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STABILITY OF STRATIFIED WALTERS (MODEL B') FLUID IN POROUS MEDIUM IN THE PRESENCE OF SUSPENDED PARTICLES AND VARIABLE MAGNETIC FIELD

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ABSTRACT

The stability of stratified Walters (Model B') viscoelastic fluid in stratified porous medium is considered. separately, in presence of suspended particles (fine dust) and variable horizontal magnetic field A variable horizontal magnetic field stabilizes certain wave–number range whereas the system was unstable for all wave number in the absence of magnetic field, for unstable stratifications. The growth rates are found to decrease with the increase in kinematic viscosity, magnetic field and kinematic viscoelasticity whereas the growth rates increase with the increase in medium permeablility. The suspended particles (fine dust) do not affect the stability or unstability, as was the situation in their absence.

1. Introduction. The stability of superposed fluids under varying assumptions of hydrodynamics and hydromagnetics has been treated in detail by Chandrasekhar [1]. The medium has been assumed to be non–porous and the fluid to be Newtonian.

The effect of suspended particles on the stability of superposed fluids finds importance in geophysics and chemical engineering. Further, motivation for the stability of fluids in presence of suspended particles is the fact that knowledge concerning fluid–particle mixtures is not commensurate with their industrial and scientific importance.

Scanlon and Segal [2] have considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles. The suspended particles were thus found to destabilize the layer. Palaniswamy and Purushotham [3] have condidered the stability of the shear flow of stratified fluids with fine dust and have found that the fine dust (suspended particles) increases the region of instability.

Generally the magnetic field has a stabilizing effect on the instability, but there are a few exceptions also. For example, Kent [4] has studied the effect of a horizontal magnetic field which varies in the vertical direction on the stability of parallel flows and has shown that the system is unstable under certain conditions, while in the absence of magnetic field the system is known to be stable.

With the growing importance of non–Newtonian fluid in modern technology and industries, the investigations of such fluids are desirable. The Walters fluid (Model B') is one such fluid. Sharma and Kumar [5] have studied the steady flow and heat transfer of Walters fluid (Model B') through a porous pipe of uniform circular cross–section with small suction.

In recent years, the investigation of flow of fluids through porous media has become an important topic due to the recovery of crude oil from the pores of reservior rocks. When the fluid permeates a porous material, the gross effect is represented by the Darcy's law. As a result of this macroscopic law, the usual viscous term in the equation of Walters fluid (Model B') motion is replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\vec{V}\right]$ where μ and μ' are the viscosity and viscoelasticity of the Walter's fluid, k_1 is the medium permeability and $\vec{V}$ is the Darcian (filter) velocity of the fluid. Yadav and Ray [6] have studied the unsteady flow of n–immiscible viscoelastic [Walters (Model B')] fluids through a porous medium between two parallel plates in the presence of a transverse magnetic field.

Keeping in mind the importance of non–Newtonian fluid in modern technology and industries and owing to the importance of suspended particles and variable magnetic field in chemical engineering and geophysics, the present paper condiders the stability of stratified Walters (Model B') fluid in stratified porous medium in the presence of suspended particles and variable horizontal magnetic field separately.

2. Effect of Variable Magnetic Field.

2.1 Perturbation Equations. The initial stationary state whose stability we wish to examine is that of an incompressible, infinitely conducting, Walters (Model B') fluid of variable density, kinematic viscosity and kinematic viscoelasticity aggranged in horizontal strata in a porous medium of variable porosity and permeability and acted on by gravity force $\vec{g}$ *(0,0, –g)* and variable horizontal megnetic field $\vec{H}$ $(H_0(z), 0, 0)$. Consider an infinite horizontal layer of thickness d bounded by the planes $z = 0$ and $z=d$.

