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THERMOSOLUTAL CONVECTION IN RIVLIN-ERICKSEN COMPRESSIBLE FLUIDS IN POROUS MEDIUM

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ABSTRACT

The thermosolutal convection in Rivlin–Ericksen compressible fluids in porous medium in the absence and presence, separately, of rotation and magnetic field is considered. The compressibility stable solute gradient, magnetic field and rotation are found to have stabilizing effects. The medium permeability may have stabilizing or destabilizing effect in the presence of rotation but has always a destabililizing effect in the absence of rotation. The oscillatory modes are introduced in the presence of stable solute gradient, magnetic field and rotation which were non–existent in their absence. The conditions for the non–existence or overstability are also obtained.

1. introduction. The problem of thermosolutal convection in Rivlin–Ericksen compressible fluids in porous medium may be of importance in geophysics, ground water hydrology, soil sciences and oil recovery. The effect of a magnetic field on the stability of such a flow is of interest in geophysics, particulary in the study of Earth's core where the earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. This occurs when the generally stabilizing effect of one component in the presence of a magnetic field is reduced by diffusion which allows a release of the potential energy of an unstable component. The other application of the results of flow through a porous medium in the presence of a magnetic field is in the study of the stability of a convective flow in the geothermal region. For the geophysical problems we have to consider the effect of rotation of the Earth (i.e. the Coriolis force) which distorts the boundaries of a hexagonal convection cell in a fluid through prorous medium because the distortion plays an important role in the extraction of energy in the geothermal regions. Lapwood [6] and Westbrook [14] have studied the breakdown of the stability of a layer of fluid subject to a vertical temperature gradient in a porous medium and the possiblity of convective flow. Wooding [15] has found in the absence of viscous dissipation and rotation and considering only Darcy resistance, that convection sets in as a fairly regular cellular pattern in the horizontal.

The theoretical and experimental results on the stability of cellular convection of a fluid layer in nonporous medium, in the absence and presence of rotation and magnetic field, have been given in a treatise by Chandrasekhar [3]. Veronis [13] has investigated the problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. Nield [7] has considered the thermohaline convection in a horizontal layer of viscous fluid heated from blow and salted from above. The physics is quite similar in the Stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The Boussinesq approximation has been used in all the above studies which means that (a) in the equations for the rate of change of mass and momentum, density variations may be neglected except when they are coupled to the gravitational acceleration in the buoyancy force and (b) the fluctuations in density which appear with the advent of motion result principally from thermal effects. This approximation is well justified in the case of incompressible fluids.

The equations governing the system become quite complicated when the fluids are compressible. To simplify them, Boussinesq tried to justify the approximation for compressible fluids when the density variations arise principally from thermal effects by noting that atmospheric pressure fluctuations are much too small to produce the observed density changes. Spiegel and Veronis (1960) have simplified the set of equations governing the flow of compressible fluids under the following assumptions :

(i) the vertical demension of the fluid is much less than any scale height, as defined by them, if only motions of infinitesimal amplitudes are considered, and

(ii) the motion–induced fluctuations in density and pressure do not exceed, in order to magnitude the total static variations of these quantities, in non–linear investigations.

Under the above approximations, Spiegel and Veronis [10] have shown that the equations governing convection in a perfect gas are formally equivalent to those for an incompressible fluid if the static temperature gradient is replaced by its excess over the adiabatic and C_v is replaced by C_p. The fluid has been considered to be Newtonian in all above studies.

The Stability of a horizonatal layer of viscoelastic (Maxwell) fluid heated from below has been investigated by Vest and Arpaci (1969). Bhatia and Steiner [1] have studied the effect of a uniform rotation on the thermal instability of a Maxwell fluid and have found that rotation has a destabilizing effect in contrast to the stabilizing effect on Newtonian fluid. Bhatia and Steiner [2] have also considered the effect of magnetic field on thermal instability of a Maxwellian viscoelastic fluid and have found that the magenetic field has the stabilizing influence on Maxwell fluid just as in the case of Newtonian fluid. The thermal instability of an Oldroydian viscoelastic fluid has been considered in the presence of magnetic field by Sharma [8] and in the presence of rotation by Eltayeb [4] and Sharma [9]. Experimental demonstration by Toms and Strawbridge [12] has revealed that a dilute solution of methyl methacrylate in n–buty1 acetate agrees well with the theoretical model of Oldroyd fluid. There are many elasticoviscous fluids that cannot be characterized by Maxwell's or Oldroyd's constitutive relstions. One such class of elasticoviscousv fluids is Rivlin–Ericksen fluid. Srivastava and Singh [11] have studied the unsteady flow of a dusty elasticoviscous Rivlin–Ericksen fluid through channels of different cross–sections in the presence of time–dependent pressure gradient. The drag on a sphere oscillating in conducting dusty Rivlin–Ericksen elastico–viscous fluid has been considered by Garg et. al. [5].

