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SOME MORE INEQUALITIES INVOLVING FOX'S *H*-FUNCTION

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ABSTRACT

In the present paper, making an appeal to the results due to Jaimini and Koul (1993). We obtain some more inequalities involving Fox's *H*-function.

1. Introduction. Recently, Jaimini and Koul [2] established six inequalities involving Fox's *H*-function by employing three inequalities established by Koti [3]. Here, in the present paper, making an appeal to the results due Jaimini and Koul [2], we obtain by some more inequalities involving Fox's *H*-function.

The Fox's *H*-function of one variable is defined as :

$$H_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_p,\alpha_p)\\(b_q,\beta_q)\end{matrix}\right.\right]=\frac{1}{2\pi w}\int_L \theta(s)\,z^s\,ds,\ w=\sqrt{(-1)},$$

where

$$\theta(s)=\frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j-\alpha_j s)}{\prod_{j=m+1}^{q}\Gamma(1-b_j+\beta_j s)\prod_{j=n+1}^{q}\Gamma(a_j-\alpha_j s)}.$$

For further details, esistence convergence conditions of $H_{p,q}^{m,n}[.]$, we may refer to Srivastava et. al. [5, pp. 10-13].

2. Formulae Required.

The following integrals are required in our investigatons :

The integral due to Bajpai [1.p. 18 (2.8] is

$$\int_0^\infty x^{\varepsilon-1}J_u(x)^\rho H_{p,q}^{m,n}\left[z\,x^{2h}\left|\begin{matrix}(a_p,\alpha_p)\\(b_q,\beta_q)\end{matrix}\right.\right]dx$$

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$$= 2^{\varepsilon-1} H^{m,n+1}_{p+2,\, q}\left[2^{2h}z \left| \begin{matrix} [(2-\varepsilon-u)/2,\, h)\, ,(a_p\, ,\alpha_p)\, ,\, [(2-\varepsilon+u)/2,\, h) \\ (b_q\, ,\beta_q) \end{matrix}\right.\right] \quad \ldots (2.1)$$

where the conditions (1.1), $h > 0$, $Re\,(\varepsilon+u)+2h \min_{i \le j \le m} [Re\, b_j/\beta_j] > 0$ and $Re\,\varepsilon + 2h \max_{i \le j \le n} [Re\,(a_j-1)/\alpha_j] < 0$ are satisfied.

The another integral due to Taxak [7] is

$$\int_0^\infty y^{\delta-1} \cos y\, J_\upsilon(y)\, H^{m,n}_{p,\, q}\left[z.y^{2l} \left| \begin{matrix} (a_p\, ,\alpha_p) \\ (b_q\, ,\beta_q) \end{matrix}\right.\right] dy$$

$$= 2^{\delta-1}\sqrt{\pi}\, H^{m+1,n+1}_{p+4,\, q+1}\left[2^{2l}z \left| \begin{matrix} (1-(\delta+\upsilon)/2,\, l)\, ,(a_p\, ,\alpha_p)\, ,\, (1+(\upsilon-\delta)/2,\, l), \\ (b_q\, ,\beta_q)\, , \end{matrix}\right.\right.$$

$$\left.\left. \begin{matrix} [(1-\upsilon-\delta)/2,\, l]\, ,\, [(1+\upsilon-\delta)/2,\, l) \\ (1/2-\delta\, ,2l) \end{matrix}\right.\right] , \quad \ldots (2.2)$$

where $l > 0$, $Re\,(\delta+\upsilon+2l) \min_{i \le j \le m} Re\,[\, b_j/\beta_j] > 0$ and rest conditions of (1.1) are satisfied.

3. The main inequalities.

(I) In this section, we establish the main inequalities, for which we multiply both sides by $x^{\varepsilon-1} J_u(x)$ in inequalities due to Jamini and Koul [2, (3.1) to (3.6)], and replace z by zx^{2h} then, integrate both sides each with respect to x from 0 to ∞ ; and made the use of (2.1), we obtain the following inequalities respectively :

$$\sum_{j=0}^{k} \binom{2k}{2j} H^{3,3}_{5,3}\left[2^{2h}z \left| \begin{matrix} (1-\rho,\, \mu),(1-\nu,1)(1+\beta+2k-2j-\rho,\mu),(\frac{2-\varepsilon-u}{2},h),(\frac{2-\varepsilon+u}{2},h) \\ (0,1),\ (\beta+2k-2j-\rho,\mu),\ (\alpha+\beta+2k-\rho,\ \mu) \end{matrix}\right.\right]$$

