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SOME FINITE INTEGRALS AND FOURIER SERIES INVOLVING GENERAL POLYNOMIALS, BIORTHOGONAL POLYNOMIALS, FOX'S *H*-FUNCTION AND THE MULTIVARIABLE *H*-FUNCTION

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ABSTRACT

In this paper, we obtain four integrals and four Fourier series involving the product of the biorthogonal polynomials, the generalized polynomials $S_{q_1,\dots,q_s}^{p_1,\dots,p_s}[x_1,\dots,x_s]$, the Fox's H- function and the multivariable H-function. On account of the most general nature of the functions involved herein a very large number of known and new integrals and Fourier series involving simpler special functions follow as particular cases of our main results.

1. Introduction. The general multivariable polynomials defined by Srivastava ([2], p. 185, eq. (7)) represented in the following manner

$$S_{q_1,\dots,q_s}^{p_1,\dots,p_s}[x_1,\dots,x_s] = \sum_{k_1=0}^{[q_1/p_1]} \dots \sum_{k_s=0}^{[q_s/p_s]} \frac{(-q_1)_{p_1k_1}}{k_1!} \dots \frac{(-q_s)_{p_sk_s}}{k_s!}$$

$$L[q_1,k_1;\dots;q_s,k_s]\, x_1^{k_1}\dots x_s^{k_s} \qquad \dots (1.1)$$

where $q_m = 0,1,2,\dots : p_m$ $(m = 1,\dots,s)$ are non-zero arbitrary positive integer. The coefficients $L[q_1,k_1;\dots;q_s,k_s]$ being arbitrary constants, real or complex.

If we take $s = 1$ in the equation (1.1) and denote L [q , k] thus obtained by $L_{q,k}$, we arrive at the well known general class of polynomials $S_q^p[x]$ introduced by Srivastava ([6],p.158, eq. (1.1)].

The series representation of Fox's H-function (see [1] and [7])

$$H_{P,Q}^{M,N}\left[z\left|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right.\right] = \sum_{g=1}^{M}\sum_{G=0}^{\infty}(-1)^G\phi(\eta_g)\,z^{\eta_G}[G!\,F_g]^{-1} \dots (1.2)$$

where

$$\phi(\eta_G) = \prod_{j=1,j\neq g}^{M}\Gamma(f_j - F_j\eta_G)\prod_{j=1}^{N}\Gamma(1-e_j-E_j\eta_G)$$

$$\{ \prod_{j=M+1}^{Q} \Gamma(1-f_j+E_j\eta_G) \prod_{j=N+1}^{P} \Gamma(e_j-F_j\eta_G) \}^{-1}$$

and

$$\eta_G = (f_g + G)/F_g$$

we shall require the following biorthogonal pair of polynomials sets defined by Prabhakar and Tomar [9]

$U_n(y\,;h)$ and $V_n(y\,;h)$

where

$$U_n(y\,;h) = \sum_{j=0}^{n} (-1)^j \binom{n}{j} \frac{[(j+1)/h]_n}{(1/h)_n} \left(\frac{1-y}{2}\right)^j. \qquad \text{... (1.3)}$$

and

$$V_n(y\,;h) = \sum_{j=0}^{n} (-1)^j \binom{n}{j} \frac{(1+n)_{hj}}{(1)_{hj}} \left(\frac{1-y}{2}\right)^{hj}. \qquad \text{... (1.3)}$$

