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ON INTEGRATION OF CERTAIN PRODUCTS INVOLVING GENERAL POLYNOMIALS, FOX'S *H*-FUNCTION AND THE MULTIVARIABLE *H*-FUNCTION

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ABSTRACT

The integrals evaluated here involve the poduct of Jacobi polynomials, Fos's *H*-function, general polynomials and the multivariable *H*-fuction. The main results of our paper are quite general in nature and capable of yielding a very large number of integrals involving polynomials and various special functions occurring in the problems of mathematical analysis, applied mathematics and mathematical physics.

1. Introduction. Srivastava ([7], p. 1, Eq, (1)] has introduced the general class of polynomials

$$S_n^m[x] = \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s}, \; [n = 0, 1, 2, \dots] \qquad \dots (1)$$

where m is an arbitrary positive integer and coefficients $A_{n,s}$ $(n, s \geq 0)$ are arbitary constants, real or complex.

The generalization of the above class of polynomials given by Srivastava ([8], p. 185, Eq. (7)) defined and reqresented as :

$$S_{n_1, \dots, n_R}^{m_1, \dots, m_R}[x_1, \dots, x_s] = \sum_{s_1=0}^{[n_1/m_1]} \dots \sum_{s_R=0}^{[n_R/m_R]} \frac{(-n_1)_{m_1 s_1}}{s_1!} \dots \frac{(-n_R)_{m_R s_R}}{S_R!}$$

$$A\,[n_1, s_1; \dots; n_R, s_R]\, x_1^{s1} \dots x_R^{s_R} \qquad \dots (2)$$

and term it as a general polynomials of R variables.

The following known results will be used throughout this paper ([1] ,p. 945, p. 946 and [4], p. 172)

$$P_k^{(\beta,\lambda)}(t+\sigma)\; P_k^{(\beta,\upsilon)}(t-\sigma) = \frac{(-1)^k (1+\upsilon)_k (1+\beta)_k}{(k!)^2}$$

$$= \sum_{n=0}^{k} (-k)_r \frac{(1+\upsilon+\beta+k)_r}{(1+\upsilon)_r (1+\beta)_r} P_r^{(\beta,\beta)}(x)\, t^r \qquad \dots (3)$$

$$\sigma^k P_k^{(\beta,\beta)}\left(\frac{1-xt}{\sigma}\right) = \frac{(1+\beta)_k}{k!} \sum_{r=0}^{k} \frac{(-k)_r}{(1+\beta)_r} P_r^{(\beta,\beta)}(x)\, t^r \qquad \ldots(4)$$

$$1/\sigma\,(1-t+\sigma)^{-\beta}\,(1+t+\sigma)^{-\upsilon} = 2^{-\upsilon-\beta} \sum_{r=0}^{\infty} P_r^{(\beta,\upsilon)}(x)\, t^r \qquad \ldots(5)$$

where $\sigma = (1-2xt+t^2)^{1/2}$ ... (6)

$$\int_{-1}^{1} (1-x)^{\beta}\,(1+x)^{\eta}\,P_t^{(\beta,\upsilon)}(x)\, S_{n_1,\ldots,n_R}^{m_1,\ldots,m_R}\begin{bmatrix}(1+x)^{\rho_1}\\ \vdots \\ (1+x)^{\rho_R}\end{bmatrix}$$

$$H_{w,d}^{p,q}\left[z\,(1+x)^h \,\middle|\, \begin{matrix}(e_W, E_W)\\ (f_d, F_d)\end{matrix}\right] H\,(z_1(1+x)^{h_1},\ldots, z_r(1+x)^{h_r})\,dx$$

$$= 2^{\beta+\eta+\rho_1 s_1+\ldots+\rho_R s_R+h g_\alpha+1} \sum_{s_1=0}^{[n_1/m_1]} \cdots \sum_{s_R=0}^{[n_R/m_R]} \sum_{G=1}^{p} \sum_{\alpha=0}^{\infty}$$

$$\frac{(-n_1)_{m_1 s_1}}{s_1!} \cdots \frac{(-n_R)_{m_R s_R}}{s_R!} A\,[n_1,s_1;\ldots;n_R,s_R]\,\frac{(-1)^{\alpha}}{\alpha!\,F_G}\,\phi(g_\alpha)\,z^{g_\alpha}\,\Gamma(\beta+t+1)$$

