Jñānābha, Vol. 30, 2000

SOME RESULTS INVOLVING GENERALIZED HYPERGEOMETRIC FUNCTION, GENERALIZED PROLATE SPHEROIDAL WAVE FUNCTION, GENERALIZED POLYNOMIALS AND THE MULTI-VARIABLE *H*-FUNCTION

By

V.G. Gupta

Department of Mathematics

University of Rajasthan, Jaipur-302 004, Rajasthan

and

Suman Jain

S.S. Jain Subodh Post Graduate College, Jaipur-302 004, Rajasthan

(Received : August 30, 2000)

ABSTRACT

In this paper an integral involving generalized prolate spheroidal wave function, generalized hypergeometric function, the generalized polynomials and the (Srivastava and Panda) H-function of several complex variables has been evaluated and an expansion formula for the product of the generalized hypergeometric function, generalized polynominals and the H-function of several complex variables has been established with the application of this integral. On account of the most general nature of the functions and the polynomials occuring in these results, our finding provide interesting unifications and extensions of a large number of (new an known) results. We record here only two special cases of our main integral, first involves the product of the generalized prolate spheroidal wave fuction, the generalized hypergeometric function, the generalized polynominals and the generalized Lauricella function of several complex variables ([13], p. 454), the other one involves the product of the generalized spheroidal wave function, the generalized hypergeometric function, the general class of polynomials and the multivariable H-function. With the application of the above two special cases of integrals we can also derive two expansion formulae for the product of the functions and polynomials stated in the integrals respectively. Out of several known results which follow as special cases of our main integral and expansion formula we refer here only to the results of Mishra [7], Gupta [6],

$$, \dots y_R (1-x)^{g_R}(1+x)^{w_R}]\; H\,[z_1(1-x)^{h_1}(1+x)^{k_1}, \dots z_r (1-x)^{h_r}(1+x)^{k_r}]$$

$$= \sum_{t,p,q=0}^{\infty} 2^{\rho+\sigma-\alpha-\beta+gq+wq}\, L(y_1,\dots,y_R)\, R_{p,n}^{\alpha,\beta}(s)\, \frac{(e_M)_q\, y^q}{q!\,(f_N)_q}$$

$$\sum_{m}^{t+p} \frac{(-t-p)_m(\alpha+\beta+t+n+p+1)_m}{m!\,(\alpha+1)_m}\, H^{0,\lambda+2:(u',v');\dots;(u^{(r)},v^{(r)})}_{A+2,C+1:(B',D');\dots;(B^{(r)},D^{(r)})}$$

$$\left(\begin{matrix}[-m-gq-g_1\alpha_1-\dots-g_R\alpha_R: h_1,\dots,h_r],\ [-\sigma-wq-w_1\alpha_1-\dots-w_R\alpha_R: k_1,\dots,k_r],\\ [(c):\psi',\dots,\psi^{(r)}]:[-1-m-\rho-\sigma-gq-wq-(g_1+w_1)\alpha_1-\dots-(g_R+w_R)\alpha_R: h_1+k_1,\dots,h_r+k_r]:\\ [(a):\theta',\dots,\theta^{(r)}]:[(b'):\varphi'];\dots;[b^{(r)}:\varphi^{(r)}];\ z_1 2^{h_1+k_1},\dots,z_r 2^{h_r+k_r}\\ [(d'):\delta'];\dots;[d^{(r)}:\delta^{(r)}];\end{matrix}\right)$$

$$\left\{\phi_t^{\alpha,\beta}(s,x)/(R_{p,t}^{\alpha,\beta}(s))^2\, \frac{\Gamma(t+\alpha+p+1)\Gamma(t+p+\beta+1)}{(2t+2p+\alpha+\beta+1)\Gamma(t+p+1)\Gamma(t+p+\alpha+\beta+1)}\right\} \quad \dots (3.1)$$

