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# USE OF THE MULTIVARIABLE *H*-FUNCTION AND GENERAL POLYNOMIALS IN LINEAR FLOW OF HEAT IN AN ANISOTROPIC FINITE SOLID MOVING IN A CONDUCTING MEDIUM

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**ABSTRACT**

The use of special functions and their applications is being increasing important in technical applications. Space research, nuclear reactor, wood technology, soil mechanics and the mechanics of solids of fibrous structure give rise to several problems of the application of special functions.

The aim of this paper is to discuss the application of certain product of Fox's H-function, general polynomials, the generalized prolate spheroidal wave function and the multivariable *H*-function in obtaining formal solution of the partial differential equation related to a problem of liner flow of heat in an anisotropic finite solid moving in a conducting medium. Thus, the present study unifies and extends a number of results scattered in the literature.

**1. INTRODUCTION** We shall consider a problem of linear heat flow in an anisotropic finite solid moving in a conducting medium, given by following partial differential equation (see [6])

$$\frac{\partial}{\partial y}\left[(1-y^2)\frac{\partial U}{\partial y}\right]-\frac{pcv}{\lambda'}\frac{\partial U}{\partial y}+\frac{Q(y)}{\lambda'}=\frac{pc}{\lambda'}\frac{\partial U}{\partial t} \qquad ...(1.1)$$

with the law of conductivity $K=\lambda'(1-y^2)$, $Q(y)$ is the intensity of a continuous source of heat situated inside this solid.

Let the initial temperature of the rod be given by

$$U(y,0)=F(y) \qquad ...(1.2)$$

assuming

$$v=(\rho-\sigma)/q,$$

$$Q(y) = -(\rho + \sigma)\,\lambda' y \frac{\partial U}{\partial y} - (s', y)^2 \lambda' U,$$

$$q = pc/\lambda'$$

a solution of (1.1) takes the form

$$U(y, t) = \sum_{l=0}^{\infty} B_l \exp(-T_l t)\, \phi_l^{\rho,\sigma}(s', y) \qquad \text{...(1.3)}$$

where $B_l = (l+p)(l+p+\rho+\sigma+1)/q$.

Consider

$$F(y) = (1-y)^{\alpha-\rho}(1+y)^{\beta-\sigma} H_{P,Q}^{M,N}\left[ z(1-y)^h(1+y)^k \,\middle|\, \begin{matrix} (e_p, E_p) \\ (f_Q, F_Q) \end{matrix} \right]$$

$$S_{n_1,\ldots,n_s}^{m_1,\ldots,m_s}\left[ (1-y)^{\alpha_1 h_1}(1+y)^{\alpha_1 k_1}, \ldots, (1-y)^{\alpha_s h_s}(1+y)^{\alpha_s k_s} \right]$$

$$H_{A,\,C\,:\,[B', D'];\ldots;[B^{(r)}, D^{(r)}]}^{0,\,\lambda\,:\,(u', v');\ldots;(u^{(r)}, v^{(r)})}$$

$$\left( z_1(1-y)^{h'}(1+y)^{k'}, \ldots, z_r(1-y)^{h^{(r)}}(1+y)^{k^{(r)}} \right) \qquad \text{...(1.4)}$$

where the multivariable $H$-function of several complex variables introduced by Srivastava and Panda ([8] and [9]; see also [10], p.251). The results needed in our investigations are as follows :

The general polynomials $S_{n_1,\ldots,n_s}^{m_1,\ldots,m_s}[x_1,\ldots,x_s]$ occuring in the integral stands for the multivariable polynomials given by Srivastava ([11], p.185, eqn. (7)] defined and represented in the following slightly modified from

$$S_{n_1,\ldots,n_s}^{m_1,\ldots,m_s}[x_1,\ldots,x_s] = \sum_{\alpha_1=0}^{[n_1/m_1]} \cdots \sum_{\alpha_s=0}^{[n_s/m_s]} \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!} \cdots \frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!}$$

$$A[n_1, \alpha_1;\ldots;\ldots; n_s, \alpha_s]\, x_1^{\alpha_1} \ldots, x_s^{\alpha_s} \qquad \text{...(1.5)}$$

where $n_i = 0, 1, 2, \ldots; m_i \neq 0\ (i = 1, \ldots, s)$, $m_i$ is an arbitrary positive integer. The coefficients $A[n_i, \alpha_i;\ldots; n_s, \alpha_s]$ being arbitrary constants, real or complex.