2. The main integrals. The following integrals have been derived in this section :

$$\int_0^{\pi/2} \cos(2\rho\theta)(\sin\theta)^{\mu}\, U_n(1-2y\sin^{2\alpha}\theta\,;h)$$

$$S_{q_1,\ldots,q_s}^{p_1,\ldots,p_s}[x_1(\sin\theta)^{2w_1},\ldots,x_s(\sin\theta)^{2w_s}]\ .$$

$$H_{P,Q}^{M,N}\left[z(\sin\theta)^{2t}\,\middle|\,\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right] H(z_1(\sin\theta)^{2\beta_1},\ldots,z_r(\sin\theta)^{2\beta_r})\,d\theta$$

$$= \sum_{j=0}^{n}\sum_{g=1}^{M}\sum_{G=0}^{\infty} R(x_1,\ldots,x_s)(-1)^j\binom{n}{j}\frac{[(j+1)/h]_n\, y^j}{(1/h)_n}\frac{(-1)^G\phi(\eta_g)\,z^{\eta_G}}{F_g\,G!}\,.$$

$$\frac{\Gamma(1/2+\rho)\,\Gamma(1/2-\rho)}{2^{\mu+2\alpha j+2t\,\eta_{G+1}}}\; H_{A+2,\,C+2\,:\,(B',\,D');\ldots;(B^{(r)},\,D^{(r)})}^{0,\,\lambda+2\,:\,(u',\,v');\ldots;(u^{(r)},\,v^{(r)})}$$

$$\left(\begin{matrix}[-\mu-2\alpha j-2t\eta_G-2w_1k_1-\ldots-2w_sk_s : 2\beta_1,\ldots,2\beta_r],\ [(a):\theta',\ldots,\theta^{(r)}]:\\ [(c):\psi';\ldots;\psi^{(r)}],\ [-\mu/2\pm\rho/2-\alpha j-t\eta_G-w_1k_1-\ldots-w_sk_s:\beta_1,\ldots,\beta_r]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}];\\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];\end{matrix}\ z_1 2^{-2\beta_1},\ldots,z_r 2^{-2\beta_r}\right), \qquad \text{... (2.1)}$$

where

$$R[x_1,\ldots,x_s] = \sum_{k_1=0}^{[q_1/p_1]}\cdots\sum_{k_s=0}^{[q_s/p_s]}\prod_{m=1}^{s}\left[\frac{(-q_m)_{p_m k_m}}{k_m!}\,x_m^{k_m}\right]$$

$$\frac{L\,[q_1,k_1\,;\,\dots\,;\,q_s,k_s\,]}{2^{\,2w_1k_1+\dots+2w_sk_s}}, \qquad \dots (2.2)$$

provided that $\rho = 0,1,2,\dots;\ t>0,\ w_m>0,\ \beta_i>0,\ i=1,\dots,r\,;\ m=1,\dots,s$, $\mathrm{Re}\,(\mu+2t\,f_{j'}/F_{j'}+2\sum_{i=1}^{r}\beta_j\,d_l^{(i)}/\delta_l^{(i)})>0,\ j'=1,\dots,M\,;\ l=1,\dots,u\,(i),\ T_i>0$, $|\arg(z_i)|<T_i\pi/2\,;\ |\arg(z)|<T\pi/2,\ T>0,\ p_m\ (m=1,\dots,s)$ are non-zero arbitrary positive integer and the coefficients $L\,[q_1,k_1\,;\dots;\,q_s,k_s]$ are arbitrary constants, real or complex.

$$[\,T=\sum_{i=1}^{N}E_i-\sum_{i=N+1}^{P}E_i+\sum_{i=1}^{M}F_i-\sum_{i=M+1}^{Q}F_i\,]$$