where $L(y_1, \dots, y_R)$ is same as given in (2.2), $Re\ \rho\left(+\sum_{i=1}^{r} h_i \frac{d_j^{(i)}}{\delta_j^{(i)}}\right) > -1$, $Re\left(\sigma + \sum_{i=1}^{r} k_i \frac{d_j^{(i)}}{\delta_j^{(i)}}\right) > -1$, $h_i, k_i > 0\ \forall i \in \{1,\dots,r\}$; $g > 0$, $w = 0$, $g_{j'}>0, w_{j'}>0$, $\forall j' \in \{1,\dots,R\}$; $p_{j'}\ (j' = 1,\dots,R)$ is an arbitrary positive integer and the coefficients $A\,[q_1,\alpha_1; \dots ;q_R, \alpha_R]$ are arbitrary constants, real or complex, $T_i>0$, $|arg\ z_i| < T_i\pi/2$, $i = 1,\dots,r$, $j= 1,\dots,u^{(i)}$, $\alpha > -1$, $\beta > -1$, $M \le N$ ($M=N+1$ and $|y|<1$).

Proof. Let

$$f(x) = (1-x)^{\rho-\alpha}(1+x)^{\sigma-\beta}\ {}_MF_N\left[\begin{matrix}e_M\\ f_N\end{matrix}; y(1-x)^g(1+x)^w\right] S^{p_1,\dots,p_R}_{q_1,\dots,q_R}[y_1(1-x)^{g_1}(1+x)^{w_1}$$

$$, \dots y_R (1-x)^{g_R}(1+x)^{w_R}]\; H\,[z_1(1-x)^{h_1}(1+x)^{k_1}, \dots z_r (1-x)^{h_r}(1+x)^{k_r}]$$

$$= \sum_{t=0}^{\infty} E_t\, \phi_t^{\alpha,\beta}(s,x), \qquad (-1 < x < 1) \qquad \dots (3.2)$$

Equation (3.2) is valid since $f(x)$ is continuous and of bounded variation in the open interval (–1,1).

Now mutliply both side of (3.2) by

$(1-x)^{\alpha}(1+x)^{\beta}\,\phi_n^{\alpha,\beta}(s,x)$

and integrate with respect to x from -1 to 1. Change the order of integration and summation (which is permissible) on the right, we obtain

$$\int_{-1}^{1} (1-x)^{\rho}(1+x)^{\sigma}\,\phi_n^{\alpha,\beta}(s,x)\ {}_MF_N\left[\begin{matrix}e_M\\ f_N\end{matrix}; y(1-x)^g(1+x)^w\right] S^{p_1,\dots,p_R}_{q_1,\dots,q_R}[y_1(1-x)^{g_1}$$

Srivastava and Singh [16].

1. Introduction. The solution of the following differential equation

$$(1-x^2)\frac{d^2y}{dx^2} + [(\beta-\alpha)-(\alpha+\beta+2)\,x]\frac{dy}{dx} + [\xi(s) - s^2x^2]\,y = 0 \qquad \ldots(1.1)$$

is the generalized prolate spheroidal wave function (Gupta [5]) which has been denoted as

$$\phi_n^{\alpha,\beta}(s,x) = \sum_{p=0}^{\infty} R_{p,n}^{\alpha,\beta}(s)\, P_{p+n}^{(\alpha,\beta)}(x) \qquad \ldots(1.2)$$

where $s = 0$ and $\xi(0) = (n+p)(\alpha+\beta+n+p+1),\ p \geq 0$.