If we take $s = 1$ in equation (1.5) and denote $A[n, \alpha]$ thus obtained by $A_{n,\alpha'}$, we arrive at the well-known general class of polynomials $S_n^m[x]$ introduced by Srivastava ([12], p.1, eqn.(1)).

The series representation of Fox's $H$-function ([3] and [7]; see also [10], p.12) :

$$H_{P,Q}^{M,N}\left[ z \left| \begin{matrix} (e_p, E_p) \\ (f_Q, F_Q) \end{matrix} \right. \right] = \sum_{g=1}^{M} \sum_{G=0}^{\infty} \frac{(-1)^G \phi(\eta_G) z^{\eta_G}}{G!\, F_g} \quad ...(1.6)$$

where

$$\phi(\eta_G) = \prod_{j=1, j\neq g}^{M} \Gamma(f_j - F_j \eta_G) \prod_{j=1}^{N} \Gamma(1 - e_j - E_j \eta_G)$$

$$\left\{ \prod_{j=M+1}^{Q} \Gamma(1 - f_j - F_j \eta_G) \prod_{j=N+1}^{P} \Gamma(e_j - E_j \eta_G) \right\}^{-1}$$

and $\eta_G = (f_g + G)/F_g$ ...(1.7)

The solution of the following differential equation

$$(1 - y^2) \frac{d^2 x}{dy^2} + [(\sigma - \rho) - (\rho + \sigma + 2)y] \frac{dx}{dy} + [(s') + (s'y)^2]x = 0 \quad ...(1.8)$$

is the generalized prolate spheroidal wave function ([4], p.107, Eqn. (2.11)), which is denoted as

$$\phi_n^{\rho, \sigma}(s', y) = \sum_{a=0}^{\infty} R_{q,n}^{\rho, \sigma}(s')\, P_{a+n}^{(\rho, \sigma)}(y) \quad ...(1.9)$$

where $s' = 0$, $\xi(0) = (n + a)(\rho + \sigma + n + a + 1)$, $a \geq 0$.

The orthogonality property of the generalized prolate spheroidal wave function ([4], p.107, eqn.(3.1))

$$\int_{-1}^{1} (1 - y)^{\rho} (1 + y)^{\sigma} \phi_v^{\rho, \sigma}(s', y)\, \phi_w^{\rho, \sigma}(s'y)\, dx = N_{v, w}^{\rho, \sigma} \delta_{v, w} \quad ...(1.10)$$

where

$$N_{v, w}^{\rho, \sigma} = \sum_{a=0}^{\infty} R_{a, v}^{\rho, \sigma}(s')^2$$

$$\frac{2^{\rho+\sigma+1}\Gamma(v + \rho + a + 1)\Gamma(v + \sigma + a + 1)}{(2v + 2a + \rho + \sigma + 1)\Gamma(v + a + 1)\Gamma(v + a + \rho + \sigma + 1)}$$

and $\delta_{v, w}$ is the Kronecker delta.

## 2. THE MAIN INTEGRAL

$$\int_{-1}^{1} (1 - y)^{\alpha}(1 + y)^{\beta} \phi_w^{\rho, \sigma}(s', y)\, H_{P,Q}^{M,N}\left[ z(1 - y)^h (1 + y)^k \left| \begin{matrix} (e_p, E_p) \\ (f_Q, F_Q) \end{matrix} \right. \right]$$

$$S^{m_1,\ldots,m_s}_{n_1,\ldots,n_s}[(1-y)^{\alpha_1 h_1}(1+y)^{\alpha_1 k_1},\ldots,(1-y)^{\alpha_s h_s}(1+y)^{\alpha_s k_s}]$$