$$H_{A+2,C+2\,:\,(B',D');\ldots;(B^{(r)},D^{(r)})}^{0,\lambda+2\,:\,(u',v');\ldots;(u^{(r)},v^{(r)})}\left(\begin{matrix}[-\eta-\rho_1 s_1-\ldots-\rho_R s_R-hg_\alpha : h_1,\ldots,h_r];\\ [(c):\psi';\ldots;\psi^{(r)}] \quad ,\end{matrix}\right.$$

$$[\upsilon-\eta-\rho_1 s_1-\ldots-\rho_R s_R-hg_\alpha : h_1,\ldots,h_r],\ [(a):\theta',\ldots,\theta^{(r)}]:$$

$$[-\upsilon-\eta-\rho_1 s_1-\ldots-\rho_R s_R-hg_\alpha+t : h_1,\ldots,h_r],[-\beta-\eta-\rho_1 s_1-\ldots-\rho_R s_R-hg_\alpha-t-1 : h_1,\ldots,h_r]$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]; & 2^{h_1} z_1\\ & \vdots \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}]; & 2^{h_r} z_r\end{matrix}\right), \qquad \ldots(7)$$

where Re $(\beta) > -1$, Re $(\eta) > -1$, $'\rho',\ldots,\rho^R > 0$, $h_i > 0$, $i = 1,\ldots,r$ and $|\arg(z_i)| < \tfrac{1}{2} T_i\pi$. $T_i > 0$, $|\arg(z)| < (T\pi)/2$, $T > 0$,

$$\text{Re}\left(\eta + h\, f_l/F_l + \sum_{i=1}^{r} h_i\, d_j^{(i)}/\delta_j^{(i)}\right) > -1,$$

$m_{i'}$, $i' = 1,\ldots,R$ is an arbitrary positive integer and the coefficients $A\,[n_1,s_1;\ldots;n_R,s_R]$ are arbitrary constants, real or complex.

$$\int_{-1}^{1} (1-x)^{\eta}\,(1-x)^{\rho}\,P_t^{(\beta,\upsilon)}(x)\, S_{n_1,\ldots,n_R}^{m_1,\ldots,m_R}\begin{bmatrix}(1-x)^{\alpha_1}(1+x)^{\beta_1}\\ \vdots \\ (1-x)^{\alpha_R}(1+x)^{\beta_R}\end{bmatrix}$$

$$H_{w,d}^{p,q}\left[z\,(1+x)^h\right] H\begin{pmatrix} z_1(1-x)^{h_1}(1+x)^{k_1}\\ \vdots \\ z_r(1-x)^{h_r}(1+x)^{k_r}\end{pmatrix} dx$$

$$= 2^{\eta+\rho+(\alpha_1+\beta_1)S_1+\ldots+(\alpha_R+\beta_R)S_R+h g_\alpha+1} \sum_{s_1=0}^{[n_1/m_1]} \cdots \sum_{s_R=0}^{[n_R/m_R]}$$

$$\sum_{G=1}^{p} \sum_{\alpha=0}^{\infty} \sum_{r=0}^{t} \frac{(-n_1)_{m_1 s_1}}{s_1!} \cdots \frac{(-n_R)_{m_R s_R}}{S_R!} A[n_1,s_1;\ldots;n_R,s_R]$$

$$\frac{(-1)^{\alpha}}{\alpha!\,F_G} \phi(g_\alpha)\, z^{g_\alpha} \frac{(-t)^r (\beta+\upsilon+t+1)_r}{(\beta+1)_r\, r!} .$$

$$H^{0,\lambda+2\ :(u',v');\ldots;(u^{(r)},v^{(r)})}_{A+2,C+1\ :(B',D');\ldots;(B^{(r)},D^{(r)})}\left(\begin{matrix} [-\rho-\beta_1 s_1-\ldots-\beta_R s_R-hg_\alpha : k_1,\ldots,k_r]; \\ [(c):\psi';\ldots;\psi^{(r)}] \quad , \end{matrix}\right.$$

$$[-r-\eta-\alpha_1 s_1-\ldots-\alpha_R s_R : h_1,\ldots,h_r],\ [(a):\theta',\ldots,\theta^{(r)}]:$$

$$[-1-r-\eta-(\alpha_1+\beta_1)s_1-\ldots-(\alpha_R+\beta_R)s_R-hg_\alpha : h_1+k_1,\ldots,h_r+k_r]:$$