$$\ge \sum_{j=0}^{k-1} \binom{2k}{2j+1} H^{3,3}_{5,3}\left[2^{2h}z \left| \begin{matrix} (1-\rho,\mu),(1-\nu,1),(\beta+2k-2j-\rho,\mu),(\frac{2-\varepsilon-u}{2},h),(\frac{2-\varepsilon+u}{2},h) \\ (0,1),\ (\beta+2k-2j-\rho-1,\ \mu),\ (\alpha+\beta+2k-\rho,\ \mu) \end{matrix}\right.\right] \quad \ldots (3.1)$$

$$\sum_{j=0}^{[k'/2]} \binom{k'}{2j+1} H^{3,3}_{5,3}\left[2^{2h}z \left| \begin{matrix} (1-\rho,\, \mu),\ (1-\nu,1),(\beta+k'-2j-\rho,\mu),(\frac{2-\varepsilon-u}{2},h),(\frac{2-\varepsilon+u}{2},h) \\ (0,1),\ (\beta+k'-2j-\rho-1,\ \mu),\ (\alpha+\beta+k'-\rho,\ \mu) \end{matrix}\right.\right]$$

$$\ge \sum_{j=0}^{[k'/2]} \binom{k'}{2j} H^{3,3}_{5,3}\left[2^{2h}z \left| \begin{matrix} (1-\rho,\, \mu),\ (1-\nu,1)\ (1-\beta+k'-2j-\rho,\mu),(\frac{2-\varepsilon-u}{2},h),(\frac{2-\varepsilon+u}{2},h) \\ (0,1),\ (\beta+k'-2j-\rho-1,\ \mu),\ (\alpha+\beta+k'-\rho,\ \mu) \end{matrix}\right.\right] \quad \ldots (3.2)$$

$$\sum_{j=1}^{k} \frac{(-2k)_{2j}}{(2j)\,!} H^{3,3}_{5,3}\left[2^{2h}z \left| \begin{matrix} (1-\rho,\, \mu),\ (1-\nu,1)\ (1+\beta+2j-\rho,\mu),(\frac{2-\varepsilon-u}{2},h),(\frac{2-\varepsilon+u}{2},h) \\ (0,1),\ (\beta+2j-\rho,\mu),\ (\alpha+\beta+m-\rho,\ \mu) \end{matrix}\right.\right]$$

$$\ge \sum_{j=0}^{k-1} \frac{(-2k)_{2j+1}}{(2j+1)\,!} H^{3,3}_{5,3}\left[2^{2h}z \left| \begin{matrix} (1-\rho,\, \mu),\ (1-\nu,1)\ (2+\beta+2\delta-\rho,\mu),(\frac{2-\varepsilon-u}{2},h),(\frac{2-\varepsilon+u}{2},h) \\ (0,1),\ (1+\beta+2j-\rho,\mu),\ (\alpha+\beta+m-\rho,\ \mu) \end{matrix}\right.\right] \quad \ldots (3.3)$$

$$\sum_{j=1}^{[k'/2]} \frac{(-k')_{2j}}{(2j)!} H^{3,3}_{5,3}\left[2^{2h}z\left|\begin{matrix}(1-\rho,\mu),(1-\nu,1),(1+\beta+2j-\rho,\mu),(\frac{2-\varepsilon-u}{2},h),(\frac{2-\varepsilon+u}{2},h)\\(0,1),(\beta+2j-\rho,\mu),(\alpha+\beta+m-\rho,\mu)\end{matrix}\right.\right]$$

$$\geq \sum_{j=0}^{[k'/2]} \frac{(-k')_{2j+1}}{(2j+1)!} H^{3,3}_{5,3}\left[2^{2h}z\left|\begin{matrix}(1-\rho,\mu),(1-\nu,1)(2+\beta+2j-\rho,\mu),(\frac{2-\varepsilon-u}{2},h),(\frac{2-\varepsilon+u}{2},h)\\(0,1),(1+\beta+2j-\rho,\mu),(\alpha+\beta+m-\rho,\mu)\end{matrix}\right.\right] \quad \ldots (3.4)$$

$$\sum_{j=0}^{k}\sum_{r=0}^{n}\binom{2k}{2j}\frac{(-n)_r(\gamma-\alpha)_r}{r!(\gamma-2k-n)_r(\gamma-n-2j+r)_\lambda}$$

$$H^{1,3}_{4,2}\left[2^{2h}z\left|\begin{matrix}(1-\gamma-\lambda+n+2j-r,\mu),(1-\gamma,1),(\frac{2-\varepsilon-u}{2},h),(\frac{2-\varepsilon+u}{2},h)\\(0,1),(1-\nu+n+2j-\sigma-r,\mu),\end{matrix}\right.\right]$$