Let δρ δ*p* $\vec{V}$ (u,v,w) and $\vec{h}$ (h_x,h_y,h_z) denote respectively, the perturbations in density ρ , pressure p, velocity *(0,0,0)* and magnetic field $\vec{H}$ $(H_0(z),0,0)$;∈, k_1, ν, ν' and μ_e denote the medium porosity, the medium permeability, the kinematic viscosity, the kinematic viscoelasticity and the magnetic permeability, respectively. Then the linearized perturbation equations governing the motion of Walters (Model B') Fluid and Maxwell's equations are

$$\frac{\rho}{\in}\frac{\partial\vec{V}}{\partial t} = -\nabla\delta p + \vec{g}\,\delta\rho + \frac{\mu_e}{4\pi}[(\nabla\times\vec{h})\times\vec{H} + (\nabla\times\vec{H})\times\vec{h}] - \frac{\rho}{k_1}(\nu - \nu'\frac{\partial}{\partial t})\vec{V}, \tag{1}$$

$$\nabla.\vec{V} = 0, \tag{2}$$

$$\in\frac{\partial}{\partial t}(\delta\rho) = -w\frac{d\rho}{dz}, \tag{3}$$

$$\in\frac{\partial}{\partial t}\vec{h} = (\vec{H}.\nabla)\vec{V} - (\vec{V}.\nabla)\vec{H}, \tag{4}$$

$$\nabla.\vec{h} = 0. \tag{5}$$

Equations (1) and (2) are linearized perturbed equations of motion and continuity whereas equation (3) ensures that the density of every particle remains unchanged as we follow it with its motion. Equations (4) and (5) are linearized perturbed, Maxwell's equations.

Analysing the disturbance into normal modes, we seek solutions whose dependence on x, y and t is given by

$$exp\ (ik_x x + ik_y y + nt), \tag{6}$$

where k_x, k_y are horizontal wave numbers, $k = (k_x^2 + k_y^2)^{1/2}$ and n is, in general, a complex constant.

Equations (1)–(5), using expression (6), give

$$\frac{\rho}{\in}\left[\left(1 - \frac{\in\nu'}{k_1}\right)n + \frac{\in\nu}{k_1}\right]u = -ik_x\delta p + \frac{\mu_e h_z}{4\pi}DH_0, \tag{7}$$

$$\frac{\rho}{\in}\left[\left(1 - \frac{\in\nu'}{k_1}\right)n + \frac{\in\nu}{k_1}\right]v = -ik_y\delta p + \frac{\mu_e}{4\pi}(ik_x h_y - ik_y h_x), \tag{8}$$

$$\frac{\rho}{\in}\left[\left(1-\in\frac{\nu'}{k_1}\right)n+\frac{\in\nu}{k_1}\right]w=-D\delta p+\frac{\mu_e}{4\pi}\left(ik_xh_z-Dh_x-h_x\frac{DH_0}{H_0}\right), \tag{9}$$

$$ik_xu+ik_yv+Dw=0, \tag{10}$$

$$ik_xh_x+ik_yh_y+Dh_z=0, \tag{11}$$

$$\in nh_x=ik_xH_0u-wDH_0, \tag{12}$$

$$\in nh_y=ik_xH_0v, \tag{13}$$

$$\in nh_z=ik_xH_0w. \tag{14}$$

Eliminating u, v, h_x, h_y, h_z and δp from equations (7)–(9) and using (10)–(14), after a little algebra, we get

$$D\left[\frac{\rho}{\in}\left\{\left(1-\in\frac{\nu'}{k_1}\right)n+\frac{\in\nu}{k_1}\right\}Dw\right]-k^2\frac{\rho}{\in}\left\{\left(1-\in\frac{\nu'}{k_1}\right)n+\frac{\in\nu}{k_1}\right\}w+\frac{gk^2(D\rho)}{\in n}w$$

$$+\frac{\mu_ek_x^2}{4\pi\in n}\left\{H_0^2(D^2-k^2)w+D(H_0^2)Dw-\frac{H_0^2(Dw)(D\in)}{\in}\right\}=0, \tag{15}$$

which, on simplification, can be written as

$$\left[\frac{\rho}{\in}\left\{\left(1-\in\frac{\nu'}{k_1}\right)n+\frac{\in\nu}{k_1}\right\}\right](D^2-k^2)w+\left\{\frac{n}{\in}(D\rho)\;1\left(-\in\frac{\nu'}{k_1}\right)+\frac{n\rho}{\in^2}(D\in)-\frac{n\rho}{k_1}D\nu'\right.$$

$$\left.+\frac{n\rho\nu'}{k_1^2}Dk_1+\frac{\nu}{k_1}D\rho+\frac{\rho}{k_1}D\nu-\frac{\rho\nu}{k_1^2}Dk_1\right\}Dw+\frac{gk^2(D\rho)}{\in n}w$$

$$+\frac{\mu_ek_x^2}{4\pi\in n}\left\{H_0^2\,(D^2-k^2)w+D(H_0^2)Dw-\frac{H_0^2}{\in}(Dw)\,(D\in)\right\}=0. \tag{16}$$

Equation (16) is the general equation governing the stability of a stratified Walters (Model B') fluid in a stratified porous medium in the presence of a variable horizontal magnetic field.

