r)_{s_r}} \frac{(4^{-\rho_1} y_1)^{s_1}}{s_1!} \cdots \frac{(4^{-\rho_r} y_r)^{s_r}}{s_r!} H(s_1,\ldots,s_r),$$

where all the conditions of (5.1) are satisfied

Case III. For each $m_i=1$, $q_i=1$, $A_{n_i s_i} = \frac{(1+\alpha_i)_{n_i}}{n_i!} \frac{(1+\alpha_i+\beta_i+n_i)}{(1+\alpha_i)_{s_i}}, S_{n_i}^1(x) \to P_{n_i}^{(\alpha_i,\beta_i)}$,

$i=1,2,\ldots,r$ we derive the result from (2.1) for Jacobi polynomials ([15], p.68, Eq. (4.3.2) and [13], p. 159)

$$(8.7)\quad \int_0^L (\sin \pi x/L)^{W-1} \sin(\pi x\lambda_m/L)\prod_{i=1}^{r} P_{n_i}^{(\alpha_i,\beta_i)}\left[1-2y_i(\sin \pi x/L)^{2\rho_i}\right]$$

$$H^{0,\lambda:(\mu',\nu');\ldots;(\mu^{(n)},\nu^{(n)})}_{A,C:[B',D'];\ldots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]:[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}];\\ [(c):\psi',\ldots,\psi^{(n)}]:[(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\right.$$

$$\left. z_1(\sin \pi x/L)^{2\xi_1},\ldots,z_n(\sin \pi x/L)^{2\xi_n}\right)dx$$

$$=\frac{L\sin(\pi\lambda_m/2)}{2^{W-1}}\prod_{i=1}^{r}\frac{(1+\alpha_i)_{n_i}}{n_i!}\sum_{s_1=0}^{n_1}\cdots\sum_{s_r=0}^{n_r}\frac{(-n_1)_{s_1}\cdots(-n_r)_{s_r}\,y_1^{s_1}\cdots y_r^{s_r}}{2^{2(\rho_1 s_1+\ldots+\rho_r s_r)}s_1!\ldots s_r!}$$

$$\prod_{i=1}^{r}\frac{(1+\alpha_i+\beta_i+n_i)_{s_i}}{(1+\alpha_i)_{s_i}}H[s_1,\ldots,s_r]$$

valid if all the conditions of (2.1) are satisfied.

Then by (4.2), the solution of the problem I is given by

$$(8.8)\quad u(x,t)=\frac{\pi}{2^{W-3}}\sum_{m=1}^{\infty}\sin(\pi x/L)\exp\left\{-(\pi\lambda_m/L)^2kt\right\}\frac{\lambda_m\sin(\pi\lambda_m/2)}{[2\pi\lambda_m-\sin 2\pi\lambda_m]}$$

$$\prod_{i=1}^{r}\frac{(1+\alpha_i)}{n_i!}\sum_{s_1=0}^{[n_1/m_1]}\cdots\sum_{s_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{s_i}}{4^{\rho_i s_i}}\frac{(1+\alpha_i+\beta_i+n_i)_{s_i}}{(1+\alpha_i)_{s_i}}\frac{y_i^{s_i}}{s_i!}H(s_1,\ldots,s_r)$$

where all conditions (2.1) and (4.2) are astified.

Also expanion formula (5.1), reduces to

$$(8.9)\,(\sin \pi x/L)^{W-1}\prod_{i=1}^{r}P_{n_i}^{(\alpha_i,\beta_i)}\left(1-2y_i(\sin \pi x/L)^{\rho_i}\right)H^{0,\lambda:(\mu',\nu');\ldots;(\mu^{(n)},\nu^{(n)})}_{A,C:[B',D'];\ldots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]:\\ [(c):\psi',\ldots,\psi^{(n)}]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}];\\ [(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\; z_1(\sin \pi x/L)^{2\xi_1},\ldots,z_n(\sin \pi x/L)^{2\xi_n}\right)$$

