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ON SOME MULTIDIMENSIONAL INTEGRAL TRANSFORMS OF SRIVASTAVA AND PANDA'S *H*-FUNCTION OF SEVERAL COMPLEX VARIABLES

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ABSTRACT

In the present paper, we obtain multidimensional Laplace transforms and Whittaker transforms of Srivastava and Panda's *H*-function of several complex variables ([9], [10] ; also see [11], p. 251).

1. Introduction. Chandel [1] introduced multidimensional Laplacian operator to give integral representations of multiple hypergeometric functions of several variables $F_A^{(n)}$, $F_B^{(n)}$, $F_D^{(n)}$, of Lauricalla [7]. Chandel [2], further used this operator to give integral representations of multiple hypergeometric functions ${}^{(k)}_{(1)}E_D^{(n)}$, ${}^{(k)}_{(2)}E_D^{(n)}$. of Exton ([5],[6]). Also Chandel and Dwivedi ([3],[4]) introduced multidimensional Whittaker transforms of Lauricella multiple hypergeometric functions of sevreral variables [7], Exton ([5],[6]) and most generalized multiple hypergeometric function of Srivastava and Daoust [8] (also see Srivastava and Manocha [12, p. 64, [18], [19], [20]).

In the present paper, we further extend the above works to obtain the multidimensional Laplace transforms and Whittaker transforms of the most generalized Srivastava and Panda's *H*-function of several complex variables ([9], [10] ; also see [11, p.251]).

2. Laplacian Operator. Chandel [1] introduced multidimensional Laplacian operator

$$(2.1)\quad L_{\alpha_1,\ldots,\alpha_n}^{(\lambda,\mu)}\{\ \} = \frac{\Gamma(\alpha_1+\ldots+\alpha_n)\,\lambda^{\alpha_1+\ldots+\alpha_n+\mu}}{\Gamma(\alpha_1)\ldots\Gamma(\alpha_n)\ \Gamma(\alpha_1+\ldots+\alpha_n+\mu)} \int_0^\infty\!\!..\int_0^\infty e^{-\lambda(x_1+..+x_n)}(x_1+..+x_n)^{\mu}$$

$$x_1^{\alpha_1-1}\ldots x_n^{\alpha_n-1}\{\ \}\,dx_1\ldots dx_n,$$

where $Re\,(\alpha_j)>0,\ j=1,\ldots,n,\ Re\,(\lambda)>0$ and $Re\,(\alpha_1+\cdots+\alpha_n+\mu)>0$.
Here we give following additional applications of the above operator

$$(2.2)\quad L_{\alpha_1,\ldots,\alpha_n}^{(\lambda,\mu)}\Big\{H^{0,\gamma:(\mu',\upsilon');\ldots;(\mu^{(n)},\upsilon^{(n)})}_{A,C:(B',D');\ldots;(B^{(n)},D^{(n)})}\Big(\begin{matrix}[(a):(\theta',\ldots,\theta^{(n)})]:[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}];\\ [(c):(\psi',\ldots,\psi^{(n)})]:[(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}$$

$$u_1\,x_1^{\sigma_1}(x_1+\ldots+x_n)^{\upsilon_1},\ldots,u_n\,x_n^{\sigma_n}(x_1+\ldots+x_n)^{\upsilon_n}\Big)\Big\}$$

$$= \frac{\Gamma(\alpha_1+\ldots+\alpha_n)}{\Gamma(\alpha_1)\ldots\Gamma(\alpha_n)\ \Gamma(\alpha_1+\ldots+\alpha_n+\mu)} H^{0,\gamma+1\,:\,(\mu',\upsilon'+1);\ldots;(\mu^{(n)},\upsilon^{(n)}+1)}_{A+1,C+1\,:\,[B'+1,D'];\ldots;[B^{(n)}+1,D^{(n)}]}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}],\\ [(c):\psi',\ldots,\psi^{(n)}],\end{matrix}\right.$$