The generalized polynomials (multivariable) defined by srivastava [11] represented in the following manner :

$$S_{q_1,\ldots,q_R}^{p_1,\ldots,p_R}\begin{bmatrix} x_1 \\ \vdots \\ x_R \end{bmatrix} = S_{q_1,\ldots,q_R}^{p_1,\ldots,p_R}[x_1,\ldots,x_R]$$

$$= \sum_{\alpha_1=0}^{[q_1/p_1]} \cdots \sum_{\alpha_R=0}^{[q_R/p_R]} \frac{(-q_1)_{p_1\alpha_1}}{\alpha_1!} \cdots \frac{(-q_R)_{p_R\alpha_R}}{\alpha_R!} . A\,[q_1,\alpha_1;\ldots;q_R,\alpha_R]\; x_1^{\alpha_1}\ldots x_R^{\alpha_R} \qquad \ldots(1.3)$$

where $q_j = 0, 1, 2, \ldots ; p_j\ (j = 1, \ldots, R)$ are non-zero arbitrary positive integer. The coefficients. $A\,[q_1,\alpha_1;\ldots;q_R,\alpha_R]$ being arbitrary constants, real or complex. If we take $R = 1$ in the equation (3) and denote $A\,[q,\alpha]$ thus obtained by $A_{q,\alpha}$, we arrive at the well known general class of polynomials $S_q^p\,[x]$ intriduced by Srivastava [12].

The multivariable H-function of several complex variables occuring in the paper will be defined by Srivastava and Panda ([14], [15]) by means of the multiple Mellin-Barnes integral.

$$H\,[z_1,\ldots,z_r] =$$

$$= H_{A,\,C:\,(B',D');\ldots;(B^{(r)},D^{(r)})}^{0,\lambda\,:\,(u',v');\ldots;(u^{(r)},v^{(r)})}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]:[(b'):\varphi'];\ldots;[b^{(r)}:\varphi^{(r)}];\\ [(c):\psi',\ldots,\psi^{(r)}]:[(d'):\delta'];\ldots;[d^{(r)}:\delta^{(r)}];\end{matrix}\ z_1,\ldots,z_r\right)$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi_1(\xi_1)\ldots\phi_r(\xi_r)\,\psi(\xi_1,\ldots,\xi_r)\,z_1^{\xi_1}\ldots z_r^{\xi_r}\,d\xi_1\ldots d\xi_r,\ \omega = \sqrt{-1} \qquad \ldots(1.4)$$

For the convergence of the integral given by (1.4) and other details of the multivariable H-function, we refer to the book by Srivastava et. al. ([10], p.251-3, eqn. (C-1)-(C-5)).

The orthogonality property of the generalized prolate spheroidal wave function (Gupta [5], p. 107, eqn. (3.1).

$$\int_{-1}^{1}(1-x)^\alpha(1+x)^\beta\,\phi_n^{\alpha,\beta}(s,x)\,\phi_t^{\alpha,\beta}(s,x)\,dx = N_{n,t}^{\alpha,\beta}\,\delta_{n,t} \qquad \ldots(1.5)$$

where

$$N_{n,t}^{\alpha,\beta} = \sum_{p=0}^{\infty}(R_{p,n}^{\alpha,\beta}(s))^2\,2^{\alpha+\beta+1}\frac{\Gamma(n+\alpha+p+1)\Gamma(n+p+\beta+1)}{(2n+2p+\alpha+\beta+1)\Gamma(n+p+1)\Gamma(n+p+\alpha+\beta+1)} \qquad \ldots(1.6)$$

and $\delta_{n,t}$ is the kronecker delta.

2. The main integral.

The following integral has been evaluated in this paper :

$$\int_{-1}^{1} (1-x)^{\rho}(1+x)^{\sigma}\, \phi_n^{\alpha,\beta}(s,x)\; {}_MF_N\left[\begin{matrix} e_M \\ f_N \end{matrix} ; y(1-x)^g(1+x)^w\right] S_{q_1,\ldots,q_R}^{p_1,\ldots,p_R}[y_1(1-x)^{g_1}$$

$$(1+x)^{w_1}, \ldots y_R(1-x)^{g_R}(1+x)^{w_R}]\; H\,[z_1(1-x)^{h_1}(1+x)^{k_1}, \ldots z_r(1-x)^{h_r}(1+x)^{k_r}]dx$$