$$\left.\begin{matrix} [(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]; & z_1 2^{h_1+k_1} \\ & \vdots \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}]; & z_r 2^{h_r+k_r} \end{matrix}\right) \quad \ldots (8)$$

where Re $(\eta) > -1$, Re $(\rho) > -1$, $\alpha',\ldots,\alpha_R > 0$, $\beta',\ldots,\beta_R > 0$, $h_i > 0$, $h > 0$, $|\arg(z)| < (T\pi)/2$, $T > 0$, $k_i > 0$, $|\arg(z_i)| < 1/2\, T_i\pi$. $T_i > 0$, $i = 1,\ldots,r$,

$Re\,[\rho + h\, f_l/F_l + \sum_{i=0}^{r} k_i\, d_j^{(i)}/\delta_j^{(i)}] > -1$, $Re[\eta + \sum_{i=0}^{r} h_i\, d_j^{(i)}/\delta_j^{(i)}] > -1$,

and m_i, is an arbitrary positive integer and coefficients $A\,[n_1,s_1;\ldots;n_R,s_R]$ are arbitrary constants, real or complex. The integral (8) can be established by using known rusult [(5), p. 284, Eq. (2)].

2. Main Integrals.

$$\int_{-1}^{1} (1-x)^{\beta} (1+x)^{\eta} P_k^{(\beta,\upsilon)}(t+\sigma)\, P_k^{(\beta,\upsilon)}(t-\sigma)\, S^{m_1,\ldots,m_R}_{n_1,\ldots,n_R}\begin{bmatrix} (1+x)^{\rho_1} \\ \vdots \\ (1+x)^{\rho_R} \end{bmatrix}$$

$$H^{p,q}_{w,d}\left[z(1+x)^h \left| \begin{matrix} (e_W, E_W) \\ (f_d, F_d) \end{matrix} \right.\right] H(z_1(1+x)^{h_1}, \ldots, z_r(1+x)^{h_r})\, dx$$

$$= 2^{\beta+\eta+\rho_1 s_1+\ldots+\rho_R s_R+h g_\alpha+1} \frac{(-1)^k \Gamma(1+\beta+k)\Gamma(1+\upsilon+k)}{(k!)^2} .$$

$$\sum_{s_1=0}^{[n_1/m_1]} \cdots \sum_{s_R=0}^{[n_R/m_R]} \sum_{G=1}^{p} \sum_{\alpha=0}^{\infty} \frac{(-n_1)_{m_1 s_1}}{s_1!} \cdots \frac{(-n_R)_{m_R s_R}}{s_R!} .$$

$$A[n_1,s_1;\ldots;n_R,s_R] \frac{(-1)^{\alpha}}{\alpha!\,F_G} \phi(g_\alpha)\, z^{g_\alpha} (-k)_r (1+\beta+\upsilon+k)_r\, t^r$$

$$H^{0,\lambda+2\ :(u',v');\ldots;(u^{(r)},v^{(r)})}_{A+2,C+2\ :(B',D');\ldots;(B^{(r)},D^{(r)})}\left(\begin{matrix} [-\eta-\rho_1 s_1-\ldots-\rho_R s_R-hg_\alpha : h_1,\ldots,h_r]; \\ [(c):\psi';\ldots;\psi^{(r)}] \quad , \end{matrix}\right.$$

$$[\upsilon-\eta-\rho_1 s_1-\ldots-\rho_R s_R-hg_\alpha : h_1,\ldots,h_r],\ [(a):\theta',\ldots,\theta^{(r)}]:$$

$$[-\upsilon-\eta-\rho_1 s_1-\ldots-\rho_R s_R-hg_\alpha+t : h_1,\ldots,h_r],[-\beta-\eta-\rho_1 s_1-\ldots-\rho_R s_R-hg_\alpha-t-1 : h_1,\ldots,h_r]$$

$$[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}];\ \begin{matrix}2^{h_1}z_1\\ \vdots\\ 2^{h_r}z_r\end{matrix}\Bigg) \quad \ldots (9)$$
$$[(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];$$

valid under the same conditions as needed for (7).