$$[1-(\alpha_1+\ldots+\alpha_n+\mu):\sigma_1+\upsilon_1,\ldots,\sigma_n+\upsilon_n]:[(b'):(\phi')],[1-\alpha_1:\sigma_1];\ldots;[(b^{(n)}):(\phi^{(n)})],[1-\alpha_n:\sigma_n];$$

$$[1-(\alpha_1+\ldots+\alpha_n):\sigma_1,\ldots,\sigma_n]:[(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];$$

$$\left. u_1/\lambda^{\sigma_1+\upsilon_1},\ldots,u_n/\lambda^{\sigma_n+\upsilon_n}\right),$$

where $Re\ (\lambda) > 0$, $Re\ (\alpha_1+\ldots+\alpha_n+\mu)>0$, $Re\ (\alpha_i) > 0$, σ_i ,υ_i are positive numbers

$$\Delta_i = -\sum_{j=\gamma+1}^{A}\theta_j^{(i)} + \sum_{j=1}^{\upsilon^{(i)}}\phi_j^{(i)} - \sum_{j=\upsilon^{(i)}+1}^{B^{(i)}}\phi_j^{(i)} - \sum_{j=1}^{C}\psi_j^{(i)} + \sum_{j=1}^{\mu^{(i)}}\delta_j^{(i)} - \sum_{j=\mu^{(i)}+1}^{D^{(i)}}\delta_j^{(i)} > 0,$$

$$|\arg u_i| < \pi\Delta_i/2,\ i=1,\ldots,n,$$

and H-is Srivastava and Panda's H-function of several complex variables ([9],[10] ; also see [11], p. 251).

3. Generalized Whittaker Tarnsforms. Chandel and Dwivedi ([3],[4]) introduced and stutied generalized Whittaker transform

$$(3.1)\ \Omega^{\lambda,\mu,\upsilon}_{\alpha_1,\ldots,\alpha_n;\sigma}\{\}$$

$$= \frac{\lambda^{\sigma+\alpha_1+\ldots+\alpha_n}(\sigma+1-\mu+\alpha_1+\ldots+\alpha_n)\Gamma(\sum_{i=1}^{n}(\alpha_i))}{\Gamma(\sigma+1/2+\alpha_1+\ldots+\alpha_n\pm\upsilon)(\prod_{i=1}^{n}\Gamma(\alpha_i))}\int_0^\infty\ldots\int_0^\infty \eta_1^{\alpha_1-1}\ldots\eta_n^{\alpha_n-1}(\eta_1+\ldots+\eta_n)^\sigma$$

$$e^{-\lambda(\eta_1+\ldots+\eta_n)/2}\,W_{\mu,\upsilon}[\lambda(\eta_1+\ldots+\eta_n)]\{\}\,d\eta_1\ldots d\eta_n,$$

where $Re\ (\lambda) > 0$, $Re\ (\sigma+1/2+\alpha_1+\ldots+\alpha_n\pm\upsilon) > 0$ *and* $Re\ (\alpha_i) > 0,\ i=1,\ldots,n.$
As an application of obove transform, we derive

$$(3.2)\ \Omega^{\lambda,\mu,\upsilon}_{\alpha_1,\ldots,\alpha_n;\sigma}\left\{H^{0,\gamma\,:\,(\mu',\upsilon');\ldots;(\mu^{(n)},\upsilon^{(n)})}_{A,C\,:\,[B',D'];\ldots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]:[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}];\\ [(c):\psi',\ldots,\psi^{(n)}]:[(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\right.\right.$$

$$\left.\left. u_1\,x_1^{\sigma_1}(x_1+\ldots+x_n)^{\rho_1},\ldots,u_n\,x_n^{\sigma_n}(x_1+\ldots+x_n)^{\rho_n}\right)\right\}$$