$$= \sum_{p,q=0}^{\infty} 2^{\rho+\sigma+1+gq+wq}\, L(y_1,\ldots,y_R)\, R_{p,n}^{\alpha,\beta}(s)\, \frac{(e_M)_q\, y^q}{q!\,(f_N)_q}$$

$$\sum_{m}^{n+p} \frac{(-n-p)_m(\alpha+\beta+n+p+1)_m}{m!\,(\alpha+1)_m}\, H^{0,\lambda+2:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A+2,C+1:(B',D');\ldots;(B^{(r)},D^{(r)})}$$

$$\left(\begin{matrix}[-m-\rho-gq-g_1\alpha_1-\ldots-g_R\alpha_R: h_1,\ldots,h_r],\ [-\sigma-wq-w_1\alpha_1-\ldots-w_R\alpha_R: k_1,\ldots,k_r], \\ [(c):\psi',\ldots,\psi^{(r)}]:[-1-m-\rho-\sigma-gq-wq-(g_1+w_1)\alpha_1-\ldots-(g_R+w_R)\alpha_R: h_1+k_1,\ldots,h_r+k_r]: \\ [(a):\theta',\ldots,\theta^{(r)}]:[(b'):\varphi'];\ldots;[b^{(r)}:\varphi^{(r)}]; \\ [(d'):\delta'];\ldots;[d^{(r)}:\delta^{(r)}];\end{matrix}\; z_1 2^{h_1+k_1},\ldots, z_r 2^{h_r+k_r}\right) \quad \ldots (2.1)$$

where

$$L(y_1,\ldots,y_R) = \sum_{\alpha_1=0}^{[q_1/p_1]} \cdots \sum_{\alpha_R=0}^{[q_R/p_R]} \prod_{j'=1}^{R} \left[\frac{(-q_{j'})_{p_{j'}\alpha_{j'}}}{\alpha_{j'}!}\, y_{j'}^{\alpha_{j'}}\, 2^{(g_{j'}+w_{j'})\alpha_{j'}}\right.$$

$$A\,[q_1,\alpha_1;\ldots;q_R,\alpha_R] \quad \ldots (2.2)$$

provided that $h_i, k_i > 0\ \forall i \in \{1,\ldots,r\}$; $g > 0$, $w = 0$, $g_{j'}>0$, $w_{j'}>0$, $\forall j' \in \{1,\ldots,R\}$; $p_{i'}$, $(i' = 1,\ldots,R)$ is an arbitrary positive integer and the coefficients $A\,[q_1,\alpha_1;\ldots;q_R,\alpha_R]$ are arbitrary constant, real or complex.

$$Re\left(\rho + \sum_{i=1}^{r} h_i \frac{d_j^{(i)}}{\delta_j^{(i)}}\right) > -1,\ Re\left(\sigma + \sum_{i=1}^{r} k_i \frac{d_j^{(i)}}{\delta_j^{(i)}}\right) > -1, T_i>0,\ |\arg z_i| < 1/2$$

$T_i\pi$, $i = 1, \ldots r$, $j = 1,\ldots,u^{(i)}$, $\alpha > -1$, $\beta > -1$, $M=N$ ($M=N+1$ and $|y|<1$).

Proof. To establish (2.1), express the generalized prolate spheroidal wave function as given in (1.2), the generalized hypergeometric function as infinite series (Rainville [8], p.73. eqn. (2)), and the general polynomials with the help of equation (1.3), change the order of integration and summation (which is easily seen to be justified due to the absolute convergence of the integral and sums involved in the process) and then evaluate the inner integral by using a result of Gupta ([6], p.31, eqn. (2.2.1)), we arrive at the right hand side of (2.1).