The present paper, therefore, attempts to study the thermosolutal instability in porous medium of a layer of Rivlin–Ericksen compressible fluid heated from below and subjected to a stable solute gradient. The thermosolutal instability of a layer of rotating Rivlin–Ericksen compressible fluid in porous medium is also considered. The thermosolutal convection in a layer of finitely (electrical) conducting, Rivlin–Ericksen compressible fluid in porous medium, acted on by a uniform vertical magnetic field and a stable solute gradient is also studied. These aspects form the subject matter of the present paper keeping in view the geophysical situations.

2. Formulation of the problem and perturbation equations.

Consider an infinite horizontal layer of compressible, Rivlin–Ericksen fluid of depth d heated from below and subjected to a stable solute gradient. This layer of fluid of thickness d is heated and soluted from below so that the temperatures and solute concentrations at the bottom surface $z = 0$ are T_0 and C_0 and at the upper surface $z = d$ are T_d and C_d respectively, z–axis being taken as vertical. The gravity force $\vec{g}\ (0, 0, -g)$ pervadege the system.

Spiegel and Veronis [10] represented f as any one of the state variables ; density (ρ), pressure (p) or temprature (T) and expressed these in the form

$$f(x, y, z, t) = f_m + f_0(z) + f'(x, y, z, t), \qquad ...(1)$$

where f_m is the constant space average of $f.f_0$ is the variation in the absence of motion and f' is the fluctuation resulting from motion.

The initial state is therefore a state in which the velocity, density, pressure, temperature and solute concentration at any point in the fluid are given by

$$\vec{V} = 0,\ \rho = \rho(z),\ p = p(z),\ T = T(z),\ C = C(z), \qquad ...(2)$$

respectively, where according to Spiegel and Veronis (1960),

$$T(z) = T_0 - \beta(z),$$

$$C(z) = C_0 - \beta'(z),$$

$$p(z) = p_m - g\int_0^z (\rho_m + \rho_0)\, dz,$$

$$\rho(z) = \rho_m [1 - \alpha_m (T - T_m) + \alpha'_m (C - C_m) + K_m (p - p_m)],$$

$$\alpha_m = -\left[\frac{1}{\rho}\frac{\partial\rho}{\partial T}\right]_m, \quad \alpha'_m = -\left[\frac{1}{\rho}\frac{\partial\rho}{\partial C}\right]_m, \quad K_m = \left[\frac{1}{\rho}\frac{\partial\rho}{\partial p}\right]_m, \qquad ...(3)$$

p_m and ρ_m thus stand for the constant space distribution of p and ρ and T_0, C_0, ρ_0 stand for temperature, solute concentration and density of the fluid at the lower boundary $z = 0$.

$$\beta\,(= |dT/dz|) \quad \text{and} \quad \beta'\,(= |dC/dz|)$$

denote the uniform temperature gradient and uniform solute gradient respectively. The approximation (i) is sufficient to produce the simplified equations if only motions of infinitesimal amplitude are considered. However, in non-linear investigations it is necessary to make the additional restriction (ii). The approximation (i) means

$$d << \left(\left| \frac{1}{f_m} \frac{d f_0}{dz} \right| \right)_{\min} , \qquad ...(4)$$

let $\delta\rho$, δp, $\vec{V}(u, v, w)$, θ and γ denote respectively the parturbations in density ρ, pressure p, velocity (which is zero initially), temperature T and solute concentration C ; υ, υ', κ, κ' and g/c_p stand for the kinematic viscosity, the kinematic viscoelasticity, the thermal diffusivity, the solute diffusivity and the adiabatic gradient respectively. Then the linearized perturbation equations of motion continuity, heat conduction and solute concetration for Rivlin–Ericksen fluids in porous medium, following Spiegel and Veronis [10] , are

$$\frac{\rho_0}{\in} \frac{\partial \vec{V}}{\partial t} = -\nabla\delta p + \vec{g}\,\delta\rho - \frac{\rho_0}{k_1} \left(\upsilon + \upsilon' \frac{\partial}{\partial t}\right) \vec{V}, \qquad ...(5)$$

$$\nabla . \vec{V} = 0, \qquad ...(6)$$

$$E \frac{\partial \theta}{\partial t} = \left(\beta - \frac{g}{c_p}\right) w + \kappa' \nabla^2 \theta , \qquad ...(7)$$

$$E \frac{\partial \gamma}{\partial t} = \beta' w + \kappa' \nabla^2 \gamma , \qquad ...(8)$$

and the equation of state is

$$\rho = \rho_0 [1 - \alpha (T - T_0) + \alpha' (C - C_0)] , \qquad ...(9)$$

where $\in$ ($0 < \in < 1$) is medium porosity, k_1 is the medium permeability and $E = \in + (1 - \in) \rho_s c_s / \rho c$.