2.2 The Case of Exponentially Varying Stratifications .

Assume the stratifications in density, viscosity, viscoelasticity, medium porosity, medium permeability and magnetic field of the forms

$$\rho=\rho_0e^{\beta z},\ \mu=\mu_0e^{\beta z},\ \mu'=\mu_0'e^{\beta z},\in=\in_0e^{\beta z},k_1=k_{10}e^{\beta z},\ H_0^2=H_1^{\,2}e^{\beta z}, \tag{17}$$

where ρ_0, μ_0, μ_0', $\in_0$, k_{10}, H_1 and β are constants. Equations (17) imply that the kinematic viscosity $\nu\left(=\frac{\mu}{\rho_0}=\frac{\mu_0}{\rho_0}\right)$,the kinematic viscoelasticity $\nu'\left(=\frac{\mu'}{\rho_0}=\frac{\mu'_0}{\rho_0}\right)$ and the Alfven velocity V_A $\left(=\sqrt{\frac{\mu_e H_0^2}{4\pi\rho_0}}=\sqrt{\frac{\mu_e H_1^2}{4\pi\rho_0}}\right)$ are constant everywhere.

Using stratifications of the form (17), equation (16) transforms to

$$\left[n+\frac{\epsilon_0}{k_{10}}(\nu_0-\nu'_0 n)+\frac{k_x^2 V_A^2}{n}\right](D^2-k^2)w+\frac{gk^2\beta\, w}{n}=0. \tag{18}$$

The general solution of equation (18) is

$$w=Ae^{m_1 z}+Be^{m_2 z}, \tag{19}$$

where A, B are arbitrary constant ; m_1 , m_2 are given by

$$m_{1,2}=\pm k\left[1-\frac{g\beta}{nL}\right]^{1/2} \qquad \text{and} \qquad L=\left[n+\frac{\epsilon_0}{k_{10}}(\nu_0-\nu'_0 n)+\frac{k_x^2 V_A^2}{n}\right]. \tag{20}$$

Here we consider the fluid to be confined between two rigid planes at $z=0$ and $z=d$. The boudary conditions for the case of two rigid surfaces are

$$w=0 \qquad \text{at } z=0 \text{ and } z=d\ . \tag{21}$$

The vanishing of w at $z=0$ is satisfied by the choice

$$w=A\,(e^{m_1 z}-e^{m_2 z}),$$

while the vanishing of w at $z=d$ requires

$$e^{(m_1-m_2)d}=1 \qquad \text{i.e. } (m_1-m_2)d=2is\pi\ ,$$

where s is an integer.

Inserting the values of m_1, m_2 from (20) in above equation, we obtain

$$1-\frac{g\beta}{nL}=-\frac{s^2\pi^2}{k^2d^2}\quad ,$$

which on simplification gives

$$\left(1-\frac{\epsilon_0\nu'_0}{k_{10}}\right)n^2+\frac{\epsilon_0\nu_0}{k_{10}}\,n+\left[k_x^2 V_A^2-\frac{gk^2\beta\, d^2}{k^2d^2+s^2\pi^2}\right]=0\ . \tag{22}$$

If $\beta<0$ (stable stratification) and $k_{10}>\epsilon_0\,\upsilon'_0$, equation (22) does not admit any positive root of n and so the system is always stable for disturbances of all wave numbers. However, the system is unstable for

$k_{10} < \epsilon_0 \upsilon'_0$. Thus for stable stratification, the system is stable for $k_{10} > \epsilon_0 \upsilon'_0$ and unstable for $k_{10} > \epsilon_0 \upsilon'_0$. This is in contrast to the Newtonian fluid where the system is always stable for stable stratification (Chandrasekhar [1]).