$$=\frac{\pi}{2^{W-3}}\sum_{m=1}^{\infty}\frac{\sin(\pi\lambda_m x/L)\,\lambda_m\sin(\pi\lambda_m/2)}{[2\pi\lambda_m-\sin 2\pi\lambda_m]}\prod_{i=1}^{r}\frac{(1+\alpha_i)_{n_i}}{n_i!}\sum_{s_1=0}^{n_1}\cdots\sum_{s_r=0}^{n_r}\prod_{i=1}^{r}\frac{(-n_i)_{s_i}}{4^{\rho_i s_i}}$$

$$\frac{(1+\alpha_i+\beta_i+n_i)_{s_1}}{(1+\alpha_i)_{s_i}}\frac{y_i^{s_i}}{s_i!}H(s_1,\ldots,s_r),$$

valid if all the conditions of (2.1) and (4.2) are satisfied.

Case IV. Choosing $\lambda=A$, $\mu^{(i)}=1, \nu^{(i)}=B^{(i)}$, replacing $D^{(i)}$ by $D^{(i)}+1$, $i=1,\ldots,n$, integral (2.1) reduces to

$$(8.10)\quad \int_0^L (\sin\pi x/L)^{W-1}\sin(\pi x\lambda_m/L)\prod_{i=1}^r S_{n_i}^{m_i} y_i(\sin\pi x/L)^{2\rho_i}\; F_{C,D';\ldots;D^{(n)}}^{A:B';\ldots;B^{(n)}}$$

$$\left(\begin{matrix}[1-(a):\theta',\ldots,\theta^{(n)}]:[1-(b'):\phi'];\ldots;[1-(b^{(n)}):\phi^{(n)}];\\ [1-(c):\psi',\ldots,\psi^{(n)}]:[1-(d'):\delta'];\ldots;[1-(d^{(n)}):\delta^{(n)}];\end{matrix}\, z_1(\sin\pi x/L)^{2\xi_1},\ldots,z_n(\sin\pi x/L)^{2\xi_n}\right)dx$$

$$=\frac{L\sin(\pi\lambda_m/2)}{2^{W-1}}\sum_{s_1=0}^{[n_1/m_1]}\cdots\sum_{s_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_is_i}}{4^{\rho_is_i}}\frac{A_{n_i,s_i}}{s_i!}y_i^{s_i}\; F_{C+2,:D';\ldots;D^{(n)}}^{A+1:B';\ldots;B^{(n)}}$$

$$\left(\begin{matrix}[1-(a):\theta',\ldots,\theta^{(n)}],[W+2(\rho_1s_1+\ldots+\rho_rs_r):2\xi_1,\ldots,2\xi_n]:\\ [1-(c):\psi',\ldots,\psi^{(n)}],\left[\frac{1+W+2(\rho_1s_1+\ldots+\rho_rs_r)\pm\lambda_m}{2}:\xi_1,\ldots,\xi_n\right]:\end{matrix}\;\begin{matrix}[1-(b'):\phi'];\ldots;[1-(b^{(n)}):\phi^{(n)}];\\ [1-(d'):\delta'];\ldots;[1-(d^{(n)}):\delta^{(n)}];\end{matrix}\, z_1/4^{\xi_1}\ldots z_n/4^{\xi_n}\right)$$

$$\frac{\Gamma(W+2(\rho_1s_1+\ldots+\rho_rs_r))}{\Gamma\left(\frac{1+W+2(\rho_1s_1+\ldots+\rho_rs_r)\pm\lambda_m}{2}\right)},$$

where all conditions of (2.1) are satisfied.