$$= \frac{\Gamma(\sigma+1-\mu+\alpha_1+\ldots+\alpha_n)\,\Gamma(\alpha_1+\ldots+\alpha_n)}{\Gamma(\sigma+1/2+\alpha_1+\ldots+\alpha_n\pm\upsilon)\,\Gamma(\alpha_1+\ldots+\alpha_n)} H^{0,\gamma+2\,:\,(\mu',\upsilon'+1);\ldots;(\mu^{(n)},\upsilon^{(n)}+1)}_{A+2,C+2\,:\,[B'+1,D'];\ldots;[B^{(n)}+1,\,D^{(n)}]}$$

$$\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}],:[1/2-(\sigma\pm\upsilon):\sigma_1+\rho_1,\ldots,\sigma_n+\rho_n];\\ [(c):\psi',\ldots,\psi^{(n)}],[\mu-(\sigma+\alpha_1+\ldots+\alpha_n):\sigma_1+\rho_1,\ldots,\sigma_n+\rho_n],[1-(\alpha_1+\ldots+\alpha_n):\sigma_1,\ldots,\sigma_n];\end{matrix}\right.$$

$$\begin{matrix}[(b'):(\phi')],[1-\alpha_1:\sigma_1];\ldots;[(b^{(n)}):(\phi^{(n)})],[1-\alpha_n:\sigma_n];\\ [(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\quad \left. u_1/\lambda^{\sigma_1+\upsilon_1},\ldots,u_n/\lambda^{\sigma_n+\upsilon_n}\right)$$

provided $Re\ (\lambda) > 0$, $Re\ (\sigma+1/2+\alpha_1+\ldots+\alpha_n\pm\upsilon) > 0$, $Re\ (\alpha_i) > 0$, σ_i ,ρ_i are positive numbers

$$\Delta_i = -\sum_{j=\gamma+1}^{A}\theta_j^{(i)} + \sum_{j=1}^{\upsilon^{(i)}}\phi_j^{(i)} - \sum_{j=\upsilon^{(i)}+1}^{B^{(i)}}\phi_j^{(i)} - \sum_{j=1}^{C}\psi_j^{(i)} + \sum_{j=1}^{\mu^{(i)}}\delta_j^{(i)} - \sum_{j=\mu^{(i)}+1}^{D^{(i)}}\delta_j^{(i)} > 0,$$

and

$$|arg\, u_i| < \pi\, \Delta_i/2\,,\ i = 1,\dots,n.$$

Special cases of (3.2)

Case (a) *For* $\sigma_1 = \dots = \sigma_n = 0$, we derive from (3.2)

$$(3.3)\ \Omega^{\lambda,\mu,\upsilon}_{\alpha_1,\dots,\alpha_n;\sigma}\left\{H^{0,\gamma\ :\ (\mu',\upsilon');\dots;(\mu^{(n)},\upsilon^{(n)})}_{A,C\ :\ [B',D'];\dots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]:[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}];\\ [(c):\psi',\dots,\psi^{(n)}]:[(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\ u_1(x_1+\dots+x_n)^{\rho_1},\dots,u_n(x_1+\dots+x_n)^{\rho_n}\right)\right\}$$

$$= \frac{\Gamma(\sigma+1-\mu+\alpha_1+\dots+\alpha_n)\,\Gamma(\alpha_1+\dots+\alpha_n)}{\Gamma(\sigma+1/2+\alpha_1+\dots+\alpha_n\pm\upsilon)\,\Gamma(\alpha_1)\dots\Gamma(\alpha_n)}\, H^{0,\gamma+2\ :\ (\mu',\upsilon');\dots;(\mu^{(n)},\upsilon^{(n)})}_{A+2,C+1\ :\ [B',D'];\dots;[B^{(n)},\ D^{(n)}]}$$