If $\beta > 0$ (stable stratification) and $k_{10} > \epsilon_0 \upsilon'_0$. the system is stable or unstable according as

$$k_x^2 V_A^2 \gtrless \frac{gk^2\beta\, d^2}{k^2d^2+s^2\pi^2} \quad . \tag{23}$$

The system is clearly unstable in the absence of a magnetic field. However, the system can be completely stabilized by large enough magnetic field and kinematic viscoelasticity as can be seen from equation (23), if

$$V_A^2 > \frac{gk^2\beta\, d^2/k_x^2}{(k^2d^2+s^2\pi^2)} \qquad \text{and } k_{10} > \epsilon_0 \nu_0' \;,$$

Thus if $\beta > 0$ and $k_x^2 V_A^2 < \dfrac{gk^2\beta\, d^2/k_x^2}{(k^2d^2+s^2\pi^2)}$, $k_{10} > \epsilon_0 \nu_0'$,

equation (22) has atleast one positive root. Let n_0 denote the positive root of equation (22) . Then

$$\left(1-\frac{\epsilon_0 \nu_0'}{k_{10}}\right) n_0^2 + \frac{\epsilon_0 \nu_0}{k_{10}} n_0 + \left[k_x^2 V_A^2 - \frac{gk^2\beta\, d^2}{k^2d^2+s^2\pi^2}\right] = 0. \tag{24}$$

To find the role of kinematic viscosity, kinematic viscoelasticity, medium permeability and magnetic field on the growth rate of unstable modes, we examine the natures of $\dfrac{dn_0}{d\nu_0}, \dfrac{dn_0}{d\nu_0'}, \dfrac{dn_0}{k_{10}}$ and $\dfrac{dn_0}{dV_A}$.

Equation (24) yields

$$\frac{dn_0}{d\nu_0} = -\frac{\epsilon_0 n_0}{2n_0 (k_{10} - \epsilon_0 \nu_0') + \epsilon_0 \nu_0} \;, \tag{25}$$

$$\frac{dn_0}{d\nu_0'} = -\frac{\epsilon_0 n_0^2}{2n_0 (k_{10} - \epsilon_0 \nu_0') + \epsilon_0 \nu_0} \;, \tag{26}$$

$$\frac{dn_0}{k_{10}} = -\frac{\epsilon_0 n_0 (\nu_0 - \nu_0' n_0)}{k_{10}[2n_0(k_{10} - \epsilon_0 \nu_0') + \epsilon_0 \nu_0]}, \tag{27}$$

$$\frac{dn_0}{d\nu_A} = -\frac{2k_x^2 V_A}{2n_0(k_{10} - \epsilon_0 \nu_0') + \epsilon_0 \nu_0}, \tag{28}$$

It is evident from equation (25) that $\frac{dn_0}{d\nu_0}$ is negative or positive depending upon whether the denominator in equation (25) is positive or negative. The growth rates, therefore, decrease as well as increase with the increase in kinematic viscosity of the fluid.

From equation (26), if, in addintion to $k^2 > \frac{k_x^2 V_A(k^2 d^2 + s^2\pi^2)}{g\beta\, d^2}$

and $k_{10} > \epsilon_0 \upsilon'_0$. which are sufficient conditions for instability, we have the condition

$$\left(\frac{2n_0 k_{10}}{\epsilon_0} + \nu_0\right) > \nu_0', \tag{29}$$

$\frac{dn_0}{d\nu_0'}$ is always negative. The growth rates, therefore, decrease with the increase in kinematic viscoalasticity. However, the growth rates increase with the increase in kinematic viscoelasticity, if

$$\left(\frac{2n_0 k_{10}}{\epsilon_0} + \nu_0\right) < \nu_0', \tag{30}$$

for then $\frac{dn_0}{d\nu_0'}$ is positive.