$$\int_0^{\pi/2}\cos(\rho\theta)\,(\cos\theta)^{\mu}\,U_n\,(1-2y\cos^{2\alpha}\theta\,;h)$$
$$S_{q_1,\dots,q_s}^{p_1,\dots,p_s}\,[x_1(\cos\theta)^{2w_1},\dots,x_s(\cos\theta)^{2w_s}].\ H_{P,Q}^{M,N}\left[z\,(\cos\theta)^{2t}\left|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right.\right]$$
$$H\,(z_1(\cos\theta)^{2\beta_1},\dots,z_r(\cos\theta)^{2\beta_r})\,d\theta$$
$$=\sum_{j=1}^{n}\sum_{g=1}^{M}\sum_{G=0}^{\infty}R\,(x_1,\dots,x_s)\,(-1)^j\binom{n}{j}\frac{[(j+1)/h]_n\,y^j}{(1/h)_n}\,\frac{(-1)^G\,\phi\,(\eta_g)\,z^{\eta_G}}{F_g\,G\,!}.$$
$$\frac{\pi\Gamma\,(\rho+1)}{2^{\mu+2\alpha j+2t\,\eta_G+1}}\,H^{0,\lambda+2\,:(u',v')\,;\dots;\,(u^{(r)},v^{(r)})}_{A+2,\,C+2\,:\,(B',D')\,;\dots;\,(B^{(r)},D^{(r)})}$$
$$\left(\begin{matrix}[(a):\theta',\dots,\theta^{(r)}]\,:\\ [(c):\psi';\dots;\psi^{(r)}]\quad,[-\mu/2\pm\rho/2-\alpha j-t\eta_G-w_1k_1-\dots-w_sk_s:\beta_1,\dots,\beta_r]\,:\end{matrix}\right.$$
$$\left.\begin{matrix}[(b'):\phi'\,];\dots;\,[(b^{(r)}):\phi^{(r)}]\,;\\ [(d'):\delta']\,;\dots;\,[(d^{(r)}):\delta^{(r)}]\,;\end{matrix}\ z_1 2^{-2\beta_1},\dots,\ z_r 2^{-2\beta_r}\right), \qquad \dots (2.3)$$

where $R\,(x_1,\dots,x_s)$ is defined by (2.2) and the other conditions of validity are the same as given in (2.1).

$$\int_0^{\pi/2}\cos(2\rho\theta)\,(\sin\theta)^{\mu}\,V_n\,(1-2y\sin^{2\alpha}\theta\,;h)$$
$$S_{q_1,\dots,q_s}^{p_1,\dots,p_s}\,[x_1(\sin\theta)^{2w_1},\dots,x_s(\sin\theta)^{2w_s}].\ H_{P,Q}^{M,N}\left[z\,(\sin\theta)^{2t}\left|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right.\right]$$
$$H\,(z_1(\sin\theta)^{2\beta_1},\dots,z_r(\sin\theta)^{2\beta_r})\,d\theta$$
$$=\sum_{j=0}^{n}\sum_{g=1}^{M}\sum_{G=0}^{\infty}R\,(x_1,\dots,x_s)\,(-1)^j\binom{n}{j}\frac{(1+n)_{hj}\,y^{hj}}{(1)_{hj}}\,\frac{(-1)^G\,\phi\,(\eta_G)\,z^{\eta_G}}{F_g\,G\,!}.$$
$$\frac{\Gamma\,(1/2+\rho)\,\Gamma\,(1/2-\rho)}{2^{\mu+2\alpha hj+2t\,\eta_G+1}}\,H^{0,\lambda+2\,:(u',v')\,;\dots;\,(u^{(r)},v^{(r)})}_{A+2,\,C+2\,:\,(B',D')\,;\dots;\,(B^{(r)},D^{(r)})}$$

$$\left(\begin{matrix}[-\mu-2\alpha hj-2t\eta_G-2w_1k_1-\ldots-2w_sk_s:2\beta_1,\ldots,2\beta_r], [(a):\theta',\ldots,\theta^{(r)}]: \\ [(c):\psi';\ldots;\psi^{(r)}], [-\mu/2\pm\rho/2-\alpha hj-t\eta_G-w_1k_1-\ldots-w_sk_s:\beta_1,\ldots,\beta_r]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]; \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];\end{matrix}\; z_1 2^{-2\beta_1},\ldots, z_r 2^{-2\beta_r}\right), \qquad \ldots (2.4)$$

valid under the same conditions as obtainable from (2.1).