Here ρ, c and ρ_s, c_s stand for density and heat capacity of fluid and solid matrix respectively. α and α' stand for thermal coefficient of expansion and an analogous solvent coefficient. The suffix 0 refers to values at the reference level $z = 0$. the change in density $\delta\rho$ caused by the perturbations θ and γ in temperature and solute concentration, is given by

$$\delta\rho = -\rho_0 (\alpha\theta - \alpha'\gamma) \qquad ...(10)$$

Here we consider the case in which both the boundaries are free as well as perfect conductors of both heat and solute. The case of two free boundaries is little artificial but the most appropriate for stellar atmospheres and provides analytical solutions. Then the boundary conditions appropriate to the problem [Chandrasekhar [3], Lapwood [6]] are

$$w = \frac{\partial^2 w}{\partial z^2} = 0, \quad \theta = 0, \ \gamma = 0 \text{ at } z = 0 \text{ and } z = d \qquad ...(11)$$

Equations (5)–(8) with the help of equation [10] give

$$\left[\frac{1}{\epsilon}\frac{\partial}{\partial t} + \frac{1}{k_1}\left(\upsilon + \upsilon'\frac{\partial}{\partial t}\right)\right]\nabla^2 w - g\alpha\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\theta + g\alpha'\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\gamma = 0, \qquad ...(12)$$

$$\left(E\frac{\partial}{\partial t} - \kappa\nabla^2\right)\theta = \left(\beta - \frac{g}{c_p}\right)w, \qquad ...(13)$$

$$\left(E\frac{\partial}{\partial t} - \kappa'\nabla^2\right)\gamma = \beta' w, \qquad ...(14)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

3. dispersion relation and discussion.

Analyzing in terms of normal modes, we assume that the perturbation quantities are of the form

$$[\,w, \theta, \gamma\,] = [\,W(z), \Theta(z), \Gamma(z)]\, exp\,(ik_x x + ik_y y + nt), \qquad ...(15)$$

where n is the growth rate which is, in general, a complex constant ; k_x, k_y are the wave numbers in the x and y directions respectively and $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number.

Equations (12) – (14) in non– dimensional form, using expression (15), become

$$\left[\left(\frac{1}{\epsilon} + \frac{\upsilon'}{k_1}\right)\sigma + \frac{1}{P_l}\right](D^2 - a^2)\,W + \frac{g\,\alpha d^2 a^2}{\nu}\,\Theta - \frac{g\,\alpha' d^2 a^2}{\nu}\,\Gamma = 0, \qquad ...(16)$$

$$(D^2 - a^2 - p_1 E\sigma)\,\Theta = -\frac{d^2}{\kappa}\,\frac{g}{c_p}\,(G-1)\,W, \qquad ...(17)$$

$$(D^2-a^2-qE\sigma)\,\Gamma = -\frac{B'd^2}{\kappa'}\,W, \qquad ...(18)$$

where $p_1 = \upsilon/\kappa$ is the Prandtl number, $q = \upsilon/\kappa'$ is the Schmidt number, $P_l = k_1/d^2$, $a = kd$, $\sigma = nd^2/\upsilon$, $G = Cp\,\beta/g$. We have put the coordinates x, y, z in the new unit of length d and $D = d/dz$.

Eliminating (x), Γ from equations (16) – (18), we get

$$\left[\sigma\left(\frac{1}{\epsilon}+\frac{\upsilon}{k_1}\right)+\frac{1}{P_l}\right](D^2-a^2)(D^2-a^2-p_1E\sigma)\,(D^2-a^2-qE\sigma)\,W$$

$$-Ra^2\left(\frac{G-1}{G}\right)(D^2-a^2-qE\sigma)\,W + S\,a^2\,(D^2-a^2-p_1E\sigma)\,W = 0, \qquad ...(19)$$

where $R = g\alpha\beta d^4/\upsilon\kappa$ is the Rayleigh number and is $S = g\alpha'\beta' d^4/\upsilon\kappa'$ is the analogous solute Rayleigh number. The boundary conditions (11) transform to

$$W = D^2W = 0,\ (x) = 0,\ \Gamma = 0 \text{ at } z = 0 \text{ and } z = 1 \qquad ...(20)$$

Using the boundary conditions (20), it can be shown with the help of eqns. (16)–(18) that all the even order derivatives of W vanish at the boundaries. Hence the proper solution of eqn. (19) characterizing the lowest mode is

$$W = W_0 \sin\pi z, \qquad ...(21)$$

where W_0 is a constant. Substituting (21) in eqn. (19) and letting $x = a^2/\pi^2$, $R_1 = R/\pi^4$, $S_1 = S/\pi^4$, and $P = \pi^2 P_l$, we obtain the dispersion relation

$$R_1 = \left(\frac{G}{G-1}\right)\left[\frac{\left\{\frac{\sigma}{\pi^2}\left(\frac{1}{\epsilon}+\frac{\upsilon'}{k_1}\right)+\frac{1}{P}\right\}(1+x)\left(1+x+Ep_1\frac{\sigma}{\pi^2}\right)}{x} + S_1\frac{\left(1+x+Ep_1\frac{\sigma}{\pi^2}\right)}{\left(1+x+Eq\frac{\sigma}{\pi^2}\right)}\right] \qquad ...(22)$$