3. Expansion Formula.

$$(1-x)^{\rho-\alpha}(1+x)^{\sigma-\beta}\; {}_MF_N\left[\begin{matrix} e_M \\ f_N \end{matrix} ; y(1-x)^g(1+x)^w\right] S_{q_1,\ldots,q_R}^{p_1,\ldots,p_R}[y_1(1-x)^{g_1}(1+x)^{w_1}$$

$(1+x)^{w_1}, \ldots y_R (1-x)^{g_R}(1+x)^{w_R}]\, H\, [z_1(1-x)^{h_1}(1+x)^{k_1}, \ldots z_r (1-x)^{h_r} (1+x)^{k_r}]dx$

$$= \sum_{l=0}^{\infty} E_l \int_{-1}^{1} (1-x)^{\alpha}(1+x)^{\beta}\, \phi_n^{\alpha,\beta}(s,x), \phi_l^{\alpha,\beta}(s,x)\, dx \quad \ldots (3.3)$$

Using the orthogonality property for the generalized prolate spheroidal wave functions (1.5) on the right-hand side and the result (2.1) on the left hand side of (3.3) we obtain

$$E_l = \sum_{p,q=0}^{\infty} 2^{\rho+\sigma+1+gq+wq} L(y_1,\ldots,y_R)\, R_{p,n}^{\alpha,\beta}(s)\, \frac{(e_M)_q y^q}{q!\,(f_N)_q}$$

$$\sum_{m}^{l+p} \frac{(-l-p)_m(\alpha+\beta+l+p+1)_m}{m!\,(\alpha+1)_m} H^{0,\lambda+2:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A+2,C+1:(B',D');\ldots;(B^{(r)},D^{(r)})}$$

$$\left(\begin{matrix} [-m-\rho-gq-g_1\alpha_1-\ldots-g_R\alpha_R: h_1,\ldots,h_r],\ [-\sigma-wq-w_1\alpha_1-\ldots-w_R\alpha_R: k_1,\ldots,k_r], \\ [(c):\psi',\ldots,\psi^{(r)}]:[-1-m-\rho-\sigma-gq-wq-(g_1+w_1)\alpha_1-\ldots-(g_R+w_R)\alpha_R: h_1+k_1,\ldots,h_r+k_r]: \end{matrix}\right.$$

$$\left.\begin{matrix} [(a):\theta',\ldots,\theta^{(r)}]:[(b'):\varphi'];\ldots;[b^{(r)}:\varphi^{(r)}]; \\ [(d'):\delta'];\ldots;[d^{(r)}:\delta^{(r)}]; \end{matrix}\ z_1 2^{h_1+k_1},\ldots, z_r 2^{h_r+k_r}\right) / \{N_{n,l}^{\alpha,\beta}\delta_{n,l}\} \quad (3.4)$$

with the help of (3.3) and (3.4) in view of (1.6), the expansion formula (3.1) is obtained.

4. Special Cases.

(i) Letting $\lambda = A$, $u^{(i)}=1$, $v^{(i)}=B^{(i)}$, $D^{(i)}=D^{(i)}+1\ \forall i=1,\ldots,r$ in (2.1), the multivariable H-function transforrms to the generalized Lauricella function of several complex variables ([13],p. 454).

$$\int_{-1}^{1} (1-x)^{\rho}(1+x)^{\sigma}\, \phi_n^{\alpha,\beta}(s,x)\, {}_MF_N\left[\begin{matrix} e_M \\ f_N \end{matrix}; y(1-x)^{g}(1+x)^{w}\right] S^{p_1,\ldots,p_R}_{q_1,\ldots,q_R}[y_1(1-x)^{g_1}(1+x)^{w_1}$$

$$,\ldots y_R(1-x)^{g_R}(1+x)^{w_R}]\, F^{A:B';\ldots;B^{(r)}}_{C:D';\ldots;D^{(r)}}\left(\begin{matrix} [1-(a):\theta',\ldots,\theta^{(r)}]:[1-(b'):\varphi'];\ldots;[1-(b^{(r)}):\varphi^{(r)}]; \\ [1-(c):\psi',\ldots,\psi^{(r)}]:[1-(d'):\delta'];\ldots;[1-(d^{(r)}):\delta^{(r)}]; \end{matrix}\right.$$