$$\int_0^{\pi/2} cos\,(\rho\theta)\,(cos\,\theta)^{\mu}\,V_n\,(1-2y\,cos^{2\alpha}\theta\,;h)$$

$$S^{p_1,\ldots,p_s}_{q_1,\ldots,q_s}[x_1\,(cos\,\theta)^{2w_1},\ldots,x_s\,(cos\,\theta)^{2w_s}].\; H^{M,N}_{P,Q}\left[z\,(cos\,\theta)^{2t}\middle|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right]$$

$$H\,(z_1(cos\,\theta)^{2\beta_1},\ldots,z_r(cos\,\theta)^{2\beta_1})\,d\,\theta$$

$$=\sum_{j=0}^{n}\sum_{g=1}^{M}\sum_{G=0}^{\infty} R\,(x_1,\ldots,x_s)\,\frac{(-1)^j\binom{n}{j}[(1+n)_{hj}\,y^{hj}}{(1)_{hj}}\,\frac{(-1)^G\,\phi\,(\eta_g)\,z^{\eta_G}}{F_g\,G\,!}.$$

$$\frac{\pi\,\Gamma\,(\rho+1)}{2^{\mu+2\alpha hj+2t\,\eta_G+1}}\; H^{0,\,\lambda+2\,:(u',\,v');\ldots;(u^{(r)},\,v^{(r)})}_{A+2,\,C+2\,:(B',\,D');\ldots;(B^{(r)},\,D^{(r)})}$$

$$\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]: \\ [(c):\psi';\ldots;\psi^{(r)}], [-\mu/2\pm\rho/2-\alpha hj-t\eta_G-w_1k_1-\ldots-w_sk_s:\beta_1,\ldots,\beta_r]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]; \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];\end{matrix}\; z_1 2^{-2\beta_1},\ldots, z_r 2^{-2\beta_r}\right), \qquad \ldots (2.5)$$

which holds good for the same conditions as obtainable from (2.1).

Proof . To obtain (2.1), express the biorthogonal polynomials $U_n\,(1-2y\,sin^{2\alpha}\theta\,;h)$ as given by (1.3), a general polynomials occurring in the integrand of (2.1) as defined in (1.1), series representation of the Fox's *H*-function by (1.2) and the multi variable *H*-function defined by ([3] and [4], see also [5]), interchange the order of summations and integration (which is permissible under the conditions stated above), evaluate the inner integral with the help of a result recently obtained in ([8], eq. (2.3.5)), we arrive at the desired results.

The integrals (2.3) through (2.5) can be developed in the similar manner with the help of known integrals ([8], eq. (2.3.5) and eq. (2.3.6)).

3. Fourier Series.

$$(sin\,\theta)^{\mu}\,U_n\,(1-2y\,cos^{2\alpha}\theta\,;h)\,H^{M,N}_{P,Q}\left[z\,(sin\,\theta)^{2t}\middle|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right]$$

$$S^{p_1, \dots, p_s}_{q_1, \dots, q_s}\left[x_1 (\sin\theta)^{2w_1}, \dots, x_s (\sin\theta)^{2w_s}\right]$$

$$H\left(z_1(\cos\theta)^{2\beta_1}, \dots, z_r(\cos\theta)^{2\beta_1}\right) d\theta$$

$$= \frac{1}{\pi\, 2^{\mu-1}} \sum_{v=0}^{\infty} \Gamma(1/2+v)\,\Gamma(1/2-v) \cos 2v\theta \sum_{j=0}^{n} \sum_{g=1}^{M} \sum_{G=0}^{\infty}$$

$$R(x_1, \dots, x_s)\,(-1)^j \binom{n}{j} \frac{[(j+1)/h]_n\, y^j}{(1/h)_n} \frac{(-1)^G \phi(\eta_g)\, z^{\eta_G}}{F_g\, G!\, 2^{2\alpha j+2t\,\eta_G}}$$