$$\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}],[1/2-(\sigma\pm\upsilon):\rho_1,\dots,\rho_n]:[(b'):(\phi')];\dots;[(b^{(n)}):(\phi^{(n)})];\\ [(c):\psi',\dots,\psi^{(n)}],[\mu-(\sigma+\alpha_1+\dots+\alpha_n):\rho_1,\dots,\rho_n]:[(d'):(\delta')];\dots;[(d^{(n)}):(\delta^{(n)})];\end{matrix}\ u_1/\lambda^{\rho_1},\dots,u_n/\lambda^{\rho_n}\right).$$

where $Re\,(\lambda) > 0$, $Re\,(\sigma + 1/2 + \alpha_1+\dots+\alpha_n \pm \upsilon) > 0$, $Re\,(\alpha_i) > 0$, σ_i, ρ_i are positive numbers

$$\Delta_i = -\sum_{j=\gamma+1}^{A}\theta_j^{(i)} + \sum_{j=1}^{\upsilon^{(i)}}\phi_j^{(i)} - \sum_{j=\upsilon^{(i)}+1}^{B^{(i)}}\phi_j^{(i)} - \sum_{j=1}^{C}\psi_j^{(i)} + \sum_{j=1}^{\mu^{(i)}}\delta_j^{(i)} - \sum_{j=\mu^{(i)}+1}^{D^{(i)}}\delta_j^{(i)} > 0,$$

and $|arg\, u_i| < \pi\,\Delta_i/2\,;\ i = 1,\dots,n.$

Case (b) For $\rho_1 = \dots = \rho_n = 0$, (3.2) further gives

$$(3.4)\ \Omega^{\lambda,\mu,\upsilon}_{\alpha_1,\dots,\alpha_n;\sigma}\left\{H^{0,\gamma\ :\ (\mu',\upsilon');\dots;(\mu^{(n)},\upsilon^{(n)})}_{A,C\ :\ [B',D'];\dots;[B^{(n)},D^{(n)}]}\left(\begin{matrix}[(a):\theta';\dots,\theta^{(n)}]:[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}];\\ [(c):\psi';\dots;\psi^{(n)}]:[(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\ u_1x_1^{\sigma_1},\dots,u_nx_n^{\sigma_n}\right)\right\}$$

$$= \frac{\Gamma(\sigma+1-\mu+\alpha_1+\dots+\alpha_n)\,\Gamma(\alpha_1+\dots+\alpha_n)}{\Gamma(\sigma+1/2+\alpha_1+\dots+\alpha_n\pm\upsilon)\,\Gamma(\alpha_1)\dots\Gamma(\alpha_n)}\, H^{0,\gamma+2\ :\ (\mu',\upsilon'+1);\dots;(\mu^{(n)},\upsilon^{(n)}+1)}_{A+2,C+2\ :\ [B'+1,D'];\dots;[B^{(n)}+1,\ D^{(n)}]}$$

$$\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}],[1/2-(\sigma\pm\upsilon):\sigma_1,\dots,\sigma_n]:\\ [(c):\psi',\dots,\psi^{(n)}],[\mu-(\sigma+\alpha_1+\dots+\alpha_n):\sigma_1,\dots,\sigma_n];[1-(\alpha_1+\dots+\alpha_n):\sigma_1,\dots,\sigma_n]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'],[1-\alpha_1:\sigma_1];\dots;[(b^{(n)}):\phi^{(n)}],[1-\alpha_n:\sigma_n];\\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\ u_1/\lambda^{\sigma_1},\dots,u_n/\lambda^{\sigma_n}\right).$$