For the stationary convection, $\sigma = 0$ and eqn. (22) reduces to

$$R_1 = \left(\frac{G}{G-1}\right)\left[\frac{(1+x)^2}{xp} + S_1\right] \qquad ...(23)$$

If the non–dimensional numbers S, P and G accounting for the solute gradient, medium permeability and compressibility effects be kept as fixed, then we find that

$$\bar{R}_c = \left(\frac{G}{G-1}\right) R'_{G/(G-1)} , \qquad \text{...(24)}$$

Where R'_c and $\bar{R}_c$ denote respectively the critical Rayleigh numbers in the absence and presence of compressibility. It is evident from eqn. (24) that the effect of compressibility is to postpone the onset of thermal instability and thus we obtain a stabilizing effect of compressibility. The cases $G<1$ and $G=1$ are irrelevant hare as they correspond to negative and infinite values of critical Rayleigh numbers in the presence of compressibility. To find the role of medium permeability, we examine the nature of dR_1/dp. It follows from eqn. (23) that

$$\frac{dR_1}{dP} = \left(\frac{G}{G-1}\right) \frac{(1+x)^2}{xP^2} , \qquad \text{...(25)}$$

which is always negative for $G>1$. The medium permeability, therefore, has a destabilizing effect on the system.

Multiplying eqn. (16) by W^*, the complex conjugate of W. integrating over the range of z, and making use of eqns. (17) and (18) we obtain

$$\left[\sigma\left(\frac{1}{\epsilon} + \frac{v'}{k_1}\right) + \frac{1}{P_l}\right] I_1 + \frac{g\,\alpha'\kappa' a^2}{\nu\beta'} (I_4 + qE\sigma^* I_5) = \frac{c_p\,\alpha\kappa a^2}{\nu(G-1)} (I_2 + p_1 E\sigma^* I_3), \quad (26)$$

where

$$I_1 = \int_0^1 (\,|DW|^2 + a^2\,|W|^2)\,dz,$$

$$I_2 = \int_0^1 (\,|D\Theta|^2 + a^2\,|\Theta|^2)\,dz,$$

$$I_3 = \int_0^1 |\Theta|^2\,dz,$$

$$I_4 = \int_0^1 (\,|D\Gamma|^2 + a^2\,|\Gamma|^2)\,dz,$$

$$I_5 = \int_0^1 |\Gamma|^2\,dz, \qquad \text{...(27)}$$

which are all positive definite. Putting and $\sigma = \sigma_r + i\,\sigma_i$ and then equating real and imaginary parts of eqn. (26), we obtain

$$\sigma_r \left[-\frac{c_p\,\alpha\,\kappa\,a^2}{\nu(G-1)} p_1 E I_3 + \frac{g\,\alpha'\kappa' a^2}{\nu\beta'} qEI_5 + \left(\frac{1}{\epsilon} + \frac{v'}{k_1}\right) I_1\right]$$

$$= -\left[-\frac{c_p \alpha \kappa a^2}{\nu (G-1)} I_2 + \frac{g\ \alpha' \kappa' a^2}{\nu \beta'} I_4 + \frac{I_1}{P_l} \right] \qquad ...(28)$$

and

$$\sigma_i \left[\frac{c_p \alpha \kappa a^2}{\nu (G-1)} p_1 E I_3 - \frac{g\ \alpha' \kappa' a^2}{\nu \beta'} q E I_5 + I_1 \left(\frac{1}{\in} + \frac{v'}{k_1} \right) \right] = 0 \qquad ...(29)$$

Equation (28) yields that σ_r may be positive or negative i.e. there may be instability or stability in the presence of solute gradient and porous medium. Equation (29) yields that $\sigma_i = 0$ or $\sigma_i \neq 0$ which means that the modes may be non–oscillatory or oscillatory. In the absence of solute gradient, eqn. (29) reduces to

$$\sigma_i \left[\left(\frac{1}{\in} + \frac{v'}{k_1} \right) I_1 + \frac{c_p \alpha \kappa a^2}{\nu (G-1)} p_1 E I_3 \right] = 0 \quad , \qquad ...(30)$$

and the term in brackets is positive definite if $G > 1$. Thus $\sigma_i = 0$, which means that oscillatory modes are not allowed in the absence of stable solute gradient if $G > 1$. The presence of stable solute gradient, thus, introduces oscillatory modes in thermosolutal convection in Rivlin–Ericksen compressible fluids in porous medium for $Cp\beta/g > 1$. Equation (28) yields that σ_r is negative if $G<1$. The system is there fore stable for $c_p\beta/g > 1$.