$$H^{0,\lambda+1\,:(u', v');\dots;(u^{(r)}, v^{(r)})}_{A+2,\, C+2\,:(B', D');\dots;(B^{(r)}, D^{(r)})}$$

$$\left(\begin{matrix}[-\mu-2\alpha j-2t\eta_G - 2w_1k_1 - \dots - 2w_sk_s : 2\beta_1, \dots, 2\beta_r],\ [(a):\theta', \dots, \theta^{(r)}]: \\ [(c):\psi'; \dots; \psi^{(r)}],\ [-\mu/2 \pm \nu/2 - \alpha j - t\eta_G - w_1k_1 - \dots - w_sk_s : \beta_1, \dots, \beta_r]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\dots;[(b^{(r)}):\phi^{(r)}]; \\ [(d'):\delta'];\dots;[(d^{(r)}):\delta^{(r)}];\end{matrix}\ z_1 2^{-2\beta_1}, \dots, z_r 2^{-2\beta_r}\right), \qquad \dots (3.1)$$

where $R(x_1, \dots, x_s)$ is given by (2.2), provided that $v = 0,1,2,\dots;\ t > 0$, $w_m > 0,\ \beta_i > 0,\ m = 1,\dots,s;\ i = 1,\dots,r;$ $\text{Re}\,(\mu+2t\, f_{j'}/F_{j'} + 2\sum_{i=1}^{r} \beta_i\, d_l^{(i)}/\delta_l^{(i)}) > 0,\ j'=1,\dots,M;\ l = 1,\dots,u(i),\ T_i > 0,$ $|arg(z_i)| < T_i\pi/2;\ |arg(z)| < T\pi/2,\ T > 0,\ p_m\ (m = 1,\dots,s)$ are non-zero arbitrary positive integer and the coefficients $L[q_1,k_1;\dots;q_s,k_s]$ are arbitrary constants, real or complex.

$$(\cos\theta)^{\mu}\, U_n(1-2y\cos^{2\alpha}\theta;\, h)\, H^{M,N}_{P,Q}\left[z(\cos\theta)^{2t}\,\middle|\,\begin{matrix}(e_P, E_P) \\ (f_Q, F_Q)\end{matrix}\right]$$

$$S^{p_1, \dots, p_s}_{q_1, \dots, q_s}\left[x_1 (\cos\theta)^{2w_1}, \dots, x_s (\cos\theta)^{2w_s}\right]$$

$$H\left(z_1(\cos\theta)^{2\beta_1}, \dots, z_r(\cos\theta)^{2\beta_1}\right)$$

$$= \frac{1}{2^{\mu-1}} \sum_{v=0}^{\infty} \Gamma(v+1) \cos v\theta \sum_{j=0}^{n} \sum_{g=1}^{M} \sum_{G=0}^{\infty}$$

$$R(x_1, \dots, x_s)\,(-1)^j \binom{n}{j} \frac{[(j+1)/h]_n\, y^j}{(1/h)_n} \frac{(-1)^G \phi(\eta_g)\, z^{\eta_G}}{F_g\, G!\, 2^{2\alpha j+2t\,\eta_G}}$$

$$H^{0,\lambda+2\ :(u',v');...;(u^{(r)},v^{(r)})}_{A+2,\,C+2\ :(B',D');...;(B^{(r)},D^{(r)})}$$

$$\left(\begin{matrix}[(a):\theta',\dots,\theta^{(r)}]: \\ [(c):\psi';...;\psi^{(r)}]\quad ,[-\mu/2\pm\nu/2-\alpha j-t\eta_G-w_1k_1-...-w_sk_s:\beta_1,...,\beta_r]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];...;[(b^{(r)}):\phi^{(r)}]; \\ [(d'):\delta'];...;[(d^{(r)}):\delta^{(r)}];\end{matrix}\ z_1 2^{-2\beta_1},...,\ z_r 2^{-2\beta_r}\right), \qquad ...(3.2)$$

valid under the same conditions as needed for (3.1).