Then the solútion (4.2) of problem I, is given by

$$(8.11)\quad u(x,t)=\frac{\pi}{2^{W-3}}\sum_{m=1}^{\infty}\sin(\lambda_m\pi x/L)\exp\{-(\pi\lambda_m/L)^2kt\}\frac{\lambda_m\sin(\pi\lambda_m/2)}{[2\pi\lambda_m-\sin 2\pi\lambda_m]}$$

$$\sum_{s_1=0}^{[n_1/m_1]}\cdots\sum_{s_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_is_i}}{4^{\rho_is_i}}\frac{A_{n_i,s_i}y_i^{s_i}}{s_i!}\frac{\Gamma(W+2(\rho_1s_1+\ldots+\rho_rs_r))}{\Gamma\left(\frac{1+W+2(\rho_1s_1+\ldots+\rho_rs_r)\pm\lambda_m}{2}\right)}$$

$$F_{C+2,:D';\ldots;D^{(n)}}^{A+1:B';\ldots;B^{(n)}}\left(\begin{matrix}[1-(a):\theta',\ldots,\theta^{(n)}],[W+2(\rho_1s_1+\ldots+\rho_rs_r):2\xi_1,\ldots,2\xi_n]:\\ [1-(c):\psi',\ldots,\psi^{(n)}],\left[\frac{1+W+2(\rho_1s_1+\ldots+\rho_rs_r)\pm\lambda_m}{2}:\xi_1,\ldots,\xi_n\right]:\end{matrix}\right.$$

$$\left.\begin{matrix}[1-(b'):\phi'];\ldots;[1-(b^{(n)}):\phi^{(n)}];\\ [1-(d'):\delta'];\ldots;[1-(d^{(n)}):\delta^{(n)}];\end{matrix}\, z_1/4^{\xi_1},\ldots,z_n/4^{\xi_n}\right)$$

provided that all conditions of (2.1) and (4.2) are satisfied.

Also expansion formula (5.1) reduces to

$$(8.12)\quad (\sin\pi x/L)^{W-1}S_{n_1}^{m_1}\left[y_1(\sin\pi x/L)^{2\rho_1}\right]\ldots S_{n_r}^{m_r}\left[y_r(\sin\pi x/L)^{2\rho_r}\right]F_{C:D';\ldots;D^{(n)}}^{A:B';\ldots;B^{(n)}}$$

$$\left(\begin{matrix} \left[1-(a):\theta',\ldots,\theta^{(n)}\right]:\left[1-(b'):\phi'\right];\ldots;\left[1-\left(b^{(n)}\right):\phi^{(n)}\right]; \\ \left[1-(c):\psi',\ldots,\psi^{(n)}\right]:\left[1-(d'):\delta'\right];\ldots;\left[1-\left(d^{(n)}\right):\delta^{(n)}\right]; \end{matrix} z_1(\sin \pi x/L)^{2\xi_1},\ldots,z_n(\sin \pi x/L)^{2\xi_n} \right) dx$$

$$= \frac{\pi}{2^{W-3}} \sum_{m=1}^{\infty} \frac{\sin(\pi\lambda_m x/L)\ \lambda_m \sin(\pi\lambda_m/2)}{\left[2\pi\lambda_m - \sin 2\pi\lambda_m\right]} \sum_{s_1=0}^{[n_1/m_1]} \cdots \sum_{s_r=0}^{[n_r/m_r]} \prod_{i=1}^{r} \frac{(-n_i)_{m_i s_i}}{4^{\rho_i s_i}} \frac{A_{n_i,s_i} y_i^{s_i}}{s_i\,!}$$

$$\frac{\Gamma\left(W + 2(\rho_1 s_1 + \ldots + \rho_r s_r)\right)}{\Gamma\left(\dfrac{1 + W + 2(\rho_1 s_1 + \ldots + \rho_r s_r) \pm \lambda_m}{2}\right)} F^{A+1:B';\ldots;B^{(n)}}_{C+2,:D';\ldots;D^{(n)}}$$