$$\int_{-1}^{1}(1-x)^{\beta}(1+x)^{\eta}P_k^{(\beta,\beta)}\left(\frac{1-xt}{\sigma}\right)S_{n_1,\ldots,n_R}^{m_1,\ldots,m_R}\begin{bmatrix}(1+x)^{\rho_1}\\ \vdots\\ (1+x)^{\rho_R}\end{bmatrix}$$

$$H_{w,d}^{p,q}\left[z(1+x)^h\right]H\left(z_1(1+x)^{h_1},\ldots,z_r(1+x)^{h_r}\right)dx$$

$$=2^{\beta+\eta+\rho_1 s_1+\ldots+\rho_R s_R+hg_\alpha+1}\frac{\Gamma(1+\beta+k)}{k!}.$$

$$\sum_{r=0}^{(k)}\sum_{s_1=0}^{[n_1/m_1]}\ldots\sum_{s_R=0}^{[n_R/m_R]}\sum_{G=1}^{p}\sum_{\alpha=0}^{\infty}(-k)_r t^r\frac{(-n_1)_{m_1 s_1}}{s_1!}\ldots\frac{(-n_R)_{m_R s_R}}{s_R!}.$$

$$A[n_1,s_1;\ldots;n_R,s_R]\frac{(-1)^\alpha}{\alpha!\,F_G}\phi(g_\alpha)z^{g_\alpha}$$

$$H_{A+2,C+2:(B',D');\ldots;(B^{(r)},D^{(r)})}^{0,\lambda+2:(u',v');\ldots;(u^{(r)},v^{(r)})}\Bigg(\begin{matrix}[-\eta-\rho_1 s_1-\ldots-\rho_R s_R-hg_\alpha:h_1,\ldots,h_r];\\ [(c):\psi';\ldots;\psi^{(r)}],\end{matrix}$$

$$\begin{matrix}[\upsilon-\eta-\rho_1 s_1-\ldots-\rho_R s_R-hg_\alpha:h_1,\ldots,h_r],[(a):\theta',\ldots,\theta^{(r)}]:\\ [-\upsilon-\eta-\rho_1 s_1-\ldots-\rho_R s_R-hg_\alpha+t:h_1,\ldots,h_r]:[-\beta-\eta-\rho_1 s_1-\ldots-\rho_R s_R-hg_\alpha-t-1:h_1,\ldots,h_r]\end{matrix}$$

$$\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}];\\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];\end{matrix}\ \begin{matrix}2^{h_1}z_1\\ \vdots\\ 2^{h_r}z_r\end{matrix}\Bigg) \quad \ldots (10)$$

which holds true under the same conditions as needed in (7).

$$\int_{-1}^{1}1/\sigma\,(1-t+\sigma)^{-\beta}(1+t+\sigma)^{-\upsilon}(1-x)^{\beta}(1+x)^{\eta}\ S_{n_1,\ldots,n_R}^{m_1,\ldots,m_R}\begin{bmatrix}(1+x)^{\rho_1}\\ \vdots\\ (1+x)^{\rho_R}\end{bmatrix}$$

$$H_{w,d}^{p,q}\left[z(1+x)^h\middle|\begin{matrix}(e_w,E_w)\\(f_d,F_d)\end{matrix}\right]\quad H\left(z_1(1+x)^{h_1},\ldots,z_r(1+x)^{h_r}\right)dx$$

$$=2^{\eta+hg_\alpha+\rho_1 s_1+\ldots+\rho_R s_R-\beta+1}\sum_{r=0}^{\infty}\sum_{s_1=0}^{[n_1/m_1]}\ldots\sum_{s_R=0}^{[n_R/m_R]}\sum_{G=1}^{p}\sum_{\alpha=0}^{\infty}$$

$$\Gamma(1+\beta+r)t^r\frac{(-n_1)_{m_1 s_1}}{s_1!}\ldots\frac{(-n_R)_{m_R s_R}}{s_R!}.A[n_1,s_1;\ldots;n_R,s_R]\frac{(-1)^\alpha}{\alpha!\,F_G}\phi(g_\alpha)z^{g_\alpha}$$

$$H_{A+2,C+2:(B',D');\ldots;(B^{(r)},D^{(r)})}^{0,\lambda+2:(u',v');\ldots;(u^{(r)},v^{(r)})}\Bigg(\begin{matrix}[-\eta-\rho_1 s_1-\ldots-\rho_R s_R-hg_\alpha:h_1,\ldots,h_r];\\ [(c):\psi';\ldots;\psi^{(r)}],\end{matrix}$$