$$(\sin\theta)^{\mu}\,V_n\,(1-2y\sin^{2\alpha}\theta\,;h)\,H^{M,N}_{P,Q}\left[z(\sin\theta)^{2t}\middle|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right]$$

$$S^{p_1,\dots,p_s}_{q_1,\dots,q_s}\left[x_1(\sin\theta)^{2w_1},\dots,x_s(\sin\theta)^{2w_s}\right]$$

$$H\,(z_1(\sin\theta)^{2\beta_1},...,z_r(\sin\theta)^{2\beta_r})$$

$$=\frac{1}{\pi\,2^{\mu-1}}\sum_{v=0}^{\infty}\Gamma(1/2+v)\,\Gamma(1/2-v)\cos 2v\theta\sum_{j=0}^{n}\sum_{g=1}^{M}\sum_{G=0}^{\infty}R\,(x_1,\dots,x_s)$$

$$\frac{(-1)^j\binom{n}{j}[(1+n)_{hj}\,y^{hj}}{(1)_{hj}}\ \frac{(-1)^G\,\phi(\eta_g)\,z^{\eta_G}}{F_g\,G!\,2^{2\alpha hj+2t\eta_G}}$$

$$H^{0,\lambda+1\ :(u',v');...;(u^{(r)},v^{(r)})}_{A+1,\,C+2\ :(B',D');...;(B^{(r)},D^{(r)})}$$

$$\left(\begin{matrix}[-\mu-2\alpha hj-2t\eta_G-2w_1k_1-...-2w_sk_s:2\beta_1,...,2\beta_r]\ ;[(a):\theta',\dots,\theta^{(r)}]: \\ [(c):\psi';...;\psi^{(r)}]\quad ,[-\mu/2\pm\nu/2-\alpha hj-t\eta_G-w_1k_1-...-w_sk_s:\beta_1,...,\beta_r]: \\ \begin{matrix}[(b'):\phi'];...;[(b^{(r)}):\phi^{(r)}]; \\ [(d'):\delta'];...;[(d^{(r)}):\delta^{(r)}];\end{matrix}\ z_1 2^{-2\beta_1},...,\ z_r 2^{-2\beta_r}\end{matrix}\right), \qquad ...(3.3)$$

which holds good for the same conditions as needed for (3.1).

$$(\cos\theta)^{\mu}\,V_n\,(1-2y\cos^{2\alpha}\theta\,;h)\,H^{M,N}_{P,Q}\left[z(\cos\theta)^{2t}\middle|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right]$$

$$S^{p_1,\dots,p_s}_{q_1,\dots,q_s}[x_1(\cos\theta)^{2w_1},\dots,x_s(\cos\theta)^{2w_s}].$$

$$H\,(z_1(\cos\theta)^{2\beta_1},...,z_r(\cos\theta)^{2\beta_r})$$

$$= \frac{1}{2^{\mu-1}} \sum_{\upsilon=0}^{\infty} \Gamma(\upsilon+1) \cos \upsilon\theta \sum_{j=0}^{n} \sum_{g=1}^{M} \sum_{G=0}^{\infty}$$

$$R(x_1, \ldots, x_s) \frac{(-1)^j \binom{n}{j} [(1+n)_{hj} y^{hj}}{(1)_{hj}} \frac{(-1)^G \phi(\eta_G) z^{\eta_G}}{F_g G! \, 2^{2\alpha hj+2t\eta_G}}$$

$$H^{0, \lambda \; :(u', v'); \ldots; (u^{(r)}, v^{(r)})}_{A, C+2 : (B', D'); \ldots; (B^{(r)}, D^{(r)})}$$