Put $\sigma/\pi^2 = i\sigma_1$, it being remembered that σ may be complex. Since for overstability, we wish to determine the critical Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which (22) will admit of solutions with σ_1 real. Putting $\sigma/\pi^2 = i\sigma_1$ where σ_1 is real in eqn. (22), and equating real and imaginary parts of the resulting equation, we obtain

$$R_1 \left(\frac{G-1}{G}\right) (b-1) = \frac{1}{P} (b^2 - E^2 q p_1 c) - c \left(\frac{1}{\in} + \frac{v'}{k_1} \right) E b\ (p_1 + q) + S_1 (b-1) \qquad (31)$$

$$R_1 \left(\frac{G-1}{G}\right) E q (b-1) = b \left(\frac{1}{\in} + \frac{v'}{k_1} \right) (b^2 - E^2 q p_1 c) + \frac{E b^2 (p_1 + q)}{p} + S_1 (b-1) E p_1 \qquad (32)$$

where $1+x = b$ and $\sigma_1^2 = c$.

Eliminating R_1 between (31) and (32), we obtain

$$c = -\frac{b^2 \left[\frac{E p_1}{P} + b \left(\frac{1}{\in} + \frac{v'}{k_1} \right) \right] + S_1 E (b-1) (p_1 - q)}{E^2 q^2 \left[\frac{E p_1}{P} + b \left(\frac{1}{\in} + \frac{v'}{k_1} \right) \right]} \qquad ...(33)$$

Since σ_1 is real, $c\ (=\sigma_1^2)$ must be positive for overstability. Thus is impossible, if $p_1 > q$. Hence $p_1 > q$ or $\kappa < \kappa'$ is a sufficient condition for the nonexistence of overstability, the violation of which does not necessarily imply occurrence of overstability.

4. Effect of rotation. In the present section, we consider an infinite horizontal layer of compressible and Rivlin–Ericksen elastico viscous fluid of depth d in porous medium heated from below and subjected to a stable solute gradient. The fluid is acted on by a uniform rotation $\vec{\Omega}\ (0,0,\Omega)$ and gravity force $\vec{g}\ (0,0,-g)$. Then the linearized perturbed equations of motion are

$$\frac{\rho_0}{\in}\frac{\partial \vec{V}}{\partial t} = -\nabla(\delta p) - \vec{g}\,\rho_0(\alpha\theta - \alpha'\gamma) - \frac{\rho_0}{k_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)\vec{V} + \frac{2\rho_0}{\in}(\vec{V}\times\vec{\Omega}), \quad (34)$$

whereas eqns. (6)–(9) remain unaltered.
Eqns. (6)–(8) and (34) give

$$\left[\frac{1}{\in}\frac{\partial}{\partial t} + \frac{1}{k_1}\left(\nu+\nu'\frac{\partial}{\partial t}\right)\right]\nabla^2 w - g\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(\alpha\theta \square \alpha'\gamma) + \frac{2\Omega}{\in}\frac{\partial \mathfrak{Z}}{\partial z} = 0, \quad (35)$$

$$\left[\frac{1}{\in}\frac{\partial}{\partial t} + \frac{1}{k_1}\left(\nu+\nu'\frac{\partial}{\partial t}\right)\right]\xi = \frac{2\Omega}{\in}\frac{\partial \varpi}{\partial z}, \quad \ldots (36)$$

together with equations (13) and (14). $\mathfrak{Z} = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}$ stands for the z–component of vorticity.

The non–dimensional forms of eqns. (35) and (36) are

$$\left[\left(\frac{1}{\in} + \frac{\nu'}{k_1}\right)\sigma + \frac{1}{P_l}\right](D^2 - a^2)\,W + \frac{2\,\Omega\, d^3}{\nu\in}DZ - \frac{g\,d^2 a^2}{\nu}(\alpha \circledast + \alpha\Gamma) = 0, \quad \ldots (37)$$

$$\left[\left(\frac{1}{\in} + \frac{\nu'}{k_1}\right)\sigma + \frac{1}{P_l}\right]Z = \frac{2\,\Omega\, d^3}{\nu\in}DW, \quad \ldots (38)$$

where we have written

$$\varsigma = Z(z)\exp(ik_x x + ik_y y + nt). \quad \ldots (39)$$

Eliminating Z, $\circledast$ and Γ from eqns. (17). (18), (37) and (38), we obtain

$$(D^2 - a^2 - p_1 E\sigma)\,(D^2 - a^2 - qE\sigma)\left[\left\{\sigma\left(\frac{1}{\in} + \frac{\nu'}{k_1}\right) + \frac{1}{P_l}\right\}^2 (D^2 - a^2) + T_A \in^{-2} D^2\right]W$$

$$= a^2\left[\sigma\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)+\frac{1}{P_l}\right]\left[R\left(\frac{G-1}{G}\right)(D^2-a^2-qE\sigma)-S(D^2-a^2-p_1E\sigma)\right], \quad ...(40)$$

where $T_A = 4\Omega^2 d^4/\nu^2$ is the Taylor number. Here also we consider the case of two free boundaries maintained at fixed temperatures and solute concentrations. The dimensionless boundary conditions appropriate for the problem are

$$W = D^2W = 0,\ \circledast = 0,\ \Gamma = 0 \text{ and } Dz = 0 \text{ at } z = 0 \text{ and } z = 1 \quad ...(41)$$

