Jñānābha, Vol. 28, 1998

INTEGRALS ASSOCIATED WITH GENERAL CLASS OF POLYNOMIALS, GENERALIZED LAURICELLA'S FUNCTION AND THE *H*-FUNCTION OF SEVERAL COMPLEX VARIABLES

By

V.B.L. Chaurasia and Anju Godika

Department of Mathematics

University of Rajasthan, Jaipur 302004, Rajasthan, India

(Received : August 3, 1997)

ABSTRACT

In this paper, we present some finite integrals involving a general class of polynomials, the (Srivastava - Daust) generalized Lauricella function and the (Srivastava- Panda) *H*- function of several complex variables. By assigning suitable values to the parameters, the results can be reduced to many known and unknown results.

1. Introduction. We derive the following integral transformation for the multivariable *H*-function defined by Srivastava and Panda [7].

$$\int_0^1 x^{\varepsilon-1}(1-x)^{\beta} F^{\nu:A';\ldots;A^{(s)};\,0;0}_{\sigma:B';\ldots;B^{(s)};\,1;1}\left(\begin{matrix}[(\alpha_\nu):\eta';\ldots;\eta^{(s)},\gamma,\gamma]:[(l'):\rho'];\ldots;\\ [(\beta_\sigma):\xi';\ldots;\xi^{(s)},\mu,\mu]:[(m'):\Im'];\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(l^{(s)}):\rho^{(s)}];\ldots;\ldots;\\ [(m^{(s)}):\Im^{(s)}];[\alpha+1:1];[\beta+1:1];\end{matrix}\quad z'_1,\ldots,z'_r,\ldots,z'_s,-xt,(1-x)t\right)$$

$$S^{m'_1;\ldots;m'_r}_{n'_1;\ldots;n'_R}(y_1x^{k_1};\ldots;y_Rx^{k_R}).\,H(z_1x^{h_1};\ldots;z_rx^{h_r})\,dx$$

$$=\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{\nu}(\alpha_j)_{n\gamma_j}}{(\alpha+1)_n(\beta+1)_n\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}}$$

$$F^{\nu:A';\ldots;A^{(s)}}_{\sigma:B';\ldots;B^{(s)}}\left(\begin{matrix}[(\alpha_\nu+n\gamma_\nu):\eta';\ldots;\eta^{(s)}]:[(l'):\rho'];\ldots;[(l^{(s)}):\rho^{(s)}];\\ [(\beta_\sigma+n\mu_\sigma):\xi';\ldots;\xi^{(s)}]:[(m'):\Im'];\ldots;[(m^{(s)}):\Im^{(s)}];\end{matrix}\right.$$

$$\left.z'_1,\ldots,z'_s,\right)\frac{(-t)^n\,\Gamma(\beta+n+1)}{n!}$$

$$\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_R=0}^{[n_R/m_R]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_R)_{m_R\alpha_R}}{\alpha_R!}$$

$$A_{n_1\alpha_1};\dots;A_{n_R\alpha_R}\,y_1^{\alpha_1};\dots;y_R^{\alpha_R}$$

$$H^{0,\lambda+2:(u',v');\dots;(u^{(r)},v^{(r)})}_{A+2,C+2:(B',D');\dots;(B^{(r)},D^{(r)})}$$

$$\left(\begin{matrix}[1-\varepsilon-k_1\alpha_1-\dots-k_R\alpha_R:h_1,\dots,h_r],\ [1-\varepsilon-k_1\alpha_1-\dots-k_R\alpha_R:h_1,\dots,h_r]\\ [(c):\psi';\dots;\psi^{(r)}],\ [1+\alpha-\varepsilon+n-k_1\alpha_1-\dots-k_R\alpha_R:h_1,\dots,h_r]\end{matrix}\right.$$

$$\begin{matrix}[(a):\theta';\dots;\theta^{(r)}] & :[(b'):\phi'];\dots;\\ [-\beta-\varepsilon-n-k_1\alpha_1-\dots-k_R\alpha_R:h_1,\dots,h_r] & :[(d'):\delta'];\dots;\end{matrix}$$

$$\left.\begin{matrix}[(b^{(r)}):\phi^{(r)}];\\ [(d^{(r)}):\delta^{(r)}];\end{matrix}\ z_1,\dots,z_r\right), \qquad \dots (1.1)$$

where $\mathrm{Re}\,(\beta)>-1$, $\mathrm{Re}\,(\varepsilon+\sum_{i=0}^{r}h_i\frac{d_j^{(i)}}{\delta_j^{(i)}})>0,\ j=1,\dots,\mu^{(i)}$;