$$\left(\begin{matrix} [(a) : \theta', \ldots, \theta^{(r)}] : \\ [(c) : \psi'; \ldots; \psi^{(r)}] \quad , [-\mu/2 \pm \nu/2 - \alpha hj - t\eta_G - w_1 k_1 - \ldots - w_s k_s : \beta_1, \ldots, \beta_r] : \\ [(b') : \phi'] ; \ldots ; [(b^{(r)}) : \phi^{(r)}] ; \\ [(d') : \delta'] ; \ldots ; [(d^{(r)}) : \delta^{(r)}] ; \end{matrix} \; z_1 2^{-2\beta_1}, \ldots, z_r 2^{-2\beta_r} \right) , \quad \ldots (3.4)$$

valid under the same conditions as needed for (3.1).

Proof . To develop the result (3.1), we first consider

$$f(\theta) -(\sin \theta)^{\mu} U_n (1-2y \sin^{2\alpha} \theta ; h)$$

$$S^{p_1, \ldots, p_s}_{q_1, \ldots, q_s} [x_1 (\sin \theta)^{2w_1}, \ldots, x_s (\sin \theta)^{2w_s}].$$

$$H^{M, N}_{P, Q} \left[z (\sin \theta)^{2t} \middle| \begin{matrix} (e_P, E_P) \\ (f_Q, F_Q) \end{matrix} \right] H(z_1 (\sin \theta)^{2\beta_1}, \ldots, z_r (\sin \theta)^{2\beta_r})$$

$$= \sum_{\upsilon=0}^{\infty} A_\upsilon \cos 2\upsilon \theta , \; (0 < \theta < \pi/2) . \quad \ldots (3.5)$$

Equation (3.5) is valid since $f(\theta)$ is continuous and of bounded variation in the open interval $(0,\pi/2)$. Now multiply both the sides of (3.5) by $\cos (2 \rho\theta)$, then integrate with respect to θ from 0 to $\pi/2$ [in the case $\upsilon = \rho$] and use the result (2.1), we find

$$A_\rho = \frac{1}{\pi \, 2^{\mu-1}} \sum_{j=0}^{n} \sum_{g=1}^{M} \sum_{G=0}^{\infty} R(x_1, \ldots, x_s) \frac{(-1)^j [(j+1)/h)_n y^j}{(1/h)_n} .$$

$$\frac{(-1)^G \phi(\eta_g) z^{\eta_G}}{F_g G!} \frac{\Gamma(1/2+\rho) \Gamma(1/2-\rho)}{2^{2\alpha j+2t \eta_G}}$$

$$H^{0, \lambda+1 \; :(u', v'); \ldots; (u^{(r)}, v^{(r)})}_{A+1, C+2 : (B', D'); \ldots; (B^{(r)}, D^{(r)})}$$

$$\left(\begin{matrix} [-\mu-2\alpha j -2t\eta_G - 2w_1 k_1 - \ldots - 2w_s k_s : 2\beta_1, \ldots, 2\beta_r] \; ; [(a) : \theta', \ldots, \theta^{(r)}] : \\ [(c) : \psi'; \ldots; \psi^{(r)}] \quad , [-\mu/2 \pm \rho/2 - \alpha j - t\eta_G - w_1 k_1 - \ldots - w_s k_s : \beta_1, \ldots, \beta_r] : \end{matrix} \right.$$

$$\left.\begin{matrix}[(b') : \phi'] ; \dots ; [(b^{(r)}) : \phi^{(r)}] ; \\ [(d') : \delta'] ; \dots ; [(d^{(r)}) : \delta^{(r)}] ;\end{matrix} \quad z_1 2^{-2\beta_1}, \dots, z_r 2^{-2\beta_r} \right) , \qquad \dots (3.6)$$

where $R(x_1, \dots, x_s)$ is given by (2.2).
Now on substituting the value of A_v in (3.5), we arrive at the desired result (3.1).

The Fourier series (3.2) through (3.4) can be developed by proceeding on the similar lines as given in the proof of (3.1).

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