The proper solution of (40) satisfying (41) is given by (21). Substituting (21) in eqn. (40), we obtain the dispersion relation

$$R_1 = \left(\frac{G-1}{G}\right)\left[\frac{\left(1+x+Ep_1\frac{\sigma}{\pi^2}\right)\left[(1+x)\left\{\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)+\frac{\Omega^2}{P}\right\}+T_{A_1}\right]}{x\left[\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)\frac{\sigma}{\pi^2}+\frac{1}{P}\right]}\right]+S_1\frac{\left(1+x+Ep_1\frac{\sigma}{\pi^2}\right)}{\left(1+x+Eq\frac{\sigma}{\pi^2}\right)}, \quad ...(42)$$

whwre $T_{A_1} = T_A/\epsilon^2\pi^4$.

For stationary convection, eqn. (42) reduces to

$$R_1 = \left(\frac{G-1}{G}\right)\left[\frac{(1+x)^2}{xp}+\frac{(1+x)}{x}PT_{A_1}+S_1\right] \quad ...(43)$$

If the non–dimensional numbers G, T_{A_1}, S_1 and P accounting for the compressibility, rotation, stable solute gradient and medium permeability effects be kept as fixed, then we find that

$$\bar{\bar{R}}_c = \left(\frac{G}{G-1}\right)R''_c, \quad ...(44)$$

where R''_c and $\bar{\bar{R}}_c$ denote respectively the critical Rayleigh numbers in the absence and presence of compressibility. This clearly depicts the stabilizing effect of compressibility on the thermosolutal convection in porous medium. Eqn. (43) yields

$$\frac{dR_1}{dT_{A_1}} = \left(\frac{G}{G-1}\right)\frac{(1+x)}{x}P, \quad ...(45)$$

$$\frac{dR_1}{dS_1} = \left(\frac{G}{G-1}\right), \quad ...(46)$$

$$\frac{dR_1}{dP} = \left(\frac{G}{G-1}\right)\left(\frac{(1+x)}{x}\right)\left[T_{A_1}-\frac{(1+x)}{P^2}\right]. \quad ...(47)$$

We consider the relevant case $G > 1$. It is evident from eqns. (45) and (46) that the rotation and the solute gradient have stabilizing effects. If $T_{A_1} > (1+x)/P^2$, dR_1/dP is positive showing the stabilizing effect of medium permeability. If $T_{A_1} < (1+x)/P^2$, dR_1/dP is negative showing the destabilizing effect of medium permeability. The effect of medium permeability is destabilizing in the absence of rotation but its effect may be stabilizing also in the presence of rotation.

Multiplying eqn. (37) by W^*, the complex conjugate of W, integrating over the range of z and making use of eqns. (17), (18) and (38), we obtain

$$\left[\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)\sigma+\frac{1}{P_1}\right]I_1 + d^2\left[\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)\sigma^*+\frac{1}{P_1}\right] I_6 - \frac{c_p\alpha\kappa a^2}{v(G-1)}$$

$$[I_2 + p_1 E\sigma^* I_3] + \frac{g\,\alpha''\kappa'' a^2}{v\beta''}\,[I_4 + qE\sigma^* I_5] = 0 \qquad \text{...(48)}$$

where I_1–I_5 are given in eqns. (27) and

$$I_6 = \int_0^1 |Z|^2\, dz, \qquad \text{...(49)}$$

which are all positive definite. Putting $\sigma = \sigma_r + i\sigma_i$ and equating real and imaginary parts of eqn. (48), we obtain

$$\sigma_r\left[\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)I_1 + d^2\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)I_6 - \frac{C_p\alpha\kappa a^2}{v(G-1)}\,p_1 E I_3 + \frac{g\,\alpha''\kappa'' a^2 qE}{v\beta''}\,I_5\right]$$

$$= -\left[\frac{1}{P_1}I_1 + \frac{1}{P_1}d^2 I_6 - \frac{c_p\alpha\kappa a^2}{v(G-1)}I_2 - \frac{g\,\alpha''\kappa'' a^2}{v\beta''}I_4\right], \qquad \text{... (50)}$$

and

$$\sigma_i\left[\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right) I_1 + d^2\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)I_6 - \frac{c_p\alpha\kappa a^2}{v(G-1)}\,p_1 E I_3 - \frac{g\,\alpha''\kappa''}{v\beta''}\,qEI_5\right] = 0 \quad (51)$$

It follows from eqn. (50) that σ_r is negative if $G<1$. The system is therefore stable for $G<1$. It is evident from eqns. (51) that σ_i may be zero or nonzero meaning thereby that the modes may be non–oscillatory or oscillatory. The oscillatory modes are introduced due to the presence of rotation and solute gradient which were completely missing in their absence.