$h_i>0,\ T_i>0,\ |\arg z_i|<T_i\pi/2,\ i=1,\dots,r,\ |t|<1$
and the series on the right is convergent.

2. Proof . In order to prove (1.1), we start with the following result [6] :

$$F^{\nu:A';\dots;A^{(s)};0;0}_{\sigma:B';\dots;B^{(s)};1;1}\left(\begin{matrix}[(\alpha_\nu):\eta';\dots;\eta^{(s)},\gamma;\gamma]:[(l'):\rho'];\dots;\\ [(\beta_\sigma):\xi';\dots;\xi^{(s)},\mu;\mu]:[(m'):\Im'];\dots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(l^{(s)}):\rho^{(s)}];\dots;\dots;\\ [(m^{(s)}):\Im^{(s)}];[\alpha+1:1];[\beta+1:1];\end{matrix}\ z'_1,\dots,z'_s,-xt,(1-x)t\right)$$

$$=\sum_{n=0}^{\infty}\frac{\prod_{j=0}^{\nu}(\alpha_j)_{n\gamma_j}}{(\alpha+1)_n\,(\beta+1)_n\,\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}}\,P_n^{(\alpha,\beta)}(1-2x)$$

$$F^{\nu:A';\dots;A^{(s)}}_{\sigma:B';\dots;B^{(s)}}\left(\begin{matrix}[(\alpha_\nu+n\gamma_\nu):\eta';\dots;\eta^{(s)}]:[(l'):\rho'];\dots;[(l^{(s)}):\rho^{(s)}];\\ [(\beta_\sigma+n\mu_\sigma):\xi';\dots;\xi^{(s)}]:[(m'):\Im'];\dots;[(m^{(s)}):\Im^{(s)}];\end{matrix}\right.$$

$$\left.z'_1,\dots,z'_s\right)t^n, \qquad \dots (2.1)$$

where $\gamma_i\ \eta_i^{(j)},\ i=1,\dots,\sigma$ and $j=1,\dots,s$;
$\mu_i\ \xi_j^{(j)},\ i=1,\dots,\nu$ and $j=1,\dots,s$;
$\rho_i\ \Im_k^{(j)},\ i=1,\dots,A^{(j)}$; $j=1,\dots,s$ and $k=1,\dots,B^{(j)}$

are all real and positive, (α_v) stands for the sequence of parameters $\alpha_1, \dots, \alpha_v$, $(l^{(j)})$ stands for the sequence of $A^{(j)}$ parameters $l_1^{(j)}, \dots, l^j_{A^{(j)}}, j = 1, \dots, s$, with similar interpretations for (β_σ) and $(m^{(j)})$, $j=1, \dots, s$ and

$F(z'_1, \dots, z'_s)$ represents the (Srivastava-Daoust) generalized Lauricella function of s complex varibles $z'_1, \dots, z'_s$ [5].

Now, to prove (1.1), we multiply both sides of (2.1) by

$$x^{\varepsilon-1}(1-x)^{\beta} S^{m_1; \dots ; m_r}_{n_1; \dots ; n_R}(y_1 x^{k_1}; \dots ; y_R x^{k_R}) . H(z_1 x^{h_1}; \dots ; z_r x^{h_r})$$

and integrating it with respect to x from 0 to 1, we interchange the order of integration and summations (which is justified due to the absolute convergence of the integral involved in this process) to evaluating the right side.

