Jñānābha, Vol. 30, 2000

ON FLEXIBILITY OF PRIME ASSOSYMMETRIC RINGS

By

K. Suvarna and G. Ramabhupal Reddy

Department of Mathematics, Sri Krishnadevaraya University,

Anantapur-515 003, Andhra Pradesh, India

(Received : April 25, 1999)

ABSTRACT

In this paper we show that a non-associative 2-and 3-divisible prime assosymmetric ring is flexible.

1. Introduction. E. Kleinfeld [1] introduced a class of non-associative rings called as assosymmetric rings in which the associator $(x,y,z) = (xy)z - x(yz)$ has the property $(x,y,z) = (p(x), p(y), p(z))$ for each permutation p of x,y and z. These rings are neither flexible nor power-associative. In [1] it is proved that the commutator and the associator are in the nucleus of this ring. By using these properties we show that a non-associative 2-and-3-divisible prime assosymmetric ring is flexible.

2. Preliminaries. Throughout this papr R will denote a non-associative 2-and 3-divisible assosymmetric ring. The commutator (x,y) of two elements x and y in a ring is defined by $(x,y) = xy - yx$. The nucleus N in R is the set of elements $n \in R$ such that $(n,x,y) = (x,n,y) = (x,y,n) = 0$ for all x,y in R. The center C of R is the set of elements $c \in N$ such that $(c,x) = 0$ for all x in R. A non-associative ring R is called flexible if $(x,y,x) = 0$ for all x,y in R. A ring is said to be power–associative if every subring of it generated by a single element is associative Let I be the associator ideal of R. I consists of the smallest ideal which contains all associators. R is called k-divisible if $kx = 0$ implies $x = 0$, $x \in R$ and k is a natural number.

In an arbitrary ring the following identities hold:

(1) $(wx,y,z) - (w, xy, z) + (w,x,yz) = w(x,y,z) + (w,x,y)z$

$f(w,x,y,z) = (wx,y,z) - x(w,y,z) - (x,y,z)w$

and

(2) $(xy,z) - x(y,z) - (x,z)y = (x,y,z) - (x,z,y) + (z,x,y).$

In any assosymetric ring (2) becomes

(3) $(xy,z) - x(y,z) - (x,z)y = (x,y,z)$

It is proved in [1] that in a 2-and 3-divisible assosymetric ring R the following identities hold for all w,x,y,z,t in R.

(4) $f(w,x,y,z) = 0$, that is, $(wx,y,z) = x(w,y,z) + (x,y,z)w,$

(5) $((w,x), y,z) = 0$

and

(6) $((w,x,y), z,t) = 0.$

That is, every commutator and associator is in the nucleus N. From

(3), (5) and (6), we obtain

(7) $x\,(y,z) + (x,z)\,y \subset N.$

Suppose that $n \in N$. Then with $w = n$ in (1) we get $(nx,yz) = n(x,y,z)$. Combining this with (5) yields.

(8) $(nx,y,z) = n\,(x,y,z) = (xn,y,z)$

From (7) and (8), we obtain

(9) $(y,z)\,(x,r,s) = -\,(x,z)\,(y,r,s).$

3. Main Results

Lemma 1. Let $S = \{s \in N \mid s(R,R,R) = 0\}$. Then S is an ideal of R and $S(R,R,R) = 0$

Proof. By subsituting s for n in (8), we have $(sx,y,z) = s(x,y,z) = (xs,y,z) = 0$. Thus $sR \subset N$ and $Rs \subset N$. From (6), $sw\,(x,y,z) = s.\,w\,(x,yz)$. But (1) multiplied on the left by s yields $s.w(x,yz) = -\,s(w,x,y)z = -\,s(w,x,y).\,z = 0$. Thus $sw.\,(x,y,z) = 0$. From (9), we have $(s,w)\,(x,y,z) = -\,(x,w)(s,y,z) = 0$. Combining this with $sw.(x,y,z) = 0$, we obtain $ws.(x,y,z) = 0$. Thus S is an ideal of R. The rest is obvious. This completes the proof of the lemma.

Lemma 2. $(x,y,x) \in S$.

Proof. By forming the associatiors of both sides of (1) with u and v, and using (6), we obtain

(10) $(w(x,y,z),\,u,v) + ((w,x,y)\,z,u,v) = 0.$

Interchanging y and z in (10) and subtracting the result from (10), we get

(11) $((w,x,y)\,z,u,v) = ((w,x,z)\,y,u,v).$

But $((w,x,z)\,y,u,v) = (y\,(w,x,z),\,u,v)$, because of (5). So that

(12) $((w,x,y)\,z,u,v) = (y\,(w,x,z),\,u,v)$, as a result of (11).

Also by permuting w and y in (10), we obtain $(y\,(w,x,z),u,v) + ((w,x,y)\,z,u,v) = 0$. This identity with (12) yields $2\,((w,x,y)\,z,u,v) = 0$. Thus

(13) $((w,x,y)\,z,u,v) = 0.$

From (6) we have $(x,y,x) \subset N$. Using (13) and (8), we get $0 = ((x,y,x)\,z,u,v) = (x,y,x)\,(z,u,v)$ for all x,y,z,u,v in R. Hence $(x,y,x) \in S$. This completes the proof of the lemma

Theorem. If R is a non-associative 2-and 3-divisible prime assosymmetric ring, then R is flexible.

Proof. Using lemma 1 and (1) we establish that $S.\,I = 0$. Since R is prime, either $S = 0$ or $I = 0$. If $I = 0$, R is associative. But we have assumed that R is not associative. Therefore $I \neq 0$. Hence $S = 0$. From lemma 2, $(x,y,x) \in S$. Thus $(x,y,x) = 0$. That is, R is flexible.

REFERENCES

[1] E. Kleinfeld, Assosymmetric rings. *Proc. Amer. Math. Soc.* **8** (1957), 983-986.

[2] E. Kleinfeld, Rings with (x,y,x) and commutators in the left nucleus. *Comm. in Algebra,* **16 (10)** (1988), 2023-2029.