$$H^{0,\lambda:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A,C:[B',D'];\ldots;[B^{(r)},D^{(r)}]}$$

$$\left(z_1(1-y)^{h'}(1+y)^{k'},\ldots,z_r(1-y)^{h^{(r)}}(1-y)^{h^{(r)}}(1+y)^{k^{(r)}}\right)dy$$

$$=\sum_{g=1}^{M}\sum_{v,G=0}^{\infty}\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_s=0}^{[n_s/m_s]}\frac{(-1)^G\phi(\eta_G)}{G!\,F_g}z^{\eta_G}R^{\rho,\sigma}_{v,w}(s',y)$$

$$\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!}A(n_1,\alpha_1;\ldots;n_s,\alpha_s)$$

$$2^{1+\alpha+\beta+(h+k)\eta_G+(h_1+k_1)\alpha_1,\ldots,(h_s+k_s)\alpha_s}$$

$$\sum_{u=0}^{v+w}(-v+w)_u(\rho+\sigma+v+w+1)_u\{(\rho+1)_u\,u!\}^{-1}$$

$$H^{0,\lambda+2:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A+2,C+1:[B',D'];\ldots;[B^{(r)},D^{(r)}]}$$

$$\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}],[-u-\alpha-h\eta_G-h_1\alpha_1,\ldots,h_s\alpha_s;h',\ldots,h^{(r)}],\\ [(c):\psi',\ldots,\psi^{(r)}];\end{matrix}\right.$$

$$[-\beta-k\eta_G-k_1\alpha_1,\ldots,k_s\alpha_s:k',\ldots,k^{(r)}]:$$
$$[-1-\alpha-\beta-u-(h+k)\eta_G-(h_1+k_1)\alpha_1\ldots-(h_s+k_s)\alpha_s:$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}];\\ h'+k',\ldots,h^{(r)}+k^{(r)}]:[(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];\end{matrix}\;z_1 2^{h'+k'},\ldots,z_r 2^{h^{(r)}+k^{(r)}}\right)$$

...(2.1)

where $\eta_i=0,1,2,\ldots;m_i\neq 0$ $[i=1,\ldots.s]$ $m_i$ is an arbitrary positive integer. The cofficients $A[n_1,\alpha_1;\ldots;n_s,\alpha_s]$ being arbitrary constant real or complex.

$$Re(\alpha+hf_l/F_l+\sum_{i=1}^{r}h^{(i)}d_j^{(i)}/\delta_j^{(i)})>-1$$

$$Re(\beta+kf_l/F_l+\sum_{i=1}^{r}k^{(i)}d_j^{(i)}/\delta_j^{(i)})>-1,Re(\rho)>-1,Re(\sigma)>-1,$$

$h^{(i)} > 0, k^{(i)} > 0, l = 1, \ldots, M; i = 1, \ldots, r, j = 1, \ldots, u^{(i)}, T_i > 0$

$|arg\,(z_i)| < T_i\,\pi/2$

$|arg(z)| < T\pi/2$ and $T = \left( \sum_{1}^{N} E_i - \sum_{N+1}^{P} E_i + \sum_{1}^{M} F_i - \sum_{M+1}^{Q} F_i \right)$.

Evaluation of (2.1). The integral (2.1) can be evaluated in a routine manner with the help of result Chaurasia ([10], p.31 eqn. (2.1) and (1.5)).