The critical value of R_1, denoted by R_{1c}, for the onset of instability, given by $\frac{dR_1}{dx} = 0$ is obtained at the value of x given by

$$x = (1 + P^2 T_{A_1})^{1/2} \qquad \text{...(52)}$$

which is independent of S_1 but depends on P and T_{A_1}. Substituting this value of x in (43) , the critical Rayleigh number is gives by

$$R_{1c} = \left(\frac{G}{G-1}\right)\left[\frac{1}{P}\ \{1+(1+P^2T_{A_1})^{1/2}\}^2 + S_1\right] \qquad ...(52\ a)$$

From this equation it is reaffirmed that the critical Rayleigh number increases with the increase in T_{A_1} (stable solute gradient parameter).

When $T_{A_1} = 0$ and $S_1 = 0$, we obtain

$$R_{1c} = \left(\frac{G}{G-1}\right)\frac{4}{P},$$

$$\text{or } R_c = \frac{4\pi^2}{P_l}\left(\frac{G}{G-1}\right),$$

For the case of overstability putting $\sigma/\pi^2 = i\sigma_1$, where σ_1 is real, in eqns. (42) and equating real and imaginary parts of the resiting equation, we obtain

$$R_1\left(\frac{G-1}{G}\right)(b-1)\left[\frac{b}{P}-\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)qEc\right] = (b^2-p_1qE^2c)\left[T_{A_1}+\frac{1}{P^2}-c\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)^2\right]$$

$$-\frac{2bE(p_1+q)}{P}\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)c + S_1(b-1)\left[\frac{b}{P}-p_1Ec\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)\right], \qquad ...(53)$$

$$R_1\left(\frac{G-1}{G}\right)(b-1)\left[\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)b+\frac{qE}{P}\right] = bE(p_1+q)\left[T_{A_1}+\frac{1}{P^2}-c\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)^2\right]$$

$$+\frac{2(b^2-p_1qE^2c)}{P}\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right) + S_1(b-1)\left[\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)b+\frac{p_1E}{P}\right] \qquad ...(54)$$

Eliminating R_1 between eqns. (53) and (54) we obtain

$$q^2c^2E^2\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)^2\left[\frac{Ep_1}{P}+b\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)\right]+\left[\frac{Ep_1}{P}q^2E^2\left(T_{A_1}+\frac{1}{P^2}\right)+b^2\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)\right.$$

$$\left.\left\{\frac{Ep_1}{P}+b\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)+q^2E^2b\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)\left(\frac{1}{P^2}-T_{A_1}\right)\right]c+\left[\frac{Ep_1}{P}b^2\left(T_{A_1}+\frac{1}{P^2}\right)\right.$$

$$\left.+b^3\left(\frac{1}{\epsilon}+\frac{v'}{k_1}\right)\left(\frac{1}{P^2}-T_{A_1}\right)+S_1(b-1)\frac{bE}{P^2}(p_1-q)\right] = 0 \qquad ...(55)$$

As σ_1 is real for overstability, the two values of $c = (\sigma_1)^2$ are positive. Eqn. (55) is quadratic in c and does not involve any of its roots to be positive if

$$p_1 > q \quad \text{and} \quad T_{A_1} < 1/p^2 \qquad ...(56)$$

for then, the coefficients of c^2, c and constant term are all positive and there is no change of sign in eqn. (55) and (56) would imply

$$\kappa < \kappa' \quad \text{and} \quad 4\,\Omega^2 < \nu^2/k_1^2 \qquad ...(57)$$

Thus if (57) are satisfied, overstability is impossible. $\kappa < \kappa'$ and $4\,\Omega^2 < \nu^2/k_1^2$ are, therefore, sufficient conditions for nonexistence of overstability, the violation of which do not necessarily imply occurrence of overstability.