4. Other multidimensional Whittaker transform $T^{\lambda,\mu,\upsilon}_{\beta_1,\dots,\beta_n;\sigma}$

Chandel and Dwivedi [4] introduced and studied the multidimensional Whittaker transform

$$T^{\lambda,\mu,\upsilon}_{\beta_1,\dots,\beta_n;\sigma}\ \{\ \},$$

$$= \frac{K\lambda^{\sigma+\beta_1+\dots+\beta_n}\Gamma(\sigma+1-\mu+\beta_1+\dots+\beta_n)\Gamma(\beta_1+\dots+\beta_n)}{\Gamma(\sigma+1/2\pm\upsilon+\beta_1+\dots+\beta_n)\prod_{j=1}^{\upsilon}\Gamma\beta_i}$$

$$\int_0^\infty\dots\int_0^\infty e^{-\lambda/2\,[\sum_{j=1}^{n}(\alpha_1{}^j x_1+\dots+\alpha_n{}^j x_n)]}\prod_{j=1}^{n}(\alpha_1{}^j x_1+\dots+\alpha^j_n x_n)^{\beta_j-1}[\sum_{j=1}^{n}(\alpha_1{}^j x_1+\dots+\alpha_n{}^j x_n)]^{\sigma}$$

$$W_{\mu,\upsilon}[\lambda \sum_{j=1}^{n} (\alpha_1^j x_1+\ldots+\alpha_n^j x_n)] \{\} dx_1\ldots dx_n .$$

where $Re(\lambda)>0$, $Re(\sigma + 1/2 + \beta_1+\ldots+\beta_n \pm \upsilon)>0$, $Re(\sigma + 1-\mu +\beta_1+\ldots+\beta_n)>0$, $Re(\beta_j)>0$ $j=1,\ldots,n$ and

$$K = \begin{vmatrix} \alpha_1^1 & \alpha_2^1 & \ldots & \alpha_n^1 \\ \alpha_1^2 & \alpha_2^2 & \ldots & \alpha_n^2 \\ \ldots & \ldots & \ldots & \ldots \\ \alpha_1^n & \alpha_2^n & \ldots & \alpha_n^n \end{vmatrix} \neq 0.$$

As additional applications of above transform, we derive

$$(4.2)\quad T^{\lambda,\mu,\upsilon}_{\beta_1,\ldots,\beta_n;\sigma}\Big\{H^{0,\gamma:(\mu',\upsilon');\ldots;(\mu^{(n)},\upsilon^{(n)})}_{A,C:[B',D'];\ldots;[B^{(n)},D^{(n)}]}\Big(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]:[(b'):\phi'];\ldots;[(b^{(n)}:\phi^{(n)}];\\ [(c):\psi',\ldots,\psi^{(n)}]:[(d'):\delta'];\ldots;[(d^{(n)}:\delta^{(n)}];\end{matrix}$$

$$u_1(\alpha_1^1 x_1+\ldots+\alpha_n^1 x_n)^{\eta_1},\ldots,u_n(\alpha_1^n x_1+\ldots+\alpha_n^n x_n)^{\eta_n}\Big)\Big\}$$

$$= \frac{\Gamma(\sigma+1-\mu+\beta_1+\ldots+\beta_n)\,\Gamma(\beta_1+\ldots+\beta_n)}{\Gamma(\sigma+1/2\pm\upsilon+\beta_1+\ldots+\beta_n)\,\Gamma(\beta_1)\ldots\Gamma(\beta_n)}\; H^{0,\gamma+2:(\mu',\upsilon'+1);\ldots;(\mu^{(n)},\upsilon^{(n)}+1);}_{A+2,C+2:[B'+1,D'];\ldots;[B^{(n)}+1,\,D^{(n)}];}$$

$$\Big(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)})],\ [1/2-(\sigma\pm\upsilon+\beta_1+\ldots+\beta_n):\eta_1,\ldots,\eta_n]:\\ [(c):\psi',\ldots,\psi^{(n)}],\ [\mu-(\sigma+\beta_1+\ldots+\beta_n):\eta_1,\ldots,\eta_n],\ [1-(\beta_1+\ldots+\beta_n):\eta_1,\ldots,\eta_n]:\end{matrix}$$

$$\begin{matrix}[(b'):\phi'],\ [1-\beta_1:\eta_1];\ldots;[b^{(n)}:\phi^{(n)}],[1-\beta_n:\eta_n];\\ [(d'):\delta'];\ldots;[(d^{(n)}):(\delta^{(n)})];\end{matrix}\quad u_1/\lambda^{\eta_1},\ldots,u_n/\lambda^{\eta_n}\Big),$$