It is clear from equation (27) that in addition to condition (29), we have the condition $\upsilon_0 > n_0 \upsilon'_0$, for which $\frac{dn_0}{dk_{10}}$ is negative. Thus the growth rates decrease with the increase in medium permeability due to the presence of kinematic viscoelasticity

$$\text{i. e. for} \quad \frac{2n_0 k_{10}}{\epsilon_0} + \nu_0 < \nu_0' < \frac{\nu_0}{n_0}. \tag{31}$$

However, the growth rates increase with the increase in medium permeability for

$$\frac{2n_0k_{10}}{\epsilon_0} + \nu_0 > \nu_0' \qquad \text{and} \quad n_0\nu_0' < \nu_0 \tag{31a}$$

or
$$\frac{2n_0k_{10}}{\epsilon_0} + \nu_0 < \nu_0' \qquad \text{and} \quad n_0\nu_0' > \nu_0 \tag{31b}$$

It is evident from equation (28) that the growth rates decrease with the increase in magnetic field for the sufficient condition,

$$k_{10} > \epsilon_0\nu_0' \tag{32}$$

We thus conclude the whole analysis with the following statements. The criteria determining stability or instability are independent of the effects of viscosity, viscoelasticity and medium permeability. The magnetic field stabilizes the system which is otherwise unstable in the absence of magnetic field. The viscosity and the medium permeability have damping as well as enhancing effects on the growth rates. The viscoelasticity has damping effect on the growth rates, but has enhancing effect also in the region (30).

3. Effect of Suspended Particles .

3.1 Perturbation Equations . The initial stationary state whose stability we wish to examine is that of an incompressible Walters (Model B') fluid of variable density, kinematic viscosity and kinematic viscoelasticity arranged in horizontal strata embedded by suspended particles (fine dust) in porous medium of variable porosity and permeability. Consider an infinite horizontal fluid particle layer of thickness d bounded by the planes $z = 0$ and $z=d$. The relevant equations of motion for the Walters (Model B') fluid are given by

$$\frac{\rho}{\epsilon}\left[\frac{\partial \vec{V}}{\partial t} + \frac{1}{\epsilon}(\vec{V}.\nabla)\vec{V}\right] = -\Delta p + \rho\vec{g} - \frac{\rho}{k_1}\left(\nu - \nu'\frac{\partial}{\partial t}\right)\vec{V} + \frac{KN}{\epsilon}(\vec{u} - \vec{v}) \tag{33}$$

$$\nabla.\vec{V} = 0, \tag{34}$$

$$\epsilon\frac{\partial}{\partial t}(\vec{V}.\Delta)\,\rho = 0\,, \tag{35}$$

Here $\vec{u}\,(\bar{x}, t)$ and $N\,(\bar{x}, t)$ denote the velocity and number density of the particles respectively, $K = 6\pi\rho\upsilon\eta$, where η is particle radius, is the Stokes drag coefficient, $\vec{u} = (l,r,s)$ and $\bar{x} = (x,y,z)$.

If mN is the mass of particles per unit volume, then the equa tions of motion and continuity for the particles are

$$mN\left[\frac{\partial \vec{u}}{\partial t}+\frac{1}{\epsilon}(\vec{u}.\nabla)\vec{u}\right]=KN(\vec{v}-\vec{u}) \tag{36}$$

$$\epsilon\frac{\partial N}{\partial t}+\nabla.(N\vec{u})=0\,, \tag{37}$$

The presence of particles adds an extra force term proportional to the velocity difference between particles and fluid and appears in the equation of motion (33). since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles (36). The buoyancy force on the particles is neglected. Interparticle reactions are not considered either since we assume that the distances between particles are quite large compared with their diameters. These assumptions have been used in writing the equations of motion for the particles (36).

Let $\delta\rho$, δp and $\vec{V}\,(u,v,w)$ denote respectively the perturbations in fluid density ρ, pressure p and particle velocity (0,0,0). Then the linearized perturbation equations of the fluid– particle layer become

$$\frac{\rho}{\epsilon}\frac{\partial \vec{V}}{\partial t}=-\nabla\delta p+\vec{g}\,\delta\rho-\frac{\rho}{k_1}\left(\nu-\nu'\frac{\partial}{\partial t}\right)\vec{V}+\frac{KN}{\epsilon}(\vec{u}-\vec{v}) \tag{38}$$

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\vec{u}=\vec{v}\,, \tag{39}$$

together with equations (2) and (3).