$$\geq \sum_{j=0}^{k-1}\sum_{r=0}^{n}\binom{2k}{2j+1}\frac{(-n)_r(\gamma-\alpha)_r}{r!(\gamma-2k-n)_r(\gamma-n-2j-1+r)_\lambda}$$

$$.H^{1,3}_{4,2}\left[2^{2h}z\left|\begin{matrix}(2-\gamma-\lambda+n+2j-r,\mu),(1-\nu,1),(\frac{2-\varepsilon-u}{2},h),(\frac{2-\varepsilon+u}{2},h)\\(0,1),(2-\gamma-\lambda+n+2j-\sigma-r,\mu),\end{matrix}\right.\right], \gamma>\alpha>n+2k; \quad \ldots (3.5)$$

$$\sum_{j=0}^{k'/2}\sum_{r=0}^{n}\binom{k'}{2j+1}\frac{(-n)_r(\gamma-\alpha)_r}{r!(\gamma-k'-n)_r(\gamma-n-2j+r-1)_\lambda}$$

$$.H^{1,3}_{4,2}\left[2^{2h}z\left|\begin{matrix}(2-\gamma-\lambda+n+2j-r,\mu),(1-\nu,1),(\frac{2-\varepsilon-u}{2},h),(\frac{2-\varepsilon+u}{2},h)\\(0,1),(2-\gamma-\lambda+n+2j-\sigma-r,\mu),\end{matrix}\right.\right]$$

$$\geq \sum_{j=0}^{k'/2}\sum_{r=0}^{n}\binom{k'}{2j}\frac{(-n)_r(\gamma-\alpha)_r}{r!(\gamma-k'-n)_r(\gamma-n-2j+r)_\lambda}$$

$$.H^{1,3}_{4,2}\left[2^{2h}z\left|\begin{matrix}(1-\gamma-\lambda+n+2j-r,\mu),(1-\nu,1),(\frac{2-\varepsilon-u}{2},h),(\frac{2-\varepsilon+u}{2},h)\\(0,1),(1-\gamma-\lambda+n+2j-\sigma-r,\mu),\end{matrix}\right.\right], \gamma>\alpha>n+k' \quad \ldots (3.6)$$

provided that all the conditions [2, (3.1) to (3.6)] and (2.1) are satisfied.

(II) To obtain other inequalities, in this part of the section, we multiply both sides of [2, (3.1) to (3.6)] by $y^{\delta-1} \cos y\ J_\upsilon(y)$, replace z by zy^{2l}, and integrate both sides with respect to y from 0 to ∞ and then using the result (2.2), we obtain some more following inequalities :

$$\sum_{j=0}^{k}\binom{2k}{2j} H^{4,3}_{7,4}\left[2^{2l}z\left|\begin{matrix}(1-\rho,\mu),(1-\nu,1)(1+\beta+2k-2j-\rho,\mu),\\(1/2-\delta,2l)(0,1),(\beta+2k-2j-\rho,\mu),\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(\frac{2-\delta-\upsilon}{2},l),(\frac{2+\upsilon-\delta}{2},l)(\frac{1-\upsilon-\delta}{2},l),(\frac{1+\upsilon-\delta}{2},l)\\(\alpha+\beta+2k-\rho,\mu)\end{matrix}\right]$$

$$\geq \sum_{j=0}^{k-1}\binom{2k}{2j+1} H^{4,3}_{7,4}\left[2^{2l}z\left|\begin{matrix}(1-\rho,\mu),(1-\nu,1)(\beta+2k-2j-\rho,\mu),\\(1/2-\delta,2l)(0,1),(\beta+2k-2j-\rho,\mu),\end{matrix}\right.\right.$$

$$\left. \begin{matrix} \left(\frac{2-\delta-\upsilon}{2}, l\right), \left(\frac{2+\upsilon-\delta}{2}, l\right) \left(\frac{1-\upsilon-\delta}{2}, l\right), \left(\frac{1+\upsilon-\delta}{2}, l\right) \\ (\alpha+\beta+2k-\rho, \mu) \end{matrix} \right] \quad \dots (3.7)$$

$$\sum_{j=0}^{[k'/2]} \binom{k'}{2j+1} H_{7,4}^{4,3}\left[2^{2l}z \middle| \begin{matrix} (1-\rho, \mu),(1-\nu,1)(\beta+k'-2j-\rho,\mu), \left(\frac{2-\delta-\upsilon}{2}, l\right), \left(\frac{2+\upsilon-\delta}{2}, l\right) \left(\frac{1-\upsilon-\delta}{2}, l\right), \left(\frac{1+\upsilon-\delta}{2}, l\right) \\ (1/2-\delta, 2l)\,(0,1), (\beta+2k-2j-\rho,\mu), (\alpha+\beta+2k-\rho, \mu) \end{matrix} \right]$$

