Jñānābha, Vol. 30, 2000

A RESULT ON SIMPLE ACCESSIBLE RINGS

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(Received : July 10, 1999)

ABSTRACT

In this paper we show that a simple accessible ring is either a (−1,1) ring or a commutative ring.

1. Introduction. Accessible rings were introduced by Kleinfeld [1]. A non-associative ring R is called accessible in case the following two identities hold :

(1) $(x,y,z) + (z,x,y) - (x,z,y) = 0,$

(2) $((w,x), y,z) = 0$

for all w,x,y,z in R, where the associator (x,y,z) is defined by $(x,y,z) = (xy)\,z - x(yz)$ and the commutator (x,y) is defined by $(x,y) = xy - yx$, An accessible ring is defined to be simple if it has no proper two sided ideals.

A $(-1,1)$ ring is a non-associative ring in which the following identities hold :

(3) $(x,y,z) + (x,z,y) = 0$,

(4) $(x,y,z) + (y,z,x) + (z,x,y) = 0.$

In [4] it is proved that a simple non-associative ring of char $\neq 2$ satisfying the identity $((x,y,z),w) = 0$ is either commutative or associative, Kleinfeld [2] proved that the same result still valid when this identity is replaced by more general conditions. The identity $((x,y,z)\,v,w) = 0$ holds in accessible rings under the assumption that the rings are without nilpotent elements in the center [1]. Without this assumption we show that $((x,y,y)\,v,w) = 0$ holds in accessible rings. Using this identity we show that a simple accessible ring is either (−1,1) or commutative.

2. Preliminaries. By substituting $z = x$ in (1) we obtain the flexible law $(x,y,x) = 0$. The following identities hold in accessible rings

(5) $(x,y,z) = -(z,y,x)$,

(6) $(x,y,z) = x\,(y,z) + (x,z)\,y,$

(7) $(x,y,z) + (y,z,x) + (z,x,y) = 0,$

(8) $((w,x,y),z) = 0$.

The nucleus N of R is defined as the set of all elements n in R with the property $(n,R,R) = 0$, If n is an element of the nucleus N of R, then because of the flexible law $(R,R,n) = 0$, Finally because of (1) it follows that also $(R,n,R) = 0$.

The following identity holds in an arbitrary ring :

$(wx,y,z) - (w,xy,z) + (w,x,yz) = w(x,y,z) + (w,x,y)z$.

If $w = n$ in the above equation, then it becomes

(9) $(nx,y,z) = n(x,y,z)$, n in N.

From (9) and from the fact that every commutator is in the nucleus, we get $(v,x)(x,y,z) = ((v,x)x,y,z)$.

It follows from (6) that $(v,x)x = (v,x,x)$. Consequently

$((v,x)x,y,z) = ((vx,x),y,z) = 0$, Thus

(10) $(v,x)(x,y,z) = 0$.

3. Main Result

Lemma 1. In an accessible ring R, $((x,y,y)v,w) = 0$.

Proof. Linearization of (10) becomes

(11) $(v,w)(x,y,z) = -(v,x)(w,y,z)$

By using (5), (11), (7) and (10),

$= (v,y)(w,y,x)$

$= (v,y)[-(y,x,w) - (x,w,y)]$

$= -(v,y)(y,x,w) + (v,y)(y,w,x)$

$= 0$.

That is,

(12) $(v,w)(x,y,y) = 0$.

Now from(6),(12)and(8), we get $((x,y,y)v,w) = (x,y,y)(v,w) + ((x,y,y),w)v = 0$

Lemma 2. Let R be an accessible ring, then $U = \{u \in R / (u,r) = 0 = (uR,R)\}$ is an ideal of R.

Proof. If we put $w = u$ in (8), then $((u,x,y),z) = 0$. From this it follows that $(ux.y,z) - (u.xy,z) = 0$. Then $(ux,y,z) = 0$ by the definition of U. Thus $ux \in U$. So U is a right ideal of R. Since $(u,R) = 0$, $(u,x) = 0$. That is $ux = xu \in U$. So U is a left ideal of R. Hence U is an Ideal of R.

Theorem. If R is a simple accessible ring, then R is either a $(-1,1)$ ring or a commutative ring.

Proof. From (8) and lemma 1 (x,y,y) is in U. Since U is an ideal of R and R is simple, we have either $U=0$ or $U = R$. If $U = 0$, then R is right alternative, that is $(x,y,y) = 0$.

Linearization of this yields $(x,y,z) + (x,z,y) = 0$. From this and (7) it follows that R is a $(-1,1)$ ring. If $U = R$ then R is a commutative ring.

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