where $Re(\lambda)>0$, $Re(\sigma + 1/2 + \beta_1+\ldots+\beta_n \pm \upsilon)>0$, $Re(\sigma+1-\mu+\beta_1+\ldots+\beta_n)>0$,

$$K = \begin{vmatrix} \alpha_1^1 & \alpha_2^1 & \ldots & \alpha_n^1 \\ \alpha_1^2 & \alpha_2^2 & \ldots & \alpha_n^2 \\ \ldots & \ldots & \ldots & \ldots \\ \alpha_1^n & \alpha_2^n & \ldots & \alpha_n^n \end{vmatrix} \neq 0,$$

$Re(\beta_i)>0$, η_i are positive numbers,

$$\Delta_i = -\sum_{j=\gamma+1}^{A}\theta_j^{(i)} + \sum_{j=1}^{\upsilon^{(i)}}\phi_j^{(i)} - \sum_{j=\upsilon^{(i)}+1}^{B^{(i)}}\phi_j^{(i)}$$

$$-\sum_{j=1}^{C}\psi_j^{(i)} + \sum_{j=1}^{\mu^{(i)}}\delta_j^{(i)} - \sum_{j=\mu^{(i)}+1}^{D^{(i)}}\delta_j^{(i)} > 0,$$

and $|arg\, u_i| < \pi\Delta_i/2$, $i=1,\ldots,n$.

$$(4.3)\ T^{\lambda,\mu,\upsilon}_{\beta_1,\ldots,\beta_n;\sigma}\Big\{H^{0,\gamma:(\mu',\upsilon');\ldots;(\mu^{(n)},\upsilon^{(n)})}_{A,C:[B',D'];\ldots;[B^{(n)},D^{(n)}]}\Big(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]:[(b'):\phi'];\ldots;[(b^{(n)}:\phi^{(n)}];\\ [(c):\psi',\ldots,\psi^{(n)}]:[(d'):\delta'];\ldots;[(d^{(n)}:\delta^{(n)}];\end{matrix}$$

$$u_1[\sum_{j=1}^{n}(\alpha_1^j x_1+\ldots+\alpha_n^j x_n]^{\zeta_1},\ldots,u_n[\sum_{j=1}^{n}(\alpha_1^j x_1+\ldots+\alpha_n^j x_n]^{\zeta_n}\Big)\Big\}$$

$$= \frac{\Gamma(\sigma+1-\mu+\beta_1+\ldots+\beta_n)}{\Gamma(\sigma+1/2\pm\upsilon+\beta_1+\ldots+\beta_n)}\; H^{0,\gamma+2:(\mu',\upsilon');\ldots;(\mu^{(n)},\upsilon^{(n)})}_{A+2,C+1:[B',D'];\ldots;[B^{(n)},\,D^{(n)}]}$$

$$\Big(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)})],\ [1/2-(\sigma\pm\upsilon+\beta_1+\ldots+\beta_n):\zeta_1,\ldots,\zeta_n]:[(b'):(\phi')];\ldots;[b^{(n)}):(\phi^{(n)})];\\ [(c):\psi',\ldots,\psi^{(n)}],\ [\mu-(\sigma+\beta_1+\ldots+\beta_n):\zeta_1,\ldots,\zeta_n]:[(d'):(\delta')];\ldots;[(d^{(n)}):(\delta^{(n)})];\end{matrix}$$

$u_1/\lambda^{\zeta_1},\dots,u_n/\lambda^{\zeta_n}\Big)$,

where all conditions of (4.2) are satisfied and ζ_i are positive numbers, $i=1,\dots,n$.