$$\geq \sum_{j=0}^{[k'/2]} \binom{k'}{2j} H_{7,4}^{4,3}\left[2^{2l}z \middle| \begin{matrix} (1-\rho, \mu),(1-\nu,1),(1+\beta+k'-2j-\rho,\mu), \left(\frac{2-\delta-\upsilon}{2}, l\right), \left(\frac{2+\upsilon-\delta}{2}, l\right) \left(\frac{1-\upsilon-\delta}{2}, l\right), \left(\frac{1+\upsilon-\delta}{2}, l\right) \\ (1/2-\delta, 2l)\,(0,1), (\beta+k'-2j-\rho,\mu), (\alpha+\beta+k'-\rho, \mu) \end{matrix} \right] \quad \dots (3.8)$$

$$\sum_{j=1}^{k} \frac{(-2k)_{2j}}{(2j)!} H_{7,4}^{4,3}\left[2^{2l}z \middle| \begin{matrix} (1-\rho, \mu),(1-\nu,1)(1+\beta+2j-\rho,\mu), \left(\frac{2-\delta-\upsilon}{2}, l\right), \left(\frac{2+\upsilon-\delta}{2}, l\right) \left(\frac{1-\upsilon-\delta}{2}, l\right), \left(\frac{1+\upsilon-\delta}{2}, l\right) \\ (1/2-\delta, 2l)\,(0,1), (\beta+2j-\rho,\mu), (\alpha+\beta+m-\rho, \mu) \end{matrix} \right]$$

$$\geq \sum_{j=0}^{k-1} \frac{(-2k)_{2j+1}}{(2j+1)!} H_{7,4}^{4,3}\left[2^{2l}z \middle| \begin{matrix} (1-\rho, \mu),(1-\nu,1)(2+\beta+2j-\rho,\mu), \left(\frac{2-\delta-\upsilon}{2}, l\right), \left(\frac{2+\upsilon-\delta}{2}, l\right) \left(\frac{1-\upsilon-\delta}{2}, l\right), \left(\frac{1+\upsilon-\delta}{2}, l\right) \\ (1/2-\delta, 2l)\,(0,1), (1+\beta+2j-\rho,\mu), (\alpha+\beta+m-\rho, \mu) \end{matrix} \right] \quad \dots (3.9)$$

$$\sum_{j=1}^{k'/2} \frac{(-2k')_{2j}}{(2j)!} H_{7,4}^{4,3}\left[2^{2l}z \middle| \begin{matrix} (1-\rho, \mu),(1-\nu,1)(1+\beta+2j-\rho,\mu) \left(\frac{2-\delta-\upsilon}{2}, l\right), \left(\frac{2+\upsilon-\delta}{2}, l\right) \left(\frac{1-\upsilon-\delta}{2}, l\right), \left(\frac{1+\upsilon-\delta}{2}, l\right) \\ (1/2-\delta, 2l)\,(0,1), (\beta+2j-\rho,\mu), (\alpha+\beta+m-\rho, \mu) \end{matrix} \right]$$

$$\geq \sum_{j=0}^{k'/2} \frac{(-k')_{2j+1}}{(2j+1)!} H_{7,4}^{4,3}\left[2^{2l}z \middle| \begin{matrix} (1-\rho, \mu),(1-\nu,1)(2+\beta+2j-\rho,\mu), \left(\frac{2-\delta-\upsilon}{2}, l\right), \left(\frac{2+\upsilon-\delta}{2}, l\right) \left(\frac{1-\upsilon-\delta}{2}, l\right), \left(\frac{1+\upsilon-\delta}{2}, l\right) \\ (1/2-\delta, 2l)\,(0,1), (\beta+2j-\rho,\mu), (\alpha+\beta+m-\rho, \mu) \end{matrix} \right] \quad \dots (3.10)$$

$$\sum_{j=0}^{k} \sum_{r=0}^{n} \binom{2k}{2j} \frac{(-n)_r\,(\gamma-\alpha)_r}{r!\,(\gamma-2k-n)_r\,(\gamma-n-2j+r)_\lambda} . H_{6,3}^{2,3}\left[2^{2l}z \middle| \begin{matrix} (1-\gamma-\lambda+n+2j-r, \mu), \\ (1/2-\delta, 2l), \end{matrix} \right.$$