## 3. SOLUTION OF THE PROBLEM

$$U(y, t) = \sum_{g=1}^{M} \sum_{\upsilon, G=0}^{\infty} \sum_{\alpha_1=0}^{[n_1/m_1]} \cdots \sum_{\alpha_s=0}^{[n_s/m_s]} \frac{(-1)^G\,\phi\,(\eta_G)}{G!\,F_g} \quad z^{\eta_G} R_{\upsilon, l}^{\rho, \sigma}(s', y)$$

$$2^{1+\alpha+\beta+(h+k)\,\eta_G+(h_1+k_1)\alpha_1, \ldots, (h_s+k_s)\alpha_s}$$

$$\sum_{l, u=0}^{\upsilon} \frac{(-\upsilon)_u(\rho+\sigma+\upsilon+1)_u}{(\rho+1)_u\,u} \{N_{l,1}^{\rho,\sigma}\}^{-1}\,\phi_{\lambda}^{\rho,\sigma}(s', y)$$

$$e^{-\upsilon(\upsilon+\rho+\sigma+1)t/q}\, H_{A+2, C+1 : [B', D'] ;\ldots; [B^{(r)}, D^{(r)}]}^{0, \lambda+2 : (u', v') ;\ldots; (u^{(r)}, v^{(r)})}$$

$$\left( \begin{array}{l} [-u-\alpha-h\eta_G-h_1\alpha_1, \ldots, -h_s\alpha_s; h', \ldots, h^{(r)}], [-\beta-k\eta_G-k_1\alpha_1, \ldots, k_s\alpha_s; k', \ldots k^{(r)}], \\ [(c) : \psi', \ldots, \psi^{(r)}], [-1-\alpha-\beta-u-(h+k)\eta_G-(h_1+k_1)\alpha_1 \ldots -(h_s+k_s)\alpha_s : \end{array} \right.$$

$$\left. \begin{array}{l} [(a) : \theta', \ldots, \theta^{(r)}] : [(b') : \phi'] ;\ldots; [b^{(r)} : \phi^{(r)}] ; \\ : h'+k', \ldots, h^{(r)}+k^{(r)}] : [(d') : \delta'] ;\ldots; [(d^{(r)}) : \delta^{(r)}]; \end{array} z_1\, 2^{h'+k'}, \ldots, z_r 2^{h^{(r)}+k^{(r)}} \right) \quad \ldots(3.1)$$

valid under the same conditions as given in (2.1).

**Evaluation of (3.1).** Take $t = 0$ then by (1.3), we have

$$F(y) = \sum_{l=0}^{\infty} B_l \phi_l^{\rho, \sigma}(s', y), \qquad \ldots(3.2)$$

where $F(y)$ is given by (1.4).

Equation (3.2) is valid since $F(y)$ is continuous and bounded variation in the interval (-1, 1).

Multiplying both sides of (3.2) by $(1-x)^{\rho}(1+x)^{\sigma}\phi_w^{\rho, \sigma}(s', y)$ $\rho > -1, \sigma > -1$ and integrating between -1 to +1 with respect to y and using the orthogonality property (1.9) a lemma ([5], p.57, eqn. (2)) and integral (2.1) after simplification, we arrive at the required result.

## 4. SPECIAL CASES

($i$) If we take $s = 1, m_1 = 2, A_{n_1, \alpha_1} = (-1)^{\alpha_1}$ in (2.1), we obtain

$$\int_{-1}^{1} (1-y)^{\alpha}(1+y)^{\beta}\, \phi_{w}^{\rho,\sigma}(s', y)\, H_{P,Q}^{M,N}\left[ z(1-y)^{h}(1+y)^{k} \,\middle|\, \begin{matrix} (e_p, E_p) \\ (f_Q, F_Q) \end{matrix} \right]$$

$$(1-y)^{h_1 n_1/2}\,(1+y)^{k_1 n_1/2}\, H_{n_1}\left[ \frac{1}{2\sqrt{(1-y)^{h_1}(1+y)^{k_1}}} \right]$$

$$H^{\,0,\,\lambda\,:\,(u',\,v')\,;\ldots;\,(u^{(r)},\,v^{(r)})}_{\,A,\,C\,:\,[B',\,D']\,;\ldots;\,[B^{(r)},\,D^{(r)}]}$$

$$\left( z_1(1-y)^{h'}(1+y)^{k'}, \ldots, z_r(1-y)^{h^{(r)}}(1-y)^{h^{(r)}}(1+y)^{k^{(r)}} \right) dy$$