$$[\upsilon-\eta-\rho_1 s_1-\ldots-\rho_R s_R-hg_\alpha : h_1,\ldots,h_r],\ [(a):\theta',\ldots,\theta^{(r)}]:$$
$$[-\upsilon-\eta-\rho_1 s_1-\ldots-\rho_R s_R-hg_\alpha+r : h_1,\ldots,h_r]:[-\beta-\eta-\rho_1 s_1-\ldots-\rho_R s_R-hg_\alpha-r-1 : h_1,\ldots,h_r];$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]; & 2^{h_1}z_1 \\ & \vdots \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}]; & 2^{h_r}z_r\end{matrix}\right) \qquad \ldots (11)$$

holds true under the same conditions as needed in (7).

$$\int_{-1}^{1}(1-x)^\eta(1+x)^\rho P_k^{(\beta,\upsilon)}(t+\sigma)\ P_t^{(\beta,\upsilon)}(t-\sigma)$$

$$S_{n_1,\ldots,n_R}^{m_1,\ldots,m_R}\begin{bmatrix}(1+x)^{\alpha_1}(1-x)^{\beta_1}\\ \vdots \\ (1+x)^{\alpha_R}(1-x)^{\beta_R}\end{bmatrix}$$

$$H_{w,d}^{p,q}\left[z(1+x)^h\right] H\begin{pmatrix}z_1(1-x)^{h_1}(1+x)^{k_1}\\ \vdots \\ z_r(1-x)^{h_r}(1+x)^{k_r}\end{pmatrix}dx$$

$$=2^{\eta+\rho+(\alpha_1+\beta_1)s_1+\ldots+(\alpha_R+\beta_R)s_R+hg_\alpha+1}\frac{(-1)^k\Gamma(1+\beta+k)\Gamma(1+\upsilon+k)}{(k!)^2}$$

$$\sum_{s_1=0}^{[n_1/m_1]}\ldots\sum_{s_R=0}^{[n_R/m_R]}\sum_{G=1}^{p}\sum_{\alpha=0}^{\infty}\sum_{r=0}^{k}\sum_{q'=0}^{r}\frac{(-n_1)_{m_1 s_1}}{s_1!}\ldots\frac{(-n_R)_{m_R s_R}}{s_R!}.$$

$$A[n_1,s_1;\ldots;n_R,s_R]\ \frac{(-1)^\alpha}{\alpha!\,F_G}\ \phi(g_\alpha)\,z^{g_\alpha}$$
$$\frac{(-k)_r(1+\beta+\upsilon+k)_r}{\Gamma(1+\beta+r)\Gamma(1+\upsilon+r)}\ \frac{(-r)_{q'}(1+\beta+\upsilon+r)_{q'}}{(1+\beta)_{q'}\,q'!}\ t^r$$

$$H_{A+2,C+2:(B',D');\ldots;(B^{(r)},D^{(r)})}^{0,\lambda+2:(u',v');\ldots;(u^{(r)},v^{(r)})}\left(\begin{matrix}[-\rho-\beta_1 s_1-\ldots-\beta_R s_R-hg_\alpha : k_1,\ldots,k_r],\\ [(c):\psi';\ldots;\psi^{(r)}]\qquad ,\end{matrix}\right.$$
$$[-\eta-\alpha_1 s_1-\ldots-\alpha_R s_R-q' : h_1,\ldots,h_r],\ [(a):\theta',\ldots,\theta^{(r)}]:$$
$$[-\eta-\rho-(\alpha_1+\beta_1)s_1-\ldots-(\alpha_R+\beta_R)s_R-q'-hg_\alpha-1 : h_1+k_1,\ldots,h_r+k_r]:$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]; & z_1 2^{h_1+k_1}\\ & \vdots \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}]; & z_r 2^{h_r+k_r}\end{matrix}\right) \qquad \ldots (12)$$

valid under the same conditions s obtainable from (8).