$$(1-\gamma,1),(\frac{2-\delta-\upsilon}{2},l),(\frac{2+\upsilon-\delta}{2},l),(\frac{1-\upsilon-\delta}{2},l),(\frac{1+\upsilon-\delta}{2},l) \\ (0,1),\ (1-\nu+n+2j-\sigma-r,\mu), \Big]$$

$$\geq \sum_{j=0}^{k-1} \sum_{r=0}^{n} \binom{2k}{2j+1} \frac{(-n)_r\,(\gamma-\alpha)_r}{r!\,(\gamma-2k-n)_r\,(\gamma-n-2j-1+r)_\lambda} H^{2,3}_{6,3}\left[2^{2l}z \middle| \begin{matrix}(2-\gamma-\lambda+n+2j-r,\mu),\ (1-\nu,1),(\frac{2-\delta-\upsilon}{2},l),(\frac{2+\upsilon-\delta}{2},l),(\frac{1-\upsilon-\delta}{2},l),(\frac{1+\upsilon-\delta}{2},l) \\ (1/2-\delta,2l),\ (0,1),\ (2-\gamma-\lambda+n+2j-\sigma-r,\mu)\end{matrix}\right],\ \gamma>\alpha>n+2k\ ;\ldots\ (3.11)$$

$$\sum_{j=0}^{k'/2} \sum_{r=0}^{n} \binom{k'}{2j+1} \frac{(-n)_r\,(\gamma-\alpha)_r}{r!\,(\gamma-k'-n)_r\,(\gamma-n-2j+r-1)_\lambda} H^{2,3}_{6,3}\left[2^{2l}z \middle| \begin{matrix}(2-\gamma-\lambda+n+2j-r,\mu),\ (1-\nu,1),(\frac{2-\delta-\upsilon}{2},l),(\frac{2+\upsilon-\delta}{2},l),(\frac{1-\upsilon-\delta}{2},l),(\frac{1+\upsilon-\delta}{2},l) \\ (1/2-\delta,2l),\ (0,1),\ (1-\gamma-\lambda+n+2j-\sigma-r,\mu)\end{matrix}\right]$$

$$\geq \sum_{j=0}^{k'/2} \sum_{r=0}^{n} \binom{k'}{2j} \frac{(-n)_r\,(\gamma-\alpha)_r}{r!\,(\gamma-k'-n)_r\,(\gamma-n-2j+r)_\lambda} H^{2,3}_{6,3}\left[2^{2l}z \middle| \begin{matrix}(1-\gamma-\lambda+n+2j-r,\mu),\ (1-\nu,1)(\frac{2-\delta-\upsilon}{2},l),(\frac{2+\upsilon-\delta}{2},l)\ (\frac{1-\upsilon-\delta}{2},l),(\frac{1+\upsilon-\delta}{2},l) \\ (1/2-\delta,2l),\ (0,1),\ (1-\gamma-\lambda+n+2j-\sigma-r,\mu)\end{matrix}\right],\ \gamma>\alpha>n+k' \quad \ldots\ (3.12)$$

provided that all canditions of [2, (3.1) to (3.6)] and (2.2) are satisfied.

4. Special Cases. Setting $\mu=h=1$ in (3.1), (3.2), ... (3.6) respectively, we obtain the following inequalities involving Meijer G-function [4] provided that existence and converqence conditions stated in section (3) (I) are satisfied :

$$\sum_{j=0}^{k} \binom{2k}{2j} G^{3,3}_{5,3}\left[4z \middle| \begin{matrix}(1-\rho,\ 1-\nu,\ 1+\beta+2k-2j-\rho,\ (2-\varepsilon-u)/2,\ (2-\varepsilon+u)/2 \\ 0,\ \beta+2k-2j-\rho,\ \alpha+\beta+2k-\rho\end{matrix}\right]$$

$$\geq \sum_{j=0}^{k-1} \binom{2k}{2j+1} G^{3,3}_{5,3}\left[4z \middle| \begin{matrix}(1-\rho,\ 1-\nu,\ \beta+2k-2j-\rho,\ (2-\varepsilon-u)/2,\ (2-\varepsilon+u)/2 \\ 0,\ \beta+2k-2j-\rho-1,\ \alpha+\beta+2k-\rho\end{matrix}\right] \quad \ldots\ (4.1)$$

$$\sum_{j=0}^{k'/2} \binom{k'}{2j+1} G^{3,3}_{5,3}\left[4z \middle| \begin{matrix}1-\rho,\ 1-\nu,\ \beta+k'-2j-\rho,\ (2-\varepsilon-u)/2,\ (2-\varepsilon+u)/2 \\ 0,\ \beta+k'-2j-\rho-1,\ \alpha+\beta+k'-\rho\end{matrix}\right]$$