Now we interpret the inner integral with the help of the following result [7]

$$\int_0^1 x^{\varepsilon}(1-x)^{\beta} P_n^{(\alpha,\beta)}(1-2x)\, S^{m_1; \dots ; m_r}_{n_1; \dots ; n_R}(y_1 x^{k_1}; \dots ; y_R x^{k_R}) . H(z_1 x^{h_1}; \dots ; z_r x^{h_r})\, dx$$

$$= \frac{(-t)^n \Gamma(\beta+n+1)}{n!} \sum_{\alpha_1=0}^{[n_1/m_1]} \dots \sum_{\alpha_R=0}^{[n_R/m_R]} \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1 !} \dots \frac{(-n_R)_{m_R\alpha_R}}{\alpha_R !}$$

$$A_{n_1\alpha_1}; \dots; A_{n_R\alpha_R}\, y_1^{\alpha_1}; \dots; y_R^{\alpha_R}$$

$$H^{0,\lambda+2:(u',v');\dots;(u^{(r)},v^{(r)})}_{A+2,C+2:[B',D'];\dots;[B^{(r)},D^{(r)}]} \left(\begin{matrix} [1-\varepsilon-k_1\alpha_1-\dots-k_R\alpha_R : h_1,\dots,h_r], [\alpha-\varepsilon-k_1\alpha_1-\dots-k_R\alpha_R : h_1,\dots,h_r] \\ [(c):\psi';\dots;\psi^{(r)}], [\alpha-\varepsilon+n-k_1\alpha_1-\dots-k_R\alpha_R : h_1,\dots,h_r] \end{matrix} \right.$$

$$\begin{matrix} [(a):\theta';\dots;\theta^{(r)}] & : [(b'):\phi'];\dots; \\ [-\beta-\varepsilon-n-1-k_1\alpha_1-\dots-k_R\alpha_R : h_1,\dots,h_r] & : [(d'):\delta'];\dots; \end{matrix}$$

$$\left. \begin{matrix} [(b^{(r)}):\phi^{(r)}]; \\ [(d^{(r)}):\delta^{(r)}]; \end{matrix}\; z_1,\dots, z_r \right), \quad \dots (2.2)$$

where Re $(\beta) > -1$, Re $\left(\varepsilon + \sum_{i=0}^{r} h_i \frac{d_j^{(i)}}{\delta_j^{(i)}}\right) > 0$, $h_i > 0$ and

$T_i > 0$, $|\arg z_i| < T_i\pi/2$, $i = 1, \ldots, r\,; j = 1, \ldots, u^{(i)}$

we arrive at the required result (1.1).

3. Particular Cases

(i) By assigning suitable values to the parameters and letting $\alpha_1 = \alpha', \alpha_2 = \alpha'', \gamma_1 = \gamma_2 = 1$ in (1.1), we obtain the following integral

$$\int_0^1 x^{\varepsilon-1}(1-x)^{\beta}\, S^{m'_1;\ldots;m'_R}_{n'_1;\ldots;n'_R}(y_1 x^{k_1};\ldots;y_R x^{k_R}) \,.\, H(z_1 x^{k_1};\ldots;z_r x^{k_r})$$

$$F_C^{(s+2)}[\alpha', \alpha''; w_1,\ldots,w_s, \alpha+1, \beta+1, ; z'_1,\ldots,z'_s - xt\,(1-x)\,t]\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(\alpha')_n (\alpha'')_n (-t)^n\, \Gamma(\beta+n+1)}{(\alpha+1)_n (\beta+1)_n\; n!}$$

$$F_C^{(s)}[\alpha'+n, \alpha''+n; w_1,\ldots,w_s; z'_1,\ldots,z'_s]\,.$$

$$\sum_{\alpha_1=0}^{[n_1/m_1]} \cdots \sum_{\alpha_R=0}^{[n_R/m_R]} \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!};\ldots;\frac{(-n_R)_{m_R\alpha_R}}{\alpha_R!} A_{n_1\alpha_1} \ldots A_{m_R\alpha_R}\, y_1^{\alpha_1}\ldots y_R^{\alpha_R}$$