$$\left(\begin{matrix} \left[1-(a):\theta',\ldots,\theta^{(n)}\right],\left[W+2(\rho_1 s_1+\ldots+\rho_r s_r'):2\xi_1,\ldots,2\xi_n\right]; & \left[1-(b'):\phi'\right];\ldots;\left[1-\left(b^{(n)}\right):\phi^{(n)}\right]; \\ \left[1-(c):\psi',\ldots,\psi^{(n)}\right],\left[\frac{1+W+2(\rho_1 s_1+\ldots+\rho_r s_r)\pm\lambda_m}{2}:\xi_1,\ldots,\xi_n\right]; & \left[1-(d'):\delta'\right];\ldots;\left[1-\left(d^{(n)}\right):\delta^{(n)}\right]; \end{matrix} z_1/4^{\xi_1} \ldots z_n/4^{\xi_n} \right)$$

where all the conditions of (2.1) and (4.2) are satisfied.

Case V. Further for $r=2,\ \lambda_m = (2m+1)/2$, our results (2.1), (3.8), (4.1), (4.2), (8.1), (8.2), (8.4), (8.5), (8.7), (8.8), (8.10) and (8.11) give respectively (7), (6) and correct forms of wrong results (11), (8), (12), (13), (14), (15), (16), (17), (18) and (19) due to Chaurasia and Patni [2].

This remark also suggests that all the results due to Chaurasia and Gutpa ([3]), (2.1), (2.2), (3.1), (3.3), (4.1), (4.2), (4.3), (4.4), (4.5), (4.6), (4.7), (4.8), (4.9), (4.10), (4.11) and (4.12) are wrongly expressed. Actually $2^{-2(\rho s+\rho' s')}$ should be written within the summation signs $\sum_{s=0}^{n} \sum_{s'=0}^{n'}$ in the all above results.

Case VI. Choosing $\lambda=A,\ n_i=0,\ \mu^{(i)}=1, \nu^{(i)}=B^{(i)}$, replacing $D^{(i)}$ by $D^{(i)}+1$, z_i by $-z_i$ $i=1, \ldots, n,$ all the results of this paper will reduce to the results of Chandel and Tiwari [1]

Similarly specializing the coefficents A_{n_i,s_i} $i=1,\ldots,n$ of polynomials $S_{n_i}^{m_i}[x_i]$ [6] and parameters of H-function of several complex variables ([10], [11], [12]), we shall get a large number of results involving various polynomials and other special functions useful in Mathmatical Analysis, Applied Mathematics and Mathematical Physics.

9. Special Cases of Problem II

Case I. For Hermite polynomials ([15], p.106, eq. (5.54), and [13], p.158), choosing

each $m_i = 2, A_{n_i,s_i} = (-1)^{s_i}, S_{n_i}^2(x) \to x^{n_i} H_{n_i}\left(\frac{1}{2\sqrt{x}}\right), i = 1,\ldots,r; \quad q_j = 2, \quad j = 1,\ldots,l$

(3.8) and (6.6) give respectively :

$$(9.1)\ f(x) = (\sin \pi x/L)^{W-1} \prod_{i=1}^{r} y_i (\sin \pi x/L)^{2\rho_i} H_{n_i}\left(\frac{1}{2\sqrt{y_i}(\sin \pi x/L)^{\rho_i}}\right)$$

$$H^{0,\lambda:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})}_{A,C:[B',D'];\dots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]:[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}];\\ [(c):\psi',\dots,\psi^{(n)}]:[(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix} z_1(\sin\pi x/L)^{2\xi_1},\dots,z_n(\sin\pi x/L)^{2\xi_n}\right)$$

and

(9.2) $$g(x)=(\sin\pi x/L)^{W'-1}\prod_{i=1}^{l}x_i(\sin\pi x/L)^{2\sigma_i}H_{p_i}\left(\frac{1}{2\sqrt{x_i}(\sin\pi x/L)^{\sigma_i}}\right)$$

$$H^{0,u:(M',N');\dots;(M^{(n)},N^{(n)})}_{v,w:[P',Q'];\dots;[P^{(n)},Q^{(n)}]}\left(\begin{matrix}[(e):E',\dots,E^{(n)}]:[(f'):F'];\dots;[(f^{(n)}):F^{(n)}];\\ [(g):G',\dots,G^{(n)}]:[(h'):H'];\dots;[(h^{(n)}):H^{(n)}];\end{matrix}\right.$$

$$\left.u_1\sin^{2\eta_1}\pi x/L,\dots,u_n\sin^{2\eta_n}\pi x/L\right)$$

Then solution (6.10) of the problem II is reduced in the form:

(9.3) $$u(x,t)=\frac{\pi}{2^{W-3}}\sum_{m=1}^{\infty}\frac{\lambda_m\sin(\pi\lambda_m x/L)\cos(\pi\lambda_m ct/L)\sin(\pi\lambda_m/2)}{[2\pi\lambda_m-\sin 2\pi\lambda_m]}$$

$$\sum_{s_1=0}^{[n_1/2]}\dots\sum_{s_r=0}^{[n_r/2]}\prod_{i=1}^{r}\frac{(-1)^{s_i}(-n_i)_{2s_i}y^{s_i}}{4^{2\rho_i s_i}s_i!}H(s_1,\dots,s_r)$$

$$+\frac{\pi L}{c.2^{W'-3}}\sum_{m=1}^{\infty}\frac{\sin(\pi\lambda_m x/L)\sin(\pi\lambda_m ct/L)\sin(\pi\lambda_m/2)}{[2\pi\lambda_m-\sin 2\pi\lambda_m]}$$

$$\sum_{s_1=0}^{[p_1/2]}\dots\sum_{s_l=0}^{[p_l/2]}\prod_{i=1}^{l}\frac{(-1)^{s_i}(-p_i)_{2s_i}x^{s_i}}{4^{2\rho_i s_i}s_i!}H(s_1,\dots,s_l)$$

where all conditins of (6.10) are satisfied.

Case II. For Laguerre polynomials ([15], p.101, Eq. 5. 1.6) and [13], p.159), choosing

each $m_i=1, A_{n_i,s_i}=\dfrac{(1+\alpha_i)_{n_i}}{n_i!}\dfrac{1}{(1+\alpha_i)_{s_i}}, S^1_{n_i}(x)\to L^{(\alpha_i)}_{n_i}, i=1,\dots,r;\ q_j=1,$

$A_{p_j,s_j}=\dfrac{(1+\beta_j)_{p_j}}{p_j!}\dfrac{1}{(1+\beta_j)_{s_j}}, S^1_{p_j}(x)\to L^{(\beta_j)}_{p_j}, j=1,\dots,l,$ (3.8) and (4.6) give respectively

(9.4) $$f(x)=(\sin\pi x/L)^{W-1}\prod_{i=1}^{r}L^{(\alpha_i)}_{n_i}\left[y_i(\sin\pi x/L)^{2\rho_i}\right]H^{0,\lambda:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})}_{A,C:[B',D'];\dots;[B^{(n)},D^{(n)}]}$$

$$\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]:[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}];\\ [(c):\psi',\dots,\psi^{(n)}]:[(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix} z_1(\sin\pi x/L)^{2\xi_1},\dots,z_n(\sin\pi x/L)^{2\xi_n}\right)$$

and

$$(9.5)\qquad g(x)=(\sin\pi x/L)^{W'-1}\prod_{i=1}^{l}L_{p_i}^{(1+\beta_i)}\left[x_i(\sin\pi x/L)^{2\sigma_i}\right]$$

$$H^{0,u:(M',N');\ldots;(M^{(n)},N^{(n)})}_{v,w:[P',Q'];\ldots;[P^{(n)},Q^{(n)}]}\left(\begin{matrix}[(e):E',\ldots,E^{(n)}]:[(f'):F'];\ldots;[(f^{(n)}):F^{(n)}];\\ [(g):G',\ldots,G^{(n)}]:[(h'):H'];\ldots;[(h^{(n)}):H^{(n)}];\end{matrix}\right.$$