$$\sum_{g=1}^{M} \sum_{v,\,G=0}^{\infty} \sum_{\alpha_1=0}^{[n_1/2]} \frac{(-1)^{G}\phi(\eta_G)}{G!\,F_g}\, z^{\eta_G} R_{v,w}^{\rho,\sigma}(s', y)\, \frac{(-n_1)_{2\alpha_1}(-1)^{\alpha_1}}{\alpha_1!}$$

$$2^{1+\alpha+\beta+(h+k)\eta_G+(h_1+k_1)\alpha_1} \sum_{u=0}^{v+w} (-v+w)_u(\rho+\sigma+v+w+1)_u$$

$$\{(\rho+1)_u\, u!\}^{-1}\, H^{\,0,\,\lambda+2\,:\,(u',\,v')\,;\ldots;\,(u^{(r)},\,v^{(r)})}_{\,A+2,\,C+1\,:\,[B',\,D']\,;\ldots;\,[B^{(r)},\,D^{(r)}]}$$

$$\left( \begin{matrix} [-u-\alpha+h\eta_G-h_1\alpha_1 : h', \ldots, h^{(r)}],\ [-\beta-k\eta_G-k_1\alpha_1 ; k', \ldots, k^{(r)}], \\ [(c):\psi', \ldots, \psi^{(r)}],\ [-1-\alpha-\beta-u-(h+k)\eta_G-(h+k)-(h_1+k_1)\alpha_1 : h'+k', \ldots, h^{(r)}+k^{(r)}] : \\ [(a):\theta', \ldots, \theta^{(r)}] : [(b'):\phi'] ;\ldots; [(b^{(r)}):\phi^{(r)}]; \\ : [(d'):\delta'] ;\ldots; [(d^{(r)}):\delta^{(r)}]; \end{matrix} \; z_1\, 2^{h'+k'}, \ldots, z_r\, 2^{h^{(r)}+k^{(r)}} \right) \quad \ldots(4.1)$$

valid under the same conditions as given by (2.1).

($ii$) On taking $s = 1, m_1 = 1$ and $A_{n_1, \alpha_1} \binom{f_1+f'}{n_1} \dfrac{(f'+g'+n_1+1)_{\alpha_1}}{(f'+1)_{\alpha_1}}$

in (2.1), we get

$$\int_{-1}^{1} (1-y)^{\alpha}(1+y)^{\beta}\, \phi_{w}^{\rho,\sigma}(s', y)\, H_{P,Q}^{M,N}\left[ z(1-y)^{h}(1+y)^{k} \,\middle|\, \begin{matrix} (e_p, E_p) \\ (f_Q, F_Q) \end{matrix} \right]$$

$$P_{n_1}^{(f',\,g')}\,[1-2(1-y)^{h_1}(1+y)^{k_1}]\, H^{\,0,\,\lambda\,:\,(u',\,v')\,;\ldots;\,(u^{(r)},\,v^{(r)})}_{\,A,\,C\,:\,[B',\,D']\,;\ldots;\,[B^{(r)},\,D^{(r)}]}$$

$$\left(z_1(1-y)^{h'}(1+y)^{k'},\dots,z_r(1-y)^{h^{(r)}}(1-y)^{h^{(r)}}(1+y)^{k^{(r)}}\right)dy$$

$$\sum_{g=1}^{M}\sum_{v,G=0}^{\infty}\sum_{\alpha_1=0}^{n_1}\frac{(-1)^G\phi(\eta_G)}{G!\,F_g}z^{\eta_G}R_{v,w}^{\rho,\sigma}(s',y)\frac{(-n_1)_{\alpha_1}}{\alpha_1!}\binom{n_1+f'}{n_1}$$

$$\frac{(f'+g'+n_1+1)_{\alpha_1}}{(f'+1)_{\alpha_1}}\,.\,2^{1+\alpha+\beta+(h+k)\eta_G+(h_1+k_1)\alpha_1}$$