$$H^{0,\lambda+2:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A+2,\,C+2:(B',D');\ldots;(B^{(r)},D^{(r)})}$$

$$\left(\begin{matrix}[1-\varepsilon-k_1\alpha_1-\ldots-k_R\alpha_R : h_1,\ldots,h_r], [1-\varepsilon+\alpha-k_1\alpha_1-\ldots-k_R\alpha_R : h_1,\ldots,h_r] \\ [(c):\psi';\ldots;\psi^{(r)}], [1-\varepsilon+\alpha+n-k_1\alpha_1-\ldots-k_R\alpha_R : h_1,\ldots,h_r]\end{matrix}\right.$$

$$\begin{matrix}[(a):\theta';\ldots;\theta^{(r)}] : [(b'):\phi'];\ldots; \\ [-\beta-\varepsilon-n-k_1\alpha_1-\ldots-k_R\alpha_R : h_1,\ldots,h_r] : [(d'):\delta'];\ldots;\end{matrix}$$

$$\left.\begin{matrix}[(b^{(r)}):\phi^{(r)}]; \\ [(d^{(r)}):\delta^{(r)}];\end{matrix}\; z_1,\ldots,z_r\right), \qquad \ldots (3.1)$$

which holds under the same conditions as obtainable from (1.1).

(ii) For $r = 2$, the result in (3.1) reduces to a known result obtained by Chaurasia [1].

(iii) Putting in $z'_1 = \ldots = z'_s = 0$ in (1.1), we obtain the following integral involving generalized Kampé de Fériet function [4,5]

$$\int_0^1 x^{\varepsilon-1}(1-x)^{\beta}\, S^{m_1;\ldots;m_r}_{n_1;\ldots;n_R}(y_1 x^{k_1};\ldots;y_R x^{k_R}) \,.\, H(z_1 x^{h_1};\ldots;z_r x^{h_r})$$

$$S^{\nu:0;0}_{\sigma:1;1}\left(\begin{matrix}[(\alpha_\nu):\gamma,\gamma]:\ldots;\ldots; \\ [(\beta_\sigma):\mu,\mu]:[\alpha+1:1];[\beta+1:1]\end{matrix}\; -xt, (1-x)\,t\right) dx$$

$$= \sum_{n=0}^{\infty} \frac{\prod_{j=0}^{\nu} \Gamma(\alpha_j + n\gamma_j)\,(-t)^n\,\Gamma(\beta+n+1)}{\Gamma(\alpha+n+1)\,\Gamma(\beta+n+1)\prod_{j=0}^{\sigma}(\beta_j+n\mu_j)\,n\,!}$$

$$\sum_{\alpha_1=0}^{[n_1/m_1]} \cdots \sum_{\alpha_R=0}^{[n_R/m_R]} \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1\,!} \cdots \frac{(-n_R)_{m_R\alpha_R}}{\alpha_R\,!} A_{n_1\alpha_1} \cdots A_{n_R\alpha_R}\, y_1^{\alpha_1} \cdots y_R^{\alpha_R}$$

$$H^{0,\lambda+2:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A+2,C+2:(B',D');\ldots;(B^{(r)},D^{(r)})}$$

$$\left(\begin{matrix}[1-\varepsilon-k_1\alpha_1-\ldots-k_R\alpha_R:h_1,\ldots,h_r],\ [1-\varepsilon+\alpha-k_1\alpha_1-\ldots-k_R\alpha_R:h_1,\ldots,h_r]\\ [(c):\psi';\ldots;\psi^{(r)}],\ [1-\varepsilon+\alpha+n-k_1\alpha_1-\ldots-k_R\alpha_R:h_1,\ldots,h_r]\end{matrix}\right.$$

$$\begin{matrix}[(a):\theta';\ldots;\theta^{(r)}] & :[(b'):\phi'];\ldots;\\ [-\beta-\varepsilon-n-k_1\alpha_1-\ldots-k_R\alpha_R:h_1,\ldots,h_r] & :[(d'):\delta'];\ldots;\end{matrix}$$

$$\left.\begin{matrix}[(b^{(r)}):\phi^{(r)}];\\ [(d^{(r)}):\delta^{(r)}];\end{matrix}\ z_1,\ldots,z_r\right), \qquad \ldots (3.2)$$

which holds under the same conditions obtainable from (1.1).