$$\left.u_1\sin^{2\eta_1}\pi x/L,\ldots,u_n\sin^{2\eta_n}\pi x/L\right)$$

Then solution (6.10) of the problem is reduced to

$$(9.6)\qquad u(x,t)=\frac{\pi}{2^{W-3}}\sum_{m=1}^{\infty}\frac{\lambda_m\sin(\pi\lambda_m x/L)\cos(\pi\lambda_m ct/L)\sin(\pi\lambda_m/2)}{[2\pi\lambda_m-\sin 2\pi\lambda_m]}$$

$$\prod_{i=1}^{r}\frac{(1+\alpha_i)_{n_i}}{n_i!}\sum_{s_1=0}^{n_1}\cdots\sum_{s_r=0}^{n_r}\prod_{i=1}^{r}\frac{(-n_i)_{s_i}}{(1+\alpha_i)_{s_i}}\frac{y^{s_i}}{4^{2\rho_i s_i}s_i!}H(s_1,\ldots,s_r)$$

$$+\frac{\pi L}{c.2^{W'-3}}\sum_{m=1}^{\infty}\frac{\sin(\pi\lambda_m x/L)\sin(\pi\lambda_m ct/L)\sin(\pi\lambda_m/2)}{[2\pi\lambda_m-\sin 2\pi\lambda_m]}$$

$$\prod_{j=1}^{l}\frac{(1+\beta_j)_{p_j}}{p_j!}\sum_{s_1=0}^{p_1}\cdots\sum_{s_l=0}^{p_l}\prod_{i=1}^{l}\frac{(-p_i)_{s_i}}{(1+\beta_i)_{s_i}}\frac{x^{s_i}}{4^{\sigma_i s_i}s_i!}\overline{H}(s_1,\ldots,s_l).$$

where all conditions of (6.10) are satisfied

Case III For each $m_i=1$, $q_i=1$, $A_{n_i,s_i}=\dfrac{(1+\alpha_i)_{n_i}}{n_i!}\dfrac{(1+\alpha_i+\beta_i+n_i)_{s_i}}{(1+\alpha_i)_{s_i}}$,

$$A_{p_i,s_i}=\frac{(1+\gamma_i)_{p_i}}{p_i!}\frac{(1+\gamma_i+\delta_i+p_i)_{s_i}}{(1+\gamma_i)_{s_i}},S^1_{n_i}[x]\to P^{(\alpha_i,\beta_i)}_{n_i}(1-2x),S^1_{p_i}[x]\to P^{(\gamma_i,\delta_i)}_{p_i}(1-2x),$$

we derive for Jacobi polynomials ([15], p.68, eq. (4.32) and [13], p. 159) from the results (3.8) and (6.6) respectively

$$(9.7)\qquad f(x)=(\sin\pi x/L)^{W-1}\prod_{i=1}^{r}P^{(\alpha_i,\beta_i)}_{n_i}\left[1-2y_i(\sin\pi x/L)^{2\rho_i}\right]H^{0,\lambda:(\mu',\nu');\ldots;(\mu^{(n)},\nu^{(n)})}_{A,C:[B',D'];\ldots;[B^{(n)},D^{(n)}]}$$

$$\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]:[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}];\\ [(c):\psi',\ldots,\psi^{(n)}]:[(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\;z_1(\sin\pi x/L)^{2\xi_1},\ldots,z_n(\sin\pi x/L)^{2\xi_n}\right)$$

and

(9.8) $$g(x)=\left(\sin \pi x/L\right)^{W'-1}\prod_{i=1}^{l} P_{p_i}^{(\gamma_i,\delta_i)}\left[1-2x_i\left(\sin \pi x/L\right)^{2\sigma_i}\right]$$

$$H^{0,u:(M',N');\ldots;(M^{(n)},N^{(n)})}_{v,w:[P',Q'];\ldots;[P^{(n)},Q^{(n)}]}\left(\begin{matrix}[(e):E',\ldots,E^{(n)}]:[(f'):F'];\ldots;[(f^{(n)}):F^{(n)}];\\ [(g):G',\ldots,G^{(n)}]:[(h'):H'];\ldots;[(h^{(n)}):H^{(n)}];\end{matrix}\right.$$