$$\left. - z_1(1-x)^{h_1}(1+x)^{k_1}, \ldots z_r (1-x)^{h_r}(1+x)^{k_r}\right]dx$$

$$= \sum_{p,q=0}^{\infty} 2^{\rho+\sigma+1+gq+wq} L(y_1,\ldots,y_R)\, R_{p,n}^{\alpha,\beta}(s)\, \frac{(e_M)_q y^q}{q!\,(f_N)_q} \sum_{m=0}^{n+p}$$

$$\frac{(-n-p)_m(\alpha+\beta+n+p+1)_m}{m!\,(\alpha+1)_m}\ \frac{\Gamma(1+m+\rho+gq+g_1\alpha_1+..+g_R\alpha_R)\Gamma(1+\sigma+wq+w_1\alpha_1+\ldots+w_R\alpha_R)}{\Gamma(2+m+\rho+\sigma+gq+wq+(g_1+w_1)\alpha_1+\ldots+(g_R+w_R)\alpha_R)}$$

$$F^{A+2:B';\ldots;B^{(r)}}_{C+1:D';\ldots;D^{(r)}}\left(\begin{matrix} [1+m+\rho+gq+g_1\alpha_1+\ldots+g_R\alpha_R: h_1,\ldots,h_r],\ [1+\sigma+wq+w_1\alpha_1 \\ [1-(c):\psi',\ldots,\psi^{(r)}]:[2+m+\rho+\sigma+gq+wq+(g_1+w_1)\alpha_1+\ldots+(g_R+w_R)\alpha_R \end{matrix}\right.$$

$$\left.\begin{matrix} +\ldots+w_R\alpha_R: k_1,\ldots,k_r],[(a):\theta',\ldots,\theta^{(r)}]:[(b'):\varphi'];\ldots;[b^{(r)}:\varphi^{(r)}]; \\ :h_1+k_1,\ldots,h_r+k_r]:[1-(d'):\delta'];\ldots;[1-(d^{(r)}):\delta^{(r)}]; \end{matrix}\ -z_1 2^{h_1+k_1},\ldots,-z_r 2^{h_r+k_r}\right)$$

$$\ldots (4.1)$$

valid under the same conditions as obtainable from (2.1)

(ii) If we take $R = 1$ in (2.1) we get an integral involving the product of general class of polynomials, the generalized prolate spheroidal wave function, generalized hypergeometric function and a multivariable H-funciton

(iii) Letting $\lambda = A$, $u^{(i)}=1$, $v^{(i)}=B^{(i)}$, $D^{(i)}=D^{(i)}+1\ \forall i=1,\ldots,r$ in (3.1), we get an expansion formula for the product of hypergeometric function, generalized polynomials and the generalized Lauricella function of several complex variables ([13],p. 454)

$$(1-x)^{\rho-\alpha}(1+x)^{\sigma-\beta}\ {}_MF_N\left[\begin{matrix} e_M \\ f_N \end{matrix};\ y(1-x)^{g}(1+x)^{w}\right] S^{p_1,\ldots,p_R}_{q_1,\ldots,q_R}\left[y_1(1-x)^{g_1}(1+x)^{w_1},\ldots\right.$$

$$\left. y_R(1-x)^{g_R}(1+x)^{w_R}\right] F^{A:B';\ldots;B^{(r)}}_{C:D';\ldots;D^{(r)}}\left(\begin{matrix}[1-(a):\theta',\ldots,\theta^{(r)}]:[1-(b'):\varphi'];\ldots;[1-(b^{(r)}):\varphi^{(r)}]; \\ [1-(c):\psi',\ldots,\psi^{(r)}]:[1-(d'):\delta'];\ldots;[1-(d^{(r)}):\delta^{(r)}];\end{matrix}\right.$$