$$\geq \sum_{j=0}^{k'/2} \binom{k'}{2j} G^{3,3}_{5,3}\left[4z \middle| \begin{matrix}1-\rho,\ 1-\nu,\ 1+\beta+k'-2j-\rho,\ (2-\varepsilon-u)/2,\ (2-\varepsilon+u)/2 \\ 0,\ \beta+k'-2j-\rho-1,\ \alpha+\beta+k'-\rho\end{matrix}\right] \quad \ldots\ (4.2)$$

$$\sum_{j=1}^{k} \frac{(-2k)_{2j}}{(2j)!} G^{3,3}_{5,3}\left[4z \middle| \begin{matrix}1-\rho,\ 1-\nu,\ 1+\beta+2j-\rho,\ (2-\varepsilon-u)/2,\ (2-\varepsilon+u)/2 \\ 0,\ \beta+2j-\rho-1,\ \alpha+\beta+m-\rho\end{matrix}\right]$$

$$\geq \sum_{j=0}^{k-1} \frac{(-2k)_{2j+1}}{(2j+1)!} G^{3,3}_{5,3}\left[4z \middle| \begin{matrix} 1-\rho,\ 1-\nu,\ 2+\beta+2j-\rho,\ (2-\varepsilon-u)/2,(2-\varepsilon+u)/2 \\ 0,\ 1+\beta+2j-\rho,\ \alpha+\beta+m-\rho \end{matrix}\right] \quad \ldots (4.3)$$

$$\sum_{j=1}^{[k'/2]} \frac{(-k')_{2j}}{(2j)!} G^{3,3}_{5,3}\left[4z \middle| \begin{matrix} 1-\rho,\ 1-\nu,\ 1+\beta+2j-\rho,\ (2-\varepsilon-u)/2,(2-\varepsilon+u)/2 \\ 0,\ \beta+2j-\rho,\ \alpha+\beta+m-\rho \end{matrix}\right]$$

$$\geq \sum_{j=0}^{[k'/2]} \frac{(-k')_{2j+1}}{(2j+1)!} G^{3,3}_{5,3}\left[4z \middle| \begin{matrix} 1-\rho,\ 1-\nu,\ 2+\beta+2j-\rho,\ (2-\varepsilon-u)/2,(2-\varepsilon+u)/2 \\ 0,\ 1+\beta+2j-\rho,\ \alpha+\beta+m-\rho \end{matrix}\right] \quad \ldots (4.4)$$

$$\sum_{j=0}^{k} \sum_{r=0}^{n} \binom{2k}{2j} \frac{(-n)_r (\gamma-\alpha)_r}{r!\,(\gamma-2k-n)_r\,(\gamma-n-2j+r)_\lambda}$$

$$.G^{1,3}_{4,2}\left[4z \middle| \begin{matrix} 1-\gamma-\lambda+n+2j-r,\ 1-\gamma,\ (2-\varepsilon-u)/2,(2-\varepsilon+u)/2 \\ 0,\ 1-\nu+n+2j-\sigma-r \end{matrix}\right]$$

$$\geq \sum_{j=0}^{k-1} \sum_{r=0}^{n} \binom{2k}{2j+1} \frac{(-n)_r (\gamma-\alpha)_r}{r!\,(\gamma-2k-n)_r\,(\gamma-n-2j-1+r)_\lambda}$$

$$.G^{1,3}_{4,2}\left[4z \middle| \begin{matrix} 2-\gamma-\lambda+n+2j-r,\ 1-\nu,\ (2-\varepsilon-u)/2,(2-\varepsilon+u)/2 \\ 0,\ 2-\gamma-\lambda+n+2j-\sigma-r \end{matrix}\right] \quad \ldots (4.5)$$

$$\sum_{j=0}^{k'/2} \sum_{r=0}^{n} \binom{k'}{2j+1} \frac{(-n)_r (\gamma-\alpha)_r}{r!\,(\gamma-k'-n)_r\,(\gamma-n-2j+r-1)_\lambda}$$

$$.G^{1,3}_{4,2}\left[4z \middle| \begin{matrix} 2-\gamma-\lambda+n+2j-r,\ 1-\nu,\ (2-\varepsilon-u)/2,(2-\varepsilon+u)/2 \\ 0,\ 2-\gamma-\lambda+n+2j-\sigma-r \end{matrix}\right]$$

$$\geq \sum_{j=0}^{k'/2} \sum_{r=0}^{n} \binom{k'}{2j} \frac{(-n)_r (\gamma-\alpha)_r}{r!\,(\gamma-k'-n)_r\,(\gamma-n-2j+r)_\lambda}$$

$$.G^{1,3}_{4,2}\left[4z \middle| \begin{matrix} 1-\gamma-\lambda+n+2j-r,\ 1-\nu,\ (2-\varepsilon-u)/2,(2-\varepsilon+u)/2 \\ 0,1-\gamma-\lambda+n+2j-\sigma-r \end{matrix}\right] \quad \ldots (4.6)$$