$$\left. u_1 \sin^{2\eta_1}\pi x/L,\ldots,u_n \sin^{2\eta_n}\pi x/L \right).$$

The solution of the problem II, reduces to

(9.9) $$u(x,t)=\frac{\pi}{2^{W-3}}\sum_{m=1}^{\infty}\frac{\lambda_m \sin(\pi\lambda_m x/L)\cos(\pi\lambda_m ct/L)\sin(\pi\lambda_m/2)}{[2\pi\lambda_m-\sin 2\pi\lambda_m]}$$

$$\prod_{i=1}^{r}\frac{(1+\alpha_i)_{n_i}}{n_i!}\sum_{s_1=0}^{[n_1/m_1]}\cdots\sum_{s_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{s_i}(1+\alpha_i+\beta_i+n_i)_{s_i}y_i^{s_i}}{(1+\alpha_i)_{s_i}4^{\rho_i s_i}s_i!}H(s_1,\ldots,s_r)$$

$$+\frac{\pi L}{c.2^{W'-3}}\frac{\sin(\pi\lambda_m x/L)\sin(\pi\lambda_m ct/L)\sin(\pi\lambda_m/2)}{[2\pi\lambda_m-\sin 2\pi\lambda_m]}$$

$$\prod_{i=1}^{l}\frac{(1+\gamma_i)_{p_i}}{p_i!}\sum_{s_1=0}^{p_1}\cdots\sum_{s_l=0}^{p_l}\prod_{i=1}^{l}\frac{(-p_i)_{s_i}(1+\gamma_i+\delta_i+p_i)_{s_i}x_i^{s_i}}{(1+\gamma_i)_{n_i}4^{\sigma_i s_i}s_i!}\overline{H}(s_1,\ldots,s_l)$$

where all conditions of (6.10) ware satisfied.

Case IV. Choosing $\lambda=A$, $\mu^{(i)}=1, \nu^{(i)}=B^{(i)}$, replacing $D^{(i)}$ by $1-D^{(i)}$, z_i by $(-z_i)$ in (3.8); and Choosing $u=v$, $M^{(i)}=1$, $N^{(i)}=P^{(i)}$ replacing by $Q^{(i)}$ by $1-Q^{(i)}$, u_i by $(-u_i)$, $i=1,\ldots,n$ in (6.6), we derive respectively

(9.10) $$f(x)=\left(\sin \pi x/L\right)^{W-1}\prod_{i=1}^{r} S_{n_i}^{m_i}\left[y_i\left(\sin \pi x/L\right)^{2\rho_i}\right] F^{A:B';\ldots;B^{(n)}}_{C,D';\ldots;D^{(n)}}$$

$$\left(\begin{matrix}[1-(a):\theta',\ldots,\theta^{(n)}]:[1-(b'):\phi'];\ldots;[1-(b^{(n)}):\phi^{(n)}];\\ [1-(c):\psi',\ldots,\psi^{(n)}]:[1-(d'):\delta'];\ldots;[1-(d^{(n)}):\delta^{(n)}];\end{matrix} z_1\left(\sin \pi x/L\right)^{2\xi_1},\ldots,z_n\left(\sin \pi x/L\right)^{2\xi_n}\right)$$

and

(9.11) $$g(x)=\left(\sin \pi x/L\right)^{W'-1}\prod_{i=1}^{l} S_{p_i}^{q_i}\left[x_i\left(\sin \pi x/L\right)^{2\sigma_i}\right]$$

$$F^{v:P';\ldots;P^{(n)}}_{W:Q';\ldots;Q^{(n)}}\left(\begin{matrix}[1-(e):E',\ldots,E^{(n)}]:[1-(f'):F'];\ldots;[1-(f^{(n)}):F^{(n)}];\\ [1-(g):G',\ldots,G^{(n)}]:[1-(h'):H'];\ldots;[1-(h^{(n)}):H^{(n)}];\end{matrix}\right.$$