$$\left. -z_1(1-x)^{h_1}(1+x)^{k_1},\ldots,-z_r(1-x)^{h_r}(1+x)^{k_r}\right)$$

$$= \sum_{t,p,q=0}^{\infty} 2^{\rho+\sigma-\alpha-\beta+gq+wq} L(y_1,\ldots,y_R)\, R^{\alpha,\beta}_{p,t}(s)\, \frac{(e_M)_q\, y^q}{q!\,(f_N)_q} \sum_{m=0}^{n+p}$$

$$\frac{(-t-p)_m(\alpha+\beta+t+p+1)_m}{m!\,(\alpha+1)_m}\ \frac{\Gamma(1+m+\rho+gq+g_1\alpha_1+\ldots+g_R\alpha_R)\Gamma(1+\sigma+wq+w_1\alpha_1+\ldots+w_R\alpha_R)}{\Gamma(2+m+\rho+\sigma+gq+wq+(g_1+w_1)\alpha_1+\ldots+(g_R+w_R)\alpha_R)}$$

$$F^{A+2:B';\ldots;B^{(r)}}_{C+1:D';\ldots;D^{(r)}}\left(\begin{matrix}[1+m+\rho+gq+g_1\alpha_1+\ldots+g_R\alpha_R: h_1,\ldots,h_r],\ [1+\sigma+wq+w_1\alpha_1 \\ [1-(c):\psi',\ldots,\psi^{(r)}]:[2+m+\rho+\sigma+gq+wq+(g_1+w_1)\alpha_1+\ldots+(g_R+w_R)\alpha_R\end{matrix}\right.$$

$$\left.\begin{matrix}+\ldots+w_R\alpha_R: k_1,\ldots,k_r],[(a):\theta',\ldots,\theta^{(r)}]:[(b'):\varphi'];\ldots;[b^{(r)}:\varphi^{(r)}]; \\ :h_1+k_1,\ldots,h_r+k_r]:[1-(d'):\delta'];\ldots;[1-(d^{(r)}):\delta^{(r)}];\end{matrix}\ -z_1 2^{h_1+k_1},\ldots,-z_r 2^{h_r+k_r}\right)$$

$$\left\{\phi_t^{\alpha,\beta}(s,x)/(R^{\alpha,\beta}_{p,t}(s))^2\ \frac{\Gamma(t+\alpha+p+1)\Gamma(t+p+\beta+1)}{(2t+2p+\alpha+\beta+1)\Gamma(t+p+1)\Gamma(t+p+\alpha+\beta+1)}\right\} \quad \ldots (4.2)$$

valid under the same condition as obtainable from (3.1).

(iv) Taking $R = 1$ in (9), we get an expansion formula for the product of hypergeometric function, general class of polynomials and the multivariable H-function.

5. Known Results.

i. Letting $q_{j'}$, $(j' = 1,\ldots,R) = 0$ in the equation (2.1) and (3.1), we get the results obtained by Gupta ([6], p.38, eqn. (2.3.2) and p. 44 eqn. (3.5.1)).

(ii) If we take $\alpha=\beta=n=0=s$, $w_1=0$, $g_1=1$ and $y=0$ in special case (ii) above, we get an integral similar to that obtained by Srivastava and Singh with , $\xi = -1$ and $\eta = 1$ ([16],p.166, eqn. (2.2)).

(iii) Letting $y = 0$ in known result (i) we get an integral obtained by Gupta ([6], p. 50, eqn. (2.6.1)).

(iv) Setting $s = 0$, $x = 1-2\eta$ and using *Saalschutz's* theorem (Slater [9], p. 49) in known results (iii) above we get a result given by Srivastava and Panda ([15], p. 131, eqn.(2.2)).

(v) Taking $r=2$, $y=0$ and $\theta'=\theta''=\psi'=\psi''=\varphi'=\varphi''=\delta'=\delta''=1$ in eqn. (2.1) we arrive at a known result obtained by Mishra ([7],p. 158, eqn. 5.6.2).