(iv) We replace ν by $\nu+2$ and take $\nu = \sigma = 2$, also let $\alpha_1 = \alpha'$, $\alpha_2 = \alpha''$, $\alpha_3 = \alpha+1$, $\alpha_4 = \beta+1 = \beta_2$ and choose $\gamma_i = 1 = \mu_j$ where $i = 1, \ldots, \nu+2$ and $j = 1, \ldots, \sigma$ in equation (3.2.) then it reduces to a known result obtained by G.S. Olkha and V.B.L. Chaurasia [3].

(v) Now putting $\lambda = A = C = 0$ in (1.1), we derive the following result involving the product of r mutually independent H-functions

$$\int_0^1 x^{\varepsilon-1}(1-x)^{\beta}\, F^{\nu:A';\ldots;A^{(s)};0;0}_{\sigma:B';\ldots;B^{(s)};1;1}\left(\begin{matrix}[(\alpha_\nu):\eta';\ldots;\eta^{(s)},\gamma,\gamma]:[(l'):\rho'];\ldots;\\ [(\beta_\sigma):\xi';\ldots;\xi^{(s)},\mu,\mu]:[(m'):\Im'];\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(l^{(s)}):\rho^{(s)}];\ldots;\ldots;\\ [(m^{(s)}):\Im^{(s)}];[\alpha+1:1];[\beta+1:1];\end{matrix}\ z'_1,\ldots,z'_r,\ldots,z'_s,-xt,(1-x)t\right)$$

$$S^{m_1;\ldots;m_R}_{n_1;\ldots;n_R}(y_1x^{k_1};\ldots;y_Rx^{k_R}).\ \sum_{i=1}^{r} H^{u^{(i)},v^{(i)}}_{B^{(i)},D^{(i)}}[z_i x^{h_i}]\,dx$$

$$= \sum_{n=0}^{\infty} \frac{\prod_{j=0}^{\nu}(\alpha_j)_{n\gamma_j}}{(\alpha+1)_n(\beta+1)_n\prod_{j=0}^{\sigma}(\beta_j)_{n\mu_j}}\ \frac{(-t)^n\,\Gamma(\beta+n+1)}{n!}$$

$$F^{\nu:A';\ldots;A^{(s)};\,0;0}_{\sigma:B';\ldots;B^{(s)};\,1;1}\left(\begin{matrix}[(\alpha_\nu+n\gamma_\nu):\eta';\ldots;\eta^{(s)}]:[(l'):\rho'];\ldots;[(l^{(s)}):\rho^{(s)}];\\ [(\beta_\sigma+n\mu_\sigma):\xi';\ldots;\xi^{(s)}]:[(m'):\Im'];\ldots;[(m^{(s)}):\Im^{(s)}];\end{matrix}\; z'_1,\ldots,z'_s\right)\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_R=0}^{[n_R/m_R]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_R)_{m_R\alpha_R}}{\alpha_R!}$$

$$A_{n_1\alpha_1}\cdots A_{n_R\alpha_R}\,y_1^{\alpha_1}\cdots y_R^{\alpha_R}\;H^{0,\;2:(u',v');\ldots;(u^{(r)},v^{(r)})}_{2,\;2:(B',D');\ldots;(B^{(r)},D^{(r)})}$$

$$\left(\begin{matrix}[1-\varepsilon-k_1\alpha_1-\ldots-k_R\alpha_R:h_1,\ldots,h_r]\;,[1-\varepsilon+\alpha-k_1\alpha_1-\ldots-k_R\alpha_R:h_1,\ldots,h_r]\\ [(c):\psi';\ldots;\psi^{(r)}]\;,[1-\varepsilon+\alpha+n-k_1\alpha_1-\ldots-k_R\alpha_R:h_1,\ldots,h_r]\end{matrix}\right.$$