Eliminating $\vec{u}$ from equation (38) by making use of (39) and using expression (6), the resulting equation (2) and (3) yield

$$\frac{\rho}{\epsilon}\left[n+\frac{\epsilon}{k_1}(\nu-\nu' n)+\frac{mNn/\rho}{mn/K+1}\right]u=-ik_x\delta p, \tag{40}$$

$$\frac{\rho}{\epsilon}\left[n+\frac{\epsilon}{k_1}(\nu-\nu' n)+\frac{mNn/\rho}{mn/K+1}\right]v=-ik_y\delta p, \tag{41}$$

$$\frac{\rho}{\epsilon}\left[n+\frac{\epsilon}{k_1}(\nu-\nu'n)+\frac{mNn/\rho}{mn/K+1}\right]w=-D(\delta p)-g(\delta\rho), \tag{42}$$

$$ik_x x+ik_y y+Dw=0, \tag{43}$$

$$\epsilon n\delta\rho=-wD\rho\ . \tag{44}$$

Eliminating u, υ and δp from equations (40)–(42) and using (43) and (44), we obtain

$$\left[n+\frac{\epsilon}{k_1}(\nu-\nu'n)+\frac{mNn/\rho}{mn/K+1}\right](D^2-K^2)w+\left[\frac{1}{k_1}D\left\{\epsilon(\nu-\nu'n)\right\}-\frac{\epsilon(\nu-\nu'n)Dk_1}{k_1^2}\right.$$

$$\left.+\frac{mn}{mn/K+1}\left\{N\left(-\frac{1}{\rho^2}\right)D\rho+\frac{1}{\rho}DN\right\}\right]Dw+\frac{gk^2(D\rho)}{\epsilon n}w=0 \tag{45}$$

3.2 The Case of Exponentially Varying Stratifications .

Assume the stratifications in fluid density, fluid viscosity, fluid viscoelasticity, suspended particles number density, medium porosity and medium permeability of the forms

$$\rho=\rho_0 e^{\beta z},\ \mu=\mu_0 e^{\beta z},\ \mu'=\mu' e^{\beta z},\ N=N_0 e^{\beta z},\ \epsilon=\epsilon_0 e^{\beta z}, k_1=k_{10}e^{\beta z} \tag{46}$$

where ρ_0, μ_0, μ_0', N_0 ϵ_0, k_{10}, and β are constants and so kinematic viscosity $\nu=\frac{\mu}{\rho}=\frac{\mu_0}{\rho_0}$ $(=\nu_0)$and the kinematic viscoelasticity $\nu'=\frac{\mu'}{\rho}=\frac{\mu'_0}{\rho_0}$ $(=\nu_0')$are constants everywhere.

Using stratifications of the form (46), equation (45) transforms to

$$\left[n+\frac{\epsilon_0}{k_{10}}(\nu_0-\nu'_0 n)+\frac{mN_0 n/\rho_0}{mn/K+1}\right](D^2-K^2)w+\frac{gk^2\beta}{n}w=0 \tag{47}$$

The general solution of equation (43) is

$$w=Ae^{q_1 z}+Be^{q_2 z}, \tag{48}$$

where A,B are constants ; q_1 , q_2 are given by

$$q_{1,2}=\pm k\,[1-g\beta/nL_1]^{1/2} \tag{49}$$

and

$$L_1=\left[n+\frac{\epsilon_0}{k_{10}}(\nu_0-\nu'_0 n)+\frac{mN_0 n/\rho_0}{mn/K+1}\right]. \tag{50}$$

Here also we consider the fluid to be confined between two rigid planes at $z=0$ and $z=d$. The boundary conditions are

$$w = 0 \qquad \text{at } z = 0 \text{ and } z = d \tag{51}$$

The vanishing of w at $z=0$ is satisfied by the choice

$$w = A\,(e^{q_1 z} - e^{q_2 z}),$$

while the vanishing of w at $z=d$ requires $e^{(q_1-q_2)d} = 1$

i.e. $(q_1-q_2)d = 2is'\pi$.

where s' is an integer.