(vi) Setting $y=0$, $A=0=C=\lambda$, $v''=B''=d''=0, u''=D''=\delta''=1$, $z_2 \to 0$ in (2.1) and applying a known transformation formula Chaurasia ([1], P. 18, eqn. (1.5.4)), we arrive at another result given by Mishra ([7],p.157, eqn. (5.6.1)).

(vii) Setting $r=2$, $y=0$ and $\theta'=\theta''=\psi'=\psi''=\varphi'=\varphi''=\delta'=\delta''=1$ in eqn. (3.1) and using a known formula (Rainville [8],p.24, eqn. (2)), we obtain a result recorded by Mishra ([7],p.166, eqn. (5.7.5)).

(viii) Setting $r=2$, $A=0=C=$, $v''=B''=d''=0$, $u''=D''=\delta''=1$, $z_2 \to 0$ in eqn. (3.1) and using a transformation formula (Chaurasia [1],p,.18, eqn. (1.5.4)), we get an expansion formula involving Fox's H-function.

(ix) When $y=0$, the known results (viii) reduces to another known result due to Mishra ([7], p.165, eqn. (5.7.4)).

Several other interesting special cases of our results can be deduced.

Acknowledgement

The authors are thankful to Professor H.M. Srivastav, University of Victoria, Canada, for valuable suggestions in the preparation of this paper.

REFERENCES

[1] V.B.L. Chaurasia, *Investigations in Integral Transforms and Special Functions*, Ph.D. thesis, University of Rajasthan, Jaipur, India, 1976.

[2] A. Erdélyi et al, *Tables of Integral Transforms*, Vol. II, Mcgraw Hill, New York, 1954.

[3] C. Flammer, *Spheroidal Wave Functions*. Stanford Univ. Press, Stanford, 1957.

[4] C. Fox, The G and H functions as symmetrical Fourier kernels, *Tran. Amer.Math. Soc.* **98** (1961), 395-429.

[5] R.K. Gupta, Generalized prolate spheroidal wave function, *Proc. Indian Acad. Sci. Sect.* A **85** (1977), 104-114.

[6] V.G. Gupta, *A Study of multi-valued function and their applications*, Ph.D. Thesis, University of Rajasthan, Jaipur, India, 1991

[7] S.D. Mishra, *Some Problem in Transform Calculus*, Ph.D. thesis, University of Rajasthan, Jaipur, India, 1977.

[8] E.D. Rainville, *Special Functions*, Macmillan, New York, 1960.

[9] L.J. Slater, *Generalized Hypergeometric Functions*, Cambridge Univ. Prss, Cambridge, 1966.

[10] H.M. Srivastava, K.C. Gupta and S.P. Goyal, *The H- function of One and Two Variables with Applications*, South Asian Publishers, New Delhi and Madras, 1982.

[11] H.M. Srivastava, A Multilinear generating function for the Konhauser sets of biorthogonal polynomials suggested by the Laguerre polynomials, *Pacific. J. Math.* **117** (1985), 183-191.

[12] H.M. Srivastava, A Contour integral involving Fox's H-function, *Indian J. Math.* **14** (1972), 1-6.

[13] H.M. Srivastava and M.C. Daoust, Certain generalized, Neumann equations associated with the Kampé de Fériet function, *Nederl, Akad. Wetensch. Proc. Ser.* A **72** *Indag. Math.*, **31**, (1969), 449-457.

[14] H.M. Srivastava and R. Panda, Some bilateral generating functions for a class of generalized hypergeometric polynomials, *J. Reine Angew. Math.* **283/284** (1976), 265-274.

[15] H.M. Srivastava and R.Panda, Expansion theorems for the H-Function of several complex variables, *J. Reine Angew. Math.* **288** (1976), 129-145.

[16] H.M. Srivastava and N.P. Singh, The Integration of certain product of the multivariable H-function with general class of polynomials, *Rend. Circ. Mat. Palermo* (2). **32**, (1983), 157-187.