$$\begin{matrix}[(a):\theta';\ldots,\theta^{(r)}]\;:[(b'):\phi'];\ldots;\\ [-\beta-\varepsilon-n-k_1\alpha_1-\ldots-k_R\alpha_R:h_1,\ldots,h_r]:[(d'):\delta'];\ldots;\end{matrix}$$

$$\left.\begin{matrix}[(b^{(r)}):\phi^{(r)}];\\ [(d^{(r)}):\delta^{(r)}];\end{matrix}\; z_1,\ldots,z_r\right), \qquad \ldots (3.3)$$

valid under the same conditions obtainable in (1.1).

(vi) We take $r = 1$ in equation (3.3) and get the following result

$$\int_0^1 x^{\varepsilon-1}(1-x)^\beta\, F^{\nu:A';\ldots;A^{(s)};\,0;0}_{\sigma:B';\ldots;B^{(s)};\,1;1}\left(\begin{matrix}[(\alpha_\nu):\eta';\ldots;\eta^{(s)},\gamma,\gamma]:[(l'):\rho'];\ldots;\\ [(\beta_\sigma):\xi';\ldots;\xi^{(s)},\mu,\mu]:[(m'):\Im'];\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(l^{(s)}):\rho^{(s)}];\ldots;\ldots;\\ [(m^{(s)}):\Im^{(s)}];[\alpha+1:1];[\beta+1:1];\end{matrix}\; z'_1,\ldots,z'_r,\ldots,z'_s,-xt,(1-x)t\right)$$

$$S^{m_1;\ldots;m_r}_{n_1;\ldots;n_R}\,(y_1x^{k_1};\ldots;y_R\,x^{k_R}).\quad H^{u',\,v'}_{B',\,D'}\,[z_i\,x^{h_i}]\,dx$$

$$=\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{\nu}(\alpha_j)_{n\gamma_j}}{(\alpha+1)_n\,(\beta+1)_n\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}}\;F^{\nu:A';\ldots;A^{(s)};}_{\sigma:B';\ldots;B^{(s)};}$$

$$\left(\begin{matrix}[(\alpha_\nu+n\gamma_\nu):\eta';\ldots;\eta^{(s)}]:[(l'):\rho'];\ldots;[(l^{(s)}):\rho^{(s)}];\\ [(\beta_\sigma+n\mu_\sigma):\xi';\ldots;\xi^{(s)}]:[(m'):\Im'];\ldots;[(m^{(s)}):\Im^{(s)}];\end{matrix}\; z'_1,\ldots,z'_s\right)t^n$$

$$\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_R=0}^{[n_R/m_R]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_R)_{m_R\alpha_R}}{\alpha_R!}\;A_{n_1\alpha_1}\cdots A_{n_R\alpha_R}\,y_1^{\alpha_1}\cdots y_R^{\alpha_R}$$

$$H^{u', v'+2}_{B'+2, D'+2}\left[z_1\left|\begin{matrix}(1-\varepsilon-k_1\alpha_1-\ldots-k_R\alpha_R:h_1),[1-\varepsilon+\alpha-k_1\alpha_1-\ldots-k_R\alpha_R:h_1) & (b'_B,\phi'_B)\\ (d'_D,\delta'_D)\quad,\quad(1-\varepsilon+\alpha+n-k_1\alpha_1-\ldots-k_R\alpha_R:h_1) & (-\varepsilon-\beta-n-k_1\alpha_1-\ldots-k_R\alpha_R:h_1)\end{matrix}\right.\right], \quad \ldots (3.4)$$

where Re $(\beta) > -1, h_1 > 0$ Re $(\varepsilon + h_1 \frac{d_j^{(i)}}{\delta_j^{(i)}}) > 0, j = 1, \ldots, u'$;

$T_1 > 0$, $|t| < 1$, $|\arg z_1| < T_1\pi/2$ and the series on the right is convergent.