Inserting the values of q_1, q_2 from (49) in above equation, we obtain

$$1-\frac{g\beta}{nL_1} = -\frac{s^e\pi^2}{d^2},$$

which on simplification yields

$$\left(1-\frac{\epsilon_0 \nu_0'}{k_{10}}\right) n^3 + \left[\frac{K}{m}\left(1-\frac{\epsilon_0 \nu_0'}{k_{10}}\right) + \frac{\epsilon_0 \nu_0}{k_{10}} + \frac{KN_0}{\rho_0}\right] n^2 + \left[\frac{\epsilon_0 \nu_0}{k_{10}}\frac{K}{m} - \frac{gk^2\beta\, d^2}{k^2d^2+s^2\pi^2}\right] n$$

$$-\frac{gk^2\beta\, d^2}{k^2d^2+s^2\pi^2}\,\frac{K}{m} = 0. \tag{52}$$

If $\beta > 0$ (stable stratification) and $k_{10} > \epsilon_0 \nu_0'$, equation (52) does not admit any positive root of n and so the system is always stable for disturbances of all wave numbers. However, the system is unstable for $k_{10} < \epsilon_0 \nu_0'$. Thus for stable stratification, the system is stable for $k_{10} > \epsilon_0 \nu_0'$ and unstable for $k_{10} < \epsilon_0 \nu_0'$. This is in contrast to the Newtonian fluid where the system is always stable for stable stratification (Chandrasekhar [l]).

If $\beta > 0$ (unstable stratification),and $k_{10} < \epsilon_0 \nu_0'$, equation (52) has one positive root. Let n_o denote the positive root of equation (52). Then

$$\left(1-\frac{\epsilon_0 \nu_0'}{k_{10}}\right) n_o^3 + \left[\frac{K}{m}\left(1-\frac{\epsilon_0 \nu_0'}{k_{10}}\right) + \frac{\epsilon_0 \nu_0}{k_{10}} + \frac{KN_0}{\rho_0}\right] n_o^2 + \left[\frac{\epsilon_0 \nu_0}{k_{10}}\frac{K}{m} - \frac{gk^2\beta\, d^2}{k^2d^2+s^2\pi^2}\right] n_0$$

$$-\frac{gk^2\beta\, d^2}{k^2d^2+s^2\pi^2}\,\frac{K}{m} = 0. \tag{53}$$

To find the roles of fluid viscosity, fluid viscoelasticity, suspended particles number density and medium permeability on the growth rates of unstable modes, we examine the natures of $\frac{dn_0}{d\nu_0}$, $\frac{dn_0}{d\nu_0'}$, $\frac{dn_0}{dN_0}$ and $\frac{dn_0}{dk_{10}}$ analytically. Equation (35) yields.

$$\frac{dn_0}{d\nu_0} = -\frac{\epsilon_0 n_0 (n_0 + K/m)}{k_{10}\left[3n_0^2\left(1-\frac{\epsilon_0\nu_0'}{k_{10}}\right)+2n_0\left(\frac{K}{m}-\frac{\epsilon_0\nu_0'}{k_{10}}\frac{K}{m}+\frac{\epsilon_0\nu_0}{k_{10}}+\frac{KN_0}{\rho_0}\right)+\left(\frac{\epsilon_0\nu_0 K}{k_{10}m}-\frac{gk^2\beta d^2}{k^2d^2+s^2\pi^2}\right)\right]} \quad (54)$$

$$\frac{dn_0}{d\nu_0'} = \frac{\epsilon_0 n_0^2 (n_0 + K/m)}{k_{10}\left[3n_0^2\left(1-\frac{\epsilon_0\nu_0'}{k_{10}}\right)+2n_0\left(\frac{K}{m}-\frac{\epsilon_0\nu_0'}{k_{10}}\frac{K}{m}+\frac{\epsilon_0\nu_0}{k_{10}}+\frac{KN_0}{\rho_0}\right)+\left(\frac{\epsilon_0\nu_0 K}{k_{10}m}-\frac{gk^2\beta d^2}{k^2d^2+s^2\pi^2}\right)\right]} \quad (55)$$

$$\frac{dn_0}{dN_0} = -\frac{n_0 K/\rho_0}{\left[3n_0\left(1-\frac{\epsilon_0\nu_0'}{k_{10}}\right)+2\left(\frac{K}{m}-\frac{\epsilon_0\nu_0'}{k_{10}}\frac{K}{m}\right)+\left(\frac{\epsilon_0\nu_0}{k_{10}}+\frac{KN_0}{\rho_0}\right)\right]}, \quad (56)$$