$$u_1 \sin^{2\eta_1} \pi x/L, \ldots, u_n \sin^{2\eta_n} \pi x/L \Bigg).$$

Then solution (6.10) of the problem II is given by

$$(9.12) \qquad u(x,t) = \frac{\pi}{2^{W-3}} \sum_{m=1}^{\infty} \frac{\lambda_m \sin(\pi\lambda_m x/L) \cos(\pi\lambda_m ct/L) \sin(\pi\lambda_m/2)}{[2\pi\lambda_m - \sin 2\pi\lambda_m]}$$

$$\sum_{s_1=0}^{[n_1/m_1]} \cdots \sum_{s_r=0}^{[n_r/m_r]} \prod_{i=1}^{r} \frac{(-n_i)_{m_i s_i} A_{n_i,s_i}}{4^{\rho_i s_i}} \frac{y_i^{s_i}}{s_i!} F^{A+1:B^1;\ldots;B^{(n)}}_{C+2,:D^1;\ldots;D^{(n)}}$$

$$\left(\begin{matrix} [1-(a):\theta',\ldots,\theta^{(n)}],[W+2(\rho_1 s_1+\ldots+\rho_r s_r):2\xi_1,\ldots,2\xi_n]: & [1-(b'):\phi'];\ldots;[1-(b^{(n)}):\phi^{(n)}]; \\ [1-(c):\psi',\ldots,\psi^{(n)}],\left[\frac{1+W+2(\rho_1 s_1+\ldots+\rho_r s_r)\pm\lambda_m}{2}:\xi_1,\ldots,\xi_n\right]: & [1-(d'):\delta'];\ldots;[1-(d^{(n)}):\delta^{(n)}]; \end{matrix} z_1/4^{\xi_1} \ldots z_n/4^{\xi_n} \right)$$

$$+ \frac{\pi L}{c.2^{W'-3}} \sum_{m=1}^{\infty} \frac{\sin(\pi\lambda_m x/L) \sin(\pi\lambda_m ct/L) \sin(\pi\lambda_m/2)}{[2\pi\lambda_m - \sin 2\pi\lambda_m]} \sum_{s_1=0}^{[p_1/q_1]} \cdots \sum_{s_l=0}^{[p_l/q_l]}$$

$$\prod_{i=1}^{l} \frac{(-p_i)_{q_i s_i} A_{p_i,s_i}}{4^{\sigma_i s_i}} \frac{x_i^{s_i}}{s_i!} F^{U+1:P^1;\ldots;P^{(n)}}_{W+2,:Q^1;\ldots;Q^{(n)}} \left(\begin{matrix} [1-(e):E',\ldots,E^{(n)}],[1+W'+2(\sigma_1 s_1+\ldots+\sigma_l s_l):2\eta_1,\ldots,2\eta_n]: \\ [1-(g):G',\ldots,G^{(n)}],\left[\frac{1+W'+2(\sigma_1 s_1+\ldots+\sigma_l s_l)\pm\lambda_m}{2}:\eta_1,\ldots,\eta_n\right]: \end{matrix} \right.$$

$$\left. \begin{matrix} [1-(f'):F'];\ldots;[1-(f^{(n)}):F^{(n)}]; \\ [1-(h'):H'];\ldots;[1-(h^{(n)}):H^{(n)}]; \end{matrix} 4^{-\eta_1} u_1 \ldots 4^{-\eta_n} u_n \right)$$

valid if all the conditions of (6.10) are satisfied

Similarly specialising the coefficients A_{n_i,s_i} of polynomials $S_{n_i}^{m_i}[x]$ [6] and parameters of H-function of several complex variables ([10],[11], [12]), we shall get a very large number of results involving various polynomials others special functions useful in mathematical analysis, applied mathematics and mathematical physics.

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