$$\sum_{u=0}^{v+w}(-v+w)_u(\rho+\sigma+v+w+1)_u\{(\rho+1)_u\,u!\}^{-1}$$

$$H\,{}^{0,\lambda+2:(u',v');\dots;(u^{(r)},v^{(r)})}_{A+2,C+1:[B',D'];\dots;[B^{(r)},D^{(r)}]}$$

$$\left(\begin{matrix}[-u-\alpha-h\eta_G-h_1\alpha_1;h',\dots,h^{(k)}],[-\beta-k\eta_G-k_1\alpha_1:k',\dots,k^{(r)}]\\ [(c):\psi',\dots,\psi^{(r)}],[-1-\alpha-\beta-u-(h+k)\eta_G-(h_1+k_1)\alpha_1:h'+k',\dots,h^{(r)}+k^{(r)}]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(a):\theta',\dots,\theta^{(r)}]:[(b'):\phi'];\dots;[b^{(r)}:\phi^{(\rho)}];\\ [(d'):\delta'];\dots;[(d^{(r)}):\delta^{(r)}];\end{matrix}\;z_1 2^{h'+k'},\dots,z_r 2^{h^{(r)}+k^{(r)}}\right) \qquad \dots(4.2)$$

valid under the same conditions as required by (2.1).

(*iii*) Let $s=1$, $m_1=1$ and $A_{n_1,\alpha_1}\binom{n_1+f'}{n_1}\dfrac{1}{(f'+1)_{\alpha_1}}$ in (2.1), we get

$$\int_{-1}^{1}(1-y)^{\alpha}(1+y)^{\beta}\phi_w^{\rho,\sigma}(s',y)H_{P,Q}^{M,N}\left[z(1-y)^h(1+y)^k\,\middle|\,\begin{matrix}(e_p,E_p)\\(f_Q,F_Q)\end{matrix}\right]$$

$$L_{n_1}^{(\alpha_1)}(1-y)^{h_1}(1+y)^{k_1}H\,{}^{0,\lambda:(u',v');\dots;(u^{(r)},v^{(r)})}_{A,C:[B',D'];\dots;[B^{(r)},D^{(r)}]}$$

$$\left(z_1(1-y)^{h'}(1+y)^{k'},\dots,z_r(1-y)^{h^{(r)}}(1-y)^{h^{(r)}}(1+y)^{k^{(r)}}\right)dy$$

$$\sum_{g=1}^{M}\sum_{G=0}^{\infty}\sum_{\alpha_1=0}^{n_1}\frac{(-1)^G\phi(\eta_G)}{G!\,F_g}z^{\eta_G}R_{v,w}^{\rho,\sigma}(s',y)\frac{(-n_1)_{\alpha_1}\binom{n_1+f'}{n_1}}{\alpha_1!\,(f'+1)_{\alpha_1}}$$

$$2^{1+\alpha+\beta+(h+k)\eta_G+(h_1+k_1)\alpha_1}\sum_{u=0}^{v+w}(-v+w)_u(\rho+\sigma+v+w+1)_u$$

$$\{(\rho+1)_u\, u!\}^{-1} H^{0,\lambda+2\,:\,(u',v');\ldots;(u^{(r)},v^{(r)})}_{A+2,C+1\,:\,[B',D'];\ldots;[B^{(r)},D^{(r)}]}$$

$$\left(\begin{matrix}[-u-\alpha-h\eta_G-h_1\alpha_1 : h',\ldots,h^{(r)}],[-\beta-k\eta_G-k_1\alpha_1 : k',\ldots,k^{(r)}],[(a):\theta',\ldots,\theta^{(r)}]: \\ [(c):\psi',\ldots,\psi^{(r)}],[-1-\alpha-\beta-u-(h+k)\eta_G-(h_1+k_1)\alpha_1 : h'+k',\ldots,h^{(r)}+k^{(r)}]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]; \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];\end{matrix}\; z_1\, 2^{h'+k'},\ldots,z_r\, 2^{h^{(r)}+k^{(r)}}\right) \qquad \ldots(4.3)$$

valid under the same conditions as required by (2.1).