Here $(T_1 = \sum_{i=1}^{v'} \phi'_1 - \sum_{i=v'+1}^{B'} \phi'_i + \sum_{i=1}^{u'} \delta'_i - \sum_{i=u'+1}^{D'} \delta'_i)$.

(vii) Now by giving the suitable values to the parameters and letting $\alpha_1 = \alpha'$, $\alpha_2 = \alpha''$, $\gamma_1 = \gamma_2 = 1$ in (3.4), we get a known result obtained by Chaurasia [1].

(viii) Lettig $n'_1 = n'_2 = \ldots = 0 = n'_R$ in (1.1), we get a known result recently obtained by Chaurasia and Tyagi [2].

(ix) For a general class of polynomial, we take the case of Hermite polynomial [8,9] by setting $S^2_{n'_1} = x^{n'_1/2} H_{n'_1} \alpha [1/(2\sqrt{x})]$ in wich case, $m'_1 = 2$, $A_{n'_1 s'} = (-1)^{s'}$, we obtain from (1.1)

$$\int_0^1 x^{\varepsilon-1}(1-x)^{\beta} F^{\nu:A';\ldots;A^{(s)};0;0}_{\sigma:B';\ldots;B^{(s)};1;1}\left(\begin{matrix}[(\alpha_\nu):\eta';\ldots;\eta^{(s)},\gamma,\gamma]:[(l'):\rho'];\ldots;[(l^{(s)}):\rho^{(s)}];\ldots;\ldots;\\ [(\beta_\sigma):\xi';\ldots;\xi^{(s)},\mu,\mu]:[(m'):\Im'];\ldots;[(m^{(s)}):\Im^{(s)}];[\alpha+1:1];[\beta+1:1];\end{matrix}\; z'_1,\ldots,z'_r,\ldots,z'_s,-xt,(1-x)t\right)$$

$$\left[(y_1x^{k_1})^{n'_1/2} H_{n'_1}\left(\frac{1}{2\sqrt{y_1x^{k_1}}}\right)\right] H(z_1x^{h_1},\ldots,z_rx^{h_r})\,dx$$

$$= \sum_{n=0}^{\infty} \frac{\prod_{j=0}^{\nu}(\alpha_j)_{n\gamma_j}}{(\alpha+1)_n(\beta+1)_n\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}}$$

$$F^{\nu:A';\ldots;A^{(s)};0;0}_{\sigma:B';\ldots;B^{(s)};1;1}\left(\begin{matrix}[(\alpha_\nu+n\gamma_\nu):\eta';\ldots;\eta^{(s)}]:[(l'):\rho'];\ldots;[(l^{(s)}):\rho^{(s)}];\\ [(\beta_\sigma+n\mu_\sigma):\xi';\ldots;\xi^{(s)}]:[(m'):\Im'];\ldots;[(m^{(s)}):\Im^{(s)}];\end{matrix}\; z'_1,\ldots,z_s\right)t^n \sum_{s'=0}^{n'_1/2}\frac{(-1)^{s'}n'_1}{s'!\,(n'_1-2s')!}y_1^{s'}$$

$$H^{0,\,\lambda+2\,:(u',\,v');\ldots;(u^{(r)},\,v^{(r)})}_{A+2,\,C+2\,:(B',\,D');\ldots;(B^{(r)},\,D^{(r)})}\left(\begin{matrix}[1-\varepsilon-k_1s' : h_1,\ldots,h_r]\ ,\ [1-\varepsilon-k_1s' : h_1,\ldots,h_r] \\ [(c):\psi';\ldots;\psi^{(r)}]\ ,\ [1+\alpha-\varepsilon+n-k_1s' : h_1,\ldots,h_r]\end{matrix}\right.$$

$$\left.\begin{matrix}[(a):\theta';\ldots;\theta^{(r)}] : [(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]; \\ [-\beta-\varepsilon-n-k_1s' : h_1,\ldots,h_r] : [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];\end{matrix}\ z_1,\ldots,z_r\right), \quad (3.5)$$

which holds under the same conditions as those required for (1.1).