$$\frac{dn_0}{k_{10}} = -\frac{\epsilon_0 n_0 \left[n_0^2\nu_0' + n_0(\nu_0 - K/m\,\nu_0') + \nu_0 K/m\right]}{k_{10}^2\left[3n_0^2\left(1-\frac{\epsilon_0\nu_0'}{k_{10}}\right)+2n_0\left(\frac{K}{m}-\frac{\epsilon_0\nu_0'}{k_{10}}\frac{K}{m}+\frac{\epsilon_0\nu_0}{k_{10}}+\frac{KN_0}{\rho_0}\right)+\left(\frac{\epsilon_0\nu_0 K}{k_{10}m}-\frac{gk^2\beta d^2}{k^2d^2+s^2\pi^2}\right)\right]} \quad (57)$$

It is evident form equation (54) that $\frac{dn_0}{d\nu_0}$ is negative or positive depending upon whether the denominator in equation (54) is positive or negative. The growth rates, therefore, decrease as well as increase with the increase in kinematic viscosity of the fluid.

From equation (55), if, in addition to $k_{10} < \epsilon_0\nu_0'$ which is the sufficient condition for instability, we have the condition

$$\frac{gk^2\beta d^2}{k^2d^2+s^2\pi^2} > 3n_0^2\left(1-\frac{\epsilon_0\nu_0'}{k_{10}}\right)+2n_0\left[\left(1-\frac{\epsilon_0\nu_0'}{k_{10}}\right)\frac{K}{m}+\frac{\epsilon_0\nu_0}{k_{10}}+\frac{KN_0}{\rho_0}\right]+\frac{\epsilon_0\nu_0}{k_{10}}\frac{K}{m}, \quad (58)$$

$\frac{dn_0}{d\nu_0'}$ is always negative. The growth rates, therefore, decrease with the increase in kinematic viscoelasticity. However, the growth rates increase with the increase in kinematic viscoelasticity if

$$\frac{gk^2\beta d^2}{k^2d^2+s^2\pi^2} < 3n_0^2\left(1-\frac{\epsilon_0\nu_0'}{k_{10}}\right)+2n_0\left[\left(1-\frac{\epsilon_0\nu_0'}{k_{10}}\right)\frac{K}{m}+\frac{\epsilon_0\nu_0}{k_{10}}+\frac{KN_0}{\rho_0}\right]+\frac{\epsilon_0\nu_0}{k_{10}}\frac{K}{m}, \quad (59)$$

for then $\frac{dn_0}{d\nu_0'}$ is positive.

It is clear from equation (56) that for $k_{10} > \epsilon_0 \nu_0'$, $\frac{dn_0}{dN_0}$ is negative. Thus, the growth rates decrease with the increase in the suspended particles number density. However, if, in addition to $k_{10} < \epsilon_0 \nu_0'$. which is a sufficient condition for instability, we have the condition

$$\frac{\epsilon_0}{k_{10}}\left(3n_0\nu_0 + 2\nu_0'\frac{K}{m}\right) > 3n_0 + 2\,\frac{K}{m} + \frac{\epsilon_0 \nu_0}{k_{10}} + \frac{KN_0}{\rho_0}\,, \tag{60}$$

the growth rates increase with the increase in suspended particles number density for then $\frac{dn_0}{dN_0}$ is positive.

Form equation (57), if, in addition to $k_{10} < \epsilon_0 \nu_0'$ which is a sufficient condition for instability, we have either of the conditions (59) or

$$\nu_0 > \frac{K}{m}\,\nu_0'\,, \tag{61}$$

$\frac{dn_0}{dk_{10}}$ is always negative. The growth rates, therefore, decrease with the

increase in medium permeability if we have either of the conditions (58) or

$$\nu_0 < \frac{K}{m}\,\nu_0', \tag{62}$$

for then $\frac{dn_0}{dk_{10}}$ is always positive.

We thus conclude the whole analysis with the following statements. The criteria determining stability or instabiltity are independent of the effects of viscosity, viscoelasticity, suspend particles number density and medium permeability. The viscosity, the viscoelasticity and the suspended particles number density have damping as well as enhancing effects on the growth rates. The viscoelasticity has damping effect on the growth rates, but has enhancing effect also in the region (59).

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