(*iv*) On taking $\lambda = A$, $u^{(i)} = 1$, $v^{(i)} = B^{(i)}$ and $D^{(i} = D^{(i)}+1$, $\forall$ $i = 1, \ldots, r$ in (2.1), we obtain

$$\int_{-1}^{1}(1-y)^{\alpha}(1+y)^{\beta}\phi_w^{\rho,\sigma}(s',y)\, H^{M,N}_{P,Q}\left[z(1-y)^h(1+y)^k \,\middle|\, \begin{matrix}(e_p,E_p)\\(f_Q,F_Q)\end{matrix}\right]$$

$$S^{m_1,\ldots,m_s}_{n_1,\ldots,n_s}[(1-y)^{\alpha_1 h_1}(1+y)^{\alpha_1 k_1},\ldots,(1-y)^{\alpha_s h_s}(1+y)^{\alpha_s k_s}]$$

$$F^{A:B';\ldots;B^{(r)}}_{C:D';\ldots;D^{(r)}}\left(\begin{matrix}[1-(a):\theta',\ldots,\theta^{(r)}]:[1-(b'):\phi'];\ldots; \\ [1-(c):\psi',\ldots,\psi^{(r)}]:[1-(d');\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}[1-(b^{(r)}):\phi^{(r)}] \\ [1-(d^{(r)}):\delta^{(r)}]\end{matrix}\; z_1(1-y)^{h'}(1+y)^{k'},\ldots,z_r(1-y)^{h^{(r)}}(1+y)^{k^{(r)}}\right)dx$$

$$\sum_{g=1}^{M}\sum_{v,G=0}^{\infty}\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_s=0}^{[n_s/m_s]}\frac{(-1)^G\phi(\eta_G)}{G!\,F_g}z^{\eta_G}R^{\rho,\sigma}_{v,w}(s',y)$$

$$\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!}A(n_1,\alpha_1;\ldots;n_s,\alpha_s)$$

$$2^{1+\alpha+\beta+(h+k)\eta_G+(h_1+k_1)\alpha_1,\ldots,(h_s+k_s)\alpha_s}\sum_{u=0}^{v+w}(-v+w)_u$$

$$(\rho+\sigma+v+w+1)_u\,\{(\rho+1)_u\, u!\}^{-1} F^{A+1:B',\ldots,B^{(r)}}_{C:D',\ldots,D^{(r)}}$$

$$\left(\begin{matrix}[1+u+\alpha+h\eta_G+h_1\alpha_1+\ldots+h_s\alpha_s;\, h',\ldots,h^{(r)}],[1+\beta+k\eta_G+k_1\alpha_1+\ldots+k_s\alpha_s\, k',\ldots,k^{(r)}] \\ [1-(c):\psi',\ldots,\psi^{(r)}]:\end{matrix}\right.$$

$$\begin{array}{l}[1-(a):\theta',\ldots,\theta^{(r)}]: \\ [2+\alpha+\beta+u+(h+k)\eta_G+(h_1+k_1)\alpha_1,\ldots,(h_s+k_s)\alpha_s : h'+k',\ldots,h^{(r)}+k^{(r)}]: \\ \left.\begin{array}{l}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]; \\ [1-(d'):\delta'];\ldots;[1-(d^{(r)}):\delta^{(r)}];\end{array} z_1 2^{h'+k'},\ldots,z_r 2^{h'+k'},\ldots,z_r 2^{h^{(r)}+k^{(r)}}\right) \quad \ldots(4.4)\end{array}$$

valid under the same conditions as given in (2.1).

($v$) If we take $s = 1$ and $m_1 = 0$ in (2.1) it reduces to a known result of Chaurasia ([1], p.31, eqn.(2.1)).

($vi$) If $m_1 = m_2 = \ldots = m_s = 0$ and $M = 1$, $N = p$, $P = p$, $Q = q + 1$ and $h \to 0$, $k \to 0$ and after a little simplification the result in (2.1) reduces to a known result recently obtained by Chaurasia and Gupta ([2], p.78, eqn. (8)).

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