$$\int_{-1}^{1}(1-x)^\eta(1+x)^\rho\sigma^k P_k^{(\beta,\beta)}\left(\frac{1-xt}{\sigma}\right)S_{n_1,\ldots,n_R}^{m_1,\ldots,m_R}\begin{bmatrix}(1-x)^{\alpha_1}(1+x)^{\beta_1}\\ \vdots \\ (1-x)^{\alpha_R}(1+x)^{\beta_R}\end{bmatrix}$$

$$H_{w,d}^{p,q}\left[z(1+x)^h\right] H\begin{pmatrix} z_1(1-x)^{h_1}(1+x)^{k_1} \\ : \\ z_r(1-x)^{h_r}(1+x)^{k_r} \end{pmatrix} dx$$

$$= 2^{\beta+\rho+(\alpha_1+\beta_1)s_1+\ldots+(\alpha_R+\beta_R)s_R+h\,g_\alpha+1} \frac{\Gamma(1+\beta+k)}{k!}$$

$$\sum_{i=0}^{k} \sum_{s_1=0}^{[n_1/m_1]} \ldots \sum_{s_R=0}^{[n_R/m_R]} \sum_{G=1}^{p} \sum_{\alpha=0}^{\infty} \sum_{q'=0}^{r} \frac{(-k_r)\,t^r}{\Gamma(1+\beta+r)}$$

$$\frac{(-n_1)_{m_1 s_1}}{s_1!} \ldots \frac{(-n_R)_{m_R s_R}}{s_R!} A\,[n_1, s_1; \ldots; n_R, s_R]\,(r)_{q'}$$

$$\frac{(2\beta+r+1)_{q'}}{(\beta+1)_{q'}\,q'!} \frac{(-1)^\alpha}{\alpha!\,F_G} \phi(g_\alpha)\,z^{g_\alpha}$$

$$H^{0,\lambda+2\ :(u',v');\ldots;(u^{(r)},v^{(r)})}_{A+2,C+1\ :(B',D');\ldots;(B^{(r)},D^{(r)})}\left(\begin{matrix} [-\rho-\beta_1 s_1-\ldots-\beta_R s_R-hg_\alpha : k_1,\ldots,k_r], \\ [(c):\psi';\ldots;\psi^{(r)}] \quad , \end{matrix} \right.$$

$$[-\eta-\alpha_1 s_1-\ldots-\alpha_R s_R-q' : h_1,\ldots,h_r],\ [(a):\theta',\ldots,\theta^{(r)}]:$$

$$[-\eta-\rho-(\alpha_1+\beta_1)s_1-\ldots-(\alpha_R+\beta_R)s_R-q'-hg_\alpha-1 : h_1+k_1,\ldots,h_r+k_r]:$$

$$\left. \begin{matrix} [(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]; & z_1 2^{h_1+k_1} \\ & : \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}]; & z_r 2^{h_r+k_r} \end{matrix} \right), \qquad \ldots (13)$$

holds true under the same conditions as needed for (8).

$$\int_{-1}^{1} (1-x)^\beta (1-x)^\eta\, 1/\rho\, (1-t+\sigma)^{-\beta} (1+t+\sigma)^{-\upsilon}$$

$$S^{m_1,\ldots,m_R}_{n_1,\ldots,n_R}\begin{bmatrix} (1-x)^{\alpha_1}(1+x)^{\beta_1} \\ : \\ (1-x)^{\alpha_R}(1+x)^{\beta_R} \end{bmatrix}$$

$$H_{w,d}^{p,q}\left[z(1+x)^h\right] H\begin{pmatrix} z_1(1-x)^{h_1}(1+x)^{k_1} \\ : \\ z_r(1-x)^{h_r}(1+x)^{k_r} \end{pmatrix} dx$$

$$= 2^{-\beta-\upsilon-\eta+\rho+(\alpha_1+\beta_1)s_1+\ldots+(\alpha_R+\beta_R)s_R+h\,g_\alpha+1}$$

$$\sum_{r=0}^{\infty} \sum_{s_1=0}^{[n_1/m_1]} \ldots \sum_{s_R=0}^{[n_R/m_R]} \sum_{G=1}^{p} \sum_{\alpha=0}^{\infty} \sum_{q'=0}^{r} t^r \frac{(-n_1)_{m_1 s_1}}{s_1!} \ldots \frac{(-n_R)_{m_R s_R}}{s_R!}.$$

$$A\,[n_1,s_1\,;\,\dots\,;\,n_R,s_R\,]\,(r)_{q'}\;\frac{(-1)^{\alpha}}{\alpha!\,F_G}\;\phi\,(g_\alpha)\,z^{g_\alpha}\;\frac{(-r)_{q'}(\beta+\upsilon+r+1)_{q'}}{(\beta+1)_{q'}\,q'!}.$$