The results (4.2) and (4.3) can be further genealized as

$$(4.4)\ T^{\lambda,\mu,\upsilon}_{\beta_1,\dots,\beta_n;\sigma}\Big\{H^{0,\gamma:(\mu',\upsilon');\dots;(\mu^{(n)},\upsilon^{(n)})}_{A,C:[B',D'];\dots;[B^{(n)},D^{(n)}]}\Big(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]:[(b'):\phi'];\dots;[(b^{(n)}:\phi^{(n)}];\\ [(c):\psi',\dots,\psi^{(n)}]:[(d'):\delta'];\dots;[(d^{(n)}:\delta^{(n)}];\end{matrix}$$

$$u_1(\alpha_1{}^1x_1+\dots+\alpha_n{}^1x_n)^{\eta_1}/\textstyle\sum_{j=1}^{n}(\alpha_1{}^jx_1+\dots+\alpha_n{}^jx_n)^{\zeta_1},\dots,\ u_n(\alpha_1{}^nx_1+\dots+\alpha_n{}^nx_n)^{\eta_n}/\sum_{j=1}^{n}(\alpha_1{}^jx_1+\dots+\alpha_n{}^jx_n)^{\zeta_n}\Big)\Big\}$$

$$=\frac{\Gamma(\sigma+1-\mu+\beta_1+\dots+\beta_n)\ \Gamma(\beta_1+\dots+\beta_n)}{\Gamma(\sigma+1/2\pm\upsilon+\beta_1+\dots+\beta_n)\,\Gamma(\beta_1)\dots\Gamma(\beta_n)}\,H^{0,\gamma+2:(\mu',\upsilon'+1);\dots;(\mu^{(n)},\upsilon^{(n)}+1)}_{A+2,C+2:[B'+1,D'];\dots;[B^{(n)}+1,\ D^{(n)}]}$$

$$\Big(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}],[1/2-(\sigma\pm\upsilon+\beta_1+\dots+\beta_n):\eta_1+\zeta_1,\dots,\eta_n+\zeta_n]:[(b'):(\phi')],[1-\beta_1,\eta_1]\\ [(c):\psi',\dots,\psi^{(n)}],[\mu-(\beta_1+\dots+\beta_n+\sigma):\eta_1+\zeta_1,\dots,\eta_n+\zeta_n],[1-(\beta_1+\dots+\beta_n):\eta_1,\dots,\eta_n]\end{matrix}$$

$$\begin{matrix};\dots;[b^{(n)}):(\phi^{(n)})],[1-\beta_n,\eta_n];\\ [(d'):(\delta')];\dots;[(d^{(n)}):(\delta^{(n)})];\end{matrix}\quad u_1/\lambda^{\eta_1+\zeta_1},\dots,u_n/\lambda^{\eta_n+\zeta_n}\Big),$$

where η_i and ζ_i ,$i=1,\dots,n$ are all positive numbers, and all other conditions of (4.2) are satisfied.

5. Another Multidimensional Whittaker Transform : Chandel and Dwivedi [4] introduced and studied another multidimensional Whittaker transform

$$(5.1)\ H^{\mu_1,\dots,\mu_n;\upsilon_1,\dots,\upsilon_n}_{\beta_1,\dots,\beta_n}\{\}=K\prod_{i=1}^{n}\frac{\Gamma(1+\beta_i-\mu_i)}{\Gamma(1/2+\beta_i\pm\upsilon_i)}\int_0^\infty\!\!\dots\int_0^\infty exp[-1/2\sum_{i=1}^{n}(\alpha_1{}^ix_1+\dots+\alpha_n{}^ix_n)]$$

$$\prod_{i=1}^{n}(\alpha_1{}^ix_1+\dots+\alpha_n{}^ix_n)^{\beta_i-1}\,W_{\mu_i,\upsilon_i}(\alpha_1{}^ix_1+\dots+\alpha_n{}^ix_n)\ \{\ \}\ dx_1\dots dx_n,$$

where $Re\ (1+\beta_i-\mu_i)>0$, $Re\ (1/2+\beta_i\pm\upsilon_i)>0,\ i=1,\dots,n$ and

$$K=\begin{vmatrix}\alpha_1{}^1\,\alpha_2{}^1&\dots&\alpha_n{}^1\\ \alpha_1{}^2\,\alpha_2{}^2&\dots&\alpha_n{}^2\\ \dots\ \dots&\dots&\dots\\ \alpha_1{}^n\,\alpha_2{}^n&\dots&\alpha_n{}^n\end{vmatrix}\neq 0,$$