Now, putting $\mu=l=1$ in (3.7), (3.8), ..., (3.12) respectively, we also deduce the following more inequalities involving Meijer G-function [4], provided that existence and convergence conditions of 3. (II), are satisfied :

$$\sum_{j=0}^{k} \binom{2k}{2j} G^{5,3}_{7,5}\left[\frac{z}{4} \middle| \begin{matrix} 1-\rho,\ 1-\nu, 1+\beta+2k-2j-\rho, \\ (1-2\delta)/4,\ (3-2\delta)/4, 0,\ \beta+2k-2j-\rho, \end{matrix}\right.$$

$$\left. \begin{matrix} (2-\delta-\nu)/2,(2+\nu-\delta)/2,\ (1-\nu-\delta)/2,\ (1+\nu+\delta)/2 \\ \alpha+\beta+2k-\rho \end{matrix}\right]$$

$$\geq \sum_{j=0}^{k-1} \binom{2k}{2j+1} G^{5,3}_{7,5}\left[\frac{z}{4} \middle| \begin{matrix} 1-\rho,\ 1-\nu,\ \beta+2k-2j-\rho, \\ (1-2\delta)/4,\ (3-2\delta)/4,\ 0,\ \beta+2k-2j-\rho, \end{matrix}\right.$$

$$\left.\begin{matrix} (2-\delta-\nu)/2, (2+\nu-\delta)/2, (1-\nu-\delta)/2, (1+\nu-\delta)/2, \\ \alpha+\beta+2k-\rho \end{matrix}\right] \qquad \dots (4.7)$$

$$\sum_{j=0}^{[k'/2]} \binom{k'}{2j+1} G^{5,3}_{7,5}\left[\frac{z}{4}\,\middle|\, \begin{matrix} 1-\rho, 1-\nu, \beta+k'-2j-\rho, (2-\delta-\nu)/2, (2+\nu-\delta)/2, (1-\nu-\delta)/2, (1+\nu-\delta)/2 \\ (1-2\delta)/4, (3-2\delta)/4, ,0, \beta+2k-2j-\rho, \alpha+\beta+2k-\rho \end{matrix}\right]$$

$$\geq \sum_{j=0}^{[k'/2]} \binom{k'}{2j} G^{5,3}_{7,5}\left[\frac{z}{4}\,\middle|\, \begin{matrix} 1-\rho, 1-\nu, 1+\beta+k'-2j-\rho, (2-\delta-\nu)/2, (2+\nu-\delta)/2, (1-\nu-\delta)/2, (1+\nu-\delta)/2, \\ (1-2\delta)/4, (3-2\delta)/4, 0, \beta+k'-2j-\rho, \alpha+\beta+k'-\rho \end{matrix}\right] \qquad \dots (4.8)$$

$$\sum_{j=1}^{k} \frac{(-2k)_{2j}}{(2j)!} G^{5,3}_{7,5}\left[\frac{z}{4}\,\middle|\, \begin{matrix} 1-\rho, 1-\nu, 1+\beta+2j-\rho, (2-\delta-\nu)/2, (2+\nu-\delta)/2, (1-\nu-\delta)/2, (1+\nu+\delta)/2, \\ (1-2\delta)/4, (3-2\delta)/4, 0, \beta+2j-\rho, \alpha+\beta+m-\rho \end{matrix}\right]$$

$$\geq \sum_{j=0}^{k-1} \frac{(-2k)_{2j+1}}{(2j+1)!} G^{5,3}_{7,5}\left[\frac{z}{4}\,\middle|\, \begin{matrix} 1-\rho, 1-\nu, 2+\beta+2j-\rho, (2-\delta-\nu)/2, (2+\nu-\delta)/2, (1-\nu-\delta)/2, (1+\nu-\delta)/2 \\ (1-2\delta)/4, (3-2\delta)/4, 0, 1+\beta+2j-\rho, \alpha+\beta+m-\rho \end{matrix}\right] \qquad \dots (4.9)$$

$$\sum_{j=1}^{k'/2} \frac{(-k')_{2j}}{(2j)!} G^{5,3}_{7,5}\left[\frac{z}{4}\,\middle|\, \begin{matrix} 1-\rho, 1-\nu, 1+\beta+2j-\rho, (2-\delta-\nu)/2, (2+\nu-\delta)/2, (1-\nu-\delta)/2, (1+\nu-\delta)/2 \\ (1-2\delta)/4, (3-2\delta)/4, 0, \beta+2j-\rho, \alpha+\beta+m-\rho \end{matrix}\right]$$