$$H^{0,\lambda+2\;\;:(u',v')\,;\dots;(u^{(r)},v^{(r)})}_{A+2,C+1\,:\,(B',D')\,;\dots;(B^{(r)},D^{(r)})}\left(\begin{matrix}[-\rho-\beta_1 s_1-\dots-\beta_R s_R-hg_\alpha:k_1,\dots,k_r]\,;\\ [(c):\psi';\dots;\psi^{(r)}]\,,\\ [-\eta-\alpha_1 s_1-\dots-\alpha_R s_R-q':h_1,\dots,h_r]\,,\,[(a):\theta',\dots,\theta^{(r)}]:\\ [-\eta-\rho-(\alpha_1+\beta_1)s_1-\dots-(\alpha_R+\beta_R)s_R-q'-hg_\alpha-1:h_1+k_1,\dots,h_r+k_r]:\\ [(b'):\phi']\,;\dots;\,[(b^{(r)}):\phi^{(r)}]\,;\\ [(d'):\delta']\,;\dots;\,[(d^{(r)}):\delta^{(r)}]\,;\end{matrix}\;\begin{matrix}z_1 2^{h_1+k_1}\\ \vdots\\ z_r 2^{h_r+k_r}\end{matrix}\right) \quad \dots (14)$$

valid under the same conditions as obtainable from (8).

Proofs : In order to derive (9), we multiply both the sides of (3) by

$$(1-x)^{\beta}\,(1+x)^{\eta}\;S^{m_1,\dots,m_R}_{n_1,\dots,n_R}\begin{bmatrix}(1+x)^{\rho_1}\\ \vdots\\ (1+x)^{\rho_R}\end{bmatrix}\;H^{p,q}_{w,d}\left[z\,(1+x)^{h}\right]$$

$$H\,(z_1(1+x)^{h_1},\dots,z_r(1+x)^{h_r})$$

and integrationg both sides with respect to x between the limits -1 to 1, we obtain

$$\int_{-1}^{1}(1+x)^{\eta}\,(1-x)^{\beta}\,P^{(\beta,\upsilon)}_{k}(t+\sigma)\;P^{(\beta,\upsilon)}_{k}(t-\sigma)\;S^{m_1,\dots,m_R}_{n_1,\dots,n_R}\begin{bmatrix}(1+x)^{\rho_1}\\ \vdots\\ (1+x)^{\rho_R}\end{bmatrix}$$

$$H^{p,q}_{w,d}\left[z\,(1+x)^{h}\right]\,H\,(z_1(1+x)^{h_1},\dots,z_r(1+x)^{h_r})\,dx$$

$$=\int_{-1}^{1}\frac{(-1)^k(1+\upsilon)_k(1+\beta)_k}{(k!)^2}\sum_{r=0}^{k}(-k)_r\,\frac{(1+\upsilon+\beta+k)_r}{(1+\upsilon)_r(1+\beta)_r}\,P^{(\beta,\beta)}_{r}(x)\,t^{r}$$

$$(1-x)^{\beta}\,(1+x)^{\eta}\;S^{m_1,\dots,m_R}_{n_1,\dots,n_R}\begin{bmatrix}(1+x)^{\rho_1}\\ \vdots\\ (1+x)^{\rho_R}\end{bmatrix}H^{p,q}_{w,d}\left[z\,(1+x)^{h}\right]$$

$$H\,(z_1(1+x)^{h_1},\dots,z_r(1+x)^{h_r})\,dx \quad \dots (15)$$

Now, we interchange the order of integration and summation on the right hand side of (15), which is justified due to the absolute convergent of the integral involved in the process, then we evaluate the inner integral with the help. of (7).

Finally, we get the desired result by interpreting the multivariable H-function [6].

The results in (10) through (14) can be easily proved by using the known results (4) through (8).

Particular Cases.

1. Letting $n_1 = \ldots = n_R \to 0$, the results in (7) through (14) reduce to known results recently obtained by Chaurasia and Sharma ([3],pp. 53-59, eqn. (13) to (20) respectively).
2. Taking $n_1 = \ldots = n_R \to 0$, $p = d = 1$, $q = w = 0$, $h \to 0$,the integrals in (7) through (14) reduce to known results recently obtained by Chaurasia and Girdhari Lal ([2]pp. 99 –105, eqn. (1.11) to(2.6)).

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