As an its additional application, we derive

$$(5.2)\ H^{\mu_1,\dots,\mu_n;\upsilon_1,\dots,\upsilon_n}_{\beta_1,\dots,\beta_n}\Big\{H^{0,\gamma:(\mu',\upsilon');\dots;(\mu^{(n)},\upsilon^{(n)})}_{A,C:[B',D'];\dots;[B^{(n)},D^{(n)}]}\Big(\begin{matrix}[(a),:\theta',\dots,\theta^{(n)}]:\\ [(c),:\psi',\dots,\psi^{(n)}]:\end{matrix}$$

$$\begin{matrix}[(b'):\phi'];\dots;[b^{(n)}):\phi^{(n)}];\\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\ u_1(\alpha^1{}_1x_1+\dots+\alpha^1{}_nx_n)^{\eta_1},\dots,u_n(\alpha_1{}^nx_1+\dots+\alpha_n{}^nx_n)^{\eta_n}\Big)\Big\}$$

$$=\prod_{i=1}^{n}\frac{\Gamma(1+\beta_i-\mu_i)}{\Gamma(1/2+\beta_i\pm\upsilon_i)}\Big\{\mathrm{H}^{0,\gamma:(\mu':\upsilon'+2);\dots;(\mu^{(n)};\upsilon^{(n)}+2)}_{A,C:(B'+2,D'+1);\dots;(B^{(n)}+2,\ D^{(n)}+1)}\Big(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]:\\ [(c):\psi',\dots,\psi^{(n)}]:\end{matrix}$$

$$\begin{matrix}[(b'):\phi'],[1/2-(\beta_1\pm\upsilon_1):\eta_1];\dots;[b^{(n)}):\phi^{(n)}],[1/2-(\beta_n\pm\upsilon_n):\eta_n];\\ [(d'):\delta'],[\mu_1-\beta_1:\eta_1];\dots;[(d^{(n)}):(\delta^{(n)})];[(\mu_n-\beta_n:\eta_n];\end{matrix}\quad u_1,\dots,u_n\Big),$$

where $Re\,(1+\beta_i-\mu_i)>0$, $Re\,(1/2+\beta_i\pm\upsilon_i)>0$, $\eta_i>0$,

$$K=\begin{vmatrix} \alpha_1^{\,1}\,\alpha_2^{\,1} & \ldots & \alpha_n^{\,1} \\ \alpha_1^{\,2}\,\alpha_2^{\,2} & \ldots & \alpha_n^{\,2} \\ \ldots\;\ldots & \ldots & \ldots \\ \alpha_1^{\,n}\,\alpha_2^{\,n} & \ldots & \alpha_n^{\,n} \end{vmatrix}\neq 0,$$

$$\Delta_i = -\sum_{j=\gamma+1}^{A}\theta_j^{(i)}+\sum_{j=1}^{\upsilon^{(i)}}\phi_j^{(i)}-\sum_{j=\upsilon^{(i)}+1}^{B^{(i)}}\phi_j^{(i)}$$
$$-\sum_{j=1}^{C}\psi_j^{(i)}+\sum_{j=1}^{\mu^{(i)}}\delta_j^{(i)}-\sum_{j=\mu^{(i)}+1}^{D^{(i)}}\delta_j^{(i)}>0,$$

and

$|arg\,u_i|<\pi\Delta_i/2\;,i=1,..,n.$

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