$$\geq \sum_{j=0}^{[k'/2]} \frac{(-k')_{2j+1}}{(2j+1)!} G^{5,3}_{7,5}\left[\frac{z}{4}\,\middle|\, \begin{matrix} 1-\rho, 1-\nu, 2+\beta+2j-\rho, (2-\delta-\nu)/2, (2+\nu-\delta)/2, (1-\nu-\delta)/2, (1+\nu-\delta)/2 \\ (1-2\delta)/4, (3-2\delta)/4, 0, 1+\beta+2j-\rho, \alpha+\beta+m-\rho \end{matrix}\right] \qquad \dots (4.10)$$

$$\sum_{j=0}^{k} \sum_{r=0}^{n} \binom{2k}{2j} \frac{(-n)_r\,(\gamma-\alpha)_r}{r!\,(\gamma-2k-n)_r\,(\gamma-n-2j+r)_\lambda} . G^{3,3}_{6,4}\left[\frac{z}{4}\,\middle|\, \begin{matrix} 1-\gamma-\lambda+n+2j-r, 1-\nu, (2-\delta-\nu)/2, (2+\nu-\delta)/2, (1-\nu-\delta)/2, (1+\nu-\delta)/2 \\ (1-2\delta)/4, (3-2\delta)/4, 0, 1-\nu+n+2j-\sigma-r \end{matrix}\right]$$

$$\geq \sum_{j=0}^{k-1} \sum_{r=0}^{n} \binom{2k}{2j+1} \frac{(-n)_r\,(\gamma-\alpha)_r}{r!\,(\gamma-2k-n)_r\,(\gamma-n-2j-1+r)_\lambda} . G^{3,3}_{6,4}\left[\frac{z}{4}\,\middle|\, \begin{matrix} 2-\gamma-\lambda+n+2j-r, \\ (1-2\delta)/4, (3-2\delta)/4, \end{matrix}\right.$$

$$\left.\begin{matrix} 1-\nu, (2-\delta-\nu)/2, (2+\nu-\delta)/2, (1-\nu-\delta)/2, (1+\nu-\delta)/2 \\ 0,\ 2-\gamma-\lambda+n+2j-\sigma-r \end{matrix}\right] \quad \ldots (4.11)$$

$$\sum_{j=0}^{k'/2} \sum_{r=0}^{n} \binom{k'}{2j+1} \frac{(-n)_r (\gamma-\alpha)_r}{r!\,(\gamma-k'-n)_r\,(\gamma-n-2j+r-1)_\lambda} . G^{3,3}_{6,4}\left[\frac{z}{4} \,\middle|\, \begin{matrix} 2-\gamma-\lambda+n+2j-r, (1-2\delta)/4, (3-2\delta)/4, \\ \end{matrix}\right.$$

$$\left.\begin{matrix} 1-\nu, (2-\delta-\nu)/2, (2+\nu-\delta)/2, (1-\nu-\delta)/2, (1+\nu-\delta)/2 \\ 0,\ 2-\gamma-\lambda+n+2j-\sigma-r \end{matrix}\right]$$

$$\geq \sum_{j=0}^{k'/2} \sum_{r=0}^{n} \binom{k'}{2j} \frac{(-n)_r (\gamma-\alpha)_r}{r!\,(\gamma-k'-n)_r\,(\gamma-n-2j+r)_\lambda} . G^{3,3}_{6,4}\left[\frac{z}{4} \,\middle|\, \begin{matrix} 1-\gamma-\lambda+n+2j-r, (1-2\delta)/4, (3-2\delta)/4, \\ \end{matrix}\right.$$

$$\left.\begin{matrix} 1-\nu, (2-\delta-\nu)/2, (2+\nu-\delta)/2, (1-\nu-\delta)/2, (1+\nu-\delta)/2 \\ 0,\ 1-\gamma-\lambda+n+2j-\sigma-r \end{matrix}\right] \quad \ldots (4.12)$$

We may also obtain some more inequalities involving ${}_pF_q$ functions by using the result due to Mathai and Saxena [4, p.4 (1.1.10)], and making an appeal to the result due to Srivastava and Manocha [6, p.43 (6)]. Since ${}_pF_q$ can be transformed in to ${}_2F_1$ and ${}_1F_1$ functions,therefore we may get more inequalities involwing of ${}_2F_1$ and ${}_1F_1$ functions, defferent from Jamini and Koul [2].

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