5. Effect of magnetic field . In this section we consider an infinite horizontal, compressible, Rivlin–Ericksen elastico–viscous and finitely (electrically) conduction fluid layer of depth d in porous medium heated from below and subjected to a stable solute gradient. The uniform magnetic field $\vec{H}\,(0,0,H)$ pervades the system. Then the linearized perturbed equations of motion and Maxwell's equations are

$$\frac{1}{\epsilon}\frac{\partial \vec{V}}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p - \vec{g}\,(\alpha\theta - \alpha'\gamma) - \frac{1}{k_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)\vec{V} + \frac{\mu_e}{4\pi\rho_0}(\nabla\times\vec{h})\times\vec{H} \qquad ...(58)$$

$$\nabla.\vec{h} = 0 \qquad ...(59)$$

$$\epsilon\frac{\partial \vec{h}}{\partial t} = (\vec{H}.\nabla)\,\vec{V} + \epsilon\eta\,\nabla^2\vec{h}, \qquad ...(60)$$

where μ_e, η and $\vec{h}\,(hx, hy, hz)$ denote respectively, the magnetic permeability, the resistivity and the perturbation in magnetic field H. Equations (6)–(8) remain unaltered. Eqns. (6)–(8) and (58)–(60) in non dimensional form, give

$$\left[\left(\frac{1}{\epsilon} + \frac{\nu'}{k_1}\right)\sigma + \frac{1}{P_l}\right](D^2-a^2)\,W + \frac{g\,d^2a^2}{\nu}(\alpha\Theta + \alpha'\Gamma) + \frac{\mu_e H d}{4\pi\rho_0\nu}(D^2-a^2)\,DK = 0 \qquad ...(61)$$

$$(D^2-a^2-p_2\sigma)\,K = -\,(Hd/\eta\epsilon)\;DW, \qquad ...(62)$$

together with (17) and (18). Here we have written $p_2 = \nu/\eta$ and

$h_z = K(z)\,exp\,(ik_x x + ik_y y + nt)$.

Eliminating K, Θ and Γ from eqns. (17), (18), (61) and (62), we obtain

$$(D^2-a^2)(D^2-a^2-p_1E\sigma)(D^2-a^2-qE\sigma)\left[\left\{\left(\frac{1}{\epsilon} + \frac{\nu'}{k_1}\right)\sigma + \frac{1}{P_l}\right\}(D^2-a^2-p_2\sigma) - \frac{Q}{\epsilon}D^2\right]W$$

$$= (D^2 - a^2 - p_2\sigma)\left[Ra^2\left(\frac{G-1}{G}\right)(D^2 - a^2 - qE\sigma) - Sa^2(D^2 - a^2 - p_1E\sigma)\right]W, \quad ...(63)$$

where $Q = \dfrac{\mu_e H^2 d^2}{4\pi\rho_0 \nu\eta}$ is the Chandrasekhar number.

Here we consider the case of two free boundaries and the medium adjoining the fluid is a perfect conductor.

Then the boundary conditions in addition to (20) are

$DK = 0$ at $z = 0$ and 1. ... (64)

The proper solution of eqn. (63) satisfying the boundary condi tions (64) and (20) is given by (21). Substituting (21) in eqn. (63), we obtain the dispersion relation

$$R_1 = \left(\frac{G-1}{G}\right)\left[\frac{\left\{\frac{\sigma}{\pi^2}\left(\frac{1}{\in} + \frac{\upsilon'}{k_1}\right) + \frac{I}{P}\right\}(1+x)\left(1+x+Ep_1\frac{\sigma}{\pi^2}\right)}{x} + S_1\frac{\left(1+x+Ep_1\frac{\sigma}{\pi^2}\right)}{\left(1+x+Eq\frac{\sigma}{\pi^2}\right)}\right.$$

$$\left. + Q_1\frac{(1+x)(1+x+Ep_1\,\sigma/\pi^2)}{\in x(1+x+Ep_2\,\sigma/\pi^2)}\right], \quad ...(65)$$

where $Q_1 = \dfrac{Q}{\pi^2}$.

For stationary convection $\sigma = 0$ and eqn. (65) reduces to

$$R_1 = \left(\frac{G}{G-1}\right)\left[\frac{(1+x)^2}{xP} + S_1 + Q_1\frac{(1+x)}{\in x}\right] \quad ...(66)$$

If the non–dimensional nubers G, S_1, Q_1 and P accounting for the compressibility, stable solute gradient, magnetic field and medium permeability effects be kept as fixed, then we find that

$$\bar{\bar{R}}_c = \left(\frac{G}{G-1}\right)R_c''', \quad ...(67)$$

where R_c''' and $\bar{\bar{R}}_c$ stand respectively for the critical Rayleigh numbers in the absence and presence of compressibility. Thus we obtain the stabilizing effect of compressiblity Eqn. (66) gives

$$\frac{dR_1}{dQ_1} = \left(\frac{G}{G-1}\right)\frac{(1+x)}{\in x}, \quad ...(68)$$

$$\frac{dR_1}{dS_1} = \left(\frac{G}{G-1}\right), \quad ...(69)$$

$$\frac{dR_1}{dP} = -\left(\frac{G}{G-1}\right)\frac{(1+x)}{xP^2}. \quad ...(70)$$

nonexistence of overstability, the violation of which do not necessarily imply occurrence of overstability.

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