(x) For the Laguerre polynomials [8,9]. setting $s_{n'_1}(x) = L^{Q}_{n'_1}(x)$ in which case $m'_1 = 1$ and $A_{n'_1,s'} = \binom{n'_1+Q}{n'_1}\left(\frac{1}{(Q+1)_K}\right)$, we obtain (1.1) in the following form

$$\int_0^1 x^{\varepsilon-1}(1-x)^{\beta} F^{\nu\,:A';\ldots;A^{(s)};\,0;0}_{\sigma\,:B';\ldots;B^{(s)};\,1;1}\left(\begin{matrix}[(\alpha_\nu):\eta';\ldots;\eta^{(s)},\gamma,\gamma] : [(l'):\rho'];\ldots;[(l^{(s)}):\rho^{(s)}];\ldots; \\ [(\beta_\sigma):\xi';\ldots;\xi^{(s)},\mu,\mu] : [(m'):\Im'];\ldots;[(m^{(s)}):\Im^{(s)}];[\alpha+1:1];[\beta+1:1];\end{matrix}\ z'_1,\ldots,z'_r,\ldots,z'_s,-xt,(1-x)t\right)$$

$$L^{Q}_{n'_1}(y_1x^{k_1}).\,H(z_1x^{h_1};\ldots;z_rx^{h_r})\,dx$$

$$= \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{\nu}(\alpha_j)_{n\gamma_j}}{(\alpha+1)_n(\beta+1)_n\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}}$$

$$F^{\nu\,:A';\ldots;A^{(s)}}_{\sigma\,:B';\ldots;B^{(s)}}\left(\begin{matrix}[(\alpha_\nu+n\gamma_\nu):\eta';\ldots;\eta^{(s)}] : [(l'):\rho'];\ldots;[(l^{(s)}):\rho^{(s)}]; \\ [(\beta_\sigma+n\mu_\sigma):\xi';\ldots;\xi^{(s)}] : [(m'):\Im'];\ldots;[(m^{(s)}):\Im^{(s)}];\end{matrix}\ z'_1,\ldots,z'_s\right) t^n$$

$$\sum_{s'=0}^{n'_1}(-1)^{s'}\binom{n'_1+Q}{n'_1-s'}(y_1)^{s'}$$

$$H^{0,\,\lambda+2\,:(u',\,v');\ldots;(u^{(r)},\,v^{(r)})}_{A+2,\,C+2\,:(B',\,D');\ldots;(B^{(r)},\,D^{(r)})}\left(\begin{matrix}[1-\varepsilon-k_1s' : h_1,\ldots,h_r]\ ,\ [1-\varepsilon+\alpha-k_1s' : h_1,\ldots,h_r], \\ [(c):\psi';\ldots;\psi^{(r)}]\ ,\ [1-\varepsilon+\alpha-k_1s' : h_1,\ldots,h_r],\end{matrix}\right.$$

$$\left(\begin{matrix}[(a):\theta';...,\theta^{(r)}] & :[(b'):\phi'];...;[(b^{(r)}):\phi^{(r)}]; \\ [-\beta-\varepsilon-n-k_1s':h_1,...,h_r] & :[(d'):\delta'];...;[(d^{(r)}):\delta^{(r)}]; \end{matrix} z_1,...,z_r\right), \quad ...(3.6)$$

valid under the same conditions as given in equation (1.1).

By assigning suitable values to the parameters, generalized Lauricella's function and the multivariable H-function may be transformed into Appell's functions, Bessel functions, E-functions, G-functions, Kampé de Fériet functions, Laurucella's functions and other higher transcendental functions in one or more arguments. Thus, we can find the integral transformations for various other functions of one or more variables as special cases of our result.

ACKNOWLEDGEMENT

The authors are highly thankful to Professor H.M. Srivastava of the University of Victoria, Victoria, Canada, for his kind help and many valuable suggestions in the preparation and improvement